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Electromagnetic Modeling in the Spectral Domain for Polarimetric Radar Applications
Theoretical Framework

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Abstract

A vectorial Fourier-based technique for electromagnetic (EM) wave reconstruction with application to polarimetric airborne and spaceborne radar data exploitation is presented. The approach provides a comprehensive picture of scattering through an electromagnetic-based formulation of equivalent sources and introduction of vector scattering functions. The method is different from conventional modeling techniques for SAR applications as result of the full electromagnetic treatment of field interaction with the scatterer, the possibility of introducing new and controllable feature classes for target classification, and the accurate decomposition of source impulse response function that avoids potential errors (e.g., loss of coherent information) associated with the spherical phase approximations. In addition to including the potential for high quality image formation, the capability of extracting target or scatterer information, such as coherent radar cross section (RCS) is explored. It is highlighted how target image and information processing can be optimized and tailored for a desired scenario (e.g., the required information type, the nature of the targets) by applying the developed model and devising appropriate inversion techniques to aid data processing in support of exploitation.

Résumé

Le document présente une technique vectorielle à transformée de Fourier permettant de reconstituer des ondes électromagnétiques (EM) par l'application à l'exploitation des données de radars polarimétriques aéroportés et spatiaux. Cette technique procure une image globale de la diffusion par la formulation de sources équivalentes, selon des principes électromagnétiques, et le recours à des fonctions vectorielles de diffusion. Elle diffère des techniques classiques de modélisation des applications SAR en raison du traitement électromagnétique complet de l'interaction des champs avec le diffuseur, de la possibilité d'établir des ensembles de caractéristiques nouveaux et contrôlables pour la classification des cibles et de la décomposition précise de la fonction de réponse impulsive des sources, qui élimine les erreurs possibles (p. ex. la perte d'information cohérente) liées aux approximations de phase sphérique. Outre la formation d'images de grande qualité, la possibilité d'extraire de l'information sur les cibles ou les diffuseurs, comme la surface équivalente radar (SER) cohérente, est également examinée. Le document fait ressortir comment le traitement des images des cibles et de l'information sur les cibles peut être optimisé et adapté en fonction d'un scénario désiré (p. ex., le type de l'information requise et la nature des cibles) par l'application du modèle élaboré et la conception de techniques d'inversion appropriées facilitant le traitement des données à l'appui de l'exploitation.

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Executive summary

Polarimetric radar or SAR (Synthetic Aperture Radar) detection is a powerful remote sensing technique for potential military applications concerning target recognition, identification and classification. The polarimetric approach is potent as it effectively explores radar target features (e.g., structure, change) by making use of the full vector properties of the electromagnetic (EM) field while interacting with the target scatterers. Accordingly, exploitation of polarimetric radar data, using appropriate algorithms, results in an enhanced target detection and classification capability compared to single channel, single polarization data exploitation. Such improved functionality is desired for consistent target detection, reduction of misclassification probability, reduction of false alarm rate (FAR), and reliable target recognition.

It is evident from the characteristic description of radar polarimetry that the analysis of target scattering ought to be based on electromagnetic theory. A full electromagnetic approach to modeling the scattered vector fields and characterize the targets introduces new challenges and requirements for more rigorous algorithm development. On the other hand, the theoretical framework cannot be very complicated from a practical application point of view. The optimal trade-off between these two aspects, i.e., accuracy and feasibility, is an important issue that has not been systematically addressed in the literature and is the underlying motivation for this work.

Considering the diversity of features characterizing a radar or SAR scenario for remote sensing (e.g., application types, required analysis types, target and clutter types), development of a unique optimized approach to model the scattering for polarimetric applications is a huge if not insurmountable task. The intention is not to accomplish such here as this is not realistic. Instead, the objective is to construct a theoretical framework capable of accurately modeling the process of vector EM field interaction with the target scatterer. This general formulation will be tailored based on the requirements for the application classes of interest and will be applied for simulations and experimental purposes in later works. The contributions of this work can be viewed in two folds:

The first contribution is a thorough survey of the literature in polarimetric remote sensing for modeling of EM scattering and characterizing the target scatterer. The outcome signifies nature, objectives, required deliverables, major classes and capabilities/limitations of the existing analytic techniques for radar target analysis. In parallel, a study is performed to identify robust and efficient methods to model scattering and diffraction (as an EM phenomenon) that belong to different areas within the electromagnetic realm (e.g., microwave structures) but have similar theoretical foundations. The overall objective from the above investigations is to derive a set of key requirements for the modeling formulation, and at the same time, propose a feasible implementation of the associated algorithms.

The second contribution of this work is the development of a theoretical approach to model the target scattering (based on the key requirements identified) that is different from conventional methods in two ways. *One aspect* is the rigorous analysis (usually lacking in typical analyses) and application of source Green's function through a four-dimensional (i.e.,

space-time) Fourier decomposition. When applying a coherent approach for target identification, this accurate spectral treatment of Green's function with respect to both phase and amplitude provides the means for more accurate modeling of EM field scattering by the target scatterer. *The other aspect* is the representation and/or classification of target scatterers by vector current functions. This approach, which is based on the concept of equivalent sources in electromagnetic theory, is close to the roots of EM scattering governed by Maxwell's equations. Technically, the latter is on the same basis as the conventional scattering matrix analysis due to formulation of scattering-induced sources. However, the present approach is novel (to the best of author's knowledge in the field of polarimetric SAR scattering modeling for target classification) and appealing from the following points of view.

To classify the target scatterer, the associated vector current function in the space or spectral domain is decomposed into a set of basis functions that can be viewed as features. It is possible to perform this decomposition using different basis sets where each set represents a class of features or sub-features. Considering the generic nature of problem formulation, efficient techniques may be adopted from other areas (e.g., RF, microwaves) for such target characteristic function decompositions. As can be seen, there exist degrees of freedom and the possibility of defining various basis sets and number of members or basis functions for each set. The described potential of controlling the nature and number of features for target decomposition based on the application requirements, alleviates the misclassification problems associated with coarse classification/segmentation encountered through current coherent target decomposition techniques. It is also conceivable that study of the induced equivalent current source's behavior on the scatterer, yield a more consistent and tangible understanding of the scatterer's physics. Finally, the source functions derived through the present approach may be readily applied to calculate the associated EM field quantities (such as coherent radar cross section) in both near and far field zones.

The present methodology provides a core theoretical framework for polarimetric SAR modeling. The approach is general and should be tailored by investigating the SAR scenario of interest and designing a suitable structure for required feature classes. A significant advantage is the possibility of introducing various target classification or basis sets to the core formulation. Moreover, is the opportunity of adopting robust and well-proven techniques (e.g., method of moments, image theory) from other EM areas to obtain the optimum (i.e., best target representative) basis sets and associated decomposition mechanisms. The future work will be directed towards model or sub-model development for different types of applications. Various EM-based numerical techniques should be studied and properly tailored for desired applications. Convenient experimental schemes must still be designed to validate and tune the model.

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Sommaire

La détection par radar polarimétrique ou SAR (radar à synthèse d'ouverture) constitue une puissante technique de télédétection pour les applications militaires possibles liées à la reconnaissance, à l'identification et à la classification des cibles. La technique polarimétrique s'avère particulièrement efficace du fait qu'elle examine avec soin les caractéristiques des cibles radar (comme les structures et les changements) en tirant parti de toutes les propriétés vectorielles des champs électromagnétiques (EM) durant leur interaction avec les diffuseurs de cibles. L'exploitation des données de radar polarimétrique au moyen d'algorithmes appropriés permet donc d'améliorer la détection et la classification des cibles comparativement à l'exploitation des données avec canal unique et polarisation unique. Cette amélioration est souhaitable pour la détection uniforme des cibles, la réduction de la probabilité d'erreurs de classification, la réduction du taux de fausses alarmes (TFA) et la reconnaissance fiable des cibles.

D'après la description caractéristique de la polarimétrie radar, il est évident que l'analyse de la diffusion des cibles devrait se fonder sur la théorie électromagnétique. Une technique totalement électromagnétique de modélisation des champs vectoriels diffusés et de caractérisation des cibles suscite de nouveaux défis et impose de nouvelles exigences en vue de l'établissement d'algorithmes plus rigoureux. Par ailleurs, le cadre théorique ne peut pas être très compliqué du point de vue de l'application pratique. Le compromis optimal entre ces deux aspects, soit la précision et la faisabilité, représente un facteur important qui n'a pas encore été traité de façon systématique dans la documentation publiée et qui constitue la motivation sous-jacente à ces travaux.

Étant donné la diversité des caractéristiques des radars ou des scénarios SAR de télédétection (p. ex., types d'applications, types d'analyses requises et types de cibles et de clutter), l'établissement d'une méthode optimisée unique permettant de modéliser la diffusion des applications polarimétriques constitue une tâche immense, voire un obstacle insurmontable. Il serait irréaliste de tenter l'adoption d'une telle méthode. L'objectif consiste plutôt à établir un cadre théorique permettant de modéliser avec exactitude le processus d'interaction des champs EM vectoriels avec le diffuseur de cible. Cette formulation générale sera adaptée aux exigences des classes d'applications d'intérêt et sera appliquée dans des travaux ultérieurs à des fins expérimentales et de simulation. Les contributions de ces travaux sont doubles.

La première contribution prend la forme d'un examen approfondi de la documentation en télédétection polarimétrique pour la modélisation de la diffusion EM et la caractérisation du diffuseur de cible. Le résultat fait ressortir la nature, les objectifs, les produits livrables requis ainsi que les principales classes et capacités/limites des techniques analytiques existantes pour l'analyse des cibles radar. Parallèlement, une étude vise à déterminer des méthodes robustes et efficaces de modélisation de la diffusion et de la diffraction (en tant que phénomènes EM) ayant trait à différents aspects de l'électromagnétisme (p. ex. les structures hyperfréquences) mais partageant des fondements théoriques similaires. Ces recherches ont pour objectif général d'établir un ensemble d'exigences clés pour la formulation de la modélisation et, simultanément, à proposer un moyen de mise en œuvre des algorithmes connexes.

À titre de seconde contribution, ces travaux permettent d'élaborer une méthode théorique servant à modéliser la diffusion des cibles (d'après les exigences clés établies), qui diffère des méthodes habituelles à deux égards. *En premier lieu*, cette différence tient à l'exécution d'une analyse rigoureuse (d'une rigueur supérieure à celle des analyses typiques) et à l'application de la fonction de Green pour la source, par une décomposition de Fourier quadridimensionnelle (espace-temps). Si l'on recourt à une méthode cohérente d'identification des cibles, ce traitement spectral précis de la fonction de Green par rapport à la phase et à l'amplitude permet une modélisation plus précise de la diffusion du champ EM par le diffuseur de cible. *En second lieu*, cette méthode se différencie par la représentation et/ou la classification des diffuseurs de cibles au moyen de fonctions vectorielles de courant. Une telle façon de procéder, fondée sur le principe des sources équivalentes dans la théorie de l'électromagnétisme, se rapproche des éléments de la diffusion électromagnétique régis par les équations de Maxwell. Sur le plan technique, elle se trouve sur le même pied que l'analyse matricielle classique de la diffusion, à cause de la formulation de sources induites par la diffusion. La méthode actuelle est toutefois novatrice (autant qu'en sache l'auteur dans le domaine de la modélisation de la diffusion par SAR polarimétrique pour la classification des cibles) et intéressante pour les raisons décrites ci-dessous.

Pour la classification des diffuseurs de cibles, la fonction vectorielle de courant connexe dans le domaine spatial ou spectral est décomposée en un ensemble de fonctions de base pouvant être considérées comme des caractéristiques. Il est possible d'effectuer cette décomposition à l'aide de différents ensembles de base, chaque ensemble représentant une classe de caractéristiques ou de sous-caractéristiques. Compte tenu de la nature générique de la formulation du problème, il est possible d'adopter des techniques efficaces issues d'autres secteurs (RF, micro-ondes, etc.) pour effectuer de telles décompositions de fonctions caractéristiques des cibles. Comme on peut le constater, il existe des degrés de liberté et la possibilité de définir divers ensembles de base ainsi qu'un certain nombre de membres ou fonctions de base pour chaque ensemble. La possibilité décrite de déterminer la nature et le nombre de caractéristiques de décomposition des cibles en fonction des exigences de l'application atténue les problèmes de classification liés à la classification/segmentation approximative, qui sont typiques des techniques actuelles de décomposition cohérente des cibles. On peut aussi concevoir que l'étude de l'incidence d'une source de courant équivalent induit sur le diffuseur procure une compréhension plus uniforme et tangible de la physique du diffuseur. Enfin, les fonctions de source liées à la méthode actuelle peuvent s'appliquer directement au calcul des valeurs des champs EM connexes (comme la surface équivalente de radar cohérent), tant en champ proche qu'en champ lointain.

La présente méthode fournit un cadre théorique central pour la modélisation par SAR polarimétrique. Cette méthode est générale et devrait être adaptée par l'examen du scénario SAR d'intérêt ainsi que par la conception d'une structure appropriée applicable aux classes de caractéristiques requises. Un avantage significatif tient à la possibilité de recourir à diverses classifications de cibles ou à différents ensembles de base pour la formulation centrale. Il est également possible d'adopter des techniques robustes ayant fait leurs preuves (méthode des moments, théorie des images, etc.) dans d'autres secteurs de l'EM afin d'obtenir les ensembles de base optimaux (le plus représentatifs des cibles) et les mécanismes de décomposition correspondants. Les travaux futurs viseront l'établissement d'un modèle ou d'un sous-modèle pour les différents types d'applications. Diverses techniques numériques fondées sur l'électromagnétisme devraient être étudiées et adaptées correctement aux applications

désirées. Il reste encore à concevoir des structures expérimentales appropriées pour valider et raffiner le modèle.

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1.   Introduction

Radar polarimetry is a fast growing and powerful remote sensing technique. It is anticipated that the majority of new radars will have polarimetric or scattering matrix capability [1]. The strength of the polarimetric approach is due to many important and key features of radar targets being vectorial in nature [2]. Hence, making use of the inherent vector property of electromagnetic field (EM) interactions with the scatterer, results in a more comprehensive picture of the target. There exists a considerable volume of fundamental work in classical physics that explores different aspects of electromagnetic wave scattering and diffraction by various objects that support the polarimetric approach. There is also significant research literature on polarimetric applications and their advantages for active microwave remote sensing [3]-[6].

Polarimetric detection in radar technology is used to create high-resolution images [7]-[9] and provide better target detection capabilities in comparison with non-polarimetric or single channel operation [10]-[13]. The latter, which amounts to the analysis of a detected signal to determine the EM characteristic (i.e., the EM signature) of the target for target identification, is an important radar application. To properly correlate the received EM signal(s) with a particular object or scatterer (or combination of scatterers), an EM-based formulation of scattering and propagation should be applied.

An important and early insight to radar polarimetry for target detection [14] was to represent a target as a polarization transformer and to express its coherent properties with a 2×2 scattering matrix. This concept was developed by many researchers [15] and entered a new phase for systematic coherent radar detection with Huynen's work on radar target phenomenology [16]-[17], which led to target decomposition techniques. These techniques represent a key theoretical approach to target recognition in current radar imagery. The essence of target decomposition theory is to express the average scattering mechanism in terms of independent scattering elements and to associate each of those with a known physical mechanism or scatterer [18]. The described association can be on a coherent target detection (CTD) [17], [19]-[20], or on a partially coherent/incoherent target detection (PCTD) basis [17]-[18]. In a CTD methodology, the scattering matrix is fully polarimetric. In a PCTD approach, however, the covariance or coherency matrices contain the polarimetric information of partially polarized scattered waves. One widely used CTD technique is Cameron's elemental decomposition of maximum symmetric scattering into scatterer classes of trihedral, dihedral, dipole, cylinder, narrow diplane and quarter wave for each resolution cell [20]. There are certain inherent problems associated with Cameron's classification for polarimetric SAR (Synthetic Aperture Radar) applications, including coarse segmentation and erroneous effects due to scattering segmentation of non-coherent scattering (under a coherent assumption). To alleviate these problematic effects, new methods have been proposed to improve the segmentation process and to make use of coherence assessment to validate the coherent nature of scattering for the target under investigation [21]. The incoherent techniques (i.e., PCTD) provide physical insight to the nature of a partially coherent scattered field and the degree of polarization through Kennaugh, Mueller, covariance or coherency matrix definitions [17]. These coherency indicators provide valuable information on the nature and class of the scatterer [22].

In the present work, the main objective is to establish a relatively general and accurate framework for modeling of EM propagation and scattering for airborne or spaceborne polarimetric applications. The theoretical approach is different from conventional techniques as it combines a multidimensional Fourier analysis (i.e., decomposition) of space Green's function and inverse analysis of scattering using the EM theory and the *principle of equivalent sources*. Due to Fourier transformation of the exact source impulse response, coherent information is included in the formulation. The spectral nature of the analysis also provides insights into the Doppler effects through the space-frequency mapping mechanism between scatterer and observation. In addition, the forward nature of the model is advantageous for the purpose of target identification.

The other novel aspect of the current method is the classification of target scatterers through the introduction of vector current functions. The latter, which are based on the concept of scattering-induced equivalent sources in electromagnetic theory, is close to the roots of EM scattering governed by Maxwell's equations. To classify the target scatterer, the associated vector current function in the space or spectral domain is decomposed into a set of basis functions that can be viewed as features. Various basis sets may be used for the required decomposition with each set representing a class of features or sub-features. Due to the generic nature of this formulation, well-known analytic and numerical techniques can be adopted from other areas (e.g., RF, microwaves) for such characteristic function projections into the desired feature space. The possibility of defining various basis sets and number of members or basis functions for each set, indicates the potential of controlling and/or designing the nature and number of features for target decomposition. Such capability can be used to lower the misclassification probability associated with coarse classification or segmentation encountered through current coherent target decomposition techniques. Furthermore, study of the induced equivalent current source's behavior on a scatterer, can yield a more tangible understanding of the scatterer's physics. Finally, the source vector functions in the present formulation may be readily applied to compute the associated EM field quantities (such as coherent radar cross section) in both near and far field zones.

An important issue considered throughout this work and future related works is to identify and examine various aspects of target recognition and imaging via their contributions in the analytic model. Upon identification and classification of these analytic entities or blocks, comparisons can be made with the experimental results and other techniques (e.g., simulations) for the respective observables to validate the model. Furthermore, potential existing analytical tools suitable for characterization of a specific block may be applied or modified. One important category is the target scattering information with key observables such as the scatterer's EM signature and its radar cross section (RCS).

According to the described goals and structure of present methodology, one may outline the overall objectives as follows:

I. To develop a general Fourier-based formulation of electromagnetic propagation and scattering for polarimetric SAR applications;
II. To extract target scattering characteristics from the model and connect them to conventional features, e.g., the scattering matrix and the scatterer's RCS;

III. To investigate inversion techniques in the spectral domain and potentials of tailoring/optimizing them for certain types of applications;

IV. To study the feasibility of a systematic target classification and/or decomposition in the spectral domain;

V. To explore frequency domain selective techniques for adaptive and targeted scattering analysis;

VI. To study the effects, significance and feasibility of an optimum collection path design by using the EM model; and

VII. To explore the possibility of a multi-incident-angle analysis for target recognition enhancement.

Achievement of these objectives is a substantial task. However, these goals signify the research units or blocks to be investigated throughout the present and future work. Formulation of EM scattering in the Fourier domain (I), which is at the core of the investigation, is presented, as well as certain considerations on scattering characteristics (II) and potential inversion techniques (III). A more in-depth EM analysis of coherent scattering (e.g., RCS) and optimization of inversion techniques will be included in a later report. Target classification and decomposition in the spectral domain (IV), potential adaptive techniques in the spectral domain based on the state matrix and an analytic interpolation formulation [23]-[24] to enhance target recognition (V), collection path design considerations (VI) and, multi-look analysis of scatterer (VII) using the spectral scattering model are under study and will be reported in future communications.

Governing electromagnetic field equations which are at the core of any EM field analysis are described in the next section. The polarimetric model is formulated in Section 3, upon the introduction of equivalent sources and derivation of the scattered EM field due to the induced sources. The general relation between the described scattering-induced sources and scattering matrix parameters are also established in Section 3. Section 4 is devoted to the spectral analysis of scattering through Fourier decomposition of exact point source Green's function. In particular, the important system kernels are derived that relate the polarimetric SAR observations to the spectral distribution of scatterer. Bandwidth properties are investigated in Section 5. In this section, a procedure to map the windowing effect from space to frequency domain is analytically detailed that provides an invaluable tool to investigate effective bandwidth and resolution. It is also highlighted how the target distribution in Fourier space may be explored to examine various radar incident and aspect angles by virtue of the described mapping. The RCS derivations based on the knowledge of target vector distribution are presented in Section 6. Section 7 includes concluding remarks and direction for future research.

2.   General electromagnetic formulation

Electromagnetic fields in a linear, isotropic and inhomogeneous medium are given by the Maxwell's equations:

$$\nabla \times \left(\frac{\nabla \times \mathbf{E}}{\mu_r} \right) - k^2 \varepsilon_r \mathbf{E} = -j\omega\mu_0 \mathbf{J}_{src} \qquad (1)$$

$$\nabla \times \left(\frac{\nabla \times \mathbf{H}}{\varepsilon_r} \right) - k^2 \mu_r \mathbf{H} = \nabla \times \left(\frac{\mathbf{J}_{src}}{\varepsilon_r} \right) \qquad (2)$$

In (1)-(2), $\mathbf{E}, \mathbf{H}$ are the complex electric and magnetic fields with time dependence $e^{j\omega t}$, ω is the temporal frequency, ε_r and μ_r (ε_0 and μ_0) are the relative (free space) permittivity and permeability, $k = \omega/c$ is the free space wavenumber, c denotes the speed of light, and $\mathbf{J}_{src}$ represents all the current sources within the medium of interest. The differential operator denoted by ∇ is the Nabla or Hamilton operator.

For scattering analysis, it is convenient to convert the above equations into relations for an equivalent scattering problem in the homogeneous medium through introduction of equivalent induced polarization sources [25]-[26]. Using the *principle of volumetric equivalence,* (1)-(2) can be written as:

$$\nabla \times (\nabla \times \mathbf{E}) - k^2 \mathbf{E} = -j\omega\mu_0 \mathbf{J} - \nabla \times \mathbf{K} \qquad (3)$$

$$\nabla \times (\nabla \times \mathbf{H}) - k^2 \mathbf{H} = \nabla \times \mathbf{J} - j\omega\varepsilon_0 \mathbf{K} \qquad (4)$$

with the equivalent electric and magnetic current sources given by:

$$\mathbf{J} = \mathbf{J}_{src} + j\omega\varepsilon_0 (\varepsilon_r - 1)\mathbf{E} \qquad (5)$$

$$\mathbf{K} = j\omega\mu_0(\mu_r - 1)\mathbf{H} \tag{6}$$

Equations (3)-(6) are general and provide a solid theoretical point of reference for EM propagation and scattering modeling for radar (polarimetric in particular) applications. A comprehensive description of scattering can be obtained by satisfying the boundary conditions and/or identifying the sources for each scenario.

3. Polarimetric radar EM model

3.1 Field solution

To develop a scattering model for polarimetric radar applications, an electromagnetic theory is employed. Maxwell's equations (3)-(4) are governing equations for the total electric and magnetic fields which include both the incident $(\mathbf{E}^i, \mathbf{H}^i)$ and scattered

$(\mathbf{E}^s, \mathbf{H}^s)$ fields. For polarimetric radars, a convenient scattering equation can be written for the electric field as:

$$\nabla \times (\nabla \times \mathbf{E}^s) - k^2 \mathbf{E}^s = -j\omega\mu_0 \mathbf{J}_s - \nabla \times \mathbf{K}_s \qquad (7)$$

In derivation of (7), we note that the incident fields satisfy the vector Helmholtz equations:

$$\nabla^2 \mathbf{E}^i + k^2 \mathbf{E}^i = 0 \qquad (8)$$

$$\nabla^2 \mathbf{H}^i + k^2 \mathbf{H}^i = 0 \qquad (9)$$

Note that vectors $\mathbf{J}_s$ and $\mathbf{K}_s$ in (7) are electric and magnetic currents that represent equivalent induced sources due to scattering.

Although modeling of the above equivalent sources introduces a challenge for certain applications, the approach provides a robust theoretical tool for the analysis of scattering. Another attractive feature is the possibility of incorporating medium effects in definition of the equivalent sources (i.e., in equations (5)-(6)).

A general solution to Maxwell's equations (e.g., (7)) for the scattered field at the observation point $\mathbf{r}$, can be found by taking advantage of the well-known three-dimensional Green's function,

$$G(\mathbf{r}) = \frac{e^{-jk|\mathbf{r}|}}{4\pi|\mathbf{r}|} \qquad\qquad (10)$$

as:

$$\mathbf{E}_{rec}{}^{s}(\mathbf{r},\omega) = \frac{1}{j\omega\varepsilon_0}(k^2 + \nabla\nabla\cdot)[G * \mathbf{J}_s] - \nabla\times[G * \mathbf{K}_s] \qquad (11)$$

where the asterisk (*) denotes three-dimensional space convolution and, as before, the time dependence is $e^{j\omega t}$.

Making use of the properties of the free-space Green's function, the far-field expression (i.e., the radiation gauge) becomes:

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{r},\omega) = \frac{k^2}{j\omega\varepsilon_0}(G * \mathbf{J}_s)_T - (\nabla\times(G * \mathbf{K}_s))_T \qquad (12)$$

where the subscript T denotes the transversal component in a plane orthogonal to the direction of radiation or line of sight (LOS). The radiation field in (12) can also be written as:

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{r},\omega) = jk\{\eta[(G * \mathbf{J}_s)\times\hat{\mathbf{r}}]\times\hat{\mathbf{r}} + (G * \mathbf{K}_s)\times\hat{\mathbf{r}}\} \qquad (13)$$

where $\eta = \dfrac{k}{\omega\varepsilon_0} = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ is the intrinsic impedance of the medium (free space) and $\hat{\mathbf{r}}$ is the unit vector in the direction of radiation (i.e., observation).

3.2 Equivalent sources

There is no constraint on the type of induced current sources in (13); in general they can be volumetric. For scattering analysis in radar applications, however, it is convenient to formulate the problem in terms of surface distributed sources. Valid electromagnetic formulations of such boundary-value problems provide a comprehensive picture of scattering [25]-[28]. The main objective of these formalisms is to derive the overall field (initial and induced or scattered) by solving respective integro-differential equations for fields and source distributions when the required boundary conditions are enforced. It is crucial to use proper boundary conditions for integro-differential formulations in order to produce unique field solutions. In particular, care should be taken in defining equivalent sources when fields are non-vanishing infinitesimally close to the boundary surface. The latter may cause erroneous results even for infinitesimally thin layers of p.e.c. (perfect electric conductor) or p.m.c. (perfect magnetic conductor).

As was addressed, the fundamental theorem of electromagnetics generally known as the *equivalence principle* is a powerful concept that is rich in physical insight and provides theoretical ground for relating the fields to scattering through definition of equivalent induced sources. The concept of equivalent or substitute sources is a well known approach that can be traced back to Huyghens [29]. This concept was further developed by Schelkunoff [30], whose equivalence principle asserts that use of surface equivalent sources derived from the overall field values produces true fields at all points outside the surface and null fields inside. The Stratton–Chu formulation [31] provides the same result when it is used to express the electric and magnetic fields in terms of surface integrals [32]. One can apply the described surface equivalence principle for target scattering modeling when the necessary boundary conditions are satisfied.

Assume S is the scattering surface of the target or scatterer with electromagnetic fields $\mathbf{E}_{tot}$ and $\mathbf{H}_{tot}$ uniquely defined at this surface. Employing Schelkunoff's equivalence principle, we may write the electric and magnetic currents as:

$$\mathbf{J}_s(\mathbf{r}_s) = \hat{\mathbf{n}}_s(\mathbf{r}_s) \times \mathbf{H}_{tot}(\mathbf{r}_s) \tag{14}$$

$$\mathbf{K}_s(\mathbf{r}_s) = -\hat{\mathbf{n}}_s(\mathbf{r}_s) \times \mathbf{E}_{tot}(\mathbf{r}_s) \tag{15}$$

where $\hat{\mathbf{n}}_s(\mathbf{r}_s)$ denotes the unit (outward) normal vector to the surface S at scattering point $\mathbf{r}_s$. The total fields $\mathbf{E}_{tot}$ and $\mathbf{H}_{tot}$ at any point of observation space can be considered as the sum of the incident fields, i.e., the electromagnetic fields when there is no scattering, plus secondary or scattering fields:

$$\mathbf{E}_{tot}(\mathbf{r}) = \mathbf{E}^i(\mathbf{r}) + \mathbf{E}^s(\mathbf{r}) \tag{16}$$

$$\mathbf{H}_{tot}(\mathbf{r}) = \mathbf{H}^i(\mathbf{r}) + \mathbf{H}^s(\mathbf{r}) \tag{17}$$

In general, one can set:

$$\mathbf{E}_{tot}(\mathbf{r}) = [\vec{\mathbf{I}} + \vec{\mathbf{S}}_E(\mathbf{r})] \cdot \mathbf{E}^i(\mathbf{r}) \tag{18}$$

$$\mathbf{H}_{tot}(\mathbf{r}) = [\vec{\mathbf{I}} + \vec{\mathbf{S}}_H(\mathbf{r})] \cdot \mathbf{H}^i(\mathbf{r}) \tag{19}$$

where $\vec{\mathbf{S}}_E$ and $\vec{\mathbf{S}}_H$ are dyadic quantities representing general electric and magnetic field scattering, and $\vec{\mathbf{I}}$ is the unitary dyadic.

In theory, one may define the equivalent sources at any arbitrary surface S in space as:

$$\mathbf{J}_s(\mathbf{r}_s) = \hat{\mathbf{n}}_s(\mathbf{r}_s) \times \{[\vec{\mathbf{I}} + \vec{\mathbf{S}}_H(\mathbf{r}_s)] \cdot \mathbf{H}^i(\mathbf{r}_s)\} \tag{20}$$

$$\mathbf{K}_s(\mathbf{r}_s) = -\hat{\mathbf{n}}_s(\mathbf{r}_s) \times \{[\vec{\mathbf{I}} + \vec{\mathbf{S}}_E(\mathbf{r}_s)] \cdot \mathbf{E}^i(\mathbf{r}_s)\} \tag{21}$$

The sources in (20)-(21) produce true fields outside the surface S and a null field inside.

Defining the general scattering from electric incident to magnetic scattering $\vec{\mathbf{Y}}_{HE}$ with:

$$\mathbf{H}^s = \vec{\mathbf{Y}}_{HE} \cdot \mathbf{E}^i \tag{22}$$

one may recast (20) as:

$$\mathbf{J}_s(\mathbf{r}_s) = \hat{\mathbf{n}}_s(\mathbf{r}_s) \times \left\{ \frac{\hat{\mathbf{r}}_i}{\eta} \times \mathbf{E}^i(\mathbf{r}_s) + \ddot{\mathbf{Y}}_{HE}(\mathbf{r}_s) \cdot \mathbf{E}^i(\mathbf{r}_s) \right\} \qquad (23)$$

In (23), $\hat{\mathbf{r}}_i$ is a unit vector specifying the propagation direction of the incident EM wave $(\mathbf{E}^i, \mathbf{H}^i)$. Application of (21) and (23) in (13) yields an expression for the scattered or the echoed field at the space-time observation point $P_{rec}(\mathbf{r}, t)$.

At this stage, no particular form is assumed for $\ddot{\mathbf{S}}_E$, $\ddot{\mathbf{S}}_H$ or $\ddot{\mathbf{Y}}_{HE}$ which are simply reflecting the general scattering mechanism, i.e., the difference between the overall field and the initial or incident field. As will be discussed, however, these quantities can be directly related to the polarimetric scattering matrix for a backscattering scenario.

3.3   Radar field model

By defining the right-handed radar system unit vectors $(\hat{\mathbf{h}}, \hat{\mathbf{v}}, \hat{\mathbf{r}}_i)$, the known incident field polarization at any point in space is represented by:

$$\hat{\mathbf{u}}_{hv}(\mathbf{r}) = \cos(\psi(\mathbf{r}))\hat{\mathbf{h}} + \sin(\psi(\mathbf{r}))\hat{\mathbf{v}} \qquad (24)$$

where $\psi(\mathbf{r})$ denotes the polarization angle. That is:

$$\mathbf{E}^i(\mathbf{r}) = E^i(\mathbf{r})\hat{\mathbf{u}}_{hv}(\mathbf{r}) \qquad (25)$$

Use of (25) in (21) and (23) yields:

$$\mathbf{K}_s = -\hat{\mathbf{n}}_s \times [(\vec{\mathbf{I}} + \vec{\mathbf{S}}_E) \cdot \hat{\mathbf{u}}_{hv}] E^i \tag{26}$$

$$\mathbf{J}_s = \hat{\mathbf{n}}_s \times \left[\frac{\hat{\mathbf{r}}_i}{\eta} \times \hat{\mathbf{u}}_{hv} + \vec{\mathbf{Y}}_{HE} \cdot \hat{\mathbf{u}}_{hv} \right] E^i \tag{27}$$

All quantities in (26)-(27) are computed at the scattering point $(\mathbf{r}_s)$. The vector terms in (26)-(27):

$$\mathbf{W} = -\hat{\mathbf{n}}_s \times [(\vec{\mathbf{I}} + \vec{\mathbf{S}}_E) \cdot \hat{\mathbf{u}}_{hv}] \tag{28}$$

$$\mathbf{F} = \hat{\mathbf{n}}_s \times \left[\frac{\hat{\mathbf{r}}_i}{\eta} \times \hat{\mathbf{u}}_{hv} + \vec{\mathbf{Y}}_{HE} \cdot \hat{\mathbf{u}}_{hv} \right] \tag{29}$$

can be viewed as (and hereafter referred to as) a polarimetric radar target function.

Some considerations must be made before completing the present discussion. The target functions in (28)-(29) represent the normalized equivalent sources due to scattering. As a result, these vectors contain the coherent scattering information and can be analyzed to estimate the scattering matrix. In fact, for a backscattering scenario where targets are modeled as reflectors, the overall scattering mechanisms in (28)-(29) reduce to conventional polarimetric scattering quantities. Since all components of these equations are known, except the scattering quantities, target reflection functions (or equivalent source functions) can be connected to the scattering parameters. The latter provides a means for studying scattering characteristics and other target properties, such as the coherent RCS, within the same analysis.

Although possible to handle analytically, throughout the rest of this analysis magnetic current sources (28) will be neglected (i.e., we take the electric reflector assumption). It will also be useful to consider the following. If the scattering matrix for the electric field in a transversal plane (or slant plane) is defined by:

$$\begin{pmatrix} E_h{}^s \\ E_v{}^s \end{pmatrix} = \begin{pmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{pmatrix} \begin{pmatrix} E_h{}^i \\ E_v{}^i \end{pmatrix} = \vec{\vec{S}} \cdot \mathbf{E}^i \tag{30}$$

and if,

$$\begin{pmatrix} H_h{}^s \\ H_v{}^s \end{pmatrix} = \begin{pmatrix} Y_{hh} & Y_{hv} \\ Y_{vh} & Y_{vv} \end{pmatrix} \begin{pmatrix} E_h{}^i \\ E_v{}^i \end{pmatrix} = \vec{\vec{Y}} \cdot \mathbf{E}^i \tag{31}$$

one gets (see (8)-(9)),

$$\vec{\vec{Y}} = \frac{1}{\eta} \, \sigma_2 \cdot \vec{\vec{S}} \tag{32}$$

where

$$\sigma_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{33}$$

is a Pauli spin matrix.

Making use of (13), (27) and (29) gives the expression for the scattered field:

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{r},\omega) = jk\,\eta\,E^i \left[(G * \mathbf{F}) \times \hat{\mathbf{r}} \right] \times \hat{\mathbf{r}} \tag{34}$$

The convolution in (34) is performed over the coordinates of the scatterer. Hence, the integral form of (34) using (10) can be written as:

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{r}, \omega) = jk\,\eta \iiint E^i (\mathbf{F} \times \hat{\mathbf{r}} \times \hat{\mathbf{r}}) \frac{e^{-jk|\mathbf{r}-\mathbf{r}_s|}}{4\pi\,|\mathbf{r}-\mathbf{r}_s|}\,d\mathbf{r}_s \qquad (35)$$

The integration in (35) is a three-dimensional volume integral whereas the equivalent sources are two–dimensional surface currents. This, however, does not introduce inconsistency or loss of generality if the currents are properly defined. For instance, if the current surface is defined by a normal vector $\hat{\mathbf{n}}(\mathbf{r}_s)$ and coordinates (x_{t1}, x_{t2}, x_n), the 3-dimensional source distribution becomes $\mathbf{J}(x_{t1}, x_{t2}, x_n)\delta(x_n - x_{ns})$. In the latter, $\delta(x)$ represents the Dirac delta function.

For convenience, henceforth, the target or scatterer coordinates are denoted by $\mathbf{r}$, the observation coordinates by $\mathbf{u}$, and the unit radial vector by $\hat{\mathbf{u}}$. Moreover, the transversal normalized current $\mathbf{F} \times \hat{\mathbf{r}} \times \hat{\mathbf{r}}$ (or $\mathbf{F} \times \hat{\mathbf{u}} \times \hat{\mathbf{u}}$) is represented by $\mathbf{F}_T$. Accordingly:

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{u}, \omega) = \frac{jk\eta}{4\pi} \iiint E^i \mathbf{F}_T \frac{e^{-jk|\mathbf{u}-\mathbf{r}|}}{|\mathbf{u}-\mathbf{r}|}\,d\mathbf{r} \qquad (36)$$

4. Spectral domain modeling

Multidimensional Fourier analysis of the backscattered field (36), provides valuable insight to the nature of the scatterer. Prior to such a transformation, certain considerations are made.

In the present formulation, the centre of spatial coordinates is assumed to be the centre of target or scatterer coordinate system, i.e., the target is centred at the origin of the coordinate system $\mathbf{r} = (x, y, z) = (0,0,0)$. The incident field (E^i) effects are evaluated at the target position $\mathbf{r}$ whereas the signal or wave originates from the radar position $\mathbf{u}$. In order to account for this delay, one may consider the incident field (at the target) as:

$$E^i = E^i(\omega)e^{-jk|\mathbf{r}-\mathbf{u}|} \tag{37}$$

where $E^i(\omega)$ is the original transmitted waveform's spectrum. Thus,

$$\mathbf{E}_{rec}{}^{sr}(\mathbf{u},\omega) = \frac{jk\eta}{4\pi} E^i(\omega) \iiint \mathbf{F}_T(\mathbf{r},\omega) \frac{e^{-j2k|\mathbf{u}-\mathbf{r}|}}{|\mathbf{u}-\mathbf{r}|} d\mathbf{r} \tag{38}$$

Furthermore, assuming the amplitude variations associated with Green's function (far field) for a target area are negligible, one may use:

$$\mathbf{E}_{rec}{}^{far}(\mathbf{u},\omega) = \frac{jk\eta}{4\pi u} E^i(\omega) \iiint \mathbf{F}_T(\mathbf{r},\omega)\, e^{-j2k|\mathbf{u}-\mathbf{r}|}\, d\mathbf{r} \qquad (u = |\mathbf{u}|) \tag{39}$$

4.1 Fourier analysis

The general three-dimensional (observation) space Fourier transform of (36) is given by:

$$\widetilde{\mathbf{E}}_{rec}^{\ sr}(\mathbf{k}_u,\omega)=\widetilde{\mathbf{F}}_T(\mathbf{k}_u,\omega)\widetilde{G}_u(\mathbf{k}_u,\omega) \tag{40}$$

where $\mathbf{k}_u=(k_{ux},k_{uy},k_{uz})$ is the spatial frequency or wave vector, and

$$\widetilde{G}_u(\mathbf{k}_u,\omega)=\frac{jk\eta}{4\pi}\,E^i(\omega)\iiint\,\frac{e^{-j2k\sqrt{u_x^2+u_y^2+u_z^2}}}{\sqrt{u_x^2+u_y^2+u_z^2}}\,e^{-jk_{ux}u_x}\,e^{-jk_{uy}u_y}\,e^{-jk_{uz}u_z}\,\,du_x\,du_y\,du_z \tag{41}$$

$$\widetilde{\mathbf{F}}_T(\mathbf{k},\omega)=\iiint\,\widetilde{\mathbf{F}}_T(\mathbf{r},\omega)\,e^{-jk_x x}\,e^{-jk_y y}\,e^{-jk_z z}\,\,dx\,dy\,dz \tag{42}$$

The integration in (41) is performed over the observation space whereas the integrations in (38) and (42) are performed over the target or scatterer coordinates. As can be seen from (40)-(42), the three-dimensional space frequency mapping is represented through $\mathbf{k}_u=\mathbf{k}$.

In order to relate the observations (in either the spatial or the spectral domain) to the Fourier characteristics of the scattering source, a proper Fourier decomposition of the Green's function in (38) is required. It is possible to handle the required derivations by using the *principle of stationary phase* (PSP), as described in Appendix B. To explore more physical insights and a potential slant plain analysis for SAR applications, however, the exact Fourier analysis is carried out on the point source Green's function in separate steps for the transversal plane and the longitudinal (or vertical) dimension.

The source response function in (38):

$$G_u(\mathbf{r})=\frac{e^{-j2k|\mathbf{r}|}}{4\pi|\mathbf{r}|} \tag{43}$$

satisfies the scalar Helmholtz equation:

$$(\nabla^2+4k^2)G_u(\mathbf{r})=-\delta(\mathbf{r}) \tag{44}$$

Hence, the three-dimensional Fourier transform satisfies the following equation:

$$(k_x{}^2 + k_y{}^2 + k_z{}^2 - 4k^2)\widetilde{G}_u(k_x, k_y, k_z) = 1 \tag{45}$$

 The polar spectrum of the Green's function can be obtained by the inverse Fourier integration over the complex k_z plane. By applying the residue theorem, one gets:

$$\widetilde{G}_{u\rho}(k_x, k_y, z) = \frac{j \exp\left(j\sqrt{4k^2 - \rho^2}\, |z|\right)}{2\sqrt{4k^2 - \rho^2}} \tag{46}$$

where (ρ, φ) are the spectral polar coordinates of (k_x, k_y), i.e.,

$$k_x = \rho\cos(\varphi) \tag{47a}$$

$$k_y = \rho\sin(\varphi) \tag{47b}$$

$$k_x{}^2 + k_y{}^2 = \rho^2 \tag{47c}$$

Next, we apply (46) in (38) when making use of the generalized Parseval's theorem to obtain:

$$\mathbf{S}_{rec}(\mathbf{u}, \omega) = E^i(\omega) \iiint \widetilde{\mathbf{F}}_T(k_x, k_y, z, \omega) \frac{\exp\left(j\sqrt{4k^2 - \rho^2}\, |u_z - z|\right)}{\sqrt{4k^2 - \rho^2}} e^{-jk_x u_x} e^{-jk_y u_y}\, dk_x dk_y dz \tag{48}$$

where the normalized received signal is defined by:

$$\mathbf{S}_{rec}(\mathbf{u}, \omega) = -\frac{8\pi^2}{k\eta}\, \mathbf{E}_{rec}{}^{sr}(\mathbf{u}, \omega) \tag{49}$$

Hence for $u_z - z > 0$, which introduces no loss of generality,

$$\mathbf{S}_{rec}(\mathbf{u},\omega) = E^i(\omega) \iint \widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega) \; \frac{\exp\!\left(j\sqrt{4k^2-\rho^2}\; u_z\right)}{\sqrt{4k^2-\rho^2}} \; e^{-jk_x u_x} e^{-jk_y u_y} \; dk_x dk_y$$

(50)

with

$$k_z = \sqrt{4k^2 - \rho^2}$$

(51)

Consider a general observation point, or in a SAR scenario, the synthetic aperture coordinates:

$$\mathbf{u} = (u_x, u_y, u_z) = (r_u \cos(\phi), r_u \sin(\phi), u_z)$$

(52)

where r_u and ϕ are polar coordinates of observation or synthetic aperture. Making use of (52) results in:

$$\mathbf{S}_{rec}(\mathbf{u},\omega) = E^i(\omega) \iint \widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega) \; \frac{\exp\!\left(j\sqrt{4k^2-\rho^2}\; u_z\right)}{\sqrt{4k^2-\rho^2}} \; \exp(-j\rho r_u \cos(\phi - \varphi)) \, \rho \, d\rho \, d\varphi$$

(53)

where k_x, k_y, k_z are defined by (47) and (51).

One can recast (53) as:

$$\mathbf{S}_{rec}(\mathbf{u},\omega) = E^i(\omega)\int \rho\, d\rho\, \frac{\exp\left(j\sqrt{4k^2-\rho^2}\; u_z\right)}{\sqrt{4k^2-\rho^2}}\, [\,\widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega)*\exp(-j\rho\, r_u\cos(\phi))\,]^{(\phi)}$$

$$(54)$$

In (54) the asterisk symbol (*) denotes the one-dimensional angular convolution (ϕ dimension) and k_x, k_y, k_z are defined by (47) where φ is replaced with ϕ.

Equation (54) relates the spectral distribution of the polarimetric target function $\widetilde{\mathbf{F}}_T$ to any arbitrary observation point (or synthetic aperture) $\mathbf{u}$ in space.

For moving platforms, as was addressed earlier, space frequency mapping to the observation spectral domain takes place in (54) (also see (40)). More specifically, taking the Fourier transform of both sides in (54) with respect to u_z :

$$\widetilde{\mathbf{S}}_{rec}(k_{uz},u_x,u_y,\omega) = E^i(\omega)\int \frac{\rho\, d\rho}{\sqrt{4k^2-\rho^2}}\, \delta(k_{uz}-\sqrt{4k^2-\rho^2}\,)[\,\widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega)*\exp(-j\rho\, r_u\cos(\phi))\,]^{(\phi)}$$

$$(55)$$

Taking advantage of the analytic properties of the delta function:

$$\int g(\rho)\delta(f(\rho)-a)d\rho = \frac{g(\rho)}{df/d\rho}\Big|_{\rho=\rho_0} \qquad (where\ \ f(\rho_0)-a=0) \qquad (56)$$

we find:

$$\widetilde{\mathbf{S}}_{rec}(k_{uz},r_u\cos(\phi),r_u\sin(\phi),\omega) = E^i(\omega)[\,\widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega)*\exp(-jk_\rho\, r_u\cos(\phi))\,]^{(\phi)} \quad (57)$$

where now,

$$k_z = k_{uz} \qquad\qquad (58)$$

$$k_x = \sqrt{4k^2 - k_{uz}^2}\, \cos(\phi) \qquad\qquad (59a)$$

$$k_y = \sqrt{4k^2 - k_{uz}^2}\, \sin(\phi) \qquad\qquad (59b)$$

with

$$k_\rho = \sqrt{4k^2 - k_{uz}^2} \qquad\qquad (60)$$

Equations (58)-(60) indicate that the mapping from the observation spectral domain to target spectral domain is, in general, non-linear. The space frequencies in (58)-(59) are located on the surface of a sphere of radius $2k$ in the spectral domain, known as the Ewald sphere.

Considering the Fourier expansion of the nonlinear phase term:

$$\exp(-js\cos(\phi)) = \sum_\alpha H_\alpha^{(2)}(s)\, e^{j\alpha(\phi - \pi/2)} \qquad\qquad (61)$$

where $H_\alpha^{(2)}(s)$ is the Hankel function of the second kind and order α, one may transform (57) to the spectral domain (α) of angular dimension ϕ (which can also be considered as an aspect angle) as:

$$\widetilde{\mathbf{S}}_{rec}(k_{uz}, \alpha, r_u, \omega) = E^i(\omega)\, \widetilde{\mathbf{F}}_T^{(\alpha)}(k_{uz}, \alpha, \omega)\, H_\alpha^{(2)}(k_\rho r_u)\, e^{-j\alpha\pi/2} \qquad (62)$$

The target function Fourier transform is defined via:

$$\widetilde{\mathbf{F}}_T^{(\alpha)}(k_{uz},\alpha,\omega)=\int_{\phi_0}\widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega)e^{-j\alpha\phi}\ d\phi \qquad (63)$$

$$\widetilde{\mathbf{F}}_T(k_x,k_y,k_z,\omega)=\sum_\alpha\widetilde{\mathbf{F}}_T^{(\alpha)}(k_{uz},\alpha,\omega)e^{j\alpha\phi} \qquad (64)$$

with (k_x,k_y,k_z) defined by (58)-(59) and, ϕ_0 representing the angular synthetic aperture domain. It is within this domain that orthogonality of Fourier components $e^{-j\alpha\phi}$ holds. Accordingly, for $\phi\in(-\pi,\pi)$, α takes on integer values.

Equation (62) (or (57)) provides an analytic means to recover the target information contained in $\mathbf{F}_T$ from the observations made through $\mathbf{S}_{rec}$. A variety of methods may be used to perform the required inverse processing which are optimal and suited for certain applications. A relatively simple and general matched filtering technique to implement the deconvolution needed for inverse processing will be addressed.

4.2 2-Dimensional slant plane Fourier analysis

As a typical SAR scenario, consider the imaging of the ground plane by a radar-carrying vehicle flying at a constant height $z=Z_h$. Although altitude variations from the ground plane $z=0$ are always present in practical operations (as included in the model developed), those effects may not be of interest for certain applications. An appropriate model for such applications can be derived from the three-dimensional formulation by imposing the associated conditions. We perform the required derivations as follows.

The target distribution at the ground level $z=0$ (i.e., ignoring the terrain variations) can be written as:

$$\mathbf{F}_T(x,y,z,\omega)=\mathbf{F}_T(x,y,\omega)\delta(z) \qquad (65)$$

Applying the three-dimensional Fourier transform of (65) in (57), one gets:

$$\tilde{\mathbf{S}}_{rec}(k_{uz}, r_u\cos(\phi), r_u\sin(\phi), \omega) = E^i(\omega)[\tilde{\mathbf{F}}_T(k_x, k_y, \omega) * \exp(-jk_\rho r_u\cos(\phi))]^{(\phi)} \quad (66)$$

The inverse Fourier transform of (66) with respect to u_z (k_{uz}) yields:

$$\tilde{\mathbf{S}}_{rec}(u_z, r_u\cos(\phi), r_u\sin(\phi), \omega) = E^i(\omega)\int dk_{uz}\, e^{jk_{uz}u_z}[\tilde{\mathbf{F}}_T(k_x, k_y, \omega) * \exp(-jk_\rho r_u\cos(\phi))]^{(\phi)} \quad (67)$$

where k_ρ is given by (60). Hence,

$$dk_{uz} = -\frac{k_\rho}{k_{uz}}\, dk_\rho \qquad (68)$$

Making use of (68) for the observation set at $u_z = Z_h$, results in:

$$\tilde{\mathbf{S}}_{rec}(Z_h, r_u\cos(\phi), r_u\sin(\phi), \omega) = E^i(\omega)\int \frac{k_\rho\, dk_\rho}{\sqrt{4k^2 - k_\rho^2}} e^{-j\sqrt{4k^2 - k_\rho^2}\, Z_h}[\tilde{\mathbf{F}}_T(k_x, k_y, \omega) * \exp(-jk_\rho r_u\cos(\phi))]^{(\phi)}$$

$$(69)$$

with,

$$k_x = k_\rho\cos(\phi) \qquad (70)$$

$$k_y = k_\rho\sin(\phi) \qquad (71)$$

Furthermore, the angular Fourier transform described in (63)-(64) may be used to transform (69) into:

$$\widetilde{\mathbf{S}}_{rec}(Z_h, \alpha, r_u, \omega) = E^i(\omega) \int k_\rho dk_\rho \frac{e^{-j\sqrt{4k^2 - k_\rho^2} Z_h}}{\sqrt{4k^2 - k_\rho^2}} \widetilde{\mathbf{F}}_T^{(\alpha)}(k_\rho, \alpha, \omega) H_\alpha^{(2)}(k_\rho r_u) e^{-j\alpha\pi/2} \quad (72)$$

As can be seen from equation (72), extraction of the ground target information $\mathbf{F}_T$ from the observation $\mathbf{S}_{rec}|_{z=Z_h}$ is more complicated than in (62) and, in principle, involves solving the integral equation. One should note, however, that the simplicity offered in (62) is due to a full three-dimensional spectral domain treatment of scattering, i.e., an extra layer of Fourier transformation in the z direction. In theory, the system defined by equation (72) can be viewed as a shift-varying system [33] with a known analytical kernel:

$$\Gamma(\omega, \omega_g) = E^i(\omega) \frac{e^{-j\sqrt{4k^2 - k_\rho^2} Z_h}}{\sqrt{4k^2 - k_\rho^2}} \quad (73)$$

where the ground plane equivalent temporal frequency ω_g is defined via:

$$k_\rho = 2\frac{\omega_g}{c} \quad (74)$$

Setting:

$$\widetilde{\mathbf{S}}_g(\alpha, r_u, \omega_g) = \left(\frac{2}{c}\right) k_\rho \widetilde{\mathbf{F}}_T^{(\alpha)}(k_\rho, \alpha) H_\alpha^{(2)}(k_\rho r_u) e^{-j\alpha\pi/2} \qquad \left(k_\rho = 2\frac{\omega_g}{c}\right) \quad (75)$$

the system equation can be written as:

$$\widetilde{\mathbf{S}}_{rec}(Z_h, \alpha, r_u, \omega) = \int_{\omega_g} \Gamma(Z_h, \omega_g, \omega) \widetilde{\mathbf{S}}_g(\alpha, r_u, \omega_g) d\omega_g \quad (76)$$

Technically, the ground plane signal (75) can be recovered by using the inverse of the system kernel function (73):

$$\widetilde{\mathbf{S}}_g(\alpha, r_u, \omega_g) = \int_{\omega} \widetilde{\mathbf{S}}_{rec}(Z_h, \alpha, r_u, \omega)\Gamma^{-1}(Z_h, \omega_g, \omega)d\omega \tag{77}$$

Application of (77) with (75) yields the desired target function. One should note that the required deconvolution to recover the ground plane signal $\widetilde{\mathbf{S}}_g$ could also be achieved through matched-filtering of the system kernel scheme.

5. Fourier properties and inversion considerations

The model developed here can be used to examine various aspects connecting the target characteristics to observations from both electromagnetics and system points of view. One important application, however, is to extract the target information from observations. According to equations (62) (or (57)) and (72) (or (62)), the latter involves inverse processing utilizing techniques ranging from matched filtering methods (as will be discussed) to more sophisticated integral equation based methods such as the method of moments (MoM).

To perform the inverse processing and conduct any spectral domain considerations (e.g., spatial resolution), bandwidth limitations associated with factors such as finite aperture and target distribution must be investigated.

5.1 Bandwidth properties

Bandwidth characteristics of the received SAR signal can be studied by making use of the windowing properties of phase-modulated (PM) signals [34]-[35]. These properties are essentially associated with the space-limited nature of the Green's function connecting the target (scatterer) and observation spaces. Since the Green's function and its spherical phase front are space-limited or windowed, the Fourier transform of the Green's function is also windowed in the space-frequency domain. To formulate the described windowing behaviour for bandwidth estimation, one can impose the space windowing conditions when deriving the instantaneous frequencies.

We will derive a general relation to map the respective spatial and spectral windowing. Prior to that, however, the following should be noted. The Fourier decomposition of the Green's function in the present formulation represents all possible components including evanescent modes. For SAR applications, the observation is normally within the far field region of the target where the evanescent or transient modes are negligible. Hence, considering the characteristic equation for the poles of the Green's function (Ewald sphere):

$$k_x{}^2 + k_y{}^2 + k_z{}^2 - 4k^2 = 0 \tag{78}$$

the overall support band in the frequency domain (infinite aperture and target distribution) is $2k$, or equally, $(k_x, k_y, k_z) \in [-2k, 2k]$.

The generalized instantaneous space frequency vector can be obtained by computing the gradient (∇) of the phase function $\Psi(\mathbf{r})$ with respect to space coordinates:

$$\mathbf{k}_{inst}(\mathbf{r}) = \nabla\Psi(\mathbf{r}) \tag{79}$$

Equation (79) provides an effective tool to evaluate the spectral domain properties of domain limited field signals. One can show that (79) satisfies the generalized stationary phase equation:

$$\delta[\Psi(\mathbf{r}) - \mathbf{k}_{inst}^{(\mathbf{r})} \cdot \mathbf{r}] = 0 \tag{80}$$

that is (in scalar format) used to determine the phase center of the phase function [35].

Application of (79) for the spherical phase front associated with the Green's function:

$$\Psi(\mathbf{u},\mathbf{r}) = 2k|\mathbf{u} - \mathbf{r}| \tag{81}$$

yields the instantaneous frequency for spectral domain considerations. Spectral domain windowing properties are examined throughout the following analysis.

Consider the SAR operation for ground plane target analysis detailed in Section 4.2. In this typical case of operation, the observation undertakes a general motion at a certain altitude $z = Z_h$. Furthermore, we assume the target has a limited distribution within a radius of r_0, i.e., $r = \sqrt{x^2 + y^2} \le r_0$ (Figure 1).

Using (79) and (81), one finds the (x, y) plane instantaneous space frequencies:

$$k_x = \frac{\partial}{\partial u_x}\left[2k\sqrt{(u_x - x)^2 + (u_y - y)^2 + Z_h^2}\right] = 2k\frac{u_x - x}{\sqrt{(u_x - x)^2 + (u_y - y)^2 + Z_h^2}} \qquad (82)$$

$$k_y = \frac{\partial}{\partial u_y}\left[2k\sqrt{(u_x - x)^2 + (u_y - y)^2 + Z_h^2}\right] = 2k\frac{u_y - y}{\sqrt{(u_x - x)^2 + (u_y - y)^2 + Z_h^2}} \qquad (83)$$

with associated polar coordinates (ρ, φ) given by:

$$\tan(\varphi) = \frac{k_y}{k_x} = \frac{u_y - y}{u_x - x} \qquad (84)$$

$$\rho = \sqrt{k_x^2 + k_y^2} = 2k\frac{\sqrt{(u_x - x)^2 + (u_y - y)^2}}{\sqrt{(u_x - x)^2 + (u_y - y)^2 + Z_h^2}} \qquad (85)$$

Define the centre slant angle associated with the centre of target distribution (centre incident angle), as given in Figure 1(a):

$$\theta_z = \tan^{-1}\left(\frac{r_u}{Z_h}\right) \qquad (86)$$

Inspection of Figure 1(b) reveals that the spectral polar angle φ defined in (84) is windowed due to the finite target distribution as:

$$\varphi \leq \varphi_0 \qquad (87)$$

where,

$$\varphi_0 = \sin^{-1}(\frac{r_0}{r_u}) \qquad\qquad r_0 \leq r_u$$

$$\quad = \pi/2 \qquad\qquad\qquad\qquad otherwise \qquad\qquad (88)$$

Making use of (85), we find the polar phase center for any given observation point (r_u):

$$\rho_0 = 2k \frac{r_u}{\sqrt{r_u^2 + Z_h^2}} = 2k\sin(\theta_z) \qquad\qquad (89)$$

with a spread that can be approximated by:

$$\Delta\rho \approx 2k\cos^2(\theta_z)\sin(\varphi_0) \qquad\qquad (90)$$

Thus the windowing function in the spectral domain for a given observation point (r_u) is:

$$W_{r_u}(\rho,\varphi,\omega) = \begin{cases} 1, & for \ \ |\rho - 2k\sin(\theta_z)| \leq 2k\cos^2(\theta_z)\sin(\varphi_0) \ \ and \ \ |\varphi| \leq \varphi_0 \\ 0, & otherwise \end{cases} \qquad (91)$$

For a more particular and/or typical case of operation, one may consider a scenario when the observation is performed along a straight line. In this case, the observation vector is represented by:

$$\mathbf{u} = (X, u_y, Z_h) \qquad\qquad (92)$$

where X denotes the constant ground range associated with the center of target distribution and, $-L \leq u_y \leq L$ represents the synthetic aperture dimension. Accordingly, L is the synthetic aperture length. The instantaneous radial frequency for the present scenario becomes:

$$\rho = 2k \frac{\sqrt{(X-x)^2 + (u_y - y)^2}}{\sqrt{(X-x)^2 + (u_y - y)^2 + Z_h^2}} \tag{93}$$

Define the instantaneous centre incident angle associated with any observation point (r_u) as:

$$\theta_z = \sin^{-1}\left(\frac{r_u}{\sqrt{r_u^2 + Z_h^2}}\right) \equiv \theta_z(u_y) \tag{94}$$

where,

$$r_u = \sqrt{X^2 + u_y^2} \tag{95}$$

The spectral windowing function for any observation point (r_u), is given by (89)-(91) when definitions (94)-(95) are applied. Hence, moving along the u_y direction from $-L$ to $+L$, i.e., $u_y \in [-L, L]$:

$$2k \sin(\theta_z(u_y)) - 2k \cos^2(\theta_z(u_y)) \sin(\varphi_0) < \rho < 2k \sin(\theta_z(u_y)) + 2k \cos^2(\theta_z(u_y)) \sin(\varphi_0) \tag{96}$$

In order to determine the window function associated with (96), consider the functions:

$$\frac{d}{d\gamma}\Omega_-(\gamma) = \frac{d}{d\gamma}[\sin(\gamma) - \cos^2(\gamma)\sin(\varphi_0)] = \cos(\gamma)[1 + 2\sin(\gamma)\sin(\varphi_0)] \qquad (97)$$

$$\frac{d}{d\gamma}\Omega_+(\gamma) = \frac{d}{d\gamma}[\sin(\gamma) + \cos^2(\gamma)\sin(\varphi_0)] = \cos(\gamma)[1 - 2\sin(\gamma)\sin(\varphi_0)] \qquad (98)$$

It can be seen that $\dfrac{d\Omega_-(\gamma)}{d\gamma}$ is positive for $0 < \gamma < \pi/2$, while $\dfrac{d\Omega_+(\gamma)}{d\gamma}$ becomes negative at:

$$\sin(\gamma_{max}) = \frac{1}{2\sin(\varphi_0)} \qquad (99)$$

The maximum function value associated with γ_{max} is:

$$\Omega_+(\gamma_{max}) = \frac{1 + 4\sin^2(\varphi_0)}{4\sin(\varphi_0)} \qquad (100)$$

Equation (97)-(100) indicate that within the interval $[\gamma_0, \gamma_L]$ ($[\gamma_0, \gamma_L] \in [0, \pi/2]$), one can write:

$$\sin(\gamma_0) - \cos^2(\gamma_0)\sin(\varphi_0) < \Omega_-(\gamma) < \sin(\gamma_L) - \cos^2(\gamma_L)\sin(\varphi_0) \qquad (101)$$

$$\sin(\gamma_0) + \cos^2(\gamma_0)\sin(\varphi_0) < \Omega_+(\gamma) < \sin(\gamma_L) + \cos^2(\gamma_L)\sin(\varphi_0) \qquad , for \ \gamma_L \leq \gamma_{max}$$

$$\sin(\gamma_0) + \cos^2(\gamma_0)\sin(\varphi_0) < \Omega_+(\gamma) < \frac{1 + 4\sin^2(\varphi_0)}{4\sin(\varphi_0)} \qquad , for \ \gamma_L > \gamma_{max}$$

$$(102)$$

Taking advantage of (101)-(102), the support band associated with (96) is as follows:

$$2k\sin(\theta_{z0}) - 2k\cos^2(\theta_{z0})\sin(\varphi_0) < \rho < 2k\sin(\theta_{zL}) + 2k\cos^2(\theta_{zL})\sin(\varphi_0) \qquad , for \ \theta_{zL} \leq \gamma_{max}$$

$$2k\sin(\theta_{z0}) - 2k\cos^2(\theta_{z0})\sin(\varphi_0) < \rho < \frac{1 + 4\sin^2(\varphi_0)}{4\sin(\varphi_0)} \qquad , for \ \theta_{zL} > \gamma_{max}$$

$$(103)$$

The respective windowing function becomes:

$$W_{r_u}(\rho,\varphi,\omega) = \begin{cases} 1, & for \ \rho \ satisfying \ (103) \ and \ |\varphi| \leq \varphi_0 \\ 0, & otherwise \end{cases}$$

$$(104)$$

where γ_{max} is defined by (99) and:

$$\theta_{z0} = \tan^{-1}(\frac{X}{Z_h}) \qquad\qquad (105)$$

$$\theta_{zL} = \tan^{-1}\left(\frac{\sqrt{X^2 + L^2}}{Z_h}\right) \qquad\qquad (106)$$

One last note in the context of windowing characteristics of phase-modulated signals is the windowing properties when the angular Fourier transform ((63)-(64)) is used. In this case, the support of the window for the angular direction ϕ in the respective spectral domain α must be determined. The latter can be done by applying the instantaneous frequency relation (79) in the angular direction and the phase definition:

$$\exp(-j\rho\, r_u \cos(\phi)) = \exp(-j\Psi) \tag{107a}$$

as,

$$\alpha_{inst} = \frac{\partial \Psi}{\partial \phi} = -\rho\, r_u \sin(\phi) \tag{107b}$$

Thus,

$$|\alpha| \le \rho\, r_u \sin(\varphi_0) \tag{108}$$

As was asserted earlier, derivations in this subsection can be effectively used to perform spectral analysis of the polarimetric radar waveform for different purposes. One typical application (as detailed) is the effective bandwidth estimation. Another potentially interesting application is the exploration of acquired data along the paths corresponding to different incident angles.

Consider the description of instantaneous frequencies given by (82)-(83), (85) and a similar description for k_z that can be easily derived in the same fashion. Making use of the above and (86), one can derive a similar relation for the incident angle in spectral domain, i.e.,

$$\tan(\theta_z) = \frac{\rho}{k_z} = \frac{\sqrt{k_x^{\,2} + k_y^{\,2}}}{k_z} \tag{109}$$

Various paths (corresponding to certain incident and aspect angles) within the 3-dimensional spectral distribution of the target scatterer may be explored in the spectral space (k_x, k_y, k_z) by using (109) and (84).

5.2 Inversion

As was addressed earlier, recovery of the target characteristic vector function requires inverse mapping according to the derived scattering model (e.g., (57), (69)). Such inverse processing normally involves deconvolution of the known system kernel or reference function and may be performed through a variety of analytic and/or numerical methods. One potentially strong candidate is the method of moments (MoM) or its variational form, *Galerkin's Method* [36]-[39]. Moment methods are suitable and numerically efficient for computations involving known analytical kernels [36]. We do not intend to discuss the analytical details of such formulations. In general, the procedure is to solve the inhomogeneous system:

$$\mathbf{L}(f) = g \tag{110}$$

where $\mathbf{L}$ is the system operator which can be viewed as the system kernel, g is the known quantity (e.g., observation), and f denotes the unknown function, e.g., scattering characteristics to be determined. Expanding f into a set of appropriate basis functions:

$$f = \sum_m a_m f_m \tag{111}$$

and defining a proper projection space $\{p_i\}$ and moments,

$$\mathbf{M}_{mi} = \langle u_m, v_i \rangle \tag{112}$$

gives:

$$\sum_m a_m \langle \mathbf{L}f_m, p_i \rangle = \sum_m a_m \, \mathrm{M}_{mi}{}^{fp} = \langle g, p_i \rangle \tag{113}$$

The resultant linear system is solved for the unknown weightings a_m. In (113), the system operator $\mathbf{L}$ is assumed to be self-adjoint. Proper selection of the basis set $\{f_m\}$, projection space $\{p_i\}$ and mechanism $\langle u, v \rangle$ (generalized inner product) is important to yield convergence and accurate results. Choosing the projection space $\{p_i\}$ to be the same as the basis functions space $\{f_m\}$, results in the variational Galerkin's method.

A formalism based on the method of moments can be developed and tailored for the present SAR inverse processing to recover the polarimetric target characteristics through knowledge of the analytic system kernel. One can explore various basis vector space design (under investigation and to be communicated) representing scattering decomposition classes for the purpose of target identification.

Relations (62) and (75) suggest that the target function in the spectral domain may be recovered by simply dividing the observation spectrum by the system kernel or reference function:

$$\widetilde{\mathbf{F}}_T{}^{(\alpha)}(k_{uz}, \alpha, \omega) = \widetilde{S}_{ref}{}^{-1}(k_\rho, \alpha, r_u, \omega)\widetilde{\mathbf{S}}_{rec}(k_{uz}, \alpha, r_u, \omega) \tag{114}$$

$$\widetilde{\mathbf{F}}_T{}^{(\alpha)}(k_\rho, \alpha) = \widetilde{S}_{gref}{}^{-1}(k_\rho, \alpha, r_u)\widetilde{\mathbf{S}}_g(\alpha, r_u, \omega_g) \tag{115}$$

where,

$$\widetilde{S}_{ref}(k_\rho, \alpha, r_u, \omega) = E^i(\omega) \, H_\alpha{}^{(2)}(k_\rho r_u) \, e^{-j\alpha\pi/2} \tag{116}$$

$$\widetilde{S}_{gref}(k_\rho, \alpha, r_u) = \left(\frac{2}{c}\right) k_\rho \, H_\alpha{}^{(2)}(k_\rho r_u) \, e^{-j\alpha\pi/2} \tag{117}$$

Technically, the operations in (114)-(115) represent the discussed deconvolution in the spectral domain with the system functions defined in (116)-(117). This recovery, however, is not practical due to the discussed (Section 5.1) finite support band of reference $(\widetilde{S}_{ref}, \widetilde{S}_{gref})$ and received $(\widetilde{\mathbf{S}}_{rec}, \widetilde{\mathbf{S}}_{g})$ signals. A practical reconstruction can be achieved by using a matched filtering scheme such as:

$$\widetilde{\mathbf{F}}_{Tm}{}^{(\alpha)}(k_{uz}, \alpha, \omega) = \widetilde{S}_{ref}^{*}(k_{\rho}, \alpha, r_u, \omega)\widetilde{\mathbf{S}}_{rec}(k_{uz}, \alpha, r_u, \omega) \tag{118}$$

$$\widetilde{\mathbf{F}}_{Tm}{}^{(\alpha)}(k_{\rho}, \alpha) = \widetilde{S}_{gref}^{*}(k_{\rho}, \alpha, r_u)\widetilde{\mathbf{S}}_{g}(\alpha, r_u, \omega_g) \tag{119}$$

where $(\widetilde{S}_{ref}^{*}, \widetilde{S}_{gref}^{*})$ are the complex conjugates of reference functions.

6. Coherent radar cross section (RCS) analysis

An attractive feature of the present approach is the possibility of direct coherent RCS derivations from the knowledge of target function $\mathbf{F}(\mathbf{r})$ (or $\mathbf{F}_T(\mathbf{r})$). The information contained in $\mathbf{F}(\mathbf{r})$ represents the electromagnetic identity of the target and, therefore, is capable of producing accurate estimate of scattering-related observables such as the coherent RCS. Furthermore, derivations of the described observables are not limited to the far field of the scatterer (or radiator) and may be carried out in the near field region by using (11) (or a proper approximation of (11)) instead of (12)-(13). Prior to such near field considerations, however, a qualified description and/or definition of RCS in the near field has to be established.

The bistatic scattering cross section for an incident plane-wave $\mathbf{E}^i$ can be expressed as [28]:

$$\sigma(\theta,\phi,\theta^i,\phi^i) = \lim_{r\to\infty} 4\pi r^2 \frac{|\mathbf{E}^{sr}(\theta,\phi)|^2}{|\mathbf{E}^i(0,0)|^2} \tag{120}$$

According to the far field formulation (36), we have:

$$\sigma(\theta,\phi,\theta^i,\phi^i) = \lim_{u\to\infty} 4\pi u^2 \left| \frac{jk\eta}{4\pi u} \iiint \mathbf{F}_T(\mathbf{r})\, e^{-jk|\mathbf{u}-\mathbf{r}|}\, d\mathbf{r} \right|^2$$

$$= \frac{k^2\eta^2}{4\pi} \left| \iiint \mathbf{F}_T(\mathbf{r})\, e^{-jk|\mathbf{u}-\mathbf{r}|}\, d\mathbf{r} \right|^2 \tag{121}$$

Applying the far field approximation:

$$|\mathbf{u}-\mathbf{r}| \approx u - \hat{\mathbf{u}}\cdot\mathbf{r} \qquad\qquad \left(\hat{\mathbf{u}} = \frac{\mathbf{u}}{u} \right) \tag{122}$$

and coordinate transformation from Cartesian to Spherical systems, we find:

$$\sigma(\theta, \phi, \theta^i, \phi^i) = \sigma_\theta(\theta, \phi) + \sigma_\phi(\theta, \phi) \tag{123}$$

where,

$$\sigma_\theta(\theta, \phi) = \frac{k^2 \eta^2}{4\pi} \left| \iiint (F_{Tx} \cos(\theta)\cos(\phi) + F_{Ty} \cos(\theta)\sin(\phi) - F_{Tz} \sin(\theta)) e^{jk\,\hat{\mathbf{u}}\cdot\mathbf{r}} \, dxdydz \right|^2 \tag{124}$$

$$\sigma_\phi(\theta, \phi) = \frac{k^2 \eta^2}{4\pi} \left| \iiint (-F_{Tx} \sin(\phi) + F_{Ty} \cos(\phi)) e^{jk\,\hat{\mathbf{u}}\cdot\mathbf{r}} \, dxdydz \right|^2 \tag{125}$$

In equations (124)-(125):

$$\mathbf{F}_T(\mathbf{r}) = \mathbf{F}_T(x, y, z) = F_{Tx}(x, y, z)\hat{\mathbf{x}} + F_{Ty}(x, y, z)\hat{\mathbf{y}} + F_{Tz}(x, y, z)\hat{\mathbf{z}} \tag{126}$$

and

$$e^{jk\,\hat{\mathbf{u}}\cdot\mathbf{r}} = \exp(jk(x\sin(\theta)\cos(\phi) + y\sin(\theta)\sin(\phi) + z\cos(\theta))) \tag{127}$$

The integrations in (124)-(125) can be analytically or numerically carried out. For a MoM formulation, the integrals are carried out for an assumed set of basis functions (limited number of analytical integrations). The RCS is then computed by summing over the above kernel moments using the weighting coefficients determined through MoM.

It is theoretically possible to transform the Green's function formulation (e.g., (13)) to the spectral domain and derive the RCS from the Fourier knowledge of the target function. This would require an equivalent RCS definition in the spectral domain.

7. Conclusion

Utilization of radar or SAR polarimetry (PolSAR) for target recognition can significantly enhance target detection, classification and recognition capabilities. To make the efficient use of such techniques for remote sensing, appropriate data exploitation algorithms should be employed. Considering the nature of a polarimetric approach that essentially is the characterization of a target or scatterer through interaction with the polarized or vectorial electromagnetic (EM) field/wave, the required modeling analysis has to be based on an electromagnetic theory. On the other hand, an EM-based approach to formulate the vector field scattering is by nature involved and complicated. Noticing the need for an optimal approach (i.e., rigorous and feasible), the formulation here is structured to satisfy the accuracy requirements by including the key EM scattering components and developing a precise system kernel. The feasibility of associated algorithm implementation is addressed by formulating a generic EM scattering problem with known (within other EM areas) analytical or numerical solutions.

In the present theoretical framework, the target scattering and classification problem is approached by a method which is new within the realm of conventional PolSAR target characterization and classification techniques. The technique which is based on the characterization of target scatterers by current vector functions (i.e., the principle of equivalent sources) and formulation of scattering using accurate Green's or system kernel functions in the spectral domain, is well founded in the field of classical electromagnetics. Introducing appropriate basis sets representing various scattering features for target function decomposition, characteristics of a target scatterer may be explored from different aspects by making use of the derived accurate system kernel. Theoretically, there is no restriction on the nature and number of features that can be exploited as long as they are modelled as basis functions. Strong and insightful techniques from other domains of computational electromagnetics, such as microwaves and millimetre-wave structures, may be adopted and tailored for appropriate feature determination, analytic expression and effective target function projection into the feature space. Examination of the current distribution behaviour on the target scatterer's surface, which is one of the outputs of current analysis, can provide good understanding of the scatterer physical characteristics (e.g., shape). The current distribution on the scatterer may be readily applied to derive near and far field characteristics and observable quantities such as RCS. The latter can also be used to extract target distribution information from RCS knowledge through inverse analysis and incorporation of a priori information. Accordingly, one may effectively utilize conventional radar measurements to validate and tune the present scattering model.

Although fundamental, the analysis of scattering developed here for polarimetric radar applications is general and should be tailored for the SAR scenario of interest. The latter is essentially done by defining the appropriate analytic basis functions that represent target features based on the nature of scattering and the scatterer. Decomposition and/or projection of the target scatterer's current function into the features (i.e., basis functions) space should also be optimized by accounting for the analytic nature of target and basis functions. The future work will be focused on such target scattering feature classifications and required analytic techniques for associated target function projections. A variety of EM computational

techniques will be considered, tailored or further developed for desired applications. Results of the available trials and experiments will be used and (if required) new experimental schemes will be designed to validate and tune the model by taking advantage of the established relations that connect measured quantities (e.g., RCS) to scattering functions. Experimental data and EM simulations for known target scatterers will be employed to investigate, derive and designate analytic characteristics to scattering features and/or classes of features.
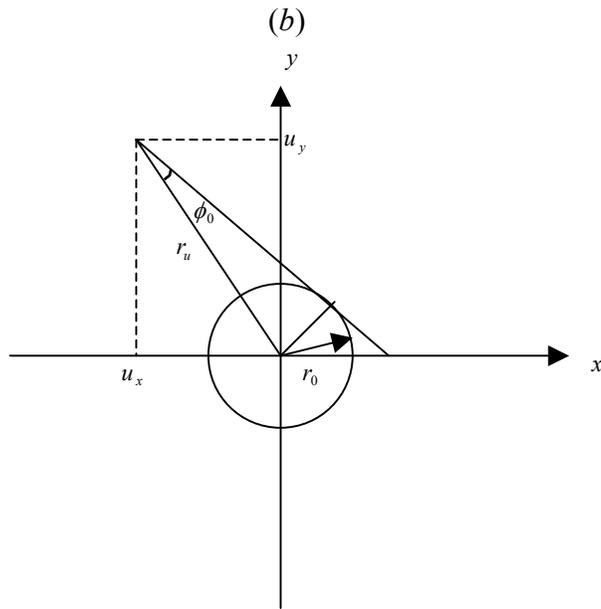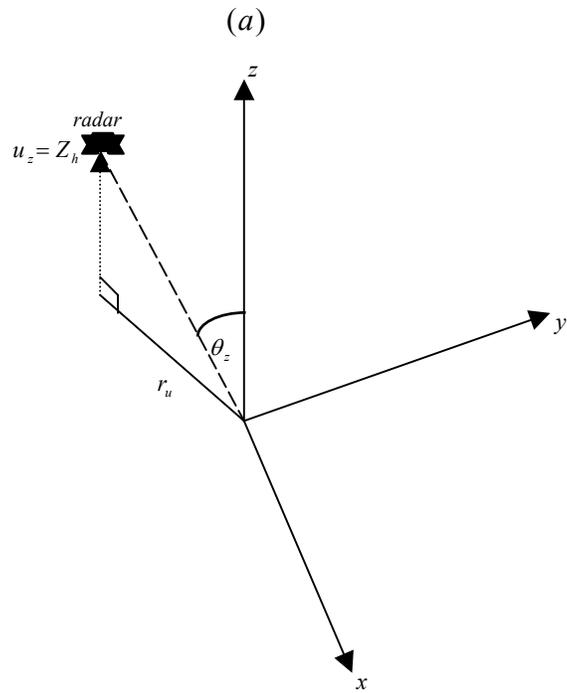
(*a*)

(*b*)

Figure 1. *(a) Radar-Target coordinate system. (b) Observation projection in ground plane with respect to ground target distribution.*

8.   References

1. F.M. Henderson, A.J. Lewis, Eds., *Principles & Applications of Imaging Radars,* Third Edition, Volume 2, John Wiley & Sons, NY, 1998, Chapter 5.
2. Können, G.P., *Polarized Light in Nature,* Cambridge University Press, Cambridge, UK, 1985.
3. Boerner, W-M., Ed., *Inverse Methods in Electromagnetic Imaging*, Proceedings of NATO Advanced Research Workshop on Inverse Methods in Electromagnetic Imaging, Bad Windsheim, Federal Republic of Germany, Sept., *NATO ASI Series C, Math. & Phys. Sci. 143.* D. Reidel Publishing Co., Dordrecht, The Netherlands, 1985.
4. Boerner, W-M., Ed., *Direct and Inverse Methods in Radar Polarimetry*, Proceedings of NATO Advanced Research Workshop on Direct and Inverse Methods in Radar Polarimetry, Bad Windsheim, Federal Republic of Germany, *NATO ASI Series C, Math. & Phys. Sci.,* D. Reidel Publishing Co., Dordrecht/Borton, 1992.
5. Van Zyl, J.J., "Calibration of Polarimetric Radar Images using only Image Parameters and Trihedral Corner Reflectors," *IEEE Trans. Geosci. Remote Sensing*, Vol. 28, pp. 337-348, 1990.
6. Van Zyl, J.J., "Unsupervised Classification of Scattering Behaviour Using Radar Polarimetry Data," *IEEE Trans. Geosci. Remote Sensing*, Vol. 27, pp. 37-45, 1989.
7. Kramer, H.J., *Observation of the Earth and its Environment Survey of (air/space-borne) Missions and Sensors,* Third Edition, Springer Verlag, Heidelberg, NY, 1996.
8. Zebker, H.A., and Van Zyl, J.J., "Imaging Radar Polarimetry: A Review," *Proceedings of the IEEE,* Vol. 79, pp. 1583-1606, 1991.
9. Zebker, H.A., Van Zyl, J.J., Durden, S.L. and Norikane, L. "Calibrated Imaging Radar Polarimetry: Technique, Examples, and Applications," *IEEE Trans. Geosci. Remote Sensing*, Vol. 29, pp. 942-961, 1991.
10. Krogager, E., "Decomposition of the Sinclair Matrix into Fundamental Components with Application to High Resolution Radar Target Imaging," *Direct and Inverse Methods in Radar Polarimetry,* Part 2, Boerner, W-M., Ed. Kluwer Academic Publishers, Dordrecht, The Netherlands, pp. 1459-1478, 1992.
11. Krogager, E., and Czyż, Z.H., "Properties of the Sphere, Diplane, Helix (Target Scattering Matrix) Decomposition," in J. Saillard *et al.,* Eds., *Proc. JIPR-3,* Nantes, France, March 21-23, 1995, pp. 106-114.
12. Krogager, E., and Madsen, S.N., "Comparison of Various Decompositions for Analysis, Interpretation, and Classification of Polarimetric SAR Images," *European Conference on Synthetic Aperture Radar, EUSAR'96,* Königswinter, Germany, March 26-28, 1996, pp. 105-108.
13. Krogager, E., Boerner, W-M., "On the Importance of Utilizing Complete Polarimetric Information in Radar Imaging and Classifications," *AGARD Symposium: Remote Sensing – A valuable source of information, AGARD Proc. ,* 528, Toulouse, France, April 22-25, 1996, pp. 17-1 – 17-12.
14. Sinclair, G., "The Transmission and Reception of Elliptically Polarized Waves," *Proceedings of the IRE,* 38, pp. 148-151.
15. Kennaugh, E.M., *Effects of the Type of Polarization on Echo Characteristics,* Antenna Laboratory, Ohio State University, Reports 389-4: 35 p, and 389-9: 39 p.

16. Huynen, J.R., *Phenomenological Theory of Radar Targets,* Ph.D. Thesis, Delft University of Technology, The Netherlands, 1970.

17. Huynen, J.R., "Measurement of the Target Scattering Matrix," *Proc. IEEE*, Vol. 53, pp. 936–946, Aug. 1965.

18. Cloude, S.R., and Pottier, E., "A Review of Target Decomposition Theorems in Radar Polarimetry," *IEEE Trans. Geosci. Remote Sensing*, Vol. 34, pp. 498–518, Mar. 1996.

19. Krogager, E., "New Decomposition of the Radar Target Scattering Matrix," *Electron. Lett.*, Vol. 26, No. 18, pp. 1525–1527, 1990.

20. Cameron, W.L., Youssef, N., and Leung, L. K., "Simulated Polarimetric Signatures of Primitive Geometrical Shapes," *IEEE Trans. Geosci. Remote Sensing*, Vol. 34, pp. 793-803, May 1996.

21. Touzi, R., and Charbonneau, F., "Characterization of Target Symmetric Scattering Using Polarimetric SARs," *IEEE Trans. Geosci. Remote Sensing*, Vol. 40, No. 11, pp. 2507-2516, Nov. 2002.

22. Cloude, S.R., and Pottier, E., "An Entropy Based Classification Scheme for Land Applications of Polarimetric SAR," *IEEE Trans. Geosci. Remote Sensing*, Vol. 35, pp. 68–78, Jan. 1997.

23. Georgiou, T.T., "Spectral Estimation Via Selective Harmonic Amplification," *IEEE Trans. Automat. Contr.,* Vol. 46, pp. 29-42, Jan. 2001.

24. Georgiou, T.T., "Spectral Analysis Based on the State Covariance: The Maximum Entropy Spectrum and Linear Fractional Parametrization," *IEEE Trans. Automat. Contr.,* Vol. 47, pp. 1811-1823, Nov. 2002.

25. Harrington, R.F., *Time-Harmonic Electromagnetic Fields,* NY: McGraw-Hill, 1961.

26. Balanis, C.A., *Advanced Engineering Electromagnetics,* NY: Wiley, 1989.

27. Chen, K.M., "A Mathematical Formulation of the Equivalence Principle," *IEEE Trans. Microwave Theory Tech.,* Vol. 37, pp. 1576-1581, 1989.

28. Peterson, A.F., Ray, S.L., and Mittra, R., *Computational Methods for Electromagnetics,* IEEE Press, NY, 1998.

29. Huyghens, C., *Traité de la Lumiére,* 1690 (English Translation, Chicago: The University of Chicago Press, 1945).

30. Schelkunoff, S.A., "Some Equivalence Theorems of Electromagnetics and their Application to Radiation Problems," *Bell System Tech., Jour.,* 15 (1936), 92-112.

31. Stratton, J.A., *Electromagnetic Theory,* NY: McGraw-Hill, 1941.

32. Elliott, R.S., *Antenna Theory and Design,* Englewood Cliffs, NJ: Prentice–Hall, 1981.

33. Soumekh, M., *Synthetic Aperture Radar Signal Processing,* NY: Wiley, 1999.

34. Soumekh, M., *Fourier Array Imaging,* Englewood Cliffs, NJ: Prentice–Hall, 1994.

35. Soumekh, M., "Reconnaissance with Slant Plane Circular SAR Imaging," *IEEE Trans. Image Proc.,* Vol. 5, pp. 1252-1265, Aug. 1996.

36. Harrington, R.F., *Field Computation by Moment Methods,* Krieger Publishing Company, Inc., Malabar, Florida, 1982.

37. Harrington, R.F., "Matrix Methods for Field Problems," *Proc. IEEE,* Vol. 55, pp. 136-149, Feb. 1967.

38. Kantorovich, L.V., Krylov, V.I., *Approximate Methods of Higher Analysis,* 4th ed., translated by C.D. Benster, John Wiley & Sons, Inc., NY, 1959.

39. Jones, D.S., "A Critique of the Variational Method in Scattering Problems," *IRE Trans.,* Vol. AP-4, pp. 297-301, 1956.

9. Annexes

9.1 Annex A: Reference frame unit vector transformation

It is often useful to express the scattered field vectors or matrices defined in the radar reference frame in terms of vectors in a universal reference frame, e.g., multi-incident-angle analysis.

Assume $(\hat{\mathbf{h}}, \hat{\mathbf{v}}, \hat{\mathbf{l}})$ as the basis set for radar reference vectors. With respect to a universal reference frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$, we have:

$$\mathbf{U}^{\hat{\mathbf{h}}\hat{\mathbf{v}}\hat{\mathbf{l}}} = \ddot{\mathbf{T}} \cdot \mathbf{U}^{\hat{\mathbf{x}}\hat{\mathbf{y}}\hat{\mathbf{z}}} \tag{A1}$$

where,

$$\mathbf{U}^{\hat{\mathbf{h}}\hat{\mathbf{v}}\hat{\mathbf{l}}} = \begin{pmatrix} \hat{\mathbf{h}} \\ \hat{\mathbf{v}} \\ \hat{\mathbf{l}} \end{pmatrix} \quad , \quad \mathbf{U}^{\hat{\mathbf{x}}\hat{\mathbf{y}}\hat{\mathbf{z}}} = \begin{pmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{pmatrix} \tag{A2}$$

$$\ddot{\mathbf{T}} = \begin{pmatrix} \cos(\theta_i)\cos(\phi) & \cos(\theta_i)\sin(\phi) & -\sin(\theta_i) \\ -\sin(\phi) & \cos(\phi) & 0 \\ \sin(\theta_i)\cos(\phi) & \sin(\theta_i)\sin(\phi) & \cos(\theta_i) \end{pmatrix} \tag{A3}$$

In (A3), θ_i denotes the incident angle and ϕ represents the polar angle in the transversal plane (to LOS).

Similarly, one finds for the scattering matrix:

$$\ddot{\mathbf{S}}_{\hat{\mathbf{h}}\hat{\mathbf{v}}\hat{\mathbf{i}}} = \ddot{\mathbf{T}}^T \cdot \ddot{\mathbf{S}}_{\hat{\mathbf{x}}\hat{\mathbf{y}}\hat{\mathbf{z}}} \cdot \ddot{\mathbf{T}} \qquad\qquad (A4)$$

9.2 Annex B: Principle of stationary phase

The principle of stationary phase (PSP) states that [1],[34], for problems involving integrations of complicated waveforms with widely-varying phase functions, the phase is variational around a certain stationary point. Considering such integrand as:

$$Integrand = f(\vec{v})\exp(j\Phi(\vec{v})) \qquad\qquad (B1)$$

where $f(\vec{v})$ is assumed to be a slowly-varying envelope, the PSP is formulated as:

$$\delta\Phi(\vec{v}) = 0 \qquad\qquad (B2)$$

Relation (B2) provides a powerful tool that can be effectively applied for integrations associated with spectral analysis of complex functions. Such complexity can be gauged by the time-bandwidth product (TBP) of the waveform.

In (B2), the stationary point $\vec{v}$ is considered to be a vector in general. For single-dimensional problems (normally the case), the stationary point satisfies:

$$\frac{\partial}{\partial v}\Phi(v)\,|_{v=v_0} = 0 \qquad\qquad (B3)$$

Expanding $\Phi(v)$ in the vicinity of v_0 :

$$\Phi(\nu) \approx \Phi(\nu_0) + \frac{(\nu - \nu_0)^2}{2!}\, \Phi''(\nu_0) \qquad \left(\Phi''(\nu_0) \equiv \frac{\partial^2}{\partial \nu^2}\, \Phi(\nu)\,|_{\nu = \nu_0}\right) \qquad\qquad (B4)$$

and using the known result,

$$\int_{-\infty}^{\infty} e^{j\alpha(\nu - \nu_0)^2}\, d\nu = \left(\frac{j\pi}{\alpha}\right)^{1/2} \qquad\qquad (B5)$$

one finds:

$$\int_{-\infty}^{\infty} f(\nu)\exp(j\Phi(\nu))\, d\nu \approx \left(\frac{j2\pi}{\Phi''(\nu_0)}\right)^{1/2} f(\nu_0)\exp(j\Phi(\nu_0)) \qquad\qquad (B6)$$

The phase function is assumed to be analytic (near ν_0) in the above derivations. The accuracy of (B6) depends on the significance of the residual terms in (B4). Effects of the residuals are inversely proportional to TBP of the waveform.

List of acronyms and initialisms

CTD   Coherent Target Detection

DND   Department of National Defence

EM   Electromagnetics

LOS   Line Of Sight

MoM   Method of Moments

PCTD   Partially Coherent Target Detection

p.e.c   perfect electric conductor

p.m.c   perfect magnetic conductor

PM   Phase Modulated

PSP   Principle of Stationary Phase

RCS   Radar Cross Section

SAR   Synthetic Aperture Radar

TBP   Time Bandwidth Product

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A vectorial Fourier-based technique for electromagnetic (EM) wave reconstruction with application to polarimetric airborne and spaceborne radar data exploitation is presented. The approach provides a comprehensive picture of scattering through an electromagnetic-based formulation of equivalent sources and introduction of vector scattering functions. The method is different from conventional modeling techniques for SAR applications as result of the full electromagnetic treatment of field interaction with the scatterer, the possibility of introducing new and controllable feature classes for target classification, and the accurate decomposition of source impulse response function that avoids potential errors (e.g., loss of coherent information) associated with the spherical phase approximations. In addition to including the potential for high quality image formation, the capability of extracting target or scatterer information, such as coherent radar cross section (RCS) is explored. It is highlighted how target image and information processing can be optimized and tailored for a desired scenario (e.g., the required information type, the nature of the targets) by applying the developed model and devising appropriate inversion techniques to aid data processing in support of exploitation.

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Polarimetric, SAR, Scattering

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