Ground Moving Target Parameter Estimation for Stripmap SAR Using the Unscented Kalman Filter

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ABSTRACT

In multi-channel SAR systems, the detection of movers is carried out using techniques based on the Wiener filter. An important problem is the estimation of the moving target parameters such as along-track velocity, across-track velocity, and azimuth displacement. In this paper, the parameter estimation problem is formulated as a novel Bayesian state estimation problem. The unscented Kalman filter is applied to the problem and it is noted that it leads to good estimates.

Keywords: Synthetic Aperture Radar, Nonlinear Filtering, Unscented Kalman FIlter

1. INTRODUCTION

The ground moving target indication (GMTI) problem is the detection of slow ground-moving targets from an airborne or spaceborne platform. This necessitates adaptive multichannel signal processing techniques, such as space-time adaptive processing (STAP) or synthetic aperture radar (SAR) MTI.[1] The signal processing outputs as well as probability of detection (Pd) and probability of false alarm (Pfa), are then the inputs to a tracker. In this paper, the focus is on stripmap SAR.

The first step in GMTI processing is the detection of movers. From SAR data, the detection can be carried out in the raw data domain as well as the image domain. These different approaches have different advantages in terms of processing as well as coherent integration gain. For instance, from space-based radar systems, the detection is usually carried out in the image doman so as to maximize gain on target.[2]

The next step is to estimate the parameters. In the STAP-based GMTI,[3–5] the outputs of the parameter estimation are (some combination of) range, bearing, and Doppler. These are then fed as input to GMTI tracker, and there has been extensive literature on GMTI tracking.[6]

In SAR-based GMTI,[7] the parameters of interest that is to be estimated are related but somewhat different.In particular, for SAR, the parameters of interest are along-track velocity, across-track velocity, and azimuth displacement.

There exist several standard techniques for parameter estimation techniques. For the estimation of the Doppler rate (along-track velocity), the parameter fit approach is the simplest but not the most accurate method. It is useful for obtaining initial estimates and for resolving phase ambiguities. A more accurate approach it to use the matched filter bank approach, although it is computationally intensive. The Doppler centroid (across-track velocity) can also be estimated using matched filter. For multi-channel systems, the along-track interferometry (ATI) phase can also be used. Finally, there are algorithms for estimating target broadside location (using direction of arrival estimation and fractional Fourier transform), however there is an ambiguity in v_y and the estimation is often poor. Further note that, target acceleration (cubic phase term) can cause serious deterioration in the performance of the parameter estimation problem.

In this paper, we use the state space approach to solve the parameter estimation problem. The model is derived in the following section, followed by a discussion of how it is different from the state-space models usually studied in GMTI tracking. This is followed by some simulation results using a Bayesian state estimation problem, the unscented Kalman filter (UKF).

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2. MODEL

The coordinate system is chosen so that the aircraft is moving along the $x-$direction with velocity v_a and at a height H. Thus, at time t, the aircraft coordinate is given by

$$\begin{bmatrix} tv_a & 0 & H \end{bmatrix}. \tag{1}$$

The target is assumed to move in the slant plane with initial location $\begin{bmatrix} x_0 & y_0 \end{bmatrix}$ and moving with $\begin{bmatrix} v_x & v_y \end{bmatrix}$. Note that the v_y is the $y-$component of the local Cartesian coordinate system, though in SAR one is interested in the component of v_y on the slant plane. Thus, at time t, the target coordinate is given by

$$\begin{bmatrix} tv_x + x_0 \\ tv_y + y_0 \\ 0 \end{bmatrix}. \tag{2}$$

The range as a function of time is given by

$$R[t] = \sqrt{H^2 + \left(-tv_a + tv_x + x_0\right)^2 + \left(v_y t + y_0\right)^2} \tag{3}$$

Expanding to second order around y_b

$$\sqrt{H^2 + y_b^2} + \frac{v_y y_b (t - t_b)}{\sqrt{H^2 + y_b^2}} + \frac{\left(H^2 \left(\left(v_a - v_x\right)^2 + v_y^2\right) + \left(v_a - v_x\right)^2 y_b^2\right)(t - t_b)^2}{2\left(H^2 + y_b^2\right)^{3/2}} \tag{4}$$

Note that, by definition, $x_0 + (v_x - v_a)t_b = 0$, and $y_b = y_0 + v_y t_b$.

By Doppler one means $\frac{2}{\lambda}\frac{d}{dt}\mathrm{R(t)}$, i.e.,

$$\frac{2\left(-v_a + v_x\right)\left(-tv_a + tv_x + x_0\right) + 2v_y\left(tv_y + y_0\right)}{\lambda\sqrt{H^2 + \left(-tv_a + tv_x + x_0\right)^2 + \left(tv_y + y_0\right)^2}} \tag{5}$$

Expanding to second order around t_b

$$\frac{2v_y y_b}{\lambda\sqrt{H^2 + y_b^2}} + \frac{\left(2H^2\left(\left(v_a - v_x\right)^2 + v_y^2\right) + 2\left(v_a - v_x\right)^2 y_b^2\right)(t - t_b)}{\lambda\left(H^2 + y_b^2\right)^{3/2}} + \frac{3v_y y_b\left(H^2\left(-\left(v_a - v_x\right)^2 - v_y^2\right) - \left(v_a - v_x\right)^2 y_b^2\right)(t - t_b)^2}{\lambda\left(H^2 + y_b^2\right)^{5/2}} \tag{6}$$

Next, consider the direction cosine as a function of time, $u(t)$. This is defined as the component of the unit vector from aircraft to target along the direction of motion (i.e., x-direction):

$$u(t) = \frac{(t - t_b)\left(-v_a + v_x\right)}{\sqrt{H^2 + (t - t_b)^2\left(v_a - v_x\right)^2 + \left((t - t_b)v_y + y_b\right)^2}} \tag{7}$$

which to second order is given by

$$u(t) = \frac{\left(-v_a + v_x\right)(t - t_b)}{\sqrt{H^2 + y_b^2}} + \frac{\left(v_a - v_x\right)v_y y_b (t - t_b)^2}{\left(H^2 + y_b^2\right)^{3/2}} \tag{8}$$

In summary, the state-space model is as follows. The states are $\begin{bmatrix} x_b & v_x & y_b & v_y \end{bmatrix}$ and are assumed to evolve according to the white noise process:

$$\{x_b, v_x, y_b, v_y\} \equiv \{x_1, x_2, x_3, x_4\} \tag{9}$$

$$\begin{bmatrix} \mathbf{x}_{1,k+1} \\ \mathbf{x}_{2,k+1} \\ \mathbf{x}_{3,k+1} \\ \mathbf{x}_{4,k+1} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_{1,k} \\ \mathbf{x}_{2,k} \\ \mathbf{x}_{3,k} \\ \mathbf{x}_{4,k} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{1,k} \\ \mathbf{v}_{2,k} \\ \mathbf{v}_{3,k} \\ \mathbf{v}_{4,k} \end{bmatrix} \tag{10}$$

3. TRADITIONAL TRACKING MODEL

It is useful to note the different tracking model used in traditional radar tracking of surface targets. Consider the case of STAP-based detection. For GMTI, STAP-based GMTI tracking based on discrete-time nearly constant velocity (CV) model.[8] For simplicity, we ignore the more general case, in which multiple models are used.

$$\mathbf{x}_k = F\mathbf{x}_{k-1} + G\mathbf{v}_{k-1}, \tag{11}$$

$$F = \begin{pmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad G = \begin{pmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{pmatrix}, \quad \mathbf{v}_{k-1} = \begin{pmatrix} \mathbf{v}_{1,k-1} \\ \mathbf{v}_{2,k-1} \end{pmatrix} \tag{12}$$

The measurements from the GMTI STAP processing are

$$\begin{bmatrix} r_k \\ \dot{r}_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} \sqrt{(\bar{x}_k)^2 + (\bar{y}_k)^2 + (\bar{z}_k)^2} \\ \frac{\bar{x}_k \dot{\bar{x}}_k + \bar{y}_k \dot{\bar{y}}_k + \bar{z}_k \dot{\bar{z}}_k}{\sqrt{\bar{x}_k^2 + \bar{y}_k^2 + \bar{z}_k^2}} \\ \arctan\left(\frac{(y_k - y_k^P)}{(x_k - x_k^P)}\right) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{1,k} \\ \mathbf{w}_{2,k} \\ \mathbf{w}_{3,k} \end{bmatrix}, \tag{13}$$

where $\bar{x}_k = x_k - x_k^P$, $\bar{y}_k = y_k - y_k^P$, and $\bar{z}_k = z_k - z_k^P$ and the dot indicates time derivatives.

First of all, note that the states, namely $\begin{bmatrix} x & v_x & y & v_y \end{bmatrix}$, are different from what we are using in the stripmap SAR parameter application. In particular, the position and velocity are coupled, as a matter of definition. Secondly, note that the measurement model is a very different function of the state variable. As a result, the filtering model being investigated in the paper is very different from that investigated in the traditional tracking literature in numerous papers.

4. SIMULATION RESULTS

We simulate the measurements with measurement interval $\Delta t = 0.05$, platform velocity $v_a = 100$ m/s and at an altitude of 3 km. The target is intially at $[-800, 20000]$ with velocity vector $[10, 5]$. The initial covariance matrix is assumed to be $P_0 = \text{diag}(10^6, 10, 1, 10)$. The range measurement standard devation is $\sigma_R = 1$ m, $\sigma_u = 0.5$ degree and the system is assumed to be operating at 5.3 GHz. The number of Monte Carlo runs is 100.

Figure 1 shows the results in the RMS error in position as well as velocity using the unscented Kalman filter (UKF) when using only range and bearing measurements. Although the initial uncertainty in position is of the order of a km, it is seen to quickly drop to under 50 m. It also converges to just under 30 m after 5 seconds. Similarly, the RMS velocity is seen to be just under 4 m/s. Figure 2 shows the normalized estimation error square (NEES). It is seen that the filter is consistent.

Next, we consider the case where all three measurements, namely, range, bearing and u, are considered. The simulation parameters as before supplemented with $\sigma_{\text{dop}} = 5$ Hz. Figure 3 shows the RMS errors in position and velocity when all three measurements are used. It is noted that there is no significant improvement in the RMS in position, but there is some noticeable improvement in RMS in velocity (from around 3.8 m/s to around 3.2 m/s).

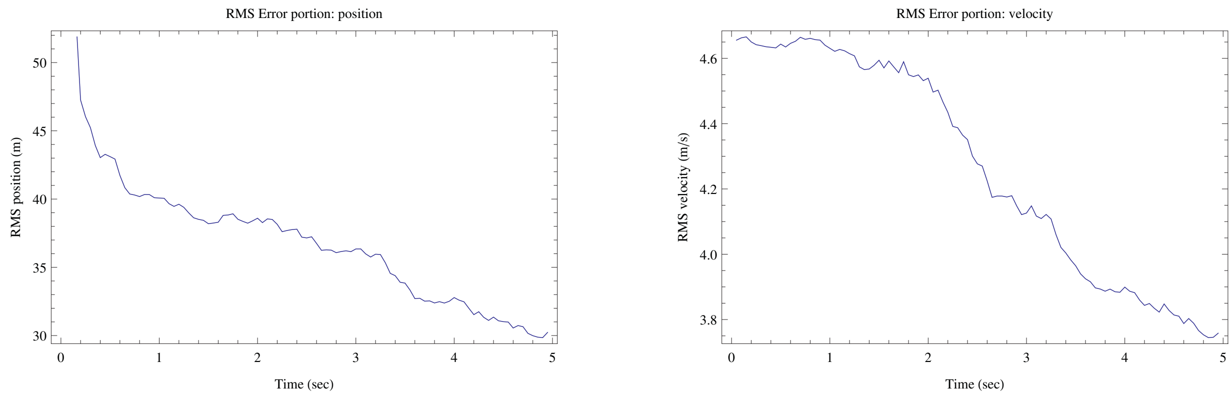

Figure 1. RMS error: range and bearing measurements

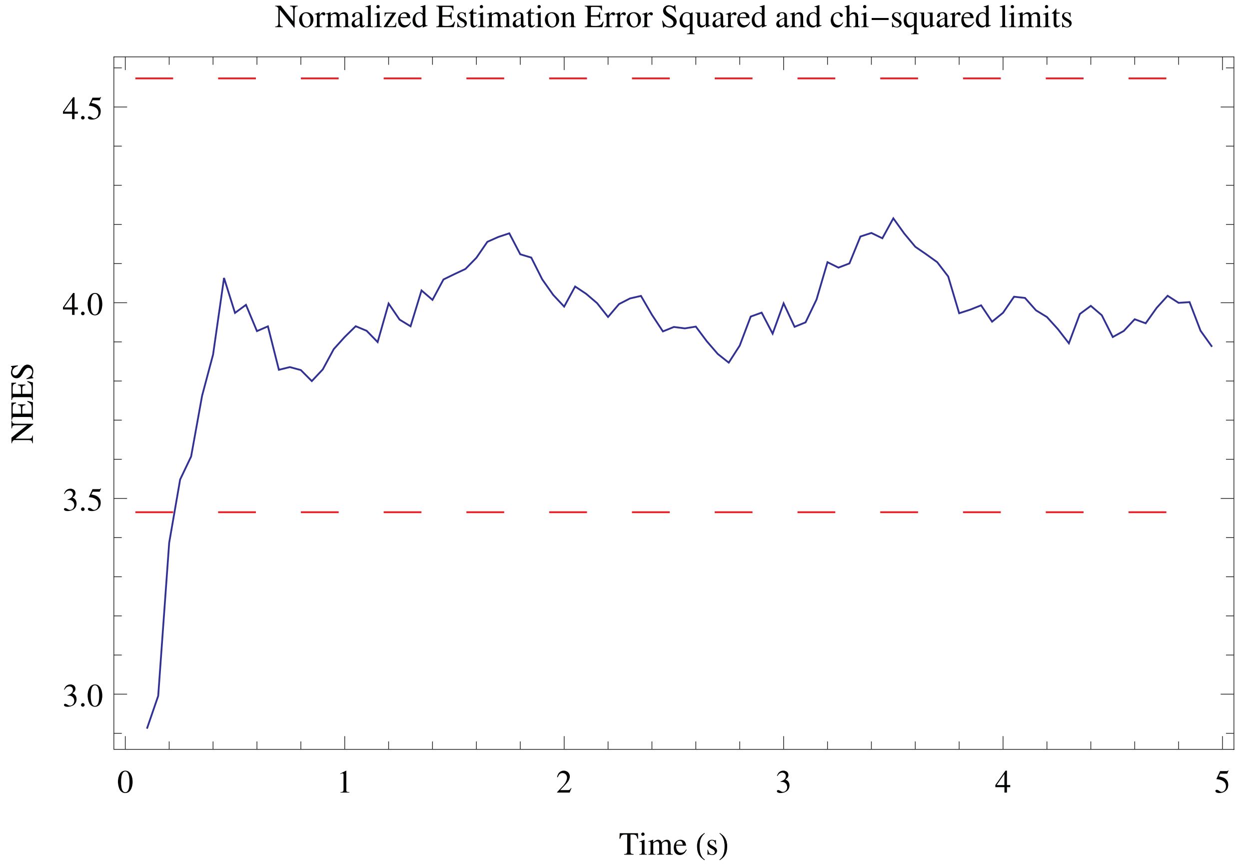

Figure 2. NEES: range and bearing

5. CONCLUSION

A Bayesian state-space approach to the stripmap SAR parameter estimation problem was investigated in this paper. It has several advantages over the traditional approaches. In particular, it is robust against model mismatch (that can be in adjustment of the pseudo noise). It is straightforward to incorporate target acceleration. The estimation process is very fast. As in traditional multi-target tracking, it is possible to incorporate false alarms, missed detections. Also, note that it provides an error bound (or a "credible interval"), rather than a

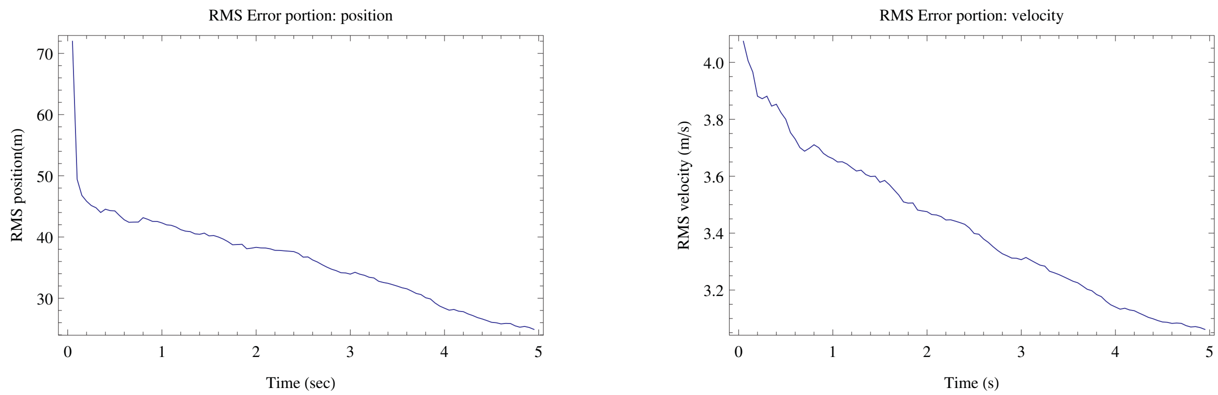

Figure 3. RMS error: range, bearing and Doppler measurements

point estimate. The consistency of the the UKF shows that the bounds obtained by the filter is actually reliable. The inclusion of Doppler measurements did not lead to an improved estimate of the positon, but it did improve the velocity estimate. In future work, better filter solutions (such as particle filters, variational filters, homptopy filters) will be investigated. Smoothing will also be investigated to improve the estimate, particularly at the earlier times.

APPENDIX A. THE UNSCENTED KALMAN FILTER (UKF)

The discrete-time system of state and measurement models considered is of the form

$$\mathbf{x}_{k+1} = f_k(\mathbf{x}_k) + \mathbf{v}_k, \quad k = 0, 1, 2, \ldots$$
$$\mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{w}_k, k = 0, 1, 2, \ldots$$

The general Bayesian filtering approach is to start with an initial density of the state vector $p(\mathbf{x}_0) \equiv p(\mathbf{x}_0|\mathbf{y}_0)$, where y_0 is the set of no measurements. The aim is to seek filtered estimates of $\mathbf{x}_k$ based on the sequence of all available measurements $\mathbf{Y}_k \equiv \{\mathbf{y}_i, i = 1, \ldots, k\}$ upto time index k. Suppose the pdf at time $k-1$, namely $p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})$ is known. Then, the *prediction density/dynamic prior pdf* is given by the Chapman-Kolmogorov equation

$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})d\mathbf{x}_{k-1}, \tag{14}$$

where $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is referred to as the transitional probability density. At time step k when measurement $\mathbf{y}_k$ becomes available, the update stage is carried out using Bayes' rule:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{Y}_k) &= p(\mathbf{x}_k|\mathbf{y}_k, \mathbf{Y}_{k-1}), \\ &= \frac{p(\mathbf{y}_k|\mathbf{x}_k, \mathbf{Y}_{k-1})p(\mathbf{x}_k|\mathbf{Y}_{k-1})}{p(\mathbf{y}_k|\mathbf{Y}_{k-1})}, \\ &= \frac{p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1})}{p(\mathbf{y}_k|\mathbf{Y}_{k-1})}. \end{aligned} \tag{15}$$

In general, this is intractable. A special case is the Kalman filter, the optimal filter in the linear Gaussian case. Specifically, for Gaussian prior and linear dynamic and measurement model with Gaussian noises, the posterior is also Gaussian and there are closed-form solutions for computing the mean vector and covariance matrix recursively.

However, in our application, the measurement model is nonlinear, so only an approximate solution is possible. There are several suboptimal nonlinear filters that are used in practice, such as the extended Kalman filter (EKF), the unscented Kalman filter (UKF),[9,10] and particle filter.[11] In this paper, the UKF was used due to its simplicity (unlike the EKF, does not require the computation of a Jacobian) and minimal computational requirements (as opposed to a particle filter in general). Like the KF and the EKF, the UKF assumes that the posterior is Gaussian. The algorithmic steps are as follows:

1. At time index $k = 0$, define the a priori initial condition: $\hat{\mathbf{x}}_{0|-1} = \mathrm{E}\{\mathbf{x}_0\}$ and $\mathbf{P}_{0|-1} = \mathrm{cov}\mathbf{x}_0$.

2. The predicted sigma points $\xi_{i,k|k-1}$ and their weights $\mathscr{W}$ are determined as follows:

$$
\begin{aligned}
\xi_{0,k|k-1} &= \hat{\mathbf{x}}_{k|k-1}, \quad \mathscr{W}_0 = \frac{\kappa}{\kappa + n_x}, \\
\xi_{i,k|k-1} &= \hat{\mathbf{x}}_{k|k-1} + \left(\sqrt{(n_x + \kappa)P_{k|k-1}}\right)_i, \quad \mathscr{W}_i = \frac{1}{2(\kappa + n_x)}, \\
\xi_{j,k|k-1} &= \hat{\mathbf{x}}_{k|k-1} - \left(\sqrt{(n_x + \kappa)P_{k|k-1}}\right)_i, \quad \mathscr{W}_{j-n_x} = \frac{1}{2(\kappa + n_x)},
\end{aligned}
\tag{16}
$$

where $i = 1, \ldots, n_x$ and $j = n_x + 1, \ldots, 2n_x$.

3. Transform the set of predicted sigma points via the measurement function

$$
\mathscr{Y}_{i,k|k-1} = \mathbf{h}_k(\xi_{i,k|k-1}), \quad i = 0, 1, \ldots, 2n_x, \tag{17}
$$

and the predicted measurement

$$
\hat{\mathbf{y}}_{k|k-1} = \sum_{i=0}^{2n_x} \mathscr{W}_i \mathscr{Z}_{i,k|k-1},
$$

$$
\begin{aligned}
\mathbf{P}_{y,k|k-1} &= \sum_{i=0}^{2n_x} \mathscr{W}_i \left(\mathscr{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}\right)\left(\mathscr{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}\right)^T + \mathbf{R}_k, \\
\mathbf{P}_{xz,k|k-1} &= \sum_{i=0}^{2n_x} \mathscr{W}_i \left(\xi_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1}\right)\left(\mathscr{Y}_{i,k|k-1} - \hat{\mathbf{y}}_{k|k-1}\right), \quad i = 0, 1, \ldots, 2n_x^T
\end{aligned}
$$

4. Update state estimate as follows:

$$
\begin{aligned}
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}'_{k|k-1} + \mathbf{K}_k\left(\mathbf{y} - \hat{\mathbf{y}}_{k|k-1}\right), \\
\mathbf{P}_{k|k} &= \mathbf{P}'_{k|k-1} - \mathbf{K}_k \mathbf{P}_{y,k|k-1}\mathbf{K}_k^T, \quad \mathbf{K}_k = \mathbf{P}_{xz,k|k-1}\left(\mathbf{P}_{z,k|k-1}\right)^{-1}.
\end{aligned}
\tag{18}
$$

where

5. Determine the set of sigma points $\xi_{i,k|k-1}$:

$$
\begin{aligned}
\xi_{0,k|k} &= \hat{\mathbf{x}}_{k|k}, \\
\xi_{i,k|k} &= \hat{\mathbf{x}}_{k|k} + \left(\sqrt{(n_x + \kappa)\,\mathbf{P}_{k|k}}\right)_i, \quad i = 1, \ldots, n_x \\
\xi_{j,k|k} &= \hat{\mathbf{x}}_{k|k} - \left(\sqrt{(n_x + \kappa)\,\mathbf{P}_{k|k}}\right)_{j-n_x} \quad j = n_x + 1, \ldots, 2n_x.
\end{aligned}
\tag{19}
$$

6. Transform the sigma points through the state equation

$$
\xi_{i,k+1|k} = \mathbf{f}_k\left(\xi_{i,k|k}\right), \quad i = 0, 1, \ldots, 2n_x \tag{20}
$$

to obtain the set of predicted sigma points that is used to compute the predicted mean and covariance matrix at time $k+1$;

$$\hat{\mathbf{x}} = \sum_{i=0}^{2n_x} \mathscr{W}_i \xi_{i,k+1|k},$$
$$\mathbf{P} = \sum_{i=0}^{2n_x} \mathscr{W}_i \left(\xi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k}\right)\left(\xi_{i,k+1|k} - \hat{\mathbf{x}}_{k+1|k}\right)^T + \mathbf{Q}_k.$$

7. Set $k = k+1$ and go to Step 2.

APPENDIX B. CONSISTENCY OF STATE ESTIMATOR

In classical parameter estimation problem, consistency of an estimator of constant parameter is defined as the stochastic convergence in the mean-square sense of the estimate to its true value. Let $\hat{\theta}(k)$ be an estimator a constant parameter θ_0, where k is the number of measurements. The condition of consistency is that

$$\lim_{k\to\infty} \mathrm{E}\{[\hat{x}(k) - \theta_0]^2\} = 0. \tag{21}$$

The estimate of the state of a dynamic system does not, in general, converge stochastically to the true state. However, in addition to the current state estimate, the associated covariance matrix is available.[12] This can be used to test for the consistency of the estimator.

Under the linear, Gaussian assumption, the conditional pdf of the state $\mathbf{x}_k$ at time index k is

$$p(\mathbf{x}_k|\mathbf{Y}_k) = N\left[\mathbf{x}_k; \hat{\mathbf{x}}_{(k|k)}, \mathbf{P}_{k|k}\right].$$

The Gaussian assumption implies that the expected value of the estimation errors achieved by the filter should match the filter-calculated covariance:

$$\mathrm{E}\{\left[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right]\left[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}\right]^T|\mathbf{Y}_k\} = \mathbf{P}_{k|k}. \tag{22}$$

A filter is then said to be consistent if it is unbiased and its state estimation errors satisfy Equation 22. It is noteworthy that this consistency property is a "finite-sample consistency" property. In other words, the estimation errors based on a finite number of samples (measurements) should be "consistent" with the filter calculated covariance matrix. In contrast, the parameter estimator consistency is an infinite sample size property.

A standard criterion for consistency of a filter is that the state errors are zero-mean (or unbiased) and compatible with the covariance as calculated by the filter. This is ideal for testing in simulations. The consistency is checked using standard hypothesis testing techniques. Let

$$\tilde{\mathbf{x}}_{k|k} = \mathbf{x}_k - \hat{\mathbf{x}}_{k|k},$$

then the normalized estimation error squared (NEES) is defined as

$$\epsilon(k) \equiv \tilde{\mathbf{x}}_{k|k} P_{k|k}^{-1} \tilde{\mathbf{x}}_{k|k}. \tag{23}$$

The filter is then consistent if the NEES $\epsilon(k)$ is chi-square distributed with n_x degrees of freedom. As a result,

$$\mathrm{E}\{\epsilon(k)\} = n_x.$$

Let N be the number of Monte Carlo simulations. Then, the sample mean

$$\tilde{\epsilon}(k) = \frac{1}{N}\sum_{i=1}^{N} \epsilon^i(k),$$

is such that $N\tilde{\epsilon}(k)$ is chi-square distributed with Nn_x degrees of freedom. Using standard statistical techniques, the state estimation errors are said to be consistent with the filter calculated covariances if

$$\tilde{\epsilon}(k) \in [r_1, r_2],$$

where the confidence interval $[r_1, r_2]$ is determined such that

$$P\{\tilde{\epsilon}(k) \in [r_1, r_2]\} = 1 - \alpha. \tag{24}$$

Thus, for 95% confidence interval, $\alpha = 0.05$, and for $n_x = 4$ and $N = 100$, the interval is $[3.46482, 4.57305]$.

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