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Discounting for Civilian and Military Projects

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DRDC – Centre for Operational Research and Analysis

GAU Journal of Social and Applied Science
ISSNs: 1305–9130

Volume 7
Issue 2
Pages 11–15

Date of Publication from Ext Publisher: December 2015

Defence Research and Development Canada

External Literature (N)
DRDC-RDDC-2018-N005
January 2018

Canada

IMPORTANT INFORMATIVE STATEMENTS

This document was reviewed for Controlled Goods by Defence Research and Development Canada (DRDC) using the Schedule to the *Defence Production Act*.

Endorsement statement: This publication has been peer-reviewed and published by the Editorial Office of Defence Research and Development Canada, an agency of the Department of National Defence of Canada. Inquiries can be sent to: Publications.DRDC-RDDC@drdc-rddc.gc.ca.

GAU Journal of Social and Applied Science, ISSNs: 1305-9130
2015, Volume 7, Issue 2: Page 11 - 15

Discounting for Civilian and Military Projects

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Abstract—The Social Discount Rate (SDR) is a crucial parameter in any economic evaluation of public projects. It is the rate at which a society is willing to convert future costs and benefits into present values. The selection of the appropriate SDR has always been a perplexing problem in Cost-Benefit Analysis. This paper develops an economic model to simultaneously derive and estimate real SDRs for civilian and military sectors. The setting is represented by a welfare function that has the Stone-Geary form and depends on consumption and military security. A Golden Rule, referred to as dg1, to find plausible values for SDRs is provided. Using the most recent data and commonly meaningful agreed-on assumptions, the real civilian and military SDRs are respectively estimated to be 2.8% and 1% for Canada.

Key words: Social Discount Rate, Cost-Benefit Analysis, Stone-Geary Model, Military Projects

1. INTRODUCTION

The Cost-Benefit Analysis is the main tool for the economic evaluation of public projects [1]. In this analysis, the Social Discount Rate (SDR) is a crucial parameter whenever costs and benefits differ in their distribution over time [2]. SDR measures the rate at which a society is willing to trade present for future consumption[3]. It is the rate to be used to calculate the present value of the costs and benefits of public policies[4]. This rate significantly affects the resource allocation and efficiency. If it is too high, distant cash flows will become negligible and future generations will face excess financial burden. If it is too low, ineffective projects will be chosen creating an inefficient allocation of resources[5]. Failure to discount implies a discount rate of zero. This would unrealistically trade one dollar in the future for one dollar received immediately [6].

The term discounting is a separate concept from inflation. It refers to the fact that a dollar received in the future is worth less than a dollar now[7]. Discounting is based on two basic reasons:

- First, consumers prefer consumption now rather than later. They generally expect to be wealthier in the future and that an extra dollar will therefore then matter less. Sometimes, they may also prefer the present to the future due to uncertainty.
- Second, capital investments enhance future production and consumption. A dollar invested in productive activities will generate additional income and hence additional consumption in the future [4].

In a simplified world without market failure such as tax or risk, the SDR would simply be the market rate of interest. When the allocation of goods and services by a free market is not efficient, determining the SDR is a complicated issue [4].

Two candidate rates are usually considered in the economic literature when determining SDRs: the opportunity cost rate and social rate of time preference [6].

1.1 The Opportunity Cost Rate

The opportunity cost rate is based on Harberger's Social Discount Rate (SDR) theory. It boils down to determining the opportunity cost of drawing funds from the private sector. That is, the benefits to society that the funds would have returned if left in the private sector [2]. For Harberger any amount invested by the government is drawn either from new private savings and/or from displaced private investments (crowding out). The social discount rate is then a weighted average of the private sector marginal productivity of capital (i.e., the before tax return on capital) and the marginal rate of time preference in the private sector (i.e., the net-of-tax yield on private savings) [8].

Detractors of the relatively high Harberger's social discount rate present three lines of criticism [9], some of them attributed to Harberger himself [10]

1. The opportunity cost rate does not make much sense from a social welfare maximization perspective.
2. Mistakes are often made in deriving the opportunity cost of a particular project from the different securities used for its financing.
3. Public expenditures contribute to the budget deficit which must be financed through debt. The financing of the additional amount of debt will in its turn crowd out an equal amount of private investment and so on [11].

1.2. The Social Rate of Time Preference

The social rate of time preference is based on Ramsey's formula[12]. Ramsey's formula is an equation that relates the SDR to three components:

1. A pure social rate of time preference, which is the discount rate on the utility of future generations;
2. the growth rate of per capita consumption; and
3. the elasticity of marginal utility of consumption.

The recent literature on discount rates seems to indicate that the appropriate SDR is the social rate of time preference. This approach is often used in the context of intergenerational choice to justify a relatively low rate [2].

In practice, countries have used very different rates and approaches in their social project appraisal. For example, in 2003 the UK switched from a 6% real rate, based mainly on cost of capital considerations, to a 3.5% real rate based entirely on the social rate of time preference[13]. Sometimes countries and organizations apply different discount rates to different public interventions. They generally use an opportunity cost rate when the financial return of the project is important and a social rate of time preference for the more standard interventions. Spain, for example, uses a 6% for transport projects but a lower 4% percent for water project[3]. The American Office of Management and Budget recommends a real rate of 7% for the productive projects and a rate between 2.5% and 3% for the other public interventions[15]. In France, a group of experts commissioned by the Ministry of Finance has recommended a reduction in the SDR from 8% to 4% for most public sector projects [16]. In Canada, the Treasury Board Secretariat recommends a SDR of 8%, based on the weighted social opportunity cost of capital method. Boardman et al. (2010) argued that this value is based on an inappropriate methodology and is too high. Using Ramsey's formula, the authors suggested that if a project is intra generational and there is no crowding out of private investment, then analysts should use an SDR of 3.5%.

The purpose of this paper is to develop an economic model to derive and estimate SDRs for civilian and military sectors. The setting is represented by a welfare function that has the Stone-Geary form and depends on consumption and military security[17]. A Golden Rule, referred to as dg1, to find plausible values for SDRs is provided. It is an equation that relates the SDR to the expected growth rate of per capita spending.

The remainder of the paper is organized as follows. In Section 2, the model is set up. In Section 3, the Golden Rule is presented and the plausible values for civilian and military SDRs are estimated. In Section 4, we briefly conclude.

2. DERIVATION OF THE SOCIAL DISCOUNT RATES

Consider a representative individual that maximises a welfare function derived from the discounted value of per-capita consumption utility $u(c)$ and per-capita security utility $v(s)$,

$$W_{c,s} = \int_0^\infty e^{-\delta t}\left(\alpha u(c_t) + (1-\alpha)v(s_t)\right)dt, \quad (1)$$

where δ denotes the rate of pure time preference, i.e. the rate at which utilities are discounted. α is referred to as the preference parameter. α governs the relationship between civilian and military spending. The welfare function has the Stone-Geary form over an infinite horizon. Security is unobservable and is estimated using per-capita military spending. As is standard, u and v have the following characteristics,

(i) u and v are twice continuously differentiable on R^+;

(ii) $u', v' > 0$, $u'', v'' < 0$;

(iii) $lim_{c\to 0}u'(0) = lim_{s\to 0}v'(0) = +\infty$, i.e., zero consumption or zero security is impossible.

By definition, the social discount rate d_x ($x = c$ for consumption and $x = s$ for security) between two successive years t and $t+1$ measures the marginal rate of substitution of x between these years[19]. It is implicitly defined by

$$e^{-d_x} = \frac{W'(x_{t+1})}{W'(x_t)} = e^{-\delta}\frac{u'(x_{t+1})}{u'(x_t)}, \ x = c, s. \quad (2)$$

Note that this is true for any given consumption path, whether optimal or not [20]. If the representative individual has the standard power utility function

$$u(x_t) = \frac{x_t^{1-\eta_x}}{1-\eta_x} \text{ with } \eta_x > 0, \ x = c, s, \quad (3)$$

which gives

$$u(x_t) = \ln(x_t) \text{ for } \eta_x = 1, \ x = c, s, \quad (4)$$

then

$$u'(x_t) = x_t^{-\eta_x}, \ x = c, s, \quad (5)$$

and Equation 2 becomes

$$e^{d_x} = e^{\delta}\left(\frac{x_{t+1}}{x_t}\right)^{\eta_x}, \ x = c, s. \quad (6)$$

Taking logs gives the Ramsey equation

$$d_x = \delta + \eta_x g_x, \ x = c, s, \quad (7)$$

where η_x is the elasticity of marginal utility of x. It is the percentage fall in the marginal utility when x increases by one per cent. η_x is given by

$$\eta_x = \frac{\frac{du'(x)}{u'(x)}}{\frac{dx}{x}} \quad (8)$$

g_x is the continuously compounded growth rate of x. g_x is given by

$$g_x = \ln\left(\frac{x_{t+1}}{x_t}\right). \quad (9)$$

It is worthwhile to note that this expression does not depend on the preference parameter α.

3. ESTIMATION OF THE SOCIAL DISCOUNT RATES

Ramsey's formula in Equation 7 expresses the SDR as a sum of two terms δ and $\eta_x g_x$. The first term is the rate of pure time preference. Also known as the utility discount rate, this parameter describes impatience. The second term is the product of two parameters: the elasticity of marginal utility (η_x) and the expected growth rate (g_x). It represents devaluation of future x caused by the combination of the two parameters. η_x indicates the percentage reduction in the marginal utility if x increases by one percent and g_x describes how fast x increases [21]. Recall that the variable x can take on two different values: either c (for consumption) or s (for security). In this section, we will estimate these parameters and provide a Golden Rule to find plausible values for SDRs in civilian and military sectors.

3.1 Pure Time Preference

Based on normative grounds, many scholars including Ramsey (1928), Stern (2007), and Dasgupta (2007) prescribed a rate close to or equal to zero. Arrow (1995) demonstrated, however, that with this prescribed value the current generation would be required to save approximately two-thirds of its income. This rate is sometimes interpreted as the instantaneous probability of death. Kula (1984), for example, adopted this approach and estimated δ to be 0.8%.

For two technical reasons we will use the reasonable value of 1% suggested by Arrow (1995) and Boardman et al. (2010). First, without a positive pure time preference the integral of the utility of consumption over an infinite future will not converge for most of the paths to be compared. Second, as noted by Koopmans (1960) and Arrow (1995), a zero pure time preference would result in an unrealistically high savings rate [3]. Recent studies seem to suggest a rate close to 1% which we use in this study.

3.2 Elasticity of marginal utility

The value of the parameter η_x could be anywhere between zero and infinity.

- If $\eta_x = 0$, then d_x is independent of growth rate g_x, signifying a complete independence between any change in consumption and willingness to invest.
- In contrast, if η_x were large and growth is positive, for example, society would completely stop to invest in future.

While there are many ways and approaches to calculate values for η_x, almost everyone agrees that this parameter would range between 1 and 2. The evidence on social preferences does not actually support high values [16]. Kula (1984), for example, suggested η_x equals 1.56 for Canada. Pearce and Ulph (1995) estimated a range from 0.7 to 1.5, with a value of 1 being defensible. Cowell and Gardiner (1999) concluded that most studies imply values of the elasticity of marginal utility of just below or just above one. Evans (2005) found η_x near 1.25 for Canada. Stern (2007) suggests η_x equals to 1.

We think that a value for η_x of 1 is reasonable and consistent with the defence economics literature. This literature uses a common methodology based on a welfare function of the Stone-Geary form. The Stone-Geary utility function is a monotonic transformation of a Cobb-Douglas type function. This linear logarithmic function implicitly entails that all elasticities of substitution are equal to unity. Assuming a value of 1 implies that if x_t doubles, for example, $u'(x_t) = x_t^{-1}$ falls to one half of the previous value.

3.3 Growth Rate of Per Capita Consumption

As suggested by Weitzman (2007), there is a large agreement to project growth at 2%. The UK government, for example, estimates that the average growth rate of consumption per head is 2%[7]. Moore et al. (2004) recommended using g = 2% for the United-States and Boardman et al. (2010) favoured the use of 1.7% for Canada.

An easy way to estimating this parameter is to regress the natural logarithm of real x on time and use the slope coefficient. We adopted this approach and fitted the two following equations

$$ln(x(t)) = g_{0x} + g_x t,\ x = c, s. \quad (10)$$

where x represents per capita consumption or security expressed in real terms, g_{0x} is a constant, g_x is the growth rate of x, and t is time.

As shown in Table 1, using the most recent data, the real civilian and military growth rates are respectively estimated to be 1.8 % and 0%. Data were observed between 1971 and 2011, inclusively. We calculated a 95% confidence interval (CI) for each growth rate. A CI gives a range within which the true growth rate is likely to reside. The civilian growth rate is statistically significant because both ends of the 95% CI are positive. In contrast, the military 95% CI includes zero, meaning that zero is the plausible value for the military growth rate.

Table 1: Results of the regression analyses

3.3 Golden Rule - Dg1

Building on the most recent data and literature, this study provides a Golden Rule to find plausible values for the civilian and military SDRs. As shown in Equation 11, this rule is an equation

that relates the SDR d_x only to the expected growth rate g_x.

$$d_x = g_x + 1,\ x = c, s. \quad (11)$$

Referred to as dg1, this rule is derived from an economic model and commonly meaningful agreed-on assumptions. As suggested by many recent studies, it is implicitly assumed that pure time preference rate and the elasticity of marginal utility are close to 1% [16].

Applying our Golden Rule, the reasonable values for civilian and military SDRs are respectively estimated to be 2.8% and 1%. The military SDR turned out to be significantly lower than civilian rate. The major reason of this gap is due to the persisting dissimilarity in real growth rates between civilian and military expenditures.

3.4 Discounting Under Uncertainty

The rate of growth in consumption could be uncertain, particularly over long horizons. When uncertainty is introduced, the standard Ramsey formula in Equation 7 can be extended to account for it. If consumption growth is, for example, independently and identically normally distributed, with mean g_x and variance ${\sigma_x}^2$, then Ramsey formula becomes

$$d_x = \delta + \eta_x g_x - \frac{1}{2}\eta_x{}^2\sigma_x{}^2,\ x = c, s, \quad (12)$$

where the last term is a precautionary effect[27][28][30]. This new term which lowers the social discount rate is likely to be negligible[23][24][29]. Kocherlakota (1996), for example, estimated it to be as small as 0.26% for the United States. In light of this evidence, sensitivity analysis around the central estimate of the discount rate seems to be more appropriate [16]. Extensive literature on this issue can be found in Heal and Millner (2013)[32].

4. CONCLUSION

In a world without any distortions, the SDR would simply be the market rate of interest. In an economy with market failure, determining the appropriate SDR is a complex issue. Two candidate rates are usually considered in the economic literature when determining SDRs: the opportunity cost rate and social rate of time preference. The recent literature on discount rates seems to indicate that the appropriate SDR is the social rate of time preference. This approach is often used in the context of intergenerational choice to justify a relatively low rate. In practice, countries have used very different rates and approaches in their social project appraisal. Sometimes they apply different discount rates to different public interventions.

The purpose of this paper is to develop an economic model to derive and estimate SDRs for civilian and military sectors. A Golden Rule, referred to as dg1, to find reasonable values for SDRs is provided. It relates the SDR to the expected growth rate of per capita spending. Results indicate that the plausible Canadian values for civilian and military SDRs are respectively 2.8% and 1%. The military SDR is lower because the real military growth rate of per capita spending is smaller. Further analysis should be undertaken to address other aspects of public project appraisals. A natural extension to this study is to estimate the shadow price of capital used in civilian and military projects.

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Discounting for Civilian and Military Projects

4. AUTHORS (last name, followed by initials – ranks, titles, etc., not to be used)

Abderrahmane, S.

5. DATE OF PUBLICATION (Month and year of publication of document.)	6a. NO. OF PAGES (Total containing information, including Annexes, Appendices, etc.)	6b. NO. OF REFS (Total cited in document.)
January 2018	5	32

7. DESCRIPTIVE NOTES (The category of the document, e.g., technical report, technical note or memorandum. If appropriate, enter the type of report, e.g., interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

External Literature (N)

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9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)	9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)

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DRDC-RDDC-2018-N005	

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