

# Final Report 2016-17: Sleep Models Optimization

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## **Abstract**

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The objective of the current task was to revise the accuracy calculation, perform linear regression analysis, and optimization for the following models: Two-Process Model of Sleep Regulation by Acherman and Borbley [1], Three-Process Model of Alertness by Akerstedt's group [2], and Sleep/Wake Model by Darwent's group [3].

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# 1 Introduction

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## 1.1 Objective

This report provides a documented trial of activities which were performed for the analysis of sleep models, fatigue performance and parameter optimization for the following sleep models: Model 1 [1], Model 2 [2], and Model 3 [3].

## 1.2 Scope

The following tasks were completed:

1. The accuracy of the sleep model was calculated with a new approach;
2. Linear regression analysis was performed for sleep models: Model 1, Model 2 and Model 3;
3. Sensitivity analysis of Model 2 and Model 1 was performed with parameter optimization;
4. The results were obtained from the models and summarized in tables.

These tasks were completed by the project team; consisting of members from both DRDC and industry.

**Table 1: Project Team Members**

Dr. Fethi Bouak	Technical Authority – DRDC Toronto Research Centre
Dr. Henry Peng	Co-Lead for DFM - DRDC Toronto Research Centre
Dr. Wenbi Wang	Co-investigator - DRDC Toronto Research Centre
Matt Lamb	Co-investigator - DRDC Toronto Research Centre
Natalia Doubova	Contractor
Vismit Joshi	CO-OP student

This report has the following sections: Introduction – this Section, Method – Section 2, Results – Section 3, and Conclusion – Section 4.

## 2 Method

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### 2.1 Accuracy

The comparison of the modeled results versus the experimental sleep in Reference [4] was based on average sleep duration. However, this approach did not include the analysis of the number of sleep episodes. To improve the comparison, a new approach was introduced and the model's accuracy was calculated as follows:

$$accuracy_i = \frac{\sum_{i=1}^N match_i}{\sum_{i=1}^N match_i + \sum_{i=1}^N missMatch_i}$$

Where

*match* = 15 minutes if the predicted and experimental sleep match;

*mismatch* = 15 minutes if the predicted sleep does not match the experimental sleep;

*N* = total number of time interval ( $\Delta t = 15$  minutes in presented calculation).

The comparison model results versus the experimental sleep with a new approach is summarized in **Table 2, Table 3, Table 4, and Table 5.**

The Model 1 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model1>.

The Model 2 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2>.

The Model 3 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model3>.

**Table 2: Accuracy in (%) of Sleep Predictions for Day Shift**

Subject	Accuracy (%)			Error (%)			Missing Data (h)
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	
227	65.41	67.51	72.25	34.59	32.49	27.75	0
228	68.02	68.34	67.32	31.98	31.66	32.68	0
229	73.72	80.32	71.67	26.28	19.68	28.33	2.5
231	70.08	75.94	68.32	29.92	24.06	31.68	0
232	77.71	85.49	80.74	22.29	14.51	19.26	0
233	74.42	82.62	75.34	25.58	17.38	24.66	0.25
234	65.86	77.52	79.50	34.14	22.48	20.50	0
235	83.55	86.71	79.29	16.45	13.29	20.71	0
236	75.40	80.46	72.84	24.60	19.54	27.16	1.75
237	73.53	75.08	71.83	26.47	24.92	28.17	7
247	80.91	86.53	76.56	19.09	13.47	23.44	0
249	50.71	68.13	63.48	49.29	31.88	36.52	6.75
250	85.77	87.32	85.61	14.23	12.68	14.39	0
251	91.75	91.30	89.64	8.25	8.70	10.36	0
253	80.24	80.43	80.23	19.76	19.57	19.77	0
255	79.46	82.07	73.44	20.54	17.93	26.56	8.75
256	90.78	90.69	86.77	9.22	9.31	13.23	0
<b>AVG</b>	75.72	80.38	76.17	24.28	19.62	23.83	
<b>SD</b>	9.83	7.29	6.93	9.83	7.29	6.93	

**Table 3: Accuracy in (%) of Sleep Predictions for Port Watch**

Subject	Accuracy (%)			Error (%)			Missing Data (h)
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	
203	72.81	73.03	69.91	27.19	26.97	30.09	0
209	83.59	84.59	83.28	16.41	15.41	16.72	0
211	70.23	69.26	73.65	29.77	30.74	26.35	6
215	60.69	65.31	56.90	39.31	34.69	43.10	0
217	53.63	60.38	54.19	46.37	39.62	45.81	0
218	60.43	59.55	48.58	39.57	40.45	51.42	0
224	42.62	51.35	47.77	57.38	48.65	52.23	0
240	74.90	83.67	72.40	25.10	16.33	27.60	0
245	60.37	65.85	60.86	39.63	34.15	39.14	0
246	65.54	66.43	64.77	34.46	33.57	35.23	0
248	67.47	73.63	65.75	32.53	26.37	34.25	0
252	47.72	53.30	45.43	52.28	46.70	54.57	0
<b>AVG</b>	63.33	67.19	61.96	36.67	32.81	38.04	
<b>SD</b>	11.16	10.04	11.28	11.16	10.04	11.28	

**Table 4: Accuracy in (%) of Sleep Predictions for Starboard Watch**

Subject	Accuracy (%)			Error (%)			Missing Data (h)
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	
205	86.73	87.21	84.64	13.27	12.79	15.36	0
210	79.35	72.58	60.38	20.65	27.42	39.62	0
213	86.67	79.78	63.64	13.33	20.22	36.36	0
214	81.62	84.84	81.40	18.38	15.16	18.60	0
216	78.04	75.96	78.93	21.96	24.04	21.07	0
219	64.23	58.75	48.30	35.77	41.25	51.70	0
221	52.19	56.39	49.27	47.81	43.61	50.73	1.75
225	78.39	79.61	77.42	21.61	20.39	22.58	0
238	64.19	65.65	53.95	35.81	34.35	46.05	0
239	63.95	67.77	60.00	36.05	32.23	40.00	0
241	72.90	68.08	53.42	27.10	31.92	46.58	0
<b>AVG</b>	73.48	72.42	64.67	26.52	27.58	35.33	
<b>SD</b>	10.49	9.64	12.91	10.49	9.64	12.91	

**Table 5: Accuracy in (%) of Sleep Predictions for Irregular Shift Workers**

Subject	Accuracy (%)			Error (%)			Missing Data (h)
	Model 1	Model 2	Model 3	Model 1	Model 2	Model 3	
202	31.51	37.67	26.93	68.49	62.33	73.07	0
207	80.82	83.40	75.31	19.18	16.60	24.69	0
208	7.52	15.87	6.68	92.48	84.13	93.32	0
212	9.44	35.77	22.55	90.56	64.23	77.45	3.5
222	5.91	24.64	17.54	94.09	75.36	82.46	1.75
<b>AVG</b>	27.04	39.47	29.80	72.96	60.53	70.20	
<b>SD</b>	28.46	23.34	23.73	28.46	23.34	23.73	

These tables can be found in the file “**sleepWorkSummaryTablesError.xlsx**”, in the sheet named “SleepAccuracyTable”.

The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\SummaryFiles>.

## 2.2 Linear Regression

Sleep and fatigue level data were collected from forty-five crew members. All participants were divided into four groups according to their working shifts: day, port, starboard and irregular. The detailed description of the experimental data is provided in Reference [4].

Based on Reference [2], it was assumed that the predicted sleepiness/alertness function can be used to predict fatigue level using a linear transformation function. To analyze the relationship for every group (Reference [4]), linear regression models were fitted with an alertness score as the independent variable and observed fatigue level as the dependent variable. The equations are shown below:

$$fatigueLevel = a + b \times S \quad \text{for Model 1}$$

$$fatigueLevel = a + b \times (S + C + U) \quad \text{for Model 2}$$

$$fatigueLevel = a + b \times f(t) \quad \text{for Model 3}$$

Experimental fatigue level is extracted from the master spreadsheet, column “H”: -  $y(t_1)_1, y(t_2)_2 \dots y(t_n)_n$ .

The master spreadsheet named “SCORE\_Individual\_data\_SUMMARY\_Test.xlsx” is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2L>

Modeled alertness (ex. Model 2):  $(S(t_1)+C(t_1)+U(t_1))_1, (S(t_n)+C(t_n)+U(t_n))_n$

Predicted fatigue level:  $y_{pred1}, y_{pred2}, \dots y_{predn}$ .

Residuals:  $y_i - y_{predi}$ .

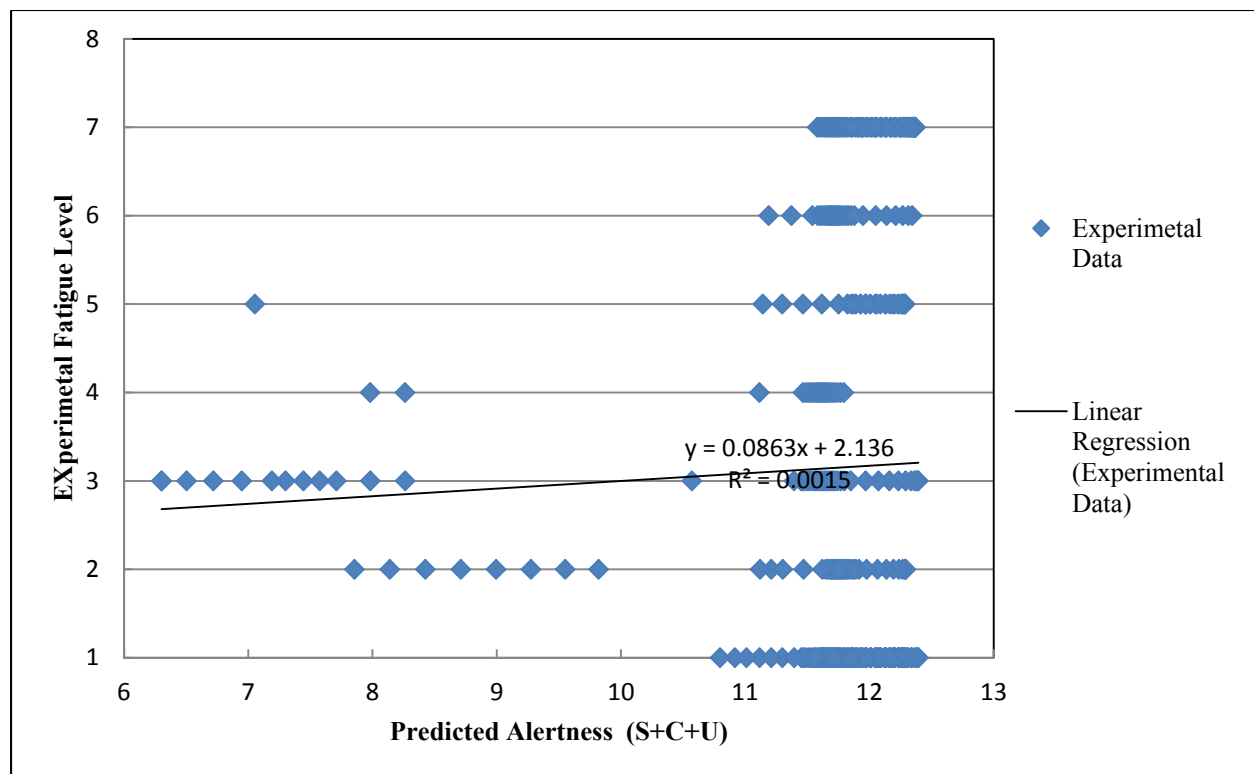
$$y_{avg} = \frac{1}{n} \sum_{i=1}^n y_i$$

$$R^2 = 1 - \frac{\sum_{j=1}^n (y_i - y_{predi})^2}{\sum_{i=1}^n (y_i - y_{avg})^2}$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - y_{predi})^2$$

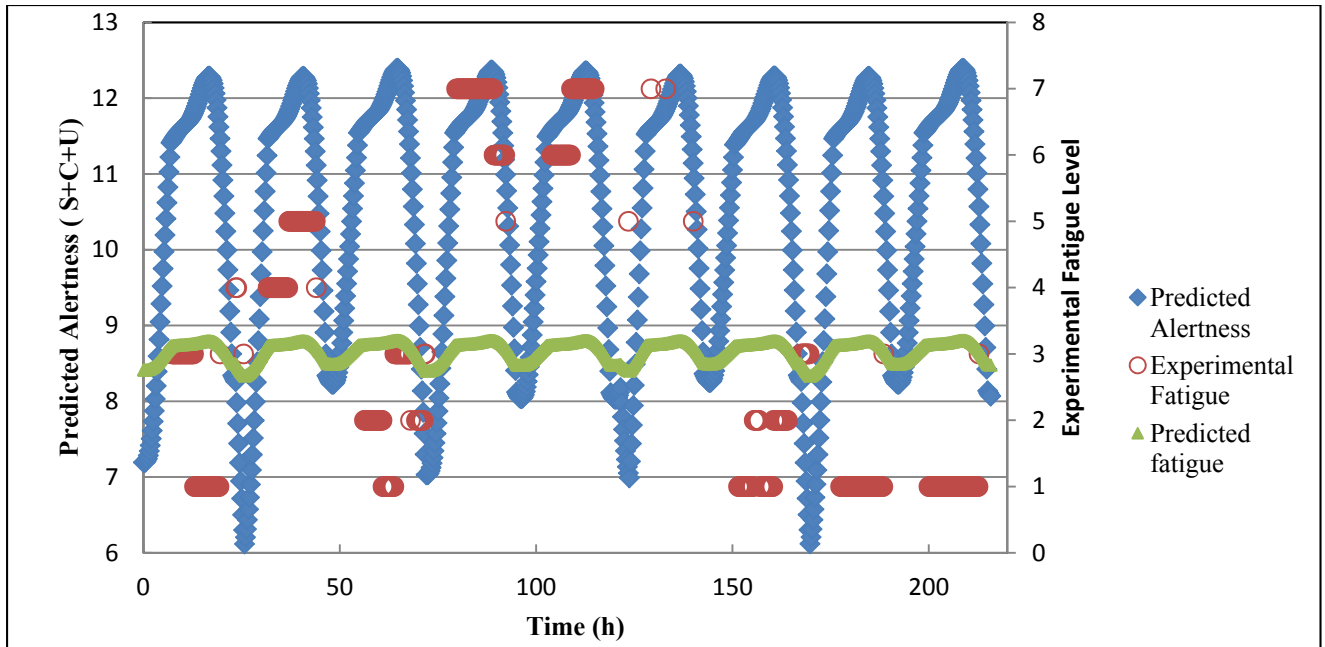
The time intervals corresponding to experimental sleep or missing fatigue values were removed from the analysis.

**Figure 1** shows the experimental fatigue level as a function of predicted alertness for Day Shift worker (participant 227) from Model 2.



**Figure 1: Fatigue Level as a function of Model 2 Predicted Alertness for Participant 227**

Observed and predicted fatigue levels, as well as predicted alertness using Model 2 for participant 227 as a function of time are presented in **Figure 2**.



**Figure 2: Predicted Alertness (Model 2), Experimental and Modeled Fatigue level as a function of time**

These figures can be found in the file “SCORE\_Individual\_data\_SUMMARY\_tmp227.xlsx”, in the sheet named “VerificationLR227”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2LRSub227>.

The Model 1 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model1LR>.

The Model 2 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2LR>.

The Model 3 files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model3LR>.

The results and the parameters of the linear regression are summarized in **Table 6**.

**Table 6: Linear Regression Results**

<b>Model 1</b>						
<b>Shift</b>	<b>a<sub>1</sub></b>	<b>b<sub>1</sub></b>	<b>R-Squared</b>	<b>R</b>	<b>MSE</b>	<b>RMSE</b>
<b>Day</b>	1.52	2.03	6.91E-02	2.63E-01	1.411	1.188
<b>Port</b>	1.59	3.23	1.05E-01	3.24E-01	2.071	1.439
<b>Stbrd</b>	2.18	1.33	2.90E-02	1.70E-01	1.491	1.221
<b>Irregular</b>	2.80	0.10	1.45E-04	1.20E-02	1.076	1.037
<b>Model 2</b>						
<b>Shift</b>	<b>a<sub>2</sub></b>	<b>b<sub>2</sub></b>	<b>R-Squared</b>	<b>R</b>	<b>MSE</b>	<b>RMSE</b>
<b>Day</b>	5.75	-0.30	5.32E-02	2.31E-01	1.435	1.198
<b>Port</b>	5.83	-0.27	2.13E-01	4.62E-01	1.820	1.349
<b>Stbrd</b>	6.21	-0.33	8.64E-02	2.94E-01	1.403	1.184
<b>Irregular</b>	2.64	0.02	3.82E-03	6.18E-02	1.072	1.036
<b>Model 3</b>						
<b>Shift</b>	<b>a<sub>3</sub></b>	<b>b<sub>3</sub></b>	<b>R-Squared</b>	<b>R</b>	<b>MSE</b>	<b>RMSE</b>
<b>Day</b>	2.28	0.53	4.85E-03	6.96E-02	1.508	1.228
<b>Port</b>	3.16	0.35	7.17E-04	2.68E-02	2.311	1.520
<b>Stbrd</b>	2.72	0.02	1.02E-05	3.19E-03	1.535	1.239
<b>Irregular</b>	2.85	0.06	1.65E-04	1.29E-02	1.076	1.037

Based on the generated results, a strong relationship between alertness and fatigue level was not observed for the collected data.

Further analysis included the linear regression model with predicted fatigue (in percent) as the independent variable and the experimental fatigue level as the dependent variable for the following sleep models: Model 1, Model 2, and Model 3.

The results are provided in **Table 7**.



**Table 7: Fatigue Linear Regression**

Model 1						
Shift	a <sub>1</sub>	b <sub>1</sub>	R-Squared	R	MSE	RMSE
Day	6.04	-0.04	4.55E-02	2.13E-01	1.446	1.203
Port	9.26	-0.07	1.79E-01	4.23E-01	1.899	1.378
Stbrd	7.28	-0.05	1.11E-01	3.33E-01	1.365	1.168
Irregular	2.87	0.00	6.40E-06	2.53E-03	1.077	1.038
Model 2						
	a <sub>2</sub>	b <sub>2</sub>	R-Squared	R	MSE	RMSE
Day	7.15	-0.05	4.39E-02	2.10E-01	1.449	1.204
Port	8.85	-0.07	1.63E-01	4.04E-01	1.936	1.391
Stbrd	7.45	-0.06	8.59E-02	2.93E-01	1.403	1.185
Irregular	2.68	0.00	2.46E-03	4.96E-02	1.074	1.036
Model 3						
	a <sub>3</sub>	b <sub>3</sub>	R-Squared	R	MSE	RMSE
Day	9.93	-0.08	5.09E-02	2.26E-01	1.438	1.199
Port	8.84	-0.07	1.80E-01	4.24E-01	1.897	1.377
Stbrd	9.15	-0.07	1.01E-01	3.18E-01	1.380	1.175
Irregular	2.81	0.00	2.17E-04	1.47E-02	1.076	1.037

These tables can be found in the file “summaryLinearRegression.xlsx”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\SummaryFiles>.

## 2.3 Model Parameters Optimization

As discussed in Reference [4], Model 2 was selected to find the impact of circadian rhythm, thresholds and additional parameters on the predicted sleep. It has been shown in Reference [4] that these parameters have a large effect on the modeled results. In addition to Model 2, parameter optimization for Model 1 was performed in order to improve the sleep accuracy.

A brief description of Model 1 and Model 2 is provided below.

### 2.3.1 Model 1

The model includes two main components: a circadian system and a sleep homeostatic system. The homeostatic sleep pressure as a function of time is as follows:

$$S_t = 1 - e^{\frac{-\Delta t}{\tau_r}} \times (1 - S_{t-\Delta t}) \text{ during wake} \quad \text{Equation 1}$$

$$S_t = e^{\frac{-\Delta t}{t_d}} \times S_{t-\Delta t} \quad \text{during sleep} \quad \text{Equation 2}$$

Where  $\Delta t$  is the time step; and  $t_r$  (18.2 h) and  $t_d$  (4.2) are the time constants for the rise and decay of the homeostatic process during wakefulness and sleep, respectively.

The circadian rhythm as a function of time is as follows:

$$C = A \sum_{k=1}^5 a_k \times \sin \frac{2k\pi}{\tau} (t - t_0) \quad \text{Equation 3}$$

Where

A	= amplitude of skewed sine wave (0.12)
$\tau$	= period of C (24 hours)
$t_0$	= the circadian phase at the beginning of the simulation (8.6 h)
$a_1$	= 0.97
$a_2$	= 0.22
$a_3$	= 0.07
$a_4$	= 0.03
$a_5$	= 0.001

The following initial conditions were used in the simulation:  $t = 24$ ;  $S = 0.49$ .

When  $S > 0.67 + C$  during rest period, it is time to sleep; when  $S < 0.17 + C$ , then it is time to wake up.

The analysis files are located in [C:\Users\Peng\\_Contractor\Documents\ndPhaseII\Model1Optimization](C:\Users\Peng_Contractor\Documents\ndPhaseII\Model1Optimization).

### 2.3.2 Model 1 Parameters Optimization

In order to maximize Model 1 sleep accuracy for every group, parameter optimization was used in the determination of the best agreement with the experimental sleep. The following parameters were optimized:  $t_r$  and  $t_d$  with upper and lower bounds obtained from Reference [5]. Furthermore, the optimization was performed with different initial parameters in order to avoid a local minimum point in a given range Reference [5].

The MATLAB optimization function “**fminsearchbnd**” with bound constraints was used for parameter optimization. The results are shown in **Table 8**.

**Table 8: Model 1- Parameter Optimization Results**

Shift	Rise Factor of S(t)	Decay Factor of S(t)	Accuracy Average (%)	Error (%)	# of Iterations
<b>Case 1.1</b>					
<b>Initial Values</b>	18.200	4.200			
<b>Bounds<sup>1</sup> (LA&gt;0)</b>	[14.09; 26.43]	[1.15; 4.2]			
<b>Day</b>	15.745	4.176	79.545	20.455	23
<b>Port</b>	15.297	4.199	64.969	35.031	20
<b>Starboard</b>	16.144	4.198	77.341	22.659	22
<b>Irregular</b>	14.095	4.195	32.413	67.587	18
<b>Case 1.2</b>					
<b>Initial Values</b>	21.380	3.000			
<b>Bounds (LA&gt;0)</b>	[14.09; 26.43]	[1.15; 4.2]			
<b>Day</b>	16.405	4.116	79.619	20.381	24
<b>Port</b>	15.319	4.197	64.969	35.031	26
<b>Starboard</b>	16.172	4.109	77.325	22.675	23
<b>Irregular</b>	14.148	4.200	32.413	67.587	20
<b>Case 1.3</b>					
<b>Initial Values</b>	18.200	4.200			
<b>Bounds (LA=0)</b>	[11.43 20.09]	[2.82 4.69]			
<b>Day</b>	15.967	4.538	79.948	20.052	28
<b>Port</b>	15.222	4.655	66.839	33.161	24
<b>Starboard</b>	16.117	4.681	77.358	22.642	22
<b>Irregular</b>	11.431	4.690	37.745	62.255	28

<sup>1</sup> The lower and upper bounds of  $t_r$  and  $t_d$  for all cases were obtained from Reference [5], Table 3.

**Table 8 (Cont'd): Model 1- Parameter Optimization Results**

Case 1. 4					
<b>Initial Values</b>	17.500	3.000			
<b>Bounds (LA=0)</b>	[11.43 20.09]	[2.82 4.69]			
<b>Day</b>	15.965	4.549	79.964	20.036	34
<b>Port</b>	16.133	4.689	66.965	33.035	24
<b>Starboard</b>	15.905	3.895	77.122	22.878	26
<b>Irregular</b>	11.431	4.690	37.745	62.255	28

This table can be found in the file “SummaryStatsFileM1.xlsx”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model1Optimization>.

### 2.3.3 Model 2

A brief description of Model 2 [2] is provided below:

$$S = la + (sw - la) \times e^{(d \times taw)} \quad \text{Equation 4}$$

$$S' = ha - (ha - ss) \times e^{(g \times tas)} \quad \text{Equation 5}$$

The original process S' (Equation 5) was modified with a “break function”. This splits the process into S'b1 (Equation 6) for the part of sleep with high homeostatic pressure and S'b2 (Equation 7) for the last part of sleep with lower pressure [2]. The “break point” in time asleep (bt) is defined a switch from S'b1 to S'b2 and is calculated as follows:

$$bt = (bl - ss)/(g \times (bl - ha)) \quad \text{Equation 6}$$

If  $tas \leq bt$ , the process S'b1 is as follows:

$$S'b1 = ss + tas * (g \times (bl - ha)) \quad \text{Equation 7}$$

In the case  $dt \geq bt$ , the following algorithm is used:

$$S'b2 = ha - (ha - bl) \times e^{(g \times (tas - bt))} \quad \text{Equation 8}$$

$$C = m1 + \alpha \times \cos\left(\frac{\pi}{12} \times (t - p)\right) \quad \text{Equation 9}$$

$$U = m2 + \alpha \times \cos(\pi/6 \times (t - (p + 3))) \quad \text{Equation 10}$$

Where

- la = low asymptote (2.4);
- sw = S at waking up;
- d = decay in alertness (-0.0353);
- taw = time since waking up, in decimal hours;
- tas = time since falling asleep, in decimal hours;
- ha = high asymptote (14.3);
- ss = S at falling asleep;
- g = recovery in alertness which is calculated as  $\ln\left(\frac{ha-14.0}{ha-7.96}\right)/8 \approx -0.381$ ;
- bl = 12.2;
- m1 = mesor (0);
- $\alpha$  = amplitude (2.5);
- p = acrophase (16.8 Reference [2]);
- m2 = mesor (-0.5);
- $S+C+U < 8.38$  – threshold for falling asleep
- $S+C+U > 11.38$  – threshold for waking up

The following initial conditions were used:

1.  $t = 24$
2.  $S'(24) = 8.76$

The analysis files are located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2OptimizationAllFiveParam>.

### 2.3.4 Model 2 Parameters Optimization

The following parameters were selected for optimization:

1. Upper and lower thresholds;
2. Acrophase (p);
3. Decay in alertness (d);
4. Recovery in alertness (g).

The default values were obtained from Reference [2] and are shown in **Table 9**.

The recovery in alertness is calculated as follows:

$$\ln\left(\frac{ha-14.0}{ha-7.96}\right)/8 \approx -0.381$$

where the denominator corresponds to a sleep duration of 8 hours.

The time constants (Reference [5]) of the recovery in alertness ranged from 1.15 to 2.91 which corresponds to g-values ranging from -0.86956 to -0.343642 respectively. The g-values fall into the logarithm value range of [3.5 and 8.878]. It has been observed from the experimental data (Reference [4]), that the sleep duration was more than 8 hours daily; therefore, the upper limit of logarithm was defined as 11 and the log range was changed to [3.5 and 11] in parameter optimization.

The time constants of decay in alertness ranged from 14.09 to 26.43 (Reference [5]). They correspond to *d-values* ranging from -0.07097 to -0.03783 respectively. Since the default of *d-value* was -0.0353 (Reference [2]), the boundaries were changed to [-0.07097; -0.0353].

The MATLAB optimization function “**fminsearchbnd**” with bound constraints was used for parameter optimization. The results are summarized in **Table 9**.

**Table 9: Model 2- Parameter Optimization Results**

Shift	Lower Threshold	Upper Threshold	Acrophase	Logarithm	Decay	Accuracy Average (%)	Error (%)	# of Iterations
<b>Case 2.1</b>								
<b>Initial Values</b>	8.38	11.38	16.8	8	-0.0353			
<b>Bounds (LA&gt;0)</b>	[8; 9]	[11; 15]	[14.62; 17]	[3.5; 11]	[-0.0709; -0.0353]			
<b>Day</b>	8.000	11.725	15.581	10.488	-0.0353	83.433	16.567	59
<b>Port</b>	8.001	12.386	15.610	10.764	-0.0358	72.465	27.535	53
<b>Starboard</b>	8.000	11.521	15.513	10.909	-0.0353	77.679	22.321	41
<b>Irregular</b>	8.004	15.000	17.000	10.906	-0.0369	64.069	35.931	64
<b>Case 2.2</b>								
<b>Initial Values</b>	8.5	13	15.8	9	-0.0353			
<b>Bounds (LA&gt;0)</b>	[8; 9]	[11; 15]	[14.62; 17]	[3.5; 11]	[-0.0709; -0.0353]			
<b>Day</b>	8.309	11.964	16.306	7.744	-0.0433	83.414	16.586	45
<b>Port</b>	8.661	12.387	16.658	9.825	-0.0652	74.552	25.448	51
<b>Starboard</b>	8.544	12.987	16.082	9.274	-0.0520	77.186	22.814	36
<b>Irregular</b>	8.401	14.990	16.998	10.939	-0.0550	68.007	31.993	47
<b>Case 2.3</b>								
<b>Initial Values</b>	8	12	15.2	10	-0.0453			
<b>Bounds (LA&gt;0)</b>	[8; 9]	[11; 15]	[14.62; 17]	[3.5; 11]	[-0.0709; -0.0353]			
<b>Day</b>	8.002	11.777	15.802	9.696	-0.0392	83.465	16.535	59
<b>Port</b>	8.199	14.389	16.984	10.587	-0.0577	74.605	25.395	53
<b>Starboard</b>	8.001	12.177	15.532	9.717	-0.0391	77.771	22.229	48
<b>Irregular</b>	8.000	14.990	16.940	10.996	-0.0505	68.702	31.298	54

These tables can be found in the file “**SummaryStatsFileM2.xlsx**”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2OptimizationAllFiveParam>.

### 2.3.5 Results of Parameter Optimization

As discussed above, a series of calculations has been performed to optimize the parameters of Model 1 and Model 2. The accuracy results are summarized in **Table 10** for Model 1 and **Table 11** for Model 2.

**Table 10: Model 1 - Parameter Optimization Results**

Shift	Average of Total Experimental Sleep (h)	Average of Total Predicted Sleep (h)					Accuracy Average (%)				
		Default Parameters <sup>2</sup>	Case 1.1	Case1.2	Case 1.3	Case 1.4	Default Parameters	Case 1.1	Case 1.2	Case 1.3	Case1. 4
<b>Day</b>	74.12	69.63	76.15	73.93	77.63	77.65	75.72	79.55	79.62	79.95	79.96
<b>Port</b>	71.94	52.25	55.81	55.81	57.81	57.10	63.33	64.97	64.97	66.84	66.96
<b>Starboard</b>	72.55	64.18	70.14	69.64	71.75	69.64	73.48	77.34	77.33	77.36	77.12
<b>Irregular</b>	76.60	39.90	48.20	48.20	58.15	58.15	27.04	32.41	32.41	37.75	37.75

**Table 10** demonstrates that the accuracy is ~3-8% higher for Day, Port, Starboard shifts with parameter optimization compared to the sleep generator with default parameters. The Model 1 accuracy is ~5-10% higher for Irregular shift.

This table can be found in the file “**SummaryStatsFileM1.xlsx**”, in the sheet named “**SummaryReportTable**”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model1Optimization>.

<sup>2</sup> The defaults values are obtained from Reference [4] as follows:  $t_r = 18.2$  h and  $t_d = 4.2$  h.



**Table 11: Model 2 - Parameter Optimization Results**

Shift	Average of Total Experimental Sleep (h)	Average of Total Predicted Sleep (h)				Accuracy Average (%)			
		Default Parameters <sup>3</sup>	Case 2.1	Case 2.2	Case 2.3	Default Parameters	Case2.1	Case 2.2	Case 2.3
<b>Day</b>	74.12	68.60	77.18	78.53	78.54	80.38	83.43	83.41	83.46
<b>Port</b>	71.94	55.48	66.31	75.85	72.63	67.19	72.46	74.55	74.60
<b>Starboard</b>	72.55	60.84	69.91	79.23	71.43	72.42	77.68	77.19	77.77
<b>Irregular</b>	76.60	52.10	77.40	90.10	86.35	39.47	64.07	68.01	68.70

**Table 11** illustrates that the accuracy of Model 2 is ~3-5% higher for Day, Port, Starboard shifts with parameter optimization compared to the sleep generator with default parameters. However, the model accuracy is significant higher for Irregular shift. The average change of accuracy between the sleep model with default parameters and Case 2.3 is found to be 29.23%.

This table can be found in the file “**SummaryStatsFileM2.xlsx**”, in the sheet named “**SummaryReportTable**”. The file is located in <C:\Users\Peng Contractor\Documents\ndPhaseII\Model2OptimizationAllFiveParam>.

<sup>3</sup> The default values are obtained from Reference [2] as follows: L=8.38; U=11.38; P=16.8; logarithm = 8; decay = -0.0353.

### 3 Results

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The results are outlined below:

- The model accuracy was calculated with a new approach and the results were summarized in **Table 2, Table 3, Table 4, and Table 5**;
- Linear regression analysis was performed for the following sleep models: Model 1, Model 2 and Model 3 with an alertness score as the independent variable and observed fatigue level as the dependent variable. The results are presented in **Table 6 and Table 7**;
- Parameter optimization results are summarized in **Table 8 and Table 9**;
- The accuracy and sleep duration for Model 1 and Model 2 using parameter optimization are provided in **Table 10 and Table 11**.

## 4 Conclusion

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Based on the generated results, the following conclusions can be drawn:

- a strong relationship between predicted alertness and experimental fatigue level was not observed for the collected data;
- the agreement between the predicted and experimental data can be improved using parameter optimization;
- Model 2 demonstrates the best prediction of sleep with optimized parameters.

## References

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