

Using Linear Model Predictive Control Via Feedback Linearization for Dynamic Encirclement

A. T. Hafez*, M. Iskandarani[†], S. N. Givigi[†], S. Yousefi*, Camille Alain Rabbath[‡], A. Beaulieu[†]

*Department of Electrical and Computer Engineering

Queens University

Kingston, ON, Canada

Email: 11edhk@queensu.ca, s.yousefi@queensu.ca

[†]Royal Military College of Canada

Kingston, ON, Canada

Email: mohamad.iskandarani@rmc.ca, sidney.givigi@rmc.ca, alain.beaulieu@rmc.ca

[‡]Defence Research and Development Canada

CFB Valcartier, QC, Canada

Email: camille.rabbath@sympatico.ca

Abstract—An Unmanned Aerial Vehicle (UAV) team formed from two or more UAVs is used in the autonomous encirclement of a stationary target in simulation. The encirclement tactic is defined as the situation in which a target is surrounded by a UAV team in formation. This tactic can be employed by a team of UAVs to neutralize a target by restricting its movement. A combination of Linear Model Predictive Control (LMPC) and Feedback Linearization (FL) is implemented on a team of UAVs in order to accomplish dynamic encirclement. The linear plant, representing each UAV, is found through System Identification then linearized using an FL technique. The contributions of this paper lay in the application of LMPC and FL to the problem of encirclement using an autonomous team of UAVs in simulation.

Index Terms—Linear Model Predictive Control, Unmanned Aerial Vehicles, Feedback Linearization

I. INTRODUCTION

UAVs are gaining great interest due to their wide area of applications in the military and civilian fields. The importance of UAVs has increased rapidly after the end of the Second World War due to their low cost of production and high flexibility in operations. Because of the increase in UAV applications, the term UAV tactics has started to appear widely and is defined as the general strategies used by individuals in a UAV team to achieve a desired outcome. In other words, it is the way UAVs act to perform a certain required mission [1]. According to the type of control of these tactics, these strategies are divided into two main types of control: centralized control where a group of UAVs receives coordinated instructions from one centralized decision maker, and decentralized control, in which each UAV in the team is responsible for making its own decisions.

Although many single UAV applications can be carried out successfully through manual operation, only autonomous systems can provide solutions to some applications, such as extensive continuous reconnaissance operations lasting weeks or months [2]. Victim search and rescue along-side reconnaissance operations are some applications where autonomous UAVs are utilized [3], thus minimizing human resources, risk of injury, and manned aerial vehicle cost. This autonomous

operation is integral to cooperative tactics utilizing teams of UAVs to accomplish a task. Some of these tasks include formation flying, such as encirclement and swarming. Accordingly, formation algorithms have been developed over many years for autonomous UAV use. For instance, Sauter et al. [4] explore a swarming algorithm on a group of unmanned vehicles responsible for infrastructure protection. Similarly, Sharma et al. [5] implemented a target capturing algorithm in order to follow a sinusoidally moving target. Also, Yamaguchi [6] implemented a feedback control law in a cooperative hunting behaviour on a mobile robot troop enclosing a target. Moreover, several recent independent results in target search [7], cooperative transportation [8], and path coordination [9], have made it clear that more complex applications, that were beyond the reach of single units, became achievable using multiple systems working cooperatively and autonomously.

Due to the effect of aerodynamic forces, the UAV dynamics are nonlinear, multivariable and are exposed to parameter uncertainties, external noise and external disturbances. The control of the UAV requires the ability of the controller to overcome these nonlinearities so that the model can reach stability, robustness, and desired dynamics properties [10]. Designing of a suitable controller to overcome these nonlinearities is a big challenge in the field of control.

Feedback Linearization technique is a common approach used in the control of nonlinear systems. Different combinations of feedback linearization and various types of controllers are used to overcome the nonlinear dynamics of UAVs during their flights in [11, 12]. For instance, in [10], a combination of an FL controller and a high order sliding mode observer is applied to a quadrotor to avoid complex nonlinear control solutions. In [13], Holger Voos uses FL to linearize the flight dynamics of a micro-quadrotor, while a combination of two control loops are used to control the altitude and velocity of the UAV. In [14], a dynamic feedback controller is used to convert the inner loop of the quadrotor into a linear, controllable, and non-interactive loop. This affects the stability and robust-

ness of the model against wind, turbulence, and parametric uncertainties. In [15], the problem of dynamic encirclement is solved on a multi-UAV system by linearizing the model using Taylor series. This resulted in eight cocontrollers running simultaneously on each vehicle. Accordingly, this paper uses Feedback Linearization in order to have only one controller for the tactic.

Model Predictive Control (MPC) can handle multivariable cases which helps in solving complex control problems [16]. In [17], How and Richards use a Decentralized MPC (DMPC) scheme in a group of UAVs to achieve collision avoidance. In [18], a Nonlinear MPC (NMPC) control scheme is used to stabilize vehicle dynamics and generate trajectories for multiple flying robots in dynamic environments. In [19], Sutton and Bitmead consider NMPC for the control of an autonomous submarine.

The main contribution of this work is solving the dynamic encirclement problem, through a combination of LMPC and FL, on a team three UAVs in simulation. On one hand, nonlinear encirclement dynamics are linearized using FL, while on the other hand, we occupy ourselves with high-level control where each team member's LMPC generates the required path necessary to respect the encirclement conditions. All encirclement tasks are accomplished autonomously by the UAV with no user input. Moreover, the control policies are implemented in a decentralized manner.

The paper is organized as follows. In Section II, the problem of dynamic encirclement is formulated, the specific control objectives are defined and the dynamic feedback linearization is introduced. In Section III, we develop our MPC and show all constraints. Section IV presents the results of our simulations while Section V concludes the work and presents some future objectives.

II. PROBLEM FORMULATION

Dynamic encirclement is defined as N UAVs encircling a stationary target. In order to simplify the problem, we consider that the UAVs act in a two dimensional space such that the height and yaw controllers have no influence on the lateral movement of the vehicles, although the problem could be extended to the three dimensional case with increased computational demands according to Sharma et al. [5]. The objective here is to create a linear version of Marasco's nonlinear design in [20], capable of being implemented in real-time on actual UAVs.

A. Problem Definition

Marasco et al. [1] presents an accurate description of UAV nonlinear dynamics while encircling a target, where each UAV in the two-dimensional Cartesian space is represented by the state vector:

$$\bar{q}_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \quad (1)$$

In order to form a dynamic encirclement around a target, a new state vector in a polar coordinate reference frame centered

at the origin is introduced. The new vector state for each UAV is as follows:

$$\bar{q}_i(t) = \begin{bmatrix} r_i(t) \\ \phi_i(t) \end{bmatrix}, = \begin{bmatrix} \|\tilde{q}_i(t)\| \\ \angle \tilde{q}_i(t) \end{bmatrix} \quad (2)$$

where i represents the number of UAV in the team, $r_i(t)$ represents the distance between the UAV and the target (equal to the required radius of dynamic encirclement at time t) and $\phi_i(t)$ represents the angle between the UAV and the target at time t . Our controller will respect the following design objectives [21]:

$$C1) \lim_{t \rightarrow \infty} |r_i(t) - R_D| = 0 \quad \forall i \leq N \quad (3)$$

$$C2) \lim_{t \rightarrow \infty} |\dot{\phi}_i(t) - \dot{\phi}_D| = 0 \quad \forall i \leq N \quad (4)$$

$$C3) \lim_{t \rightarrow \infty} |\phi_{i+1}(t) - \phi_i(t)| = \frac{2\pi}{N} \quad \forall i \leq N \quad (5)$$

Condition *C1* states that each UAV in the team maintains a desired distance from the target, while condition *C2* states that each UAV in the team maintains a desired angular velocity around the target. Finally, condition *C3* states that each member in the team spreads itself evenly in a circular formation around the target. The LMPC controller respects these constraints when accomplishing dynamic encirclement.

The same model for the UAV, as mentioned in [1], is used to describe each member of the team:

$$\bar{X}(t) = \begin{bmatrix} x(t) \\ y(t) \\ \theta(t) \\ V(t) \end{bmatrix}, \quad \bar{U}(t) = \begin{bmatrix} \omega(t) \\ a(t) \end{bmatrix} \quad (6)$$

where the vector $\bar{X}(t)$ represents the states of the aircraft while $\bar{U}(t)$ represents the inputs to the system. x and y are the cartesian coordinates; θ is the heading of the UAV; V is its speed; ω is its angular velocity and a is its acceleration. This system can be scaled to any number of UAVs. The state-equations that describe the motion of the aircraft are [1]:

$$\dot{\bar{X}}(t) = f(\bar{X}(t), \bar{U}(t)) = \begin{bmatrix} V(t) \cos \theta(t) \\ V(t) \sin \theta(t) \\ \omega(t) \\ a(t) \end{bmatrix} \quad (7)$$

Due to the nonlinearities of this system, computational complexity while implementing a MPC arises. This complexity is due to the fact that the problem is non-convex in nature and unable to run in real-time. In this paper, we propose a solution to this problem in two steps. Firstly, we use a system identification technique to isolate the UAV's nonlinear Cartesian dynamics then transform the system to yield radius and angular velocity as controllable states. Secondly, an FL technique is used to convert the nonlinear states of the quadrotor into a linear model, which permit the use of LMPC to control the UAVs in the team.

B. System Identification and State Transformation

Flight data from a Qball-X4 quadrotor is collected in order to find the process model describing the Cartesian movement of the UAV. System identification, based on a least-squares algorithm, constructs linear process models from dynamic systems, not easily found through first principles, based on input-output data. As a result, a second order system is found for both x and y . Detailed work on this may be found in [15]. Fig. 1 shows the Qball-X4 response to a step input in both x and y coordinates, along side a linear second order plant found through system identification. We can see that both responses are suitably similar for practical purposes during the fifteen second period.

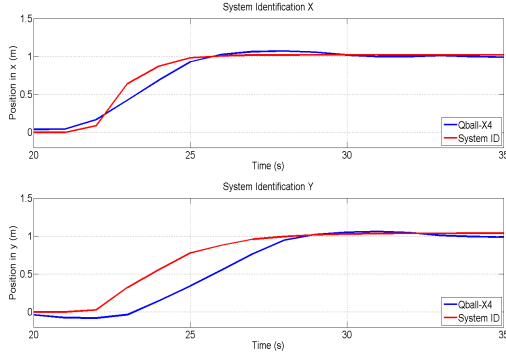


Fig. 1. Qball-X4 response to a step input in a Cartesian plane and its similar linear second order system.

The linear systems found in [15] have a state vector $[x \ y \ \dot{x} \ \dot{y}]^T$ and inputs $[X_d \ Y_d]^T$, where X_d and Y_d are the desired positions of the Qball-X4, x and y are the current positions of the Qball-X4 during flight and \dot{x} and \dot{y} are the corresponding speeds. Since the control is done using the radius of encirclement and the angular velocity a state transformation from Cartesian to Polar coordinates is applied. The resultant system is represented in the following form:

$$\dot{\bar{X}} = f(r, \theta, \dot{r}, \dot{\theta}, X_d, Y_d) \quad (8)$$

where f is a nonlinear function combining the inputs and states of the system while $\bar{X} = [\dot{r} \ \dot{\theta} \ \ddot{r} \ \ddot{\theta}]^T$ are the overall state equations. Moreover, r represents the radius of encirclement, θ the angle between the target and the encircling UAV, and $\dot{\theta}$ (or ω), is the angular velocity of the UAV. We now have a model that compared to the one designed by Marasco et al. [1] in that it outputs states, r and ω , for a UAV. The novel work is found in the fact that this new linear model may be implemented on actual UAVs running in real-time.

III. FEEDBACK LINEARIZATION

In II-B, we identify the model for the Cartesian movement of the Qball-X4 and setup the transformation to the new set of states that allows us to output the required radius and angular velocity of each UAV in the team. The objective of FL is

to linearize equation (8) and represent it in the standard state-space form. This is done by equating the state-equations found in (8) to a linear function holding the new inputs u_1 and u_2 . Isolating the old inputs X_d and Y_d , gives us the following transformation:

$$\begin{aligned} X_d = & 0.0000572((-20000\omega\dot{r}\sin(\theta)\cos(\theta)^2) \\ & + (12323r\sin(\theta)^2\cos(\theta)) - (20000\omega\dot{r}\sin(\theta)) \\ & + (12320\dot{r}\sin(\theta)^2\cos(\theta)) - (20000u_2r\sin(\theta)) \\ & - (26160\omega r\sin(\theta)) - (10000u_1r\sin(\theta)) \quad (9) \\ & - (20000\omega^2r\cos(\theta)^3) + (12320\dot{r}\cos(\theta)^3) \\ & + (10000u_1\cos(\theta)) + (12320r\cos(\theta)^3) \\ & + (10000\omega^2r\cos(\theta)) - (10000u_2\cos(\theta)) \\ & - (10000r\theta\sin(\theta)) \end{aligned}$$

$$\begin{aligned} Y_d = & -0.01608((-327\omega r\cos(\theta)) + (250\omega^2r\cos(\theta)^2\sin(\theta)) \\ & + (250\omega\dot{r}\cos(\theta)\sin(\theta)^2) + (154\omega r\cos(\theta)\sin(\theta)^2) \\ & - (125\omega^2r\sin(\theta)) - (125u_1\sin(\theta)) + (125u_2\sin(\theta)) \\ & - (250\omega\dot{r}\cos(\theta)) + (154\omega r\cos(\theta)^3) - (125u_1r\cos(\theta)) \\ & - (250u_2r\cos(\theta)) - (125r\theta\cos(\theta))) \quad (10) \end{aligned}$$

This substitution cancels the nonlinearities in equation (8) which introduces a linear system with new inputs u_1 and u_2 . We may write the resultant linear system as such:

$$\dot{\bar{X}} = g(r, \theta, \dot{r}, \dot{\theta}, u_1, u_2) \quad (11)$$

where g is a linear function found through FL substitution. The final linear process model may be represented in state-space form:

$$\dot{\bar{X}} = A\bar{X} + BU \quad ; \quad \bar{Y} = C\bar{X} \quad (12)$$

where U is $[u_1 \ u_2]^T$, \bar{X} is $[r \ \theta \ \dot{r} \ \dot{\theta}]^T$, \bar{Y} is the output vector holding r and $\omega = \dot{\theta}$ and matrices A, B, C ensure controllability and observability of the state-space. In order to summarize the FL process, Fig. 2 shows the linearization scheme of what is done so far.

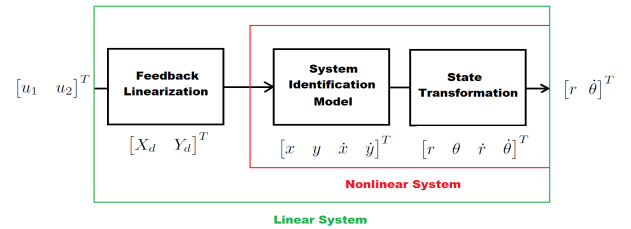


Fig. 2. Summary of system identification, state transformation and FL.

IV. CONTROL DESIGN

One of the contributions in this paper lies in the use of LMPC to accomplish the task of dynamic encirclement using multiple UAVs around a stationary target. The FL method is used to linearize the encirclement model described in the previous section. This allows us to design a suitable LMPC responsible for the dynamic encirclement of a group of UAVs in real-time. The optimization problem is convex in nature which allows for faster computational capacity compared to its nonlinear counterparts. Moreover, the UAVs accomplish the task of encircling a stationary target autonomously with the user only initiating the system.

In order to accomplish dynamic encirclement using a team of N UAVs, the conditions (C1,C2,C3) mentioned in section II must be met. Each UAV must control its radius to the target, its angular speed $w = \dot{\theta}$, and its angular separation relative to other members. This is done when each member is cognisant of the leading and lagging UAV in the formation. θ_{lead} represents the angular difference between the member being considered and the one in front of it, while θ_{lag} represents the angular difference with the one behind it. The next step of design now becomes a question of stability.

We define the following terms:

$$\Delta\theta_D = 2\pi/N \quad (13)$$

$$\Delta\theta_{i,j}(t) = \theta_j(t) - \theta_i(t) \quad (14)$$

where θ_i represents the angle of the i^{th} UAV, θ_j is the angle of the j^{th} UAV, $\Delta\theta_D$ is the desired angular separation between two UAVs, and N is the number of UAVs in the formation. Thus, the error between two of the UAVs in a team is given as follows:

$$e_i(t) = \Delta\theta_{i,j}(t) - \Delta\theta_D \quad (15)$$

The suitable Lyapunov candidate function is defined as

$$V(t) = 1/2(e_1^2(t) + e_2^2(t) + e_3^2(t) + \dots + e_N^2(t)) \quad (16)$$

$$\dot{V}(t) = e_1(t)\dot{e}_1(t) + e_2(t)\dot{e}_2(t) + \dots + e_N(t)\dot{e}_N(t)$$

By choosing the suitable condition

$$\dot{\theta}_j(t) - \dot{\theta}_i(t) = -\gamma e_i(t) \quad (17)$$

and by choosing γ as a positive constant, Lyapunov stability is achieved:

$$\dot{V}(t) = -\gamma[e_1(t)^2 + e_2(t)^2 + \dots + e_N(t)^2] \quad (18)$$

Moreover, the chosen Lyapunov candidate function allows the errors to decrease as time goes by, despite the fact that the UAVs are allowed to vary their speeds initially to achieve encirclement. We now add the desired angular speed to equation (17) using the following equation

$$\dot{\theta}_D = \frac{\dot{\theta}_1(t) + \dot{\theta}_2(t) + \dots + \dot{\theta}_N(t)}{N} \quad (19)$$

We can use equations (17) and (19) to calculate the desired angular speed for each member in the formation.

In general, the desired angular speed for each member in the

team is calculated by observing the current angular positions of the leading and lagging UAV in the formation respectively according to the following equation:

$$\dot{\theta}_{Di}(t) = \frac{3 * \dot{\theta}_D + \gamma(\theta_{lead}(t) - \theta_{lag}(t))}{N}, \forall i \in [1, N] \quad (20)$$

Finally, the state $\Delta\theta_{i,j}$, angular separation between two UAVs is added to the linear system presented in section III so that it may be included in the cost function. Moreover, the desired angular velocity $\dot{\theta}_{Di}$ is fed back as a reference to the process model. The prediction horizon and control horizon used are eight and two respectively.

V. SIMULATION RESULTS

The control strategy discussed in sections II and IV is successfully implemented in simulation on a multi-UAV team consisting of three vehicles. The objective of these simulations is to show that the LMPC policy designed is fit for dynamic encirclement on real platforms. Our requirements are a radius of encirclement of 2m, a desired angular speed of 0.15 rad/s and an angular separation of $\frac{2\pi}{3}$ rad. This section will highlight two cases: the first case considers the three UAVs starting close to the required values of radius and angular separation while the second case considers the UAVs starting at different positions. The objective of the second case is to show robustness of the proposed design.

A. Case #1: UAVs at standard initial positions

The success of one UAV encircling the stationary target prompted us to apply the system on a scalable multi-UAV platform. The objective here is to show that the UAVs may accomplish dynamic encirclement by controlling their own radius with respect to the target, their angular velocity and the angular separation considering the leading and lagging UAV in the team. In other words we take the formulation shown in Section IV, and apply it to three UAVs working together on the task of encirclement. Using multiple vehicles is a suitable and robust testbed for the designed system.

A set of three UAVs with initial positions (2, 0), (-1, 1.732) and (-1, -1.732) successfully encircle a stationary target located at the origin as seen in Fig. 3. This simulation was also run for 120 seconds. The main parameters used in simulation are desired radius of 2 m, angular velocity of 0.15 rad/s and angular separation between the three UAVs is $\Delta\theta_D = \frac{2\pi}{3}$ rad.

Fig. 4 shows that the team of UAVs is able to successfully encircle the target. The angular separation between the three vehicles is shown in Fig. 5, where the vehicles maintain their desired angular separation at $\frac{2\pi}{3}$ rad.

B. Case #2: UAVs at different initial positions

The robustness of our simulation is shown by different initial positions and angles for the UAVs. The team successfully converges to the desired radius around the target as seen in Fig. 6. The first UAV starts from a distance 2 m from the target with angle 0 rad, UAV 2 starts at a distance 1.5 m with angle separation of $\frac{5\pi}{6}$ rad, while UAV 3 starts at 2.5 m with angle

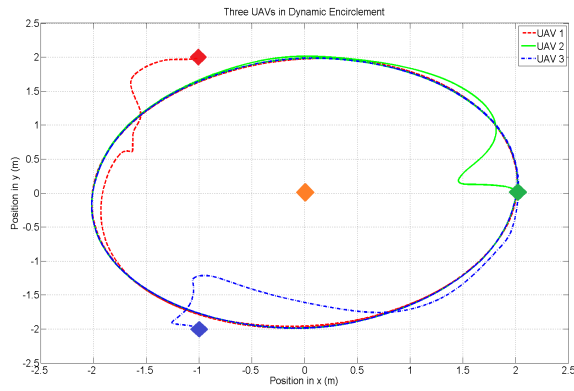


Fig. 3. Three UAVs, for Case #1, encircling stationary target. Each UAV is represented by a red, green or blue diamond while the target is shown as an orange diamond.

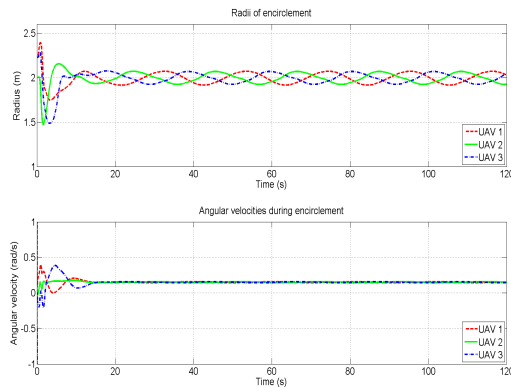


Fig. 4. The radii of encirclement and angular velocities for three UAVs encircling a stationary target at the origin. UAV 1 is shown in red and is initialized at (2,0), UAV 2 is shown in green and is initialized at (-1,1.732) with angle $\frac{2\pi}{3}$ rad from UAV 1, while UAV 3 is shown in blue and is initialized at (-1,-1.732) with angle $\frac{4\pi}{3}$ rad from UAV 1.

separation of $\frac{7\pi}{6}$ rad. The radii of encirclement and the angular velocities of the three vehicles are shown in Fig. 7, while the angles of separation between the three UAVs are shown in Fig. 8. Despite different positions, the team of UAVs successfully converges to all required values.

VI. CONCLUSION

In this paper, we show that LMPC can be successfully applied to solve the problem of dynamic encirclement of a stationary target using one or more UAVs in simulation. A system identification technique, based on least-squares algorithm, and the linearization of a Cartesian to Polar transformation via feedback linearization method are used successfully to identify the model needed. Furthermore, the solutions found in this paper are scaled to accommodate larger teams of UAVs in more complex environments.

Our results shows that applying LMPC strategy to our linearized model permits dynamic encirclement using multiple UAVs in real-time. Our simulation results show that the system

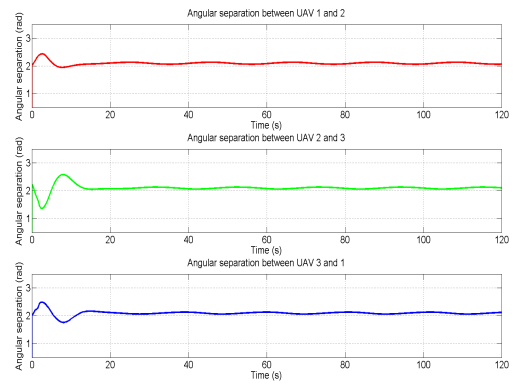


Fig. 5. Angular separation for three UAVs encircling a stationary target.

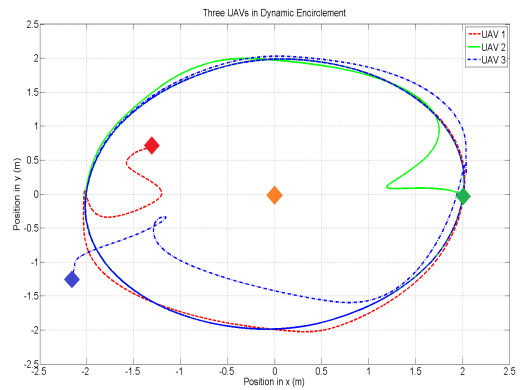


Fig. 6. Three UAVs, for Case #2, encircling stationary target. Each UAV is represented by a red, green or blue diamond while the target is shown as an orange diamond.

is stable and converges to the proper references. The use of different forms of MPC, such as Learning Based MPC, is encouraged for future research. This will increase the stability necessary for these sorts of autonomous applications. Future work on this topic will include experimental results for one or more Qball-X4 quadrotors encircling a stationary target using a combination of LMPC and feedback linearization.

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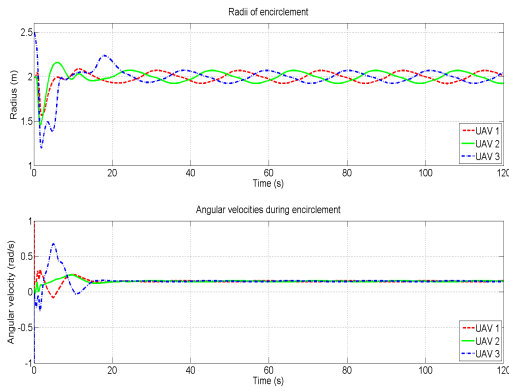


Fig. 7. The radii of encirclement and the angular velocities for three UAVs encircling a stationary target at the origin. UAV 1 is shown in red and is initialized at (2,0), UAV 2 is shown in green and is initialized at 1.5 m from the target with angle $\frac{5\pi}{6}$ rad from UAV 1, while UAV 3 is shown in blue and is initialized at 2.5 m from the target with angle $\frac{7\pi}{6}$ rad from UAV 1.

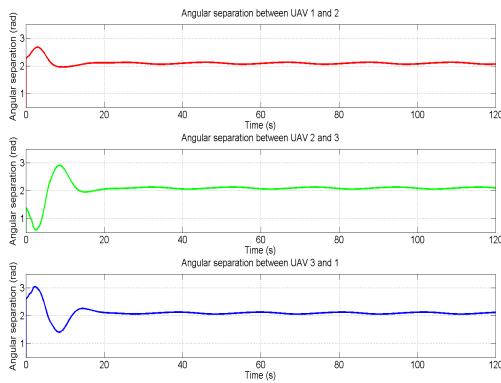


Fig. 8. Angular separation for three UAVs encircling a stationary target starting at different locations and angles. The UAVs converge to a separation of $\frac{2\pi}{3}$

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