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SECOND-ORDER ANALYSIS OF ROLL DECAY TESTS

Ross Graham - Andrew Webber

Defence Research Establishment Atlantic

Centre de Recherches pour la Défense Atlantique

Canada

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Abstract

A method for determining linear plus quadratic roll damping coefficients from roll decay tests that is believed to be accurate up to second order in the damping coefficients is described. Two computer programs are developed to estimate roll decay coefficients via chi-square fitting using first-order and second order analyses. The methods are applied to measured and simulated roll decay data, and excellent results are obtained with both codes. While the second-order results represent an improvement in both goodness-of-fit and accuracy compared with the first-order results, the latter are already so good that the improvement is considered to be of limited practical significance.

RÉSUMÉ

Le présent rapport décrit une méthode de détermination des coefficients linéaires et quadratiques d'amortissement du roulis à partir d'essais de décroissance du roulis. On estime que cette méthode fournit des coefficients avec une précision du deuxième ordre. Deux logiciels ont été élaborés pour l'estimation des coefficients de décroissance du roulis utilisant la méthode du chi-carré à partir d'analyses de premier ordre et de deuxième ordre. Les méthodes sont appliquées à des donnés de décroissance du roulis mesurées et simulées. Les deux programmes donnent d'excellents résultats. Bien que les résultats de l'analyse de deuxième ordre représentent une amélioration en ce qui concerne à la fois la qualité de l'ajustement et l'exactitude par rapport aux résultats de l'analyse de premier ordre, ces derniers résultats sont déjà si bons que l'amélioration est jugée superflue.

Contents

Notation

A	added moment of inertia in roll
b_1	non-dimensional, linear roll damping coefficient, $b_1 = B_1/(I + A)$
B_1	linear roll damping coefficient
b_2	non-dimensional, quadratic roll damping coefficient, $b_2 = B_2/(I + A)$
B_2	quadratic roll damping coefficient
B_2^L	equivalent linear roll damping coefficient, $B_2^L = \frac{8}{3\pi}\omega B_2 \phi_0$
C	roll restoring coefficient, $C = \triangle\overline{GM}$
$C(\phi)$	roll restoring moment
E_c^L	energy dissipated by the linear term $B_1\dot{\phi}$ in a roll half-cycle of constant amplitude.
E_v^L	energy dissipated by the linear term $B_1\dot{\phi}$ in a roll half-cycle of variable amplitude, using the exact linear solution obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \tilde{\phi}_N$
E^P	loss of potential energy over a half-cycle, $E^P = \frac{1}{2}C\left(\phi_N^2 - \phi_{N+1}^2\right)$
E_c^Q	energy dissipated by the quadratic term $B_2\dot{\phi}\vert\dot{\phi}\vert$ in a roll half-cycle of constant amplitude.
E_v^Q	energy dissipated by the quadratic term $B_2\dot{\phi}\vert\dot{\phi}\vert$ in a roll half-cycle of variable amplitude, using the exact linear solution obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \hat{\phi}_N$
$\overline{GM}$	metacentric height
I	roll moment of inertia
M	number of roll extrema less one
n	roll decay coefficient, $n = \omega_0 B_1/2C$ for a purely linear damping, or $n = \frac{\omega_0}{2C}\left(B_1 + \frac{8}{3\pi}\omega_0 B_2 \overline{\phi}_N\right)$ for the equivalent linear damping $B_1 + B_2^L$
$\hat{n}$	$\hat{n} = \frac{\omega_0}{2C}\left(B_1 + \frac{8}{3\pi}\omega_0 B_2 \hat{\phi}_N\right)$

$\tilde{n}$	$\tilde{n} = \frac{\omega_0}{2C}\left(B_1 + \frac{8}{3\pi}\omega_0 B_2 \tilde{\phi}_N\right)$
O	order symbol
t	time
t_N	time of occurrence of the Nth extreme point of the roll decay curve
$\triangle$	ship or model displacement
Δx	uncertainty in the quantity x
κ_1	normalized linear roll damping coefficient, $\kappa_1 = \frac{\omega_0 B_1}{2C}$
κ_2	normalized quadratic roll damping coefficient, $\kappa_2 = \frac{4\omega_0^2 B_2}{3\pi C}$
ϕ	roll angle
$\dot{\phi}$	roll velocity
$\ddot{\phi}$	roll acceleration
ϕ_0	roll amplitude
ϕ_N	absolute value of the Nth extreme point of the roll decay curve
$\overline{\phi}_N$	average roll amplitude during a decay half-cycle, $\overline{\phi}_N = (\phi_N + \phi_{N+1})/2$
$\hat{\phi}_N$	angle chosen such that the energy dissipated by $B_2\dot{\phi}\lvert\dot{\phi}\rvert$ in a half-cycle using the exact linear solution obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \hat{\phi}_N$ equals the energy dissipated by $B_2\dot{\phi}\lvert\dot{\phi}\rvert$ using the full nonlinear solution
$\tilde{\phi}_N$	angle chosen such that the energy dissipated by $B_1\dot{\phi}$ in a half-cycle using the exact linear solution obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \tilde{\phi}_N$ equals the energy dissipated by $B_1\dot{\phi}$ using the full nonlinear solution
χ^2	chi-square merit function, $\chi^2 = \sum_{N=1}^{M}[(y_N - y(\overline{\phi}_N; \kappa_1, \kappa_2))/(\Delta y_N)]^2$
ω	angular frequency
ω_0	roll natural frequency, $\omega_0 = \sqrt{C/(I+A)}$
ω_d	damped roll natural frequency, $\omega_d = \omega_0\sqrt{1-n^2}$

1 Introduction

Because of the important influence of roll damping on ship response, there is great interest in obtaining accurate estimates of linear and nonlinear roll damping coefficients from model tests. The easiest experimental method for carrying out these tests is the roll decay test in which the model is given a large initial heel angle, and the resulting roll motion is recorded as it decays. Spouge[1] has recently prepared an excellent review and analysis of available methods for obtaining roll damping coefficients from roll decay tests. The review includes discussions of the quasi-linear method, the Froude energy method[2], Roberts energy method[3], the averaging method[4], and the perturbation method[5]. All of the methods involve approximations of various sorts, as any nonlinear analysis must, and are only accurate to first order in the damping terms. Spouge considered the perturbation method to be the best, but noted that it was complex to implement, and sensitive to distortion of the first peak of the roll decay record.

This memorandum describes a method for estimating linear plus quadratic roll damping coefficients from roll decay tests that is believed to be accurate to second order in both damping terms. The method is an extension of the energy methods, in which the damping coefficients are obtained by equating the loss in potential energy over a half-cycle with the energy dissipated by the damping terms. Roberts[3] also considers nonlinear restoring terms and linear plus cubic damping. The present method can be adapted to these cases, but they are not addressed in this memorandum.

Two computer programs are developed to estimate the roll damping coefficients from the first- and second-order analyses using chi-square fitting. The use of chi-square or weighted least-squares fitting rather than the more commonly used least-squares fitting is important in this application because of the large uncertainties in the roll decay tests which occur for small roll amplitudes. The first and second order codes are applied to measured and simulated roll decay data, and an evaluation of the two programs is made.

2 First-Order Analysis

We assume a one-dimensional roll equation with linear and quadratic roll damping. Denoting roll angle by ϕ, the roll decay is described by the following equation:

$$(I + A)\ddot{\phi} + B_1\dot{\phi} + B_2\dot{\phi}|\dot{\phi}| + C(\phi) = 0 \tag{1}$$

where I is the roll moment of inertia, A is the added moment of inertia in roll, B_1 is the linear roll damping coefficient, B_2 is the quadratic roll damping coefficient and $C(\phi)$ is the roll restoring moment. The roll restoring will be assumed to be linear in the following analysis, so that

$$C(\phi) = C\phi = \Delta\overline{GM}\phi \tag{2}$$

where Δ is the displacement and $\overline{GM}$ is the metacentric height. It is convenient to rewrite Equation 1 in non-dimensional form

$$\ddot{\phi} + b_1\dot{\phi} + b_2\dot{\phi}|\dot{\phi}| + \omega_0^2\phi = 0 \tag{3}$$

where $\omega_0 = \sqrt{C/(I+A)}$ is the roll natural frequency, and $b_i = B_i/(I+A)$ for $i = 1$ or 2.

In the case of a purely linear damping, $b_2 = 0$, we can rewrite Equation 3 in the form

$$\ddot{\phi} + 2n\omega_0\dot{\phi} + \omega_0^2\phi = 0 \tag{4}$$

where $n = \omega_0 B_1/2C$ is the roll decay coefficient. Given the initial conditions $\phi(0) = \phi_0$ and $\dot{\phi}(0) = 0$ this equation has the exact solution

$$\phi = \phi_0 e^{-\omega_0 nt}\cos(\omega_d t) + \frac{n}{\sqrt{1-n^2}}\phi_0 e^{-\omega_0 nt}\sin(\omega_d t) \tag{5}$$

where $\omega_d = \omega_0\sqrt{1-n^2}$ is the damped roll natural frequency.

To first order in n, the solution can be written

$$\phi = \phi_0 e^{-\omega_0 nt}\cos(\omega_0 t) + n\phi_0 e^{-\omega_0 nt}\sin(\omega_0 t) \tag{6}$$

Let ϕ_N be the absolute value of the Nth extreme point of the roll decay curve, and assume that ϕ_N occurs at time t_N. The next extreme point, ϕ_{N+1}, occurs at time $t_N + \frac{\pi}{\omega_0}$; hence

$$\frac{\phi_{N+1}}{\phi_N} = e^{-\pi n} \tag{7}$$

and

$$\frac{1}{\pi}\ln\left(\frac{\phi_N}{\phi_{N+1}}\right) = n \tag{8}$$

Let $\overline{\phi}_N = (\phi_N + \phi_{N+1})/2$. In the case of linear damping, a plot of $(1/\pi)\ln(\phi_N/\phi_{N+1})$ versus $\overline{\phi}_N$ should be a constant with the value n.

The standard method of accomodating nonlinear damping in a linear model is by introducing the equivalent linear damping, $B_2^L\dot{\phi}$, which dissipates the same amount of energy in a roll cycle as the quadratic term $B_2\dot{\phi}|\dot{\phi}|$.

Let $\phi = \phi_0 \sin\omega t$, where ω is angular frequency, and let E_c^Q be the energy dissipated by the quadratic term in a roll half-cycle. The subscript c indicates that the roll amplitude is held constant during the half-cycle.

$$
\begin{aligned}
E_c^Q &= 2\int_0^{\phi_0} B_2\dot{\phi}|\dot{\phi}|d\phi \\
&= 2\omega^2 B_2\phi_0^3 \int_0^{\frac{\pi}{2\omega}} \cos^3\omega t\, d(\omega t) \\
&= \frac{4}{3}\omega^2 B_2\phi_0^3
\end{aligned} \tag{9}
$$

Let E_c^L be the energy dissipated by the linear term.

$$
\begin{aligned}
E_c^L &= 2\int_0^{\phi_0} B_2^L\dot{\phi}d\phi \\
&= 2\omega B_2^L\phi_0^2 \int_0^{\frac{\pi}{2\omega}} \cos^2\omega t\, d(\omega t) \\
&= \frac{\pi}{2}\omega B_2^L\phi_0^2
\end{aligned} \tag{10}
$$

Equating the expressions for E_c^Q and E_c^L we have

$$
B_2^L = \frac{8}{3\pi}\omega B_2\phi_0 \tag{11}
$$

The total equivalent linear damping is simply $B_1 + B_2^L$. Assuming that the linear solution Equation 6 is still approximately valid, Equation 8 implies

$$
\frac{1}{\pi}\ln\left(\frac{\phi_N}{\phi_{N+1}}\right) = n = \frac{\omega_0}{2C}\left(B_1 + \frac{8}{3\pi}\omega_0 B_2\overline{\phi}_N\right) \tag{12}
$$

where the equivalent linear damping is based on the average roll amplitude $\overline{\phi}_N$ during the decay half-cycle. A plot of $(1/\pi)\ln(\phi_N/\phi_{N+1})$ versus $\overline{\phi}_N$ should be a straight line with intercept $\omega_0 B_1/(2C)$ and slope $4\omega_0^2 B_2/(3\pi C)$.

In the perturbation method[5], Mathisen and Price avoid introducing an average roll amplitude and the associated approximations; however, the comparison with simulation results described in Section 4 indicates that Equation 12 is surprisingly accurate.

Using the energy approach, an equivalent result can be obtained directly by equating the loss of potential energy over a half-cycle with the energy dissipated by the damping terms.

The potential energy stored at time t_N is

$$
\int_0^{\phi_N} C\phi d\phi = \frac{1}{2}C\phi_N^2 \tag{13}
$$

and the loss of potential energy, E^P, over a half-cycle is

$$E^P = \frac{1}{2}C\left(\phi_N^2 - \phi_{N+1}^2\right) \tag{14}$$

Assuming that the roll damping is small, the energy dissipated by the linear and quadratic damping terms can be approximated by that dissipated in the half-cycle of constant amplitude $\overline{\phi}_N$. From Equations 14, 9 and 10 we obtain

$$\frac{1}{2}C\left(\phi_N^2 - \phi_{N+1}^2\right) = \frac{\pi}{2}\omega_0 B_1 \overline{\phi}_N^2 + \frac{4}{3}\omega_0^2 B_2 \overline{\phi}_N^3 \tag{15}$$

i.e.

$$\frac{\phi_N - \phi_{N+1}}{\pi\overline{\phi}_N} = \frac{\omega_0 B_1}{2C} + \frac{4\omega_0^2 B_2}{3\pi C}\overline{\phi}_N \tag{16}$$

This result is equivalent to Equation 12 to first order in the damping, as can be seen by expanding

$$\begin{aligned}
\ln\left(\frac{\phi_N}{\phi_{N+1}}\right) &= \ln\left(1 + \frac{\phi_N - \phi_{N+1}}{\phi_{N+1}}\right) \\
&= \frac{\phi_N - \phi_{N+1}}{\phi_{N+1}} + O(n^2) \\
&= \frac{\phi_N - \phi_{N+1}}{\overline{\phi}_N} + O(n^2)
\end{aligned} \tag{17}$$

where we have made use of the relation $\phi_N - \phi_{N+1} = O(n)$, which follows from Equation 16.

3 Second-Order Analysis

In this section, a more careful energy analysis is presented. The goal of this work is to develop a method of estimating linear plus quadratic damping coefficients that is accurate to second order in the damping.

We will first determine the energy dissipated in the half-cycle between ϕ_N and ϕ_{N+1} by the linear term $B_1\dot{\phi}$ using the exact linear solution (Equation 5) obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \tilde{\phi}_N$. Let E_v^L denote this energy, where the subscript v indicates that the roll amplitude varies during the half-cycle. The angle $\tilde{\phi}_N$ is chosen such that the energy dissipated by $B_1\dot{\phi}$ in the half-cycle using the linear solution equals the energy dissipated by $B_1\dot{\phi}$ using the full nonlinear solution. Both $\tilde{\phi}_N$ and the full nonlinear solution are unknown.

From Equation 5, with the initial conditions $\phi(0) = \phi_N$ and $\dot{\phi}(0) = 0$

$$\dot{\phi} = -\frac{\omega_0 \phi_N}{\sqrt{1-\tilde{n}^2}} e^{-\omega_0 \tilde{n} t} \sin(\omega_d t) \tag{18}$$

where we have introduced the notation

$$\tilde{n} = \frac{\omega_0}{2C}\left(B_1 + \frac{8}{3\pi}\omega_0 B_2 \tilde{\phi}_N\right) \tag{19}$$

Hence

$$\begin{aligned} E_v^L &= \int_{\phi_N}^{\phi_{N+1}} B_1 \dot{\phi} d\phi \\ &= \frac{B_1 \omega_0^2 \phi_N^2}{1-\tilde{n}^2} \int_0^{\pi/\omega_d} e^{-2\omega_0 \tilde{n} t} \sin^2 \omega_d t \, dt \\ &= \frac{B_1 \omega_0 \phi_N^2}{(1-\tilde{n}^2)^{3/2}} \int_0^{\pi} e^{-2\tilde{n}x/\sqrt{1-\tilde{n}^2}} \sin^2 x \, dx \end{aligned} \tag{20}$$

Using integration by parts, it can be shown that

$$\int_0^{\pi} e^{-ax} \sin^2 x \, dx = \frac{2(1-e^{-a\pi})}{a(a^2+4)} \tag{21}$$

Therefore

$$E_v^L = \frac{B_1 \omega_0 \phi_N^2}{4\tilde{n}} \left(1 - e^{-2\tilde{n}\pi/\sqrt{1-\tilde{n}^2}}\right) \tag{22}$$

Let E_v^Q be the energy dissipated by the quadratic term during the same half-cycle, using the exact linear solution obtained with the equivalent linear damping $B_1 + (8/3\pi)\omega_0 B_2 \hat{\phi}_N$, where $\hat{\phi}_N$ is chosen analogously to $\tilde{\phi}_N$, and $\hat{n}$ is defined by analogy with Equation 19.

$$\begin{aligned} E_v^Q &= \int_{\phi_N}^{\phi_{N+1}} B_2 \dot{\phi} |\dot{\phi}| d\phi \\ &= \frac{\omega_0^3 B_2 \phi_N^3}{(1-\hat{n}^2)^{3/2}} \int_0^{\pi/\omega_d} e^{-3\omega_0 \hat{n} t} \sin^3 \omega_d t \, dt \\ &= \frac{\omega_0^2 B_2 \phi_N^3}{(1-\hat{n}^2)^2} \int_0^{\pi} e^{-3\hat{n}x/\sqrt{1-\hat{n}^2}} \sin^3 x \, dx \end{aligned} \tag{23}$$

Once again, integration by parts can be used to show

$$\int_0^{\pi} e^{-bx} \sin^3 x dx = \frac{6(1+e^{-b\pi})}{(1+b^2)(9+b^2)} \tag{24}$$

Hence

$$E_v^Q = \frac{2\omega_0^2 B_2 \phi_N^3}{3(1+8\hat{n}^2)}\left(1 + e^{-3\hat{n}\pi/\sqrt{1-\hat{n}^2}}\right) \tag{25}$$

Using the relation $E^P = E_v^L + E_v^Q$, we obtain from Equations 22 and 25

$$\begin{aligned} \frac{\phi_N^2 - \phi_{N+1}^2}{2\pi\phi_N^2} &= \frac{\kappa_1}{2\pi\tilde{n}}\left(1 - e^{-2\tilde{n}\pi/\sqrt{1-\tilde{n}^2}}\right) + \\ & \quad \frac{\kappa_2\phi_N}{2(1+8\hat{n}^2)}\left(1 + e^{-3\hat{n}\pi/\sqrt{1-\hat{n}^2}}\right) \end{aligned} \tag{26}$$

and we have introduced the shorthand

$$\kappa_1 = \frac{\omega_0 B_1}{2C} \tag{27}$$

$$\kappa_2 = \frac{4\omega_0^2 B_2}{3\pi C} \tag{28}$$

We now wish to estimate the error introduced in Equation 26 by replacing $\hat{n}$ and $\tilde{n}$ by $n = \kappa_1 + \kappa_2\overline{\phi}_N$. In view of Equation 16, it is reasonable to assume that $\overline{\phi}_N - \tilde{\phi}_N = O(n)$. Replacing the $\hat{n}^2$ or $\tilde{n}^2$ terms by n^2 terms introduces an error

$$\begin{aligned} n^2 - \hat{n}^2 &= (n+\hat{n})(n-\hat{n}) \\ &= (n+\hat{n})\kappa_2(\overline{\phi}_N - \tilde{\phi}_N) \\ &= O(n^3) \end{aligned} \tag{29}$$

The error introduced in replacing the remaining $\tilde{n}$'s in Equation 26 can be estimated by expanding the exponential terms. For example

$$\begin{aligned} \frac{\kappa_1}{2\pi\tilde{n}}\left(1 - e^{-2\tilde{n}\pi/\sqrt{1-\tilde{n}^2}}\right) &\approx \frac{\kappa_1}{2\pi}\left(\frac{2\pi}{\sqrt{1-n^2}} - \frac{2\tilde{n}\pi^2}{1-n^2} + \cdots\right) \\ &= \frac{\kappa_1}{2\pi}\left(\frac{2\pi}{\sqrt{1-n^2}} - \frac{2n\pi^2}{1-n^2} + \frac{2(n-\tilde{n})\pi^2}{1-n^2} + \cdots\right) \\ &= \frac{\kappa_1}{2\pi}\left(\frac{2\pi}{\sqrt{1-n^2}} - \frac{2n\pi^2}{1-n^2} + O(n^2) + \cdots\right) \\ &= \frac{\kappa_1}{2\pi}\left(\frac{2\pi}{\sqrt{1-n^2}} - \frac{2n\pi^2}{1-n^2} + \cdots\right) + O(n^3) \end{aligned} \tag{30}$$

Hence

$$\frac{\phi_N^2 - \phi_{N+1}^2}{2\pi\phi_N^2} \approx \frac{\kappa_1}{2\pi n}\left(1 - e^{-2n\pi/\sqrt{1-n^2}}\right) + \frac{\kappa_2\phi_N}{2(1+8n^2)}\left(1 + e^{-3n\pi/\sqrt{1-n^2}}\right) + O(n^3) \tag{31}$$

Equation 31 can be used to obtain estimates of the damping coefficients κ_1 and κ_2 that are accurate to order n^3 as described in the next section. Since the right hand side of Equation 31 depends on both ϕ_N and $\overline{\phi}_N$, it is not possible to give a simple geometric interpretation of the resulting fit.

4 Sample Roll Decay Analyses

Two computer programs called FIRST and SECOND were developed to estimate the damping coefficients from Equations 12 and 31, respectively. In estimating the roll damping coefficients, it is important to use chi-square fitting (also known as weighted least-squares fitting) as opposed to the more commonly used least-squares fitting, because of the large uncertainties in the left hand sides of Equations 12 and 31 when $\phi_N - \phi_{N+1}$ is small. Let $y_N = (1/\pi)\ln(\phi_N/\phi_{N+1})$. The chi-square fit of the damping coefficients κ_1 and κ_2 is obtained by minimizing the quantity

$$\chi^2 \equiv \sum_{N=1}^{M}\left(\frac{y_N - y(\overline{\phi}_N;\kappa_1,\kappa_2)}{\Delta y_N}\right)^2 \tag{32}$$

where M is one less than the number of roll extrema and Δy_N denotes the error in y_N. (In writing down this expression, we have assumed that the uncertainty in y_N is large compared to the uncertainty in y($\overline{\phi}_N;\kappa_1,\kappa_2$). This is a reasonable assumption, as can easily be checked for sample decay tests.) We denote the absolute error in the roll peak ϕ_N by $\Delta\phi$, which we assume to be independent of N. Then the uncertainty in y_N is given by

$$\Delta y_N = \frac{\Delta\phi}{\pi}\sqrt{\frac{1}{\phi_N^2} + \frac{1}{\phi_{N+1}^2}} \tag{33}$$

Let $z_N = (\phi_N^2 - \phi_{N+1}^2)/(2\pi\phi_N^2)$. The chi-square is defined in an analogous way and the uncertainty in z_N can be estimated from

$$\Delta z_N = \frac{\Delta\phi}{\pi}\frac{\sqrt{2\phi_N^4 - \phi_N^2\phi_{N+1}^2 + \phi_{N+1}^4}}{\phi_N^3} \tag{34}$$

Development of the algorithms was straightforward for the straight line fit of Equation 12. The nonlinear chi-square fit for Equation 31 was performed using the Levenberg-Marquardt method as described in Reference 6. Nonlinear fits require a 'first guess' for the parameters to be estimated. In this case, the results of program FIRST provide the necessary information.

In addition to outputting the damping coefficients, the programs also output the quantity $\chi^2/(M-2)$ as an estimate of the goodness of fit to the data. (Since we are using a two-parameter model for roll damping, $M-2$ is the number of degrees of freedom.)

Measured roll decay records from the Institute for Marine Dynamics[7] for a 9-metre model of the DDH 280 were analyzed using the programs FIRST and SECOND. In addition, numerically simulated roll decay records generated by Dr. Jan Mathisen of Veritas Research using Equation 1 were also considered.

4.1 Analysis of Numerical Roll Decay Records

We will first discuss the results of the analysis of the numerical records, since the absence of experimental error permits a precise evaluation of the methods. The results are summarized in Table 1. The calculations shown under the 'M & P' column were computed using the method of Reference 5 by Dr. Mathisen.

Table 1: Comparison of Analyzed Roll Damping Coefficients with Simulation Input Values

Initial Amp. (degrees)	Quantity	Simulation	FIRST	SECOND	M & P
5.7	$\kappa_1 \times 10^2$	1.145	1.133	1.132	1.126
	$\kappa_2 \times 10^3$ (deg^{-1})	3.661	3.646	3.645	3.689
	$\chi^2/(M-2)$		0.064	0.054	
22.9	$\kappa_1 \times 10^2$	1.145	1.127	1.129	1.123
	$\kappa_2 \times 10^3$ (deg^{-1})	3.661	3.668	3.662	3.686
	$\chi^2/(M-2)$		0.58	0.35	
22.9	$\kappa_1 \times 10^2$	1.145	1.168	1.155	1.107
	$\kappa_2 \times 10^2$ (deg^{-1})	1.098	1.088	1.089	1.113
	$\chi^2/(M-2)$		1.20	0.41	
22.9	$\kappa_1 \times 10^2$	3.435	3.463	3.449	3.393
	$\kappa_2 \times 10^2$ (deg^{-1})	1.098	1.089	1.088	1.121
	$\chi^2/(M-2)$		0.60	0.17	

It is clear from Table 1 that all of the methods are capable of reproducing the simulation coefficients with a great deal of accuracy. Marginally, the best results were obtained with program SECOND (largest error: 1%), and the least accurate results with the M & P method (largest error: 3%); however, in view of the good results (largest error: 2%) and great simplicity of program FIRST, this method is the clear winner from the practical point of view. Comparing the normalized χ^2 coefficients for programs FIRST and SECOND, it is clear that program SECOND provides considerably better fits to the data; however, the roll damping coefficients are already predicted so well by program FIRST that the resulting improvements are of limited practical significance.

Figure 1 shows a sample plot of y_N versus $\overline{\phi}_N$ and the resulting chi-squared fit obtained using program FIRST. The high quality of the fit is evident.

4.2 Analysis of Experimental Roll Decay Records

Sample measured roll decay time histories from the roll decay tests described in Reference 7 were also analyzed. Sample plots from program FIRST are shown in Figures 2 and 3. In this case, the large increase in scatter for small roll amplitudes is evident; however, the chi-square fit works well. Roll damping coefficients computed with FIRST and SECOND are compared in Table 2.

Table 2: Comparison of Roll Damping Coefficients Estimated Using FIRST and SECOND for Experimental Results

Initial Amp. (degrees)	Quantity	FIRST	SECOND
22.	$\kappa_1 \times 10^2$	3.05	3.03
	$\kappa_2 \times 10^3$ (deg^{-1})	3.61	3.63
	$\chi^2/(M-2)$	0.060	0.043
25.	$\kappa_1 \times 10^2$	2.65	2.57
	$\kappa_2 \times 10^3$ (deg^{-1})	4.68	4.78
	$\chi^2/(M-2)$	0.59	0.41

From the normalized χ^2 coefficients for programs FIRST and SECOND, it can be seen that program SECOND provides better fits to the data. The differences in the results computed by the two programs are somewhat larger than in the case of the numerical roll decay time histories, the largest difference being about 3%.

The results of this section indicate that FIRST, SECOND, and the M & P method of Reference 5 are all capable of determining roll decay coefficients with high accuracy. A careful handling of experimental uncertainties is the key to obtaining good results. In the forms discussed here, all of methods consider only the roll extrema and throw out the rest of the roll decay curves. In situations in which the roll damping is large and only a few roll decay cycles can be obtained, there is a clear advantage in using methods which make use of the whole time history. An example of such a method is described in Reference 8.

5 Concluding Remarks

A new method for estimating linear plus quadratic roll damping coefficients from roll decay tests has been described. The method is believed to be accurate up to second order in both damping terms.

Computer programs have been developed to estimate roll decay coefficients via chi-square fits using first and second order methods. The programs were applied to measured and simulated roll decay time-histories, and excellent results were obtained with both codes. While the second order method produced better results both in terms of goodness-of-fit and accuracy, the results obtained with the first order code were already sufficiently accurate that the improvements are considered to be of limited practical significance.

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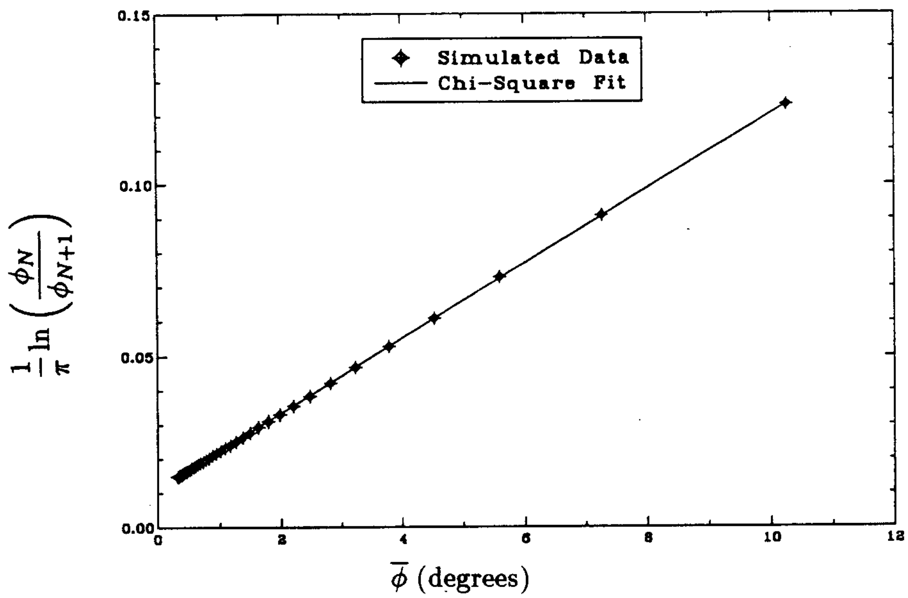

Figure 1: Roll Decay Plot, Simulated Roll Decay Time History

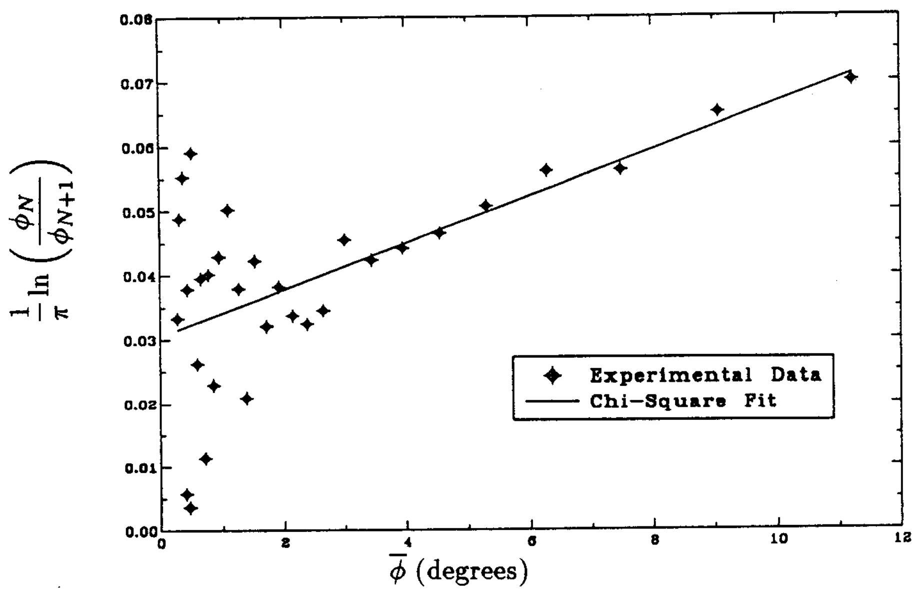

Figure 2: Roll Decay Plot, DDH 280 Model with Bilge Keel, Low $\overline{GM}$ Condition, Speed 0.94 m/sec.

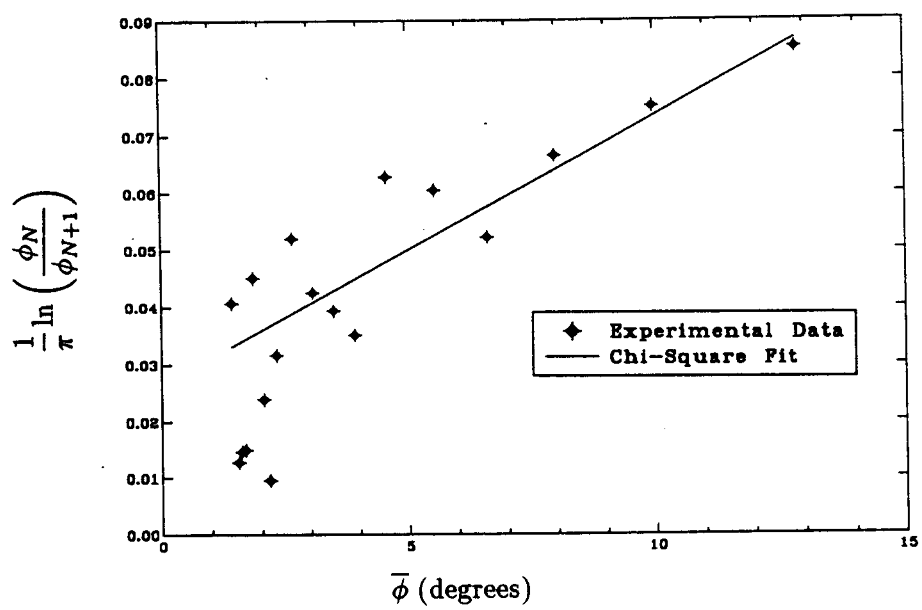

Figure 3: Roll Decay Plot, DDH 280 Model with Bilge Keel, High $\overline{GM}$ Condition, Speed 0.47 m/sec.

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A method for determining linear plus quadratic roll damping coefficients from roll decay tests that is believed to be accurate up to second order in the damping coefficients is described. Two computer programs are developed to estimate roll decay coefficients via chi-square fitting using first-order and second-order analyses. The methods are applied to measured and simulated roll decay data, and excellent results are obtained with both codes. While the second-order results represent an improvement in both goodness of fit and accuracy compared with the first-order results, the latter are already so good that the improvement is considered to be of limited practical significance.

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