Application of Bayesian multistatic localization to sea trial data

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Abstract

Four methods were used to localize an underwater target, using multistatic sonar data from a sea trial that was conducted at the Atlantic Undersea Test and Evaluation Center in February 2013. The first method is a conventional single-ping localization method. The second is a Kalman filter with the single-ping estimates of the first method used as input. The third is a relatively new single-ping localization method that consists mainly of a numerical search for a peak in a probability distribution function. This function is constructed using Bayesian principles. The fourth method is the "Bayesian pseudo-Kalman filter", which makes its first appearance in this report. It consists of modifying the Bayesian single-ping method by prior information derived from previous estimates that are interpreted as in a Kalman filter. For this data set, the results of the Bayesian methods are markedly superior to those of the more conventional methods, in terms of localization accuracy.

Significance to defence and security

The localization of underwater targets via multistatic active sonar is of interest in tactical picture compilation. By applying several mathematical methods to a set of real sonar data, this report demonstrates that accuracy in localization can depend greatly on which methods are used to interpret the data. The novel methods that are here described as "Bayesian" achieve better accuracy than the more conventional methods.

Résumé

Quatre méthodes ont été utilisées pour localiser une cible sous-marine à partir des données d'un sonar multistatique obtenues dans le cadre d'un essai en mer réalisé à l'Atlantic Undersea Test and Evaluation Center, en février 2013. La première est une méthode traditionnelle de localisation à une seule impulsion. La deuxième consiste en un filtre de Kalman qui utilise les estimations de la méthode à une seule impulsion comme intrants. La troisième est une méthode relativement nouvelle de localisation à une seule impulsion constituée principalement de la recherche numérique d'une crête dans une fonction de répartition des probabilités. Cette fonction repose sur les principes de Bayes. La quatrième méthode est celle du « pseudo-filtre de Kalman de Bayes », mentionnée pour la première fois dans le présent rapport. Elle consiste à modifier la méthode de Bayes à une seule impulsion de à partir de données existantes tirées d'estimations antérieures qui sont interprétées comme celles d'un filtre de Kalman. Pour cet ensemble de données, sur le plan de la précision de la localisation, les résultats des méthodes bayésiennes sont nettement supérieurs à ceux des méthodes plus traditionnelles.

Importance pour la défense et la sécurité

La localisation de cibles sous-marines au moyen d'un sonar actif multistatique présente un intérêt pour la compilation de la situation tactique. Grâce à l'application de plusieurs méthodes mathématiques à un ensemble de données sonar réelles, le présent rapport montre que la précision de la localisation peut dépendre beaucoup des méthodes utilisées pour interpréter les données. Les nouvelles méthodes décrites comme étant « bayésiennes » sont plus précises que les méthodes plus traditionnelles.

Table of contents

List of figures

Acknowledgements

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1 Introduction

Active sonar systems, by definition, include at least one acoustic source (transmitter) and at least one acoustic sensor (receiver). Signals from the transmitter(s) will be reflected off various objects, including targets of interest, and some of these echoes will in general be detected by the receiver(s). Such a system is said to be “multistatic” if it has more than one transmitter or more than one receiver. An active sonar system with only one transmitter and one receiver is said to be “bistatic” if the transmitter and receiver are separated and “monostatic” if the transmitter and the receiver are co-located. In this Scientific Report we consider a system with one transmitter and several receivers.

This report is concerned with localization of a target by means of the data from a single signal (“ping”). It is assumed here that each receiver provides at most one report (“contact”), and that this contact comes about from an echo off the target of interest. Thus, the problem of association is neglected here. It is also assumed that each receiver measures both the time of arrival (TOA) and the direction of arrival (DOA) of each contact. Here, we treat localization as a two-dimensional problem, with the depth of the target being neglected.

Localization of a target, under the given assumptions, can be achieved by any of several different methods. The method with which this report is primarily concerned involves subjecting a rather complicated probability distribution function (pdf) to numerical maximization [1]. This method will be described loosely as “Bayesian” because the pdf is constructed based on Bayesian principles, although a Bayesian purist might reject that label on the grounds that a point estimate is ultimately generated. For the sake of comparison, we consider also a more established localization method [2], where (a) for each receiver, an estimate of the target’s position is made based on treating the TOA and DOA measurements as exact, (b) a covariance matrix is computed for each such estimate, based on propagation-of-error principles, and (c) the single-receiver estimates pertaining to the same ping are fused as independent measurements. This latter method will be referred to as the “baseline method”, for convenience. These methods are described in more detail in Subsections 3.1 (baseline) and 3.3 (Bayesian).

These methods are applied to data from a sea trial, briefly described in Section 2, in which a moving target was subjected to pings over the course of about 40 minutes, at a rate of one ping every 30 seconds. In the data set that was provided, several contacts had been already removed by hand, having been judged to be false alarms. The remaining contacts are treated as fulfilling the assumptions described above.

Because several successive pings are used, for a single target, the question naturally arises whether tracking methods can reduce the error in localization. In order to address this question, the results from the baseline method were fed directly into a Kalman filter (Subsection 3.2), while something analogous was done in the case of the Bayesian method (Subsection 3.4). The latter tracking method, the “Bayesian pseudo-Kalman filter”, makes its first appearance in this report.

Results of applying these localization and tracking methods to the sea trial data are presented in Section 4, and concluding comments appear in Section 5.

2 Sea trial

Multistatic detection trials were conducted at the Atlantic Undersea Test and Evaluation Center (AUTEC) in the Bahamas, in February 2013 [3].

The target was an autonomous underwater vehicle (AUV) called the Arctic Explorer, built by International Submarine Engineering Ltd. (Port Coquitlam, BC). The AUV is approximately 7 m long and 0.75 m in diameter. Its top speed is about 5 knots. While submerged, it follows a preprogrammed route.

The AUV ran at a constant depth of about 100 m during the trials. It was equipped with a pinger that was tracked by a network of bottom-mounted hydrophones. The results of this tracking procedure are taken as ground truth for purposes of this study.

The source for the multistatic data was a VP-2 acoustic projector, consisting of two transducer rings arranged vertically, towed by the Canadian Forces Auxiliary Vessel QUEST. The projector transmitted hyperbolic frequency-modulated waveforms with a duration of 0.25 s and at intervals of 30 s. The waveforms were centred on a frequency of 1500 Hz, with bandwidths cycling through 100, 200, and 400 Hz. The source level was over 220 dB relative to 1 µPa at 1 m.

Six of the receivers (sonobuoys) successfully detected echoes from the target. Figure 1 shows the true track of the target (black) and the source (red), running westward, over the course of the roughly 40 minutes of data gathering, as well as the true tracks of the reporting sonobuoys (blue), drifting slightly eastward and southward, with each track shown only for the time interval during which the corresponding sonobuoy made any reports of the target.

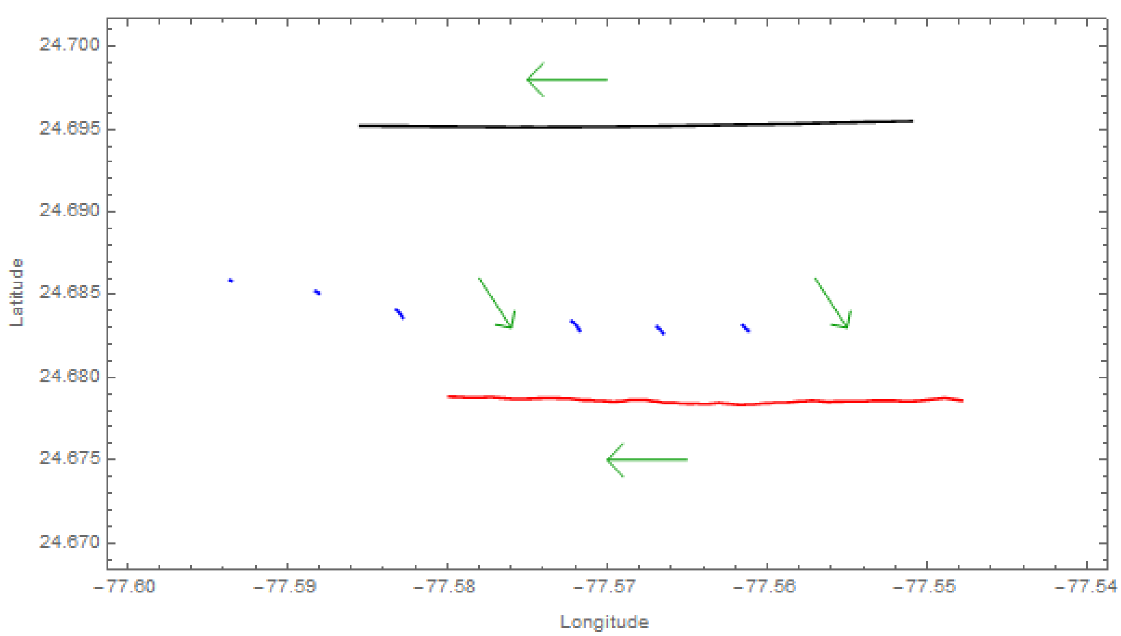

Figure 1: *True tracks of the target (black), source (red), and reporting sonobuoys (blue). Green arrows indicate the direction of motion and drift.*

Real-time data processing and recording was provided by the Maritime Acoustic Processing System (MAPS) on board the QUEST.

The processed data provided afterwards included, for each report of each ping:

- the clock time of the report;
- an identifier for which receiver was making the report;
- the theoretical time delay between the arrival of the main blast at the receiver and the arrival of the echo at the receiver, based on ground truth positions and the assumption of a constant sound speed;
- the true bearing of the target relative to the receiver;
- the true distance from the source to the target;
- the true distance from the target to the receiver;
- the true depth of the target;
- the latitude and longitude of the source;
- the latitude and longitude of the receiver;
- the measured bearing of the target relative to the receiver; and
- the measured time delay between the arrival of the main blast at the receiver and the arrival of the echo at the receiver.

Taking the reported latitude and longitude of the receiver as true, the true latitude and longitude of the target are easily derived from the latitude and longitude of the receiver, the true bearing of the target relative to the receiver, and the true distance from the target to the receiver (see [4] for an iterative method to find a point's geodetic coordinates from its Cartesian coordinates relative to another given point, based on the WGS-84 ellipsoidal earth model [5]).

3 Localization and tracking methods

This section is concerned with describing the methods used to localize the target. In each case it is assumed that we have the following quantities:

- the local speed of sound in water, c, assumed to be constant throughout the region of interest, with standard deviation σ_c
- the position of the source, $\mathbf{x}_0 = (x_0, y_0)$, with standard deviation σ_x in each dimension (we assume no correlation between the two dimensions)
- the position of the receiver, $\mathbf{x}_k = (x_k, y_k)$ for k from 1 to N, with standard deviation σ_x in each dimension (again assuming no correlation between the two dimensions)
- either the ping time t (at the source), with standard deviation σ_t, and the time of arrival (TOA) of the echo at each receiver, T_k for k from 1 to N, with standard deviation σ_T, or the TOA of the echo at each receiver relative to the ping time, $\tau_k = T_k - t$ for k from 1 to N, with standard deviation σ_τ
- the direction of arrival (DOA) of the echo at each receiver, θ_k for k from 1 to N, with standard deviation σ_θ

The formulae for the Bayesian method (Subsection 3.3) were derived with the assumption that the source and the receivers would have similar uncertainties in their positions. Hence the use of a common standard deviation σ_x for the source position and the receiver positions. In the present application, the source is a towed body while the receivers are drifting bodies, so ideally a smaller standard deviation should be applied to the source position. However, in the context of the performance comparison made in this report, it is not clear that any imbalance will be created by inflating the assumed standard deviation for the source position to be equal to that of the receiver position. In general, inflating the standard deviation representing a measurement uncertainty should have the effect of inflating the uncertainty in the final estimate, and thus reducing the relative error (as defined in Equation (62)).

3.1 Baseline single-ping localization method

The method reported by Coraluppi [2] is, for convenience, described as the "baseline method" in this report. All the equations in this subsection are based on that reference, with some changes in notation.

The fused estimate $\mathbf{X}_{\text{BL}}$ for the target's position and the corresponding covariance matrix $\mathbf{P}_{\text{BL}}$ are given by

$$\mathbf{P}_{\text{BL}} = \left(\sum_{k=1}^{N} \mathbf{P}_k^{-1} \right)^{-1}, \tag{1}$$

$$\mathbf{X}_{\mathrm{BL}} = \mathbf{P}_{\mathrm{BL}} \sum_{k=1}^{N} \mathbf{P}_k^{-1} \mathbf{X}_k, \tag{2}$$

where $\mathbf{X}_k$ and $\mathbf{P}_k$ are the target position estimate and its covariance matrix based on the information at the k^{th} receiver. In the case of two receivers, (1) and (2) above are equivalent to (39) in [2] together with the unnumbered equation just before (40). For more than two receivers, (1) and (2) are the natural generalization thereof.

Let the bearing difference between the transmitter and the target, with respect to the k^{th} receiver, be denoted α_k (see Figure 2), so that

$$\cos(\alpha_k + \theta_k) = \frac{x_0 - x_k}{\Delta_k}, \tag{3}$$

$$\sin(\alpha_k + \theta_k) = \frac{y_0 - y_k}{\Delta_k}, \tag{4}$$

where

$$\Delta_k = \sqrt{(x_0 - x_k)^2 + (y_0 - y_k)^2}. \tag{5}$$

Equations (3) and (4) should be regarded as a definition of α_k. In the case where $x_0 > x_k$, this means that

$$\alpha_k = \arctan\left(\frac{y_0 - y_k}{x_0 - x_k}\right) - \theta_k. \tag{6}$$

The target position estimate $\mathbf{X}_k$ (from the k^{th} sensor perspective) is given by

$$\mathbf{X}_k = \begin{pmatrix} X_k \\ Y_k \end{pmatrix} = \begin{pmatrix} x_k + r_k \cos\theta_k \\ y_k + r_k \sin\theta_k \end{pmatrix}, \tag{7}$$

where r_k is the estimated distance between the receiver and the target. The distance between the transmitter and the target is given in terms of r_k by the law of cosines (see Figure 2):

$$c\tau_k - r_k = \sqrt{\Delta_k^2 + r_k^2 - 2\Delta_k r_k \cos\alpha_k}. \tag{8}$$

Solving Equation (8) for r_k, we get

$$r_k = \frac{a_k}{b_k}, \tag{9}$$

where

$$a_k = c^2\tau_k^2 - \Delta_k^2, \tag{10}$$

$$b_k = 2(c\tau_k - \Delta_k \cos\alpha_k). \tag{11}$$

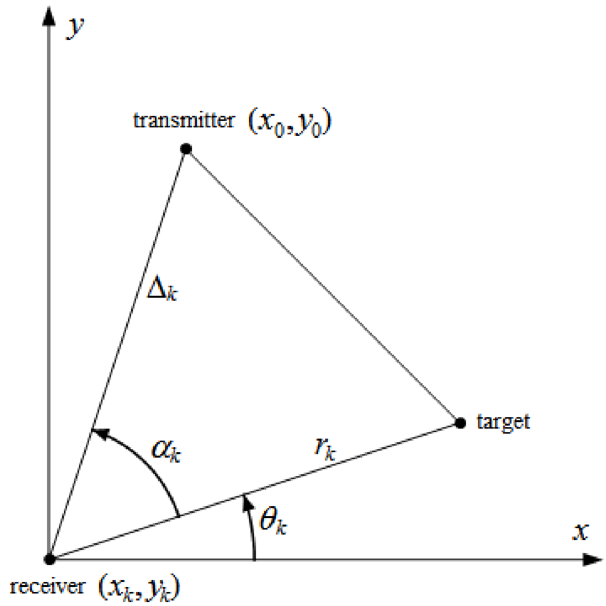

Figure 2: *Illustration of α_k, θ_k, Δ_k, and r_k.*

The covariance $\mathbf{P}_k$ is given in Annex A.

In this way, we get an estimate of the target's position, and a corresponding covariance matrix, from each ping.

3.2 Kalman filter with baseline method input

Since we are following a single target over some time, it should be possible to improve localization accuracy by means of basic tracking methods. This subsection describes the use of a Kalman filter (KF) with a four-dimensional state estimate vector. See [6] for more background on the Kalman filter.

Let the baseline method position estimate and its covariance matrix, for the j^{th} ping, be denoted $\mathbf{X}_{\text{BL}}(j)$ and $\mathbf{P}_{\text{BL}}(j)$, respectively. We count only the pings for which at least one receiver reported an echo. Let the time interval between the $(j\text{-}1)^{\text{th}}$ ping and the j^{th} ping be denoted $\Delta t(j)$, and let the KF state vector and its covariance matrix for the j^{th} ping be denoted $\hat{\mathbf{x}}(j)$ and $\mathbf{C}(j)$, respectively.

The first and second pings are used to initiate the KF:

$$\hat{\mathbf{x}}(2) = \begin{pmatrix} \mathbf{X}_{\mathrm{BL}}(2) \\ \frac{1}{\Delta t(2)}\left(\mathbf{X}_{\mathrm{BL}}(2) - \mathbf{X}_{\mathrm{BL}}(1)\right) \end{pmatrix}, \tag{12}$$

$$\mathbf{C}(2) = \begin{pmatrix} \mathbf{P}_{\mathrm{BL}}(2) & \frac{1}{\Delta t(2)}\mathbf{P}_{\mathrm{BL}}(2) \\ \frac{1}{\Delta t(2)}\mathbf{P}_{\mathrm{BL}}(2) & \frac{1}{\Delta t(2)^2}\left(\mathbf{P}_{\mathrm{BL}}(2) + \mathbf{P}_{\mathrm{BL}}(1)\right) \end{pmatrix}, \tag{13}$$

and afterwards, the state estimates and covariance matrices are found recursively. Let $\mathbf{F}(T)$ be the time-projection matrix (as a function of the time interval *T*),

$$\mathbf{F}(T) = \begin{pmatrix} \mathbf{I} & T\mathbf{I} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}, \tag{14}$$

where $\mathbf{I}$ is the 2×2 identity matrix and $\mathbf{0}$ is the 2×2 zero matrix. Let $\mathbf{Q}(T)$ be the process noise matrix (as a function of the time interval *T*),

$$\mathbf{Q}(T) = \nu \begin{pmatrix} \frac{T^3}{3}\mathbf{I} & \frac{T^2}{2}\mathbf{I} \\ \frac{T^2}{2}\mathbf{I} & T\mathbf{I} \end{pmatrix}, \tag{15}$$

for some noise parameter ν, and let $\hat{\mathbf{x}}_{\mathrm{proj}}(j)$ and $\mathbf{C}_{\mathrm{proj}}(j)$ denote the projected state estimate and covariance at the j^{th} ping, based on information up to the $(j\text{-}1)^{\mathrm{th}}$ ping. Then (for $j > 2$) we have

$$\mathbf{C}_{\mathrm{proj}}(j) = \mathbf{F}\left(\Delta t(j)\right)\mathbf{C}(j-1)\mathbf{F}^{\mathrm{T}}\left(\Delta t(j)\right) + \mathbf{Q}\left(\Delta t(j)\right), \tag{16}$$

$$\hat{\mathbf{x}}_{\mathrm{proj}}(j) = \mathbf{F}\left(\Delta t(j)\right)\hat{\mathbf{x}}(j-1), \tag{17}$$

$$\mathbf{C}(j) = \left(\mathbf{C}_{\mathrm{proj}}^{-1}(j) + \mathbf{H}^{\mathrm{T}}\mathbf{P}_{\mathrm{BL}}^{-1}(j)\mathbf{H}\right)^{-1}, \tag{18}$$

$$\hat{\mathbf{x}}(j) = \mathbf{C}(j)\left(\mathbf{C}_{\mathrm{proj}}^{-1}(j)\hat{\mathbf{x}}_{\mathrm{proj}}(j) + \mathbf{H}^{\mathrm{T}}\mathbf{P}_{\mathrm{BL}}^{-1}(j)\mathbf{X}_{\mathrm{BL}}(j)\right), \tag{19}$$

where $\mathbf{H} = (\mathbf{I} \quad \mathbf{0})$ and the superscript "T" denotes the transpose. Thus $\mathbf{X}_{\mathrm{KF}}(j) = \mathbf{H}\hat{\mathbf{x}}(j)$ is an estimate of the target's position at the j^{th} ping, and $\mathbf{P}_{\mathrm{KF}}(j) = \mathbf{H}\mathbf{C}(j)\mathbf{H}^{\mathrm{T}}$ is the corresponding covariance matrix.

3.3 Bayesian single-ping localization method

Here we construct a probability distribution function (pdf) for the target, given the measurements and the standard deviations listed at the start of Section 3. This discussion follows that in [1].

Let **X** be a (two dimensional vector) position variable referring to the unknown position of the target. Let $P(\mathbf{X})$ be taken as a shorthand for $P(\mathbf{X}|t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c)$, i.e. the probability density for **X** conditioned on the measurements. In order to construct this posterior pdf, an assumption must be made for the prior pdf. We take the prior to be minimally informative, hence uniform and unbounded, so that the posterior pdf is proportional to the corresponding likelihood density:

$$P(\mathbf{X}|t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c) \propto P(t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c|\mathbf{X}). \tag{20}$$

The likelihood density for **X** in (20) is the probability density for the measurements, under the assumption that **X** is the true target position. It can be expressed as

$$\begin{aligned} &P(t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c|\mathbf{X}) \\ &= \int \dots \int P(t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c|\mathbf{X},\tilde{t},\tilde{\mathbf{x}}_0,\{\tilde{\mathbf{x}}_k\},\tilde{c})P_{\mathrm{pr}}(\tilde{t},\tilde{\mathbf{x}}_0,\{\tilde{\mathbf{x}}_k\},\tilde{c})d\tilde{t}d\tilde{\mathbf{x}}_0 d\tilde{\mathbf{x}}_1 \dots d\tilde{\mathbf{x}}_N d\tilde{c}, \end{aligned} \tag{21}$$

where $\tilde{t}$, $\tilde{\mathbf{x}}_0$, $\tilde{\mathbf{x}}_k$, and $\tilde{c}$ are the assumed true values corresponding respectively to the measurements t, $\mathbf{x}_0$, $\mathbf{x}_k$, and c, and **X** is taken to be independent of those variables. The notation $P_{\mathrm{pr}}(\tilde{t},\tilde{\mathbf{x}}_0,\{\tilde{\mathbf{x}}_k\},\tilde{c})$ refers to the prior pdf for those variables of integration. Note that the assumed true values corresponding similarly to T_k and θ_k are constrained by $\tilde{t}$, $\tilde{\mathbf{x}}_0$, $\tilde{\mathbf{x}}_k$, and $\tilde{c}$, together with the target position **X**; therefore we do not introduce them as additional variables of integration. As we did with **X**, we take a uniform and unbounded prior for each of these integration variables. Thus we have

$$P(\mathbf{X}) \propto \int \dots \int P(t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c|\mathbf{X},\tilde{t},\tilde{\mathbf{x}}_0,\{\tilde{\mathbf{x}}_k\},\tilde{c})\, d\tilde{t}d\tilde{\mathbf{x}}_0 d\tilde{\mathbf{x}}_1 \dots d\tilde{\mathbf{x}}_N d\tilde{c}. \tag{22}$$

The use of a uniform unbounded prior for $\tilde{c}$ is questionable, but the form of the integrand ensures that only values of $\tilde{c}$ that are close to the estimate c will contribute. The integrand from (22) is given (up to a constant factor) by

$$\begin{aligned} &P(t,\{T_k\},\{\theta_k\},\mathbf{x}_0,\{\mathbf{x}_k\},c|\mathbf{X},\tilde{t},\tilde{\mathbf{x}}_0,\{\tilde{\mathbf{x}}_k\},\tilde{c}) \\ &\propto \exp\left(-\frac{1}{2}\left[\frac{(t-\tilde{t})^2}{\sigma_t^2}+\frac{(c-\tilde{c})^2}{\sigma_c^2}+\frac{(x_0-\tilde{x}_0)^2}{\sigma_x^2}+\frac{(y_0-\tilde{y}_0)^2}{\sigma_x^2}\right.\right. \\ &\left.\left.+\sum_{k=1}^{N}\left(\frac{(x_k-\tilde{x}_k)^2}{\sigma_x^2}+\frac{(y_k-\tilde{y}_k)^2}{\sigma_x^2}+\frac{\left(T_k-\tilde{T}_k\right)^2}{\sigma_T^2}+\frac{\left(\theta_k-\tilde{\theta}_k\right)^2}{\sigma_\theta^2}\right)\right]\right), \end{aligned} \tag{23}$$

where $\tilde{T}_k$ is the TOA derived from the assumed target position $\mathbf{X} = (X \quad Y)^{\mathrm{T}}$ as well as from the other assumed quantities:

$$\tilde{T}_k = \tilde{t} + \frac{\tilde{S}_0 + \tilde{S}_k}{\tilde{c}}, \tag{24}$$

$$\tilde{S}_0 = \sqrt{(\tilde{x}_0 - X)^2 + (\tilde{y}_0 - Y)^2}, \tag{25}$$

$$\tilde{S}_k = \sqrt{(\tilde{x}_k - X)^2 + (\tilde{y}_k - Y)^2}, \tag{26}$$

and similarly $\tilde{\theta}_k$ is the DOA derived from the assumed target position **X** as well as from the other assumed quantities:

$$\sin\tilde{\theta}_k = \frac{Y - \tilde{y}_k}{\tilde{S}_k}, \tag{27}$$

$$\cos\tilde{\theta}_k = \frac{X - \tilde{x}_k}{\tilde{S}_k}, \tag{28}$$

$$\left|\theta_k - \tilde{\theta}_k\right| \leq \pi. \tag{29}$$

It is desirable to find a closed-form expression for the integral in (22), so that the process of numerical peak-finding can be made practicable. An approximate expression is available, based on the assumption that the distance from the target to the nearest receiver or transmitter is large in comparison to the uncertainty in the receiver or transmitter positions. See [1] for details. Aside from a constant factor, the pdf evaluates approximately to

$$P(\mathbf{X}) \propto \frac{1}{\sqrt{A}} \exp\left(-C + \frac{B^2}{4A}\right), \tag{30}$$

where

$$\begin{aligned} C = \frac{1}{2}\sum_k \left[\frac{S_k^2\left(\theta_k - \breve{\theta}_k\right)^2}{\sigma_x^2 + S_k^2\sigma_\theta^2} + \ln\left(1 + \frac{\sigma_x^2}{S_k^2\sigma_\theta^2}\right)\right] + \frac{t^2}{2\sigma_t^2} + \frac{S_0^2}{2\sigma_x^2} + \frac{\mu}{2\alpha} - \frac{(\alpha S_0 + \sigma_x^2\lambda)^2}{2\sigma_x^2\alpha\gamma} \\ - \frac{\left(\gamma t + c\sigma_t^2(\lambda - NS_0)\right)^2}{2\sigma_t^2\gamma\gamma'}, \end{aligned} \tag{31}$$

$$\begin{aligned} B = \frac{\chi}{\alpha} - \frac{c\sigma_T^2\mu}{\alpha^2} - \frac{N\sigma_x^2}{c\alpha} - \frac{Nc\sigma_x^2(\sigma_T^2 - \sigma_t^2)}{\alpha\gamma'} \\ - \frac{(\alpha S_0 + \sigma_x^2\lambda)}{\alpha^2\gamma^2}\left(\alpha\gamma\tau + c\sigma_T^2(N\alpha S_0 - (\alpha + \gamma)\lambda)\right) \\ - \frac{\left(\gamma t + c\sigma_t^2(\lambda - NS_0)\right)}{\gamma^2\gamma'^2}(\eta(\lambda - NS_0) + c\gamma\gamma'\tau - N(N+1)c\sigma_x^2\gamma t), \end{aligned} \tag{32}$$

$$
\begin{aligned}
A = \frac{1}{2\sigma_c^2} - \frac{\sigma_T^2 \beta\mu}{2\alpha^3} - \frac{2c\sigma_T^2\chi}{\alpha^2} + \frac{\upsilon}{2\alpha} + \frac{N\sigma_x^2(3\alpha-\beta)}{4c^2\alpha^2} \\
- \frac{N\sigma_x^2(\sigma_T^2 - \sigma_t^2)}{2\alpha^2{\gamma'}^2}\Big[(N+1)\sigma_x^4 - \sigma_x^2 c^2\left((N+2)\sigma_T^2 + N\sigma_t^2\right) \\
- 3c^4\sigma_T^2(\sigma_T^2 + N\sigma_t^2)\Big] \\
- \frac{1}{2\alpha^3\gamma^3}\left[\sigma_T^2\sigma_x^2\zeta\lambda^2 + \sigma_T^2\alpha^3\delta S_0(NS_0 - 2\lambda) + \sigma_x^2\alpha^2\gamma^2\tau^2 \right. \\
\left. - 4c\sigma_T^2\alpha\gamma\tau(\alpha^2 S_0 + \sigma_x^2(\alpha+\gamma)\lambda)\right] \\
- \frac{1}{2\gamma^3{\gamma'}^3}\Big[\sigma_t^2\kappa(\lambda - NS_0)^2 - 2c(\sigma_T^2 + N\sigma_t^2)\gamma^3(2\gamma' + \delta')(\lambda - NS_0)t \\
+ 4c\sigma_t^2\gamma\gamma'\eta(\lambda - NS_0)\tau - N(N+1)\sigma_x^2\gamma^3\delta' t^2 + \gamma^3\gamma'(\gamma' + \delta')t\tau \\
+ c^2\sigma_t^2\gamma^2{\gamma'}^2\tau^2\Big],
\end{aligned}
\tag{33}
$$

$$\alpha = \sigma_x^2 + c^2\sigma_T^2, \tag{34}$$

$$\beta = \sigma_x^2 - 3c^2\sigma_T^2, \tag{35}$$

$$\gamma = (N+1)\sigma_x^2 + c^2\sigma_T^2, \tag{36}$$

$$\gamma' = \gamma + Nc^2\sigma_t^2, \tag{37}$$

$$\delta = (N+1)\sigma_x^2 - 3c^2\sigma_T^2, \tag{38}$$

$$\delta' = \delta - 3Nc^2\sigma_t^2, \tag{39}$$

$$\eta = (N+1)^2\sigma_x^4 - c^4\sigma_T^2(\sigma_T^2 + N\sigma_t^2), \tag{40}$$

$$\zeta = -(N+1)(N+2)\sigma_x^6 + 3(N^2+2N+2)\sigma_x^4 c^2\sigma_T^2 + 9(N+2)\sigma_x^2 c^4\sigma_T^4 + 10c^6\sigma_T^6, \tag{41}$$

$$
\begin{aligned}
\kappa = (N+1)^4\sigma_x^8 - 3(N+1)^3\sigma_x^6 c^2(2\sigma_T^2 + N\sigma_t^2) - 12(N+1)^2\sigma_x^4 c^4\sigma_T^2(\sigma_T^2 + N\sigma_t^2) \\
- (N+1)\sigma_x^2 c^6\sigma_T^2(\sigma_T^2 + N\sigma_t^2)(2\sigma_T^2 + N\sigma_t^2) + 3c^8\sigma_T^4(\sigma_T^2 + N\sigma_t^2)^2,
\end{aligned}
\tag{42}
$$

$$\tau = \sum_k T_k, \tag{43}$$

$$\upsilon = \sum_k T_k^2, \tag{44}$$

$$\lambda = \sum_k (cT_k - S_k), \tag{45}$$

$$\mu = \sum_k (cT_k - S_k)^2, \tag{46}$$

$$\chi = \sum_k T_k (cT_k - S_k), \tag{47}$$

$$S_0 = \sqrt{(x_0 - X)^2 + (y_0 - Y)^2}, \tag{48}$$

$$S_k = \sqrt{(x_k - X)^2 + (y_k - Y)^2}, \tag{49}$$

$$\cos\breve{\theta}_k = \frac{X - x_k}{S_k}, \tag{50}$$

$$\sin\breve{\theta}_k = \frac{Y - y_k}{S_k}, \tag{51}$$

$$\left|\theta_k - \breve{\theta}_k\right| \leq \pi. \tag{52}$$

Localization of the target then consists of finding a peak in the expression $\exp(-C + B^2/4A)/\sqrt{A}$, or equivalently a peak in the expression $-C + B^2/4A - \frac{1}{2}\ln A$, considered as a function of $\mathbf{X}$. Let $\mathbf{X}_{\mathrm{Bayes}}$ denote the position estimate derived in such a fashion.

In the implementation used to derive the results in Section 4, the maximization technique consisted of a grid search. A maximum value of $P(\mathbf{X})$ over a discrete set of positions was found, with those positions initially distributed over a wide rectangle whose corners are given by $\left(x_0 \pm c(\max\{T_k\} - t), y_0 \pm c(\max\{T_k\} - t)\right)$. Having found this maximum, the grid search function is called repeatedly, using ever smaller rectangles, each one centred on the position of the previous maximum, until the length of the rectangle is less than one metre.

To find the covariance matrix $\mathbf{P}_{\mathrm{Bayes}}$, we assume for the sake of argument that the probability distribution function is approximately Gaussian in the near vicinity of the peak, so that

$$P(\mathbf{X}) \approx K\exp\left[-\tfrac{1}{2}\left(\mathbf{X} - \mathbf{X}_{\mathrm{Bayes}}\right)^{\mathrm{T}} \mathbf{P}_{\mathrm{Bayes}}^{-1} \left(\mathbf{X} - \mathbf{X}_{\mathrm{Bayes}}\right)\right], \tag{53}$$

for some constant *K*, as long as $\left|\mathbf{X} - \mathbf{X}_{\mathrm{Bayes}}\right|$ is not much greater than the width D of the peak. In the implementation used in Section 4, the value used for the width was $D = \sqrt{\tfrac{1}{2}\mathrm{trace}(\mathbf{P}_{\mathrm{BL}})}$. If $\mathbf{e}$ is a two-dimensional unit vector, it follows from (53) that

$$\mathbf{e}^{\mathbf{T}}\mathbf{P}_{\text{Bayes}}^{-1}\mathbf{e} = \frac{2}{D^2}\ln\left(\frac{P(\mathbf{X} = \mathbf{X}_{\text{Bayes}})}{P(\mathbf{X} = \mathbf{X}_{\text{Bayes}} + D\mathbf{e})}\right). \tag{54}$$

Thus, if we define

$$g(\theta) = \frac{2}{D^2}\ln\left(\frac{P(\mathbf{X} = \mathbf{X}_{\text{Bayes}})}{P\left(\mathbf{X} = \mathbf{X}_{\text{Bayes}} + D\mathbf{u}(\theta)\right)}\right), \tag{55}$$

where $\mathbf{u}(\theta)$ denotes the unit vector $(\cos\theta \quad \sin\theta)^{\mathrm{T}}$, then we can find the covariance matrix by

$$\mathbf{P}_{\text{Bayes}} = \begin{pmatrix} \frac{1}{2}\left(g(0) + g(\pi)\right) & \frac{1}{4}\left(g\left(\frac{\pi}{4}\right) - g\left(-\frac{\pi}{4}\right) - g\left(\frac{3\pi}{4}\right) + g\left(-\frac{3\pi}{4}\right)\right) \\ \frac{1}{4}\left(g\left(\frac{\pi}{4}\right) - g\left(-\frac{\pi}{4}\right) - g\left(\frac{3\pi}{4}\right) + g\left(-\frac{3\pi}{4}\right)\right) & \frac{1}{2}\left(g\left(\frac{\pi}{2}\right) + g\left(-\frac{\pi}{2}\right)\right) \end{pmatrix}^{-1}. \tag{56}$$

3.4 Pseudo-Kalman filter with Bayesian input

The pdf given in Equations (30)–(52) is based on a uniform prior for the target's position **X**. Instead, we can use prior information derived from the estimates from the previous pings, following the principles underlying the Kalman filter.

If we initiate our pseudo-Kalman filter from the first two pings, as in Equations (12) and (13) except using $\mathbf{X}_{\text{Bayes}}$ and $\mathbf{P}_{\text{Bayes}}$ in place of $\mathbf{X}_{\text{BL}}$ and $\mathbf{P}_{\text{BL}}$, then we can project the state estimate and the covariance forward in time in the same way as in a KF, using Equations (16) and (17). The new estimate is found by following the method of Subsection 3.3, except that the pdf is now given by

$$P(\mathbf{X}) \propto \frac{1}{\sqrt{A}}\exp\left(-C + \frac{B^2}{4A} - \frac{1}{2}\left(\mathbf{X} - \mathbf{H}\hat{\mathbf{x}}_{\text{proj}}\right)^{\mathrm{T}}\left(\mathbf{H}\mathbf{C}_{\text{proj}}\mathbf{H}^{\mathrm{T}}\right)^{-1}\left(\mathbf{X} - \mathbf{H}\hat{\mathbf{x}}_{\text{proj}}\right)\right), \tag{57}$$

where the ping number dependence is here kept implicit. Thus, the quantity to be maximized is $-C + B^2/4A - \frac{1}{2}\ln A - \frac{1}{2}\left(\mathbf{X} - \mathbf{H}\hat{\mathbf{x}}_{\text{proj}}\right)^{\mathrm{T}}\left(\mathbf{H}\mathbf{C}_{\text{proj}}\mathbf{H}^{\mathrm{T}}\right)^{-1}\left(\mathbf{X} - \mathbf{H}\hat{\mathbf{x}}_{\text{proj}}\right)$.

The result of this procedure is to find an estimate $\mathbf{X}_{\text{PKF}}$ and a corresponding covariance matrix $\mathbf{P}_{\text{PKF}}$ for the position of the target at the j^{th} ping. In order to complete the tracking loop, we need something analogous to Equations (18) and (19). We take

$$\mathbf{C}(j) = \mathbf{C}_{\text{proj}}(j) - \mathbf{C}_{\text{proj}}(j)\mathbf{H}^{\mathrm{T}}\left(\mathbf{H}\mathbf{C}_{\text{proj}}(j)\mathbf{H}^{\mathrm{T}}\right)^{-1}\left(\mathbf{I} - \mathbf{P}_{\text{PKF}}(j)\left(\mathbf{H}\mathbf{C}_{\text{proj}}(j)\mathbf{H}^{\mathrm{T}}\right)^{-1}\right)\mathbf{H}\mathbf{C}_{\text{proj}}(j), \tag{58}$$

$$\hat{\mathbf{x}}(j) = \hat{\mathbf{x}}_{\text{proj}}(j) + \mathbf{C}_{\text{proj}}(j)\mathbf{H}^{\mathrm{T}}\left(\mathbf{H}\mathbf{C}_{\text{proj}}(j)\mathbf{H}^{\mathrm{T}}\right)^{-1}\left(\mathbf{X}_{\text{PKF}}(j) - \mathbf{H}\hat{\mathbf{x}}_{\text{proj}}(j)\right). \tag{59}$$

It is easy to check that $\mathbf{X}_{\text{PKF}}(j) = \mathbf{H}\hat{\mathbf{x}}(j)$ and $\mathbf{P}_{\text{PKF}}(j) = \mathbf{H}\mathbf{C}(j)\mathbf{H}^{\mathrm{T}}$. Within these two constraints, (58) and (59) give the unique track state estimate and covariance matrix that could conceivably have arisen by means of updating a Kalman filter given the values found in the previous iteration.

4 Results

For the present study, the analysis was done in Cartesian coordinates, centered on a convenient point roughly in the middle of the set of sonobuoys (latitude 24.684°N, longitude 77.575°W).

As already noted in Section 2, the provided data included the latitude and longitude of the source and of each reporting receiver. These values were transformed into Cartesian coordinates, with the third dimension neglected.

The provided data also included a theoretical time delay between the arrival of the main blast at the receiver and the arrival of the echo at the receiver. We assume that these theoretical values were derived from the reported true positions and from some measured value of the local speed of sound in water:

$$\text{delay} = \frac{s_{\text{source}\to\text{target}} + s_{\text{target}\to\text{receiver}} - s_{\text{source}\to\text{receiver}}}{c}. \tag{60}$$

Thus, by inverting Equation (60), we get a formula for c (the speed of sound) as a function of the distances and the time delay. The average value of c thus derived, for all the reports, was used as the measured value for purposes of applying the target localization methods: $c = 1527$ m/s.

For the baseline method, the total elapsed time τ_k from the production of the ping to the arrival of the echo was found by adding Δ_k/c to the reported time delay. For the Bayesian method, the ping was treated as having taken place at $t = 0$ s, so the arrival time used was $T_k = \tau_k$.

The DOA (bearings) were reported in nautical convention (clockwise from North), whereas all the formulae used here are based on DOA defined as counter clockwise from East. The transformation is trivial.

Ideally, any data set used for the comparison of localization methods would include some indication of the amount of uncertainty in the measurements. Such was not provided, so some values for these standard deviations were chosen arbitrarily. The initial standard deviations used were $\sigma_x = 20$ m, $\sigma_c = 10\ \text{m/s}$, $\sigma_\theta = 0.35$ radians, and $\sigma_\tau = \sigma_T = \sigma_t = 0.1$ s. Each of these values was then varied in order to see whether the comparison among methods was sensitive to these choices.

The process noise parameter used in the tracking methods (see Subsections 3.2 and 3.4) was $\nu = 0.01\text{m}^2/\text{s}^3$.

Altogether there were 74 pings for which at least one receiver reported an echo. The time difference between successive pings was taken to be exactly 30s, as per the design of the trial. Thus the quantity $\Delta t(j)$ (see Subsection 3.2) was always an integer multiple of 30s, the exact multiple depending on how many pings were skipped due to having no reports.

For each of the 74 pings, the estimates $\mathbf{X}_{\mathrm{BL}}$, $\mathbf{X}_{\mathrm{KF}}$, $\mathbf{X}_{\mathrm{Bayes}}$, and $\mathbf{X}_{\mathrm{PKF}}$, along with the corresponding covariance matrices, were computed as described in Section 3. (In the case of the first two pings, $\mathbf{X}_{\mathrm{KF}}$ was taken to be equal to $\mathbf{X}_{\mathrm{BL}}$ and $\mathbf{X}_{\mathrm{PKF}}$ equal to $\mathbf{X}_{\mathrm{Bayes}}$, and similarly for their covariance matrices.) The absolute and relative error for an estimate $\mathbf{X}_{\mathrm{est}}$ with covariance $\mathbf{P}_{\mathrm{est}}$ are defined as

$$E_{\mathrm{abs}} = |\mathbf{X}_{\mathrm{est}} - \mathbf{X}_{\mathrm{true}}|, \tag{61}$$

$$E_{\mathrm{rel}} = \sqrt{(\mathbf{X}_{\mathrm{est}} - \mathbf{X}_{\mathrm{true}})^{\mathrm{T}} \mathbf{P}_{\mathrm{est}}^{-1} (\mathbf{X}_{\mathrm{est}} - \mathbf{X}_{\mathrm{true}})}. \tag{62}$$

Here $\mathbf{X}_{\mathrm{est}}$ means one of $\mathbf{X}_{\mathrm{BL}}$, $\mathbf{X}_{\mathrm{KF}}$, $\mathbf{X}_{\mathrm{Bayes}}$, or $\mathbf{X}_{\mathrm{PKF}}$, while $\mathbf{P}_{\mathrm{est}}$ means the corresponding covariance, either $\mathbf{P}_{\mathrm{BL}}$, $\mathbf{P}_{\mathrm{KF}}$, $\mathbf{P}_{\mathrm{Bayes}}$, or $\mathbf{P}_{\mathrm{PKF}}$.

Figures 3 and 4 show the absolute error (in metres) and the relative error, respectively, as a function of clock time in minutes, where time zero indicates an hour of the clock. The standard deviations used here for the inputs were the initial values listed above. The RMS absolute error, considered over all the pings, was 710 m for the baseline single-ping method, 460 m for the Kalman filter with baseline method input, 270 m for the Bayesian single-ping method, and 200 m for the pseudo-Kalman filter with Bayesian input. The corresponding RMS relative error results were respectively 4.9, 5.4, 1.1 and 1.4, indicating more realistic covariance matrices being generated by the Bayesian methods.

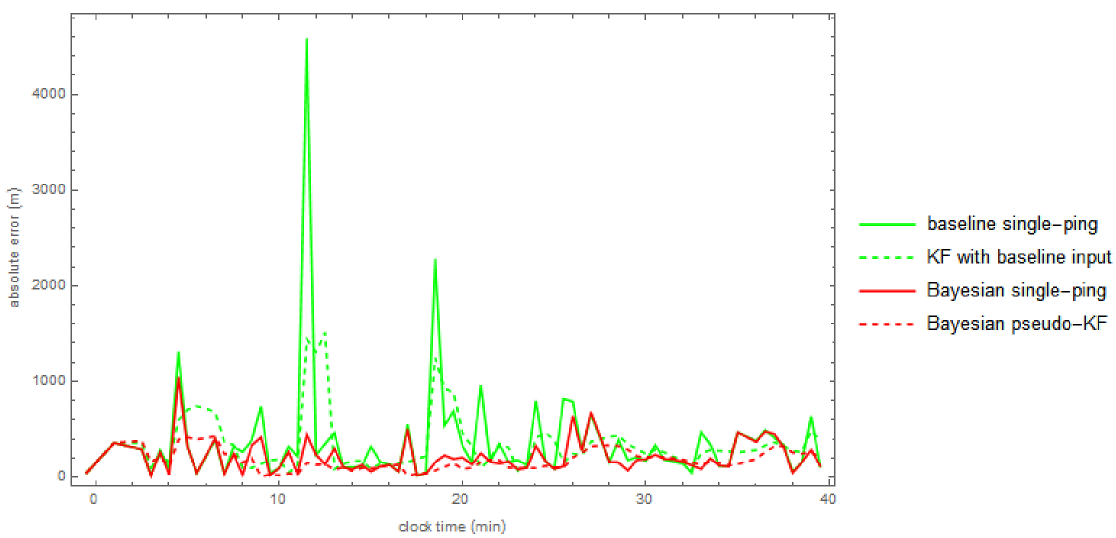

Figure 3: *Absolute error (m) in four localization methods as a function of time (min).*

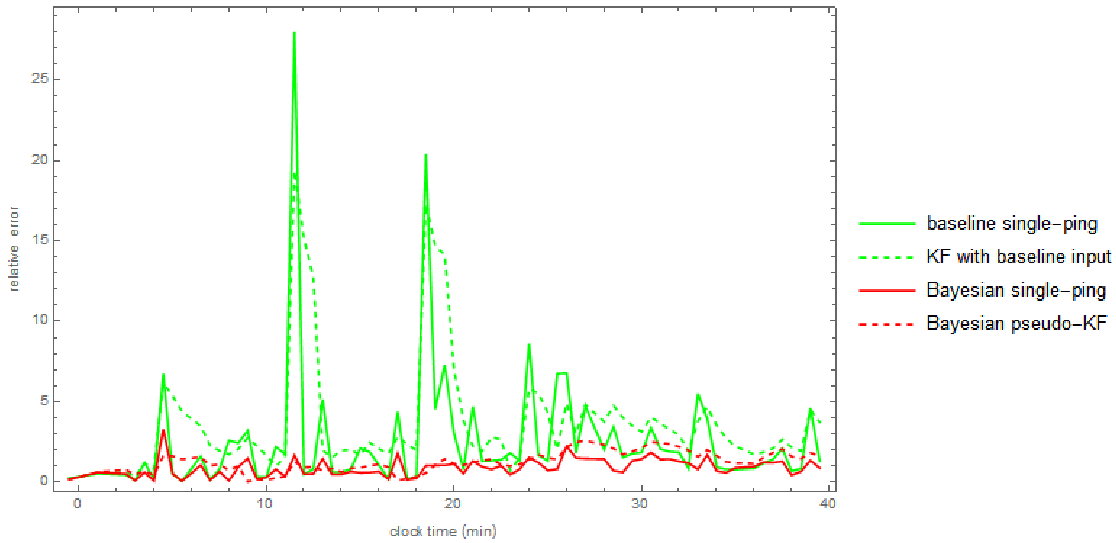

Figure 4: *Relative error in four localization methods as a function of time (min).*

The standard deviations used for the inputs were varied from their initial values in order to see how the RMS absolute and relative errors would be affected. Figures 5 and 6 show the RMS absolute error (in metres) and the RMS relative error, respectively, with σ_T varied from 0.02 s to 0.2 s, and σ_t and σ_τ both kept equal to σ_T. The figures show that the advantage of the Bayesian methods over the baseline methods is lost as these uncertainty parameters are increased.

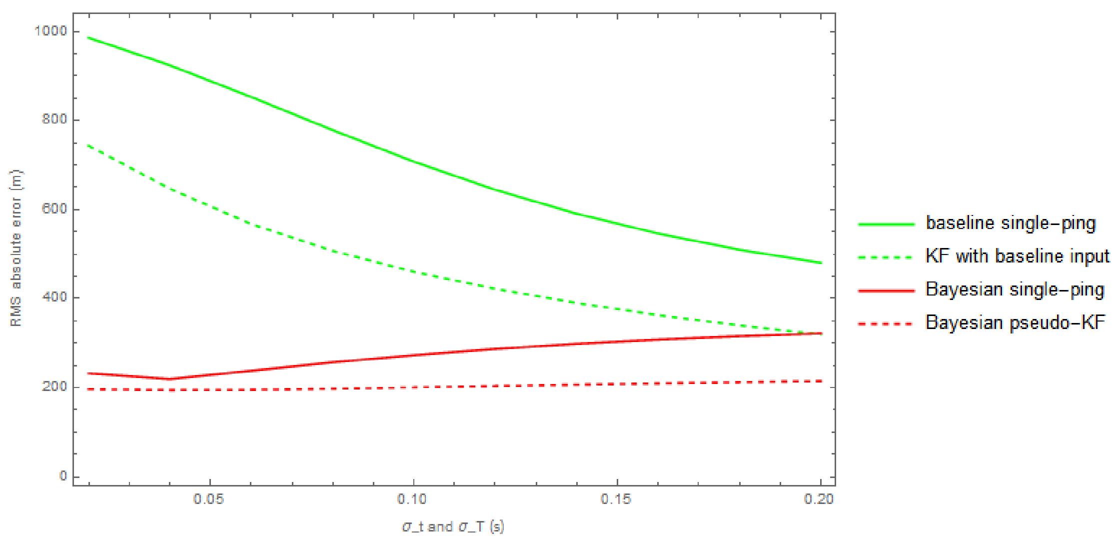

Figure 5: *RMS absolute error (m) in four localization methods as a function of the standard deviation assigned to the time measurements (s).*

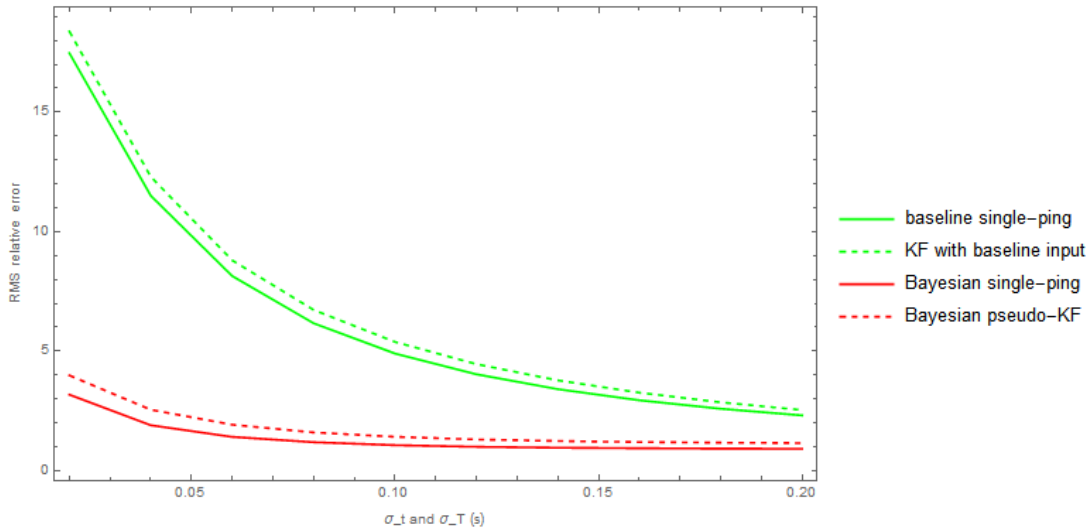

Figure 6: *RMS relative error in four localization methods as a function of the standard deviation assigned to the time measurements (s).*

Figures 7 and 8 show the RMS absolute error (in metres) and the RMS relative error, respectively, with σ_θ varied from 0.05 radians to 0.5 radians. The figures show that the advantage of the Bayesian methods over the baseline methods is lost as this uncertainty parameter is decreased.

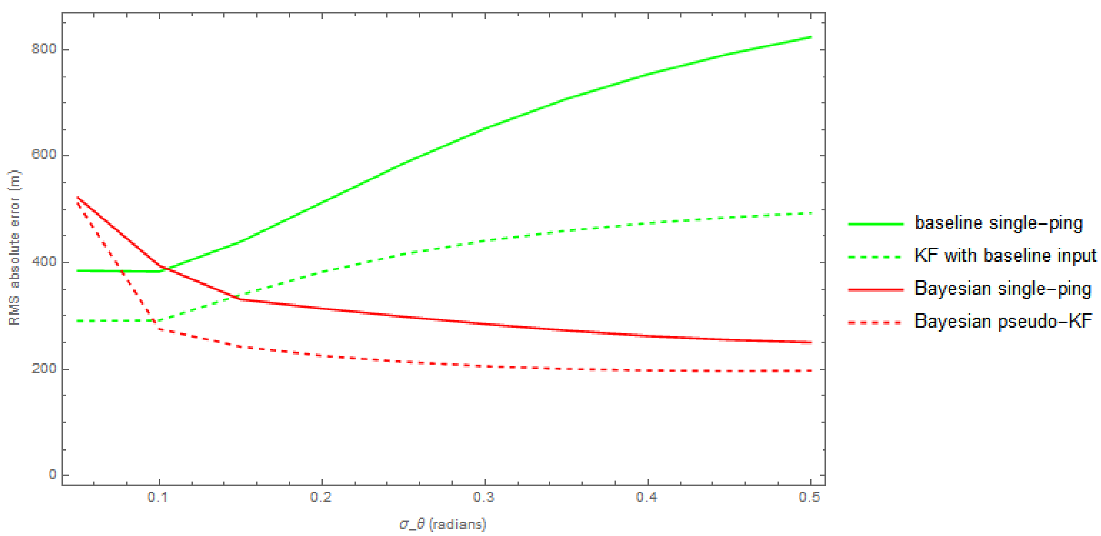

Figure 7: *RMS absolute error (m) in four localization methods as a function of the standard deviation assigned to the bearing measurements (radians).*

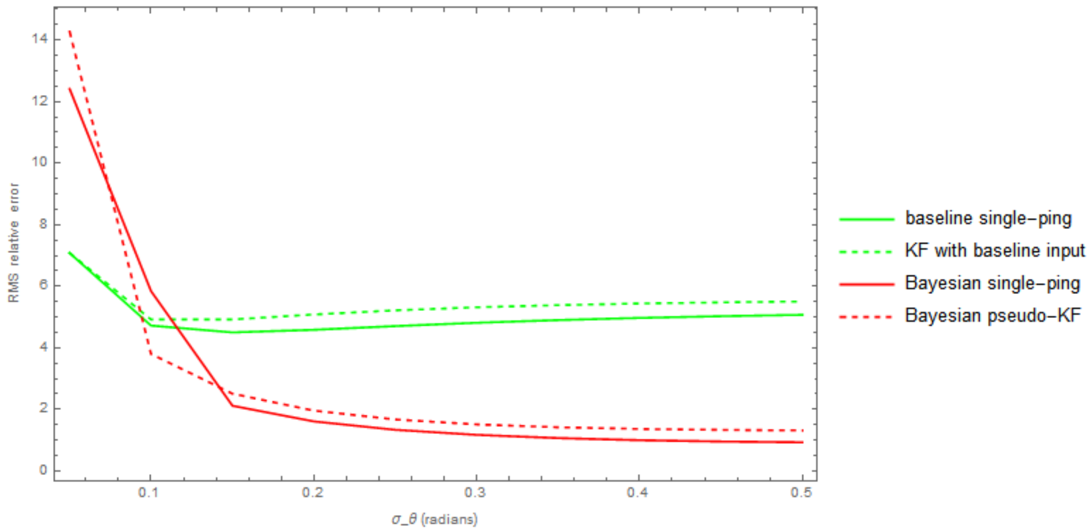

Figure 8: *RMS relative error in four localization methods as a function of the standard deviation assigned to the bearing measurements (radians).*

Figures analogous to Figures 5 through 8, but made by varying σ_x from 2 m to 30 m, simply show four nearly horizontal lines, so there was nearly no effect of that variation. A similar result holds for varying σ_c from 2 m/s to 20 m/s.

Based on the ground truth data, the actual standard deviation in the time measurement error was about 0.061 s, and the actual standard deviation in the bearing measurement error was about 0.36 radians. Since the initial comparison was made using 0.1 s for σ_T and 0.35 radians for σ_θ, the advantage of the Bayesian methods in that initial comparison is understated, relative to the advantage that is found with correct standard deviations for the measurements.

Finally, the effects of the process noise parameter ν are considered. Increasing this parameter means less filtering, so the Kalman filter results will approach the single-ping results. Decreasing this parameter is more interesting. Since the target was in fact trying to move with constant velocity, a smaller value of ν could improve the tracking results for this data set. By dropping ν all the way to zero, the RMS absolute error for the Kalman filter with baseline method input dropped from 460 m to 340 m, while the RMS relative error rose from 5.4 to 6.9. Meanwhile, the pseudo-Kalman filter with Bayesian input was affected similarly: the RMS absolute error dropped from 200 m to 170 m, while the RMS relative error rose from 1.4 to 2.8.

5 Conclusion

The Bayesian method for single-ping localization, first reported in [1], is here shown to be markedly more effective in localizing a real underwater target than the more conventional method from [2], at least for the present data set, when realistic standard deviations for the measured quantities are used.

Given that we are following a single target over time, it was natural to expect some improvement in localization by using basic tracking methods. The use of a Kalman filter with input from the baseline method did achieve some improvement over the baseline method alone, but Bayesian single-ping localization remained better. Best of all was the Bayesian pseudo-Kalman filter—that is, the Bayesian method modified by prior information derived from previous estimates interpreted as in a Kalman filter. This ranking among methods did not depend on the value used for the process noise parameter in the Kalman filter.

In the data set that was provided, some of the contacts had been already removed by hand, having been judged to be false alarms. In a more typical real-world tactical scenario, there could be plenty of false contacts and an unknown number of real targets, bringing serious challenges with regard to the association of contacts. Nevertheless, the results here can be taken as an indication that the Bayesian approach to localization could ultimately contribute to more accurate underwater picture compilation.

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Annex A The covariance in the baseline method

The covariance $\mathbf{P}_k$, corresponding to the k^{th} receiver's estimate $\mathbf{X}_k$ of the target's position in the baseline method (Subsection 3.1), is presented here. We start by writing it as

$$\mathbf{P}_k = \begin{pmatrix} \sigma_{X_k}^2 & \sigma_{X_k Y_k} \\ \sigma_{X_k Y_k} & \sigma_{Y_k}^2 \end{pmatrix}. \tag{A.1}$$

The components of this matrix are given by

$$\sigma_{X_k}^2 = \sigma_x^2 + \sigma_{r_k}^2 \cos^2\theta_k + r_k^2 \sigma_\theta^2 \sin^2\theta_k + 2\sigma_{x_k r_k}\cos\theta_k - 2r_k\sigma_{\theta_k r_k}\sin\theta_k\cos\theta_k, \tag{A.2}$$

$$\sigma_{Y_k}^2 = \sigma_x^2 + \sigma_{r_k}^2 \sin^2\theta_k + r_k^2 \sigma_\theta^2 \cos^2\theta_k + 2\sigma_{y_k r_k}\sin\theta_k + 2r_k\sigma_{\theta_k r_k}\sin\theta_k\cos\theta_k, \tag{A.3}$$

$$\begin{aligned}\sigma_{X_k Y_k} = \sigma_{x_k r_k}\sin\theta_k + \sigma_{y_k r_k}\cos\theta_k + \left(\sigma_{r_k}^2 - r_k^2\sigma_\theta^2\right)\sin\theta_k\cos\theta_k \\ + r_k\sigma_{\theta_k r_k}(\cos^2\theta_k - \sin^2\theta_k),\end{aligned} \tag{A.4}$$

where

$$\sigma_{\theta_k r_k} = \frac{2a_k\Delta_k}{b_k^2}\sigma_\theta^2 \sin\alpha_k, \tag{A.5}$$

$$\sigma_{x_k r_k} = \left[2\left(\frac{1}{b_k} - \frac{a_k\cos\alpha_k}{b_k^2\Delta_k}\right)(x_0 - x_k) - \frac{2a_k\sin\alpha_k}{b_k^2\Delta_k}(y_0 - y_k)\right]\sigma_x^2, \tag{A.6}$$

$$\sigma_{y_k r_k} = \left[2\left(\frac{1}{b_k} - \frac{a_k\cos\alpha_k}{b_k^2\Delta_k}\right)(y_0 - y_k) + \frac{2a_k\sin\alpha_k}{b_k^2\Delta_k}(x_0 - x_k)\right]\sigma_x^2, \tag{A.7}$$

$$\sigma_{r_k}^2 = \frac{b_k^2\sigma_{a_k}^2 + a_k^2\sigma_{b_k}^2 - 2a_k b_k \sigma_{a_k b_k}}{b_k^4}, \tag{A.8}$$

$$\sigma_{a_k}^2 = 4\left(c^2\tau_k^4\sigma_c^2 + c^4\tau_k^2\sigma_\tau^2 + 2\Delta_k^2\sigma_x^2\right), \tag{A.9}$$

$$\sigma_{b_k}^2 = 4\left(\tau_k^2\sigma_c^2 + c^2\sigma_\tau^2 + 2\sigma_x^2 + \Delta_k^2\sigma_\theta^2\sin^2\alpha_k\right), \tag{A.10}$$

$$\sigma_{a_k b_k} = 4\left(c\tau_k^3\sigma_c^2 + c^3\tau_k\sigma_\tau^2 + 2\Delta\sigma_x^2\cos\alpha_k\right). \tag{A.11}$$

These formulae are taken from [2]. They are simplified slightly from those in that reference because of the use of a common standard deviation for each dimension in the transmitter and receiver positions and the assumption of no correlation between the two dimensions.

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Four methods were used to localize an underwater target, using multistatic sonar data from a sea trial that was conducted at the Atlantic Undersea Test and Evaluation Center in February 2013. The first method is a conventional single-ping localization method. The second is a Kalman filter with the single-ping estimates of the first method used as input. The third is a relatively new single-ping localization method that consists mainly of a numerical search for a peak in a probability distribution function. This function is constructed using Bayesian principles. The fourth method is the “Bayesian pseudo-Kalman filter”, which makes its first appearance in this report. It consists of modifying the Bayesian single-ping method by prior information derived from previous estimates that are interpreted as in a Kalman filter. For this data set, the results of the Bayesian methods are markedly superior to those of the more conventional methods, in terms of localization accuracy.

Quatre méthodes ont été utilisées pour localiser une cible sous-marine à partir des données d’un sonar multistatique obtenues dans le cadre d’un essai en mer réalisé à l’Atlantic Undersea Test and Evaluation Center, en février 2013. La première est une méthode traditionnelle de localisation à une seule impulsion. La deuxième consiste en un filtre de Kalman qui utilise les estimations de la méthode à une seule impulsion comme intrants. La troisième est une méthode relativement nouvelle de localisation à une seule impulsion constituée principalement de la recherche numérique d’une crête dans une fonction de répartition des probabilités. Cette fonction repose sur les principes de Bayes. La quatrième méthode est celle du « pseudo-filtre de Kalman de Bayes », mentionnée pour la première fois dans le présent rapport. Elle consiste à modifier la méthode de Bayes à une seule impulsion de à partir de données existantes tirées d’estimations antérieures qui sont interprétées comme celles d’un filtre de Kalman. Pour cet ensemble de données, sur le plan de la précision de la localisation, les résultats des méthodes bayésiennes sont nettement supérieurs à ceux des méthodes plus traditionnelles.

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