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Formulation of a Mathematical Model for a Small Electric Rotorcraft UAV

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Abstract

In Applied Research Project 12pu, DRDC Suffield is investigating the use of highly manoeuvrable small rotorcraft unmanned aerial vehicles (rUAVs) to provide situational awareness to dismounted soldiers. The rUAV must provide useful information that contributes to improved situational awareness. It must do so while minimizing operational workload and allow the rUAV operator to continue with their primary tasks. Thus, operation of the rUAV must not compromise operator safety but provide battle-space awareness that provides a force multiplier to the dismounted soldier unit. A Draganflyer X8 was selected to carry an autonomy package consisting of a light power-efficient computer, LIDAR, and cameras for the purpose of developing autonomous navigation, obstacle avoidance and landing behaviours. The autonomous performance of the rUAV must increase to meet the project objective. This report presents the reasons for developing a mathematical model of the Draganflyer X8. The resulting mathematical can be used for control systems design and analysis. The mathematical model is used to frame stability and performance criteria to mathematically formulate the real-world control problem governed by the underlying physics and performance specifications. This mathematical representation of the real-world problem will then be used to select control strategies best suited for the Draganflyer X8 in achieving project performance objectives. This paper presents a mathematical model of the Draganflyer X8 from first principles highlighting the complexity involved in accurately representing system dynamics. Two recommendations are made that can be pursued to create an accurate mathematical model for control system design and analysis to achieve flight stability and performance objectives.

Résumé

Dans le cadre du projet de recherche appliqué 12pu, RDDC Suffield étudie l’utilisation de petits véhicules aériens sans pilote à voilure tournante (rUAV) hautement manœuvrables dans le but de donner la connaissance de la situation à des soldats débarqués. Le rUAV doit fournir des renseignements utiles visant à améliorer la connaissance de la situation. Il doit le faire tout en réduisant la charge de travail opérationnelle et en permettant à l'opérateur du rUAV de poursuivre ses tâches principales. Par conséquent, l’emploi du rUAV ne doit pas mettre en péril la sécurité de l’opérateur, mais fournir une connaissance de l’espace de bataille qui constitue un multiplicateur de force pour l’unité débarquée. L’appareil Draganflyer X8 a été choisi pour porter un ensemble d’autonomie comprenant un ordinateur léger à faible consommation énergétique, un LIDAR et des caméras destinés à perfectionner les comportements de navigation autonome, d’évitement des obstacles et d’atterrissage. Le rendement en autonomie du rUAV doit être amélioré pour que l’appareil puisse réaliser l’objectif du projet. Le présent rapport explique ce qui motive l’élaboration d’un modèle mathématique du Draganflyer X8. Le modèle créé peut servir à concevoir et à analyser les systèmes de commande. Ce modèle mathématique permet également de définir des critères de stabilité et de rendement servant à formuler sous forme mathématique le problème de commande constaté dans le monde réel, qui est assujetti aux principes physiques sous-jacents ainsi qu’aux spécifications de rendement. La représentation mathématique du problème réel pourra ensuite être employée pour sélectionner les stratégies de commande les mieux adaptées à l’appareil Draganflyer X8 dans l’intérêt de l’atteinte des objectifs de rendement visés par le projet. Le rapport présente donc un modèle mathématique du Draganflyer X8 partant des principes fondamentaux, modèle qui fait ressortir la complexité d’une représentation fidèle de la dynamique du système. Il renferme aussi deux recommandations susceptibles de faciliter la création d’une modèle mathématique exact aux fins de la conception et de l’analyse de systèmes de commande destinés à assurer la stabilité du vol et la réalisation des objectifs de rendement.

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Executive summary

Formulation of a Mathematical Model for a Small Electric Rotorcraft UAV

M. Trentini; C. Thibault; and H. Li; DRDC Suffield TM 2011-243; Defence R&D Canada – Suffield; December 2011.

Background: DRDC Suffield envisions autonomous systems contributing to decisive operations in the urban battle space. One of these objectives is to build a world representation of the urban battle space to improve soldier situational awareness. This paper focuses on the use of small electric rotorcraft unmanned aerial vehicles (rUAVs) to provide improved soldier situational awareness in complex urban environments. To be useful, rUAVs must be designed to improve autonomous navigation in complex urban environments. This requires increased maneuverability and interaction in complex environments. For example, the rUAV must follow building perimeters while avoiding obstacles such as power lines. This increased performance will allow the operator command the rUAV to navigate freely without constraints in the battlespace.

Results: Modern control methods are better equipped to provide rUAV control laws that operate in the face of plant uncertainty, unmodelled dynamics, external disturbances such as wind gusts while providing multiple performance metrics such as long endurance, high speed, attitude and rate responses to navigate quickly through complex environments with the aid of world representation information. This increased autonomy will reduce operator burden, allow increased endurance and speed, navigation and landing in complex environments to provide a situational awareness asset to the dismounted soldier. These techniques require an accurate mathematical model representation of the rUAV for control system design and analysis. A preliminary mathematical model of the Draganflyer X8 based on fundamental physics from first principles is presented in this paper.

Significance: The formulation of the mathematical model from first principles highlights the complexity of accurately representing the system dynamics. Recommendations are made regarding the formulation of a mathematical model that more accurately represents the Draganflyer X8. Specifically, DRDC Suffield concludes from experience with Parameter Estimation Technology, that it is the best option for producing a mathematical model that fully represents the rUAV flight dynamics.

Sommaire

Formulation d’un modèle mathématique décrivant un petit UAV électrique à voilure tournante

M. Trentini, C. Thibault et H. Li, RDDC Suffield TM 2011-243, R et D pour la défense Canada – Suffield, décembre 2011.

Contexte : RDDC Suffield prévoit que des systèmes autonomes joueront un rôle dans les opérations décisives dans l’espace de bataille urbain. L’un des objectifs visés est l’établissement d’une représentation de l’espace de bataille urbain destinée à améliorer la connaissance de la situation des soldats. Le présent rapport traite de l’utilisation de petits véhicules aériens sans pilote à voilure tournante (rUAV) pour donner une connaissance de la situation améliorée aux soldats dans des environnements urbains complexes. Pour être utiles, les rUAV doivent être conçus en fonction d’une navigation autonome améliorée dans les environnements urbains complexes, ce qui exige une manœuvrabilité accrue et davantage d’interaction dans les environnements complexes. Par exemple, un rUAV doit pouvoir longer le périmètre d’un édifice tout en évitant les obstacles tels que les fils électriques. Le rendement amélioré permettra à l’opérateur de commander les rUAV de façon à ce qu’ils se déplacent sans contrainte dans l’espace de bataille.

Résultats : Les méthodes de commande modernes sont à même de fournir aux rUAV des principes de commande aptes à fonctionner dans des contextes déterminés par l’incertitude, les dynamiques non modélisées et les perturbations externes telles que les rafales de vent tout en offrant de multiples caractéristiques de rendement telles qu’une longue endurance, une grande vitesse, une bonne assiette et une réaction rapide, qui leur permettent d’évoluer rapidement dans des environnements complexes à l’aide d’information constituant une représentation du monde. L’autonomie accrue réduira la charge de travail des opérateurs, augmentera l’endurance et la vitesse, et améliorera la navigation et l’atterrissage dans des environnements complexes. Tout cela contribuera à faire des rUAV une ressource fournissant la connaissance de la situation aux soldats débarqués. Pour y parvenir, il faut disposer d’un modèle mathématique qui représente de façon exacte le rUAV et qui oriente la conception et l’analyse du système de commande. Le présent rapport propose un modèle mathématique préliminaire du Draganflyer X8 fondé sur les principes physiques fondamentaux.

Importance : La formulation d’un modèle mathématique à partir des principes de base fait ressortir la complexité de la représentation exacte de la dynamique du système. Le rapport contient des recommandations concernant la formulation d’un modèle mathématique qui représentera plus fidèlement l’appareil Draganflyer X8. Enfin, RDDC Suffield conclut de son expérience de la technologie d’estimation de paramètres qu’il s’agit là de la meilleure option pour produire un modèle mathématique représentant de façon intégrale la dynamique de vol d’un rUAV.

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1 Introduction

The Autonomous Intelligent Systems Section (AISS) at Defence R&D Canada – Suffield (DRDC Suffield) envisions autonomous systems contributing to decisive operations in the urban battle space. In this vision, teams of unmanned ground, aerial, and marine vehicles (UGVs, UAVs, and UMVs) will gather and coordinate information, formulate plans, and complete tasks. In this scenario, smaller more highly manoeuvrable UAVs may be employed to build world representation models of the urban battle space to improve soldier situational awareness. Current tactical UAVs have been limited to high altitude overwatch which avoids obstacles, reduces operator burden, but limits the information that these systems are able to provide. In Applied Research Project 12pu, AISS is investigating the use of highly manoeuvrable small rotorcraft unmanned aerial vehicles (rUAVs) for their ability to fly low and slow. It is this capability that we wish to exploit by flying into complex urban environments to provide useful information that contributes to improved situational awareness. It must do so while minimizing operational workload and allow the rUAV operator to continue with their primary tasks. Thus, operation of the rUAV must not compromise operator safety but provide battle-space awareness that provides a force multiplier to the dismounted soldier unit. A Draganflyer X8 was selected to carry an autonomy package consisting of a light power-efficient computer, LIDAR, and cameras for the purpose of developing autonomous navigation, obstacle avoidance and mapping capabilities. The autonomous performance of the rUAV must be increased to meet the project objective. This requires operation in close proximity to obstacles in the environment for the purpose of navigating city blocks.

Control strategies implemented by commercial-off-the-shelf rUAVs typically use simple heuristic based controllers, such as PID control in the case of the Draganflyer X8 in benign environments. However, these types of controllers do not provide robust stability to wind gusts and varying sensor payload weights and mounting configurations, while maintaining and high performance specifications such as pitch, roll, yaw attitude responses or vertical and horizontal translation speeds for navigation in cluttered environments such as buildings with power lines. This report presents the reasons for developing a mathematical model of the Draganflyer X8. The resulting mathematical model can be used for control systems design and analysis. The mathematical model is used to frame stability and performance criteria to mathematically formulate the real-world control problem governed by the underlying physics and performance specifications. This mathematical representation of the real-world problem will then be used to select control strategies best suited for the Draganflyer X8 in achieving project performance objectives.

Modern control methods are better equipped to provide rUAV control laws that operate in the face of plant uncertainty, unmodelled dynamics, and external disturbances such as wind gusts [1]. These techniques require a mathematical model of the rUAV for control system design and analysis. The preliminary mathematical model of the Draganflyer X8 derived from first principles is formulated and presented in this paper. Upon formulation, it is evident that alternative methods need to be investigated if high fidelity models are the desired result. Recommendations are made for techniques that can be pursued to achieve more accurate models that more readily capture all system information.

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2 Draganflyer X8 rotorcraft

The rotorcraft selected for use in ARP 12pu is the Draganflyer X8, as shown in Figure 1. Its physical attributes are presented in this section to be used for mathematical model formulation in the following section of the paper.

Figure 1: Draganflyer X8.

Table 1: Draganflyer X8 specifications.

Width	87cm (36.25 in)
Length	87cm (36.25 in)
Top diameter	l06cm (41.8 in)
Height	32cm (12.6 in)
Helicopter weight	1700 g (60 oz)
Payload capability	1000 g (35 oz)

2.1 Actuators

The Draganflyer X8 uses eight motors, whose collective throttle is the sum of the thrusts from each motor. Pitch is achieved by increasing (or reducing) the speed of the rear motors while reducing (or increasing) the speed of the front motors. Roll is obtained similarly using the lateral motors. Yaw is produced by increasing (or decreasing) the speed of the upper motors while decreasing (or increasing) the speed of the lower motors. The total thrust is kept constant.

Rotor blade parameters are listed below [2]:

Table 2: Draganflyer X8 rotor specifications.

Upper rotor diameter	40cm (16in)
Lower rotor diameter	38cm (15in)

2.2 Sensors

The following sensors are found on the Draganflyer X8 [2]:

- a three-axis solid state microelectromechanical Systems (MEMS) gyro
- a three-axis solid state MEMS accelerometer
- three magnetometers (magnetoresistive sensors)
- a barometric pressure sensor
- a GPS Receiver

GPS is used for outdoor navigation and does not provide any internal state measurements to the rotorcraft.

3 Mathematical model for control system design and analysis

Most rUAVs have used classical control methods based on empirical tuning methods for flight control. However, to exploit the benefits of modern control methods, an accurate model of the rUAV dynamics are required. In modern control design methods the first step is an adequate mathematical model representation of the system to be controlled. The mathematical model facilitates control system design and analysis of the rUAV. The control, analysis and simulation tools offered from the mathematical model development will allow for robust stability and performance of the rUAV to be tested offering improved insight to real world performance. This paper presents the first steps in developing the required linear state space representation for modern control system design.

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4 Deriving the mathematical model of the Draganflyer X8

In control theory, knowledge about the dynamic behaviour of a given system can be acquired through its states. For a quadrotor, its attitude about all 3 axes of rotation is known with 6 states: the Euler angles roll, pitch and yaw denoted as [φ, θ, ψ], and the angular velocities around each axis of the body fixed reference [p, q, r]. Yet another 6 states are necessary: the position of the centre of gravity [x, y, z] and respective linear velocity components [u, v, w] relative to the fixed frame. In sum, the quadrotor has 12 states that describe its 6 degrees of freedom (DOF).

4.1 Kinematics equations

When analyzing the motion of aerial vehicles in 6 DOF, two coordinate frames can be defined. The moving coordinate frame is conveniently fixed to the vehicle and is called the body fixed reference. The motion of the body fixed frame is described relative to the inertial reference frame. The earth fixed reference frame can be considered to be inertial [3]. The coordinate systems and the rotorcraft diagram are shown in Figure 2. The inertial frame is the world frame, defined as the North–East–Down frame. The body fixed frame is attached to the centre of gravity of the rotorcraft, with x_B aligned with the forward direction and z_B pointing downward; the subscript B represents the body frame. Rotors 1 to 4 are the upper rotors while rotors 5 to 8 are the lower rotors. The corresponding force and moment of each motor are represented as F and M respectively. The centre of rotors 1 and 5 is on the positive x_B axis. The centre of rotors 4 and 8 is on the positive y_B axis. The centre of rotors 2 and 6 is on the negative y_B axis. The centre of rotors 3 and 7 is on the negative x_B axis.

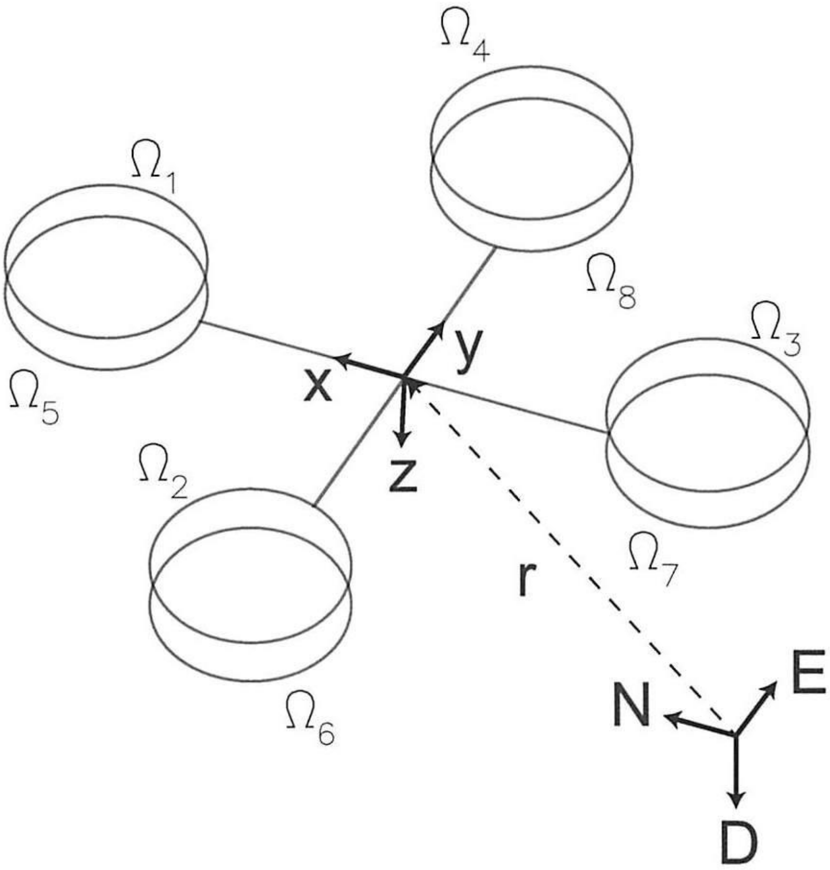

Figure 2: Coordinate frames and force/moments.

The general motion of a vehicle in 6 DOF can be described by

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T]^T, \tag{1}$$

$$\boldsymbol{\nu} = [\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^T]^T, \tag{2}$$

where

$$\boldsymbol{\eta_1} = [x, y, z]^T, \tag{3}$$

$$\boldsymbol{\eta_2} = [\phi, \theta, \psi]^T, \tag{4}$$

$$\boldsymbol{\nu}_1 = [u, v, w]^T, \tag{5}$$

$$\boldsymbol{\nu}_2 = [p, q, r]^T, \tag{6}$$

in which $\boldsymbol{\eta}$ denotes the position and orientation vector with coordinates in the earth fixed frame, $\boldsymbol{v}$ denotes the linear and angular velocity vector with coordinates in the body fixed frame.

The vehicle's path relative to the earth fixed coordinate system is given by

$$\dot{\boldsymbol{\eta}}_1 = \boldsymbol{J}_1(\boldsymbol{\eta}_2)\boldsymbol{v}_1 \tag{7}$$

where

$$\boldsymbol{J}_1(\boldsymbol{\eta}_2) = \begin{bmatrix} \cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\varphi) + \cos(\psi)\sin(\theta)\sin(\varphi) & \sin(\psi)\sin(\varphi) + \cos(\psi)\cos(\varphi)\sin(\theta) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\varphi) + \sin(\varphi)\sin(\theta)\sin(\psi) & -\cos(\psi)\sin(\varphi) + \sin(\theta)\sin(\psi)\cos(\varphi) \\ -\sin(\theta) & \cos(\theta)\sin(\varphi) & \cos(\theta)\cos(\varphi) \end{bmatrix} \tag{8}$$

The body fixed angular velocity vector $\boldsymbol{v}_2 = [p, q, r]^T$ and the Euler rate vector $\dot{\boldsymbol{\eta}}_2 = [\dot{\varphi}, \dot{\theta}, \dot{\psi}]^T$ are related through a transformation matrix $\boldsymbol{J}_2(\boldsymbol{\eta}_2)$ according to

$$\dot{\boldsymbol{\eta}}_2 = \boldsymbol{J}_2(\boldsymbol{\eta}_2)\boldsymbol{v}_2 \tag{9}$$

Where

$$\boldsymbol{J}_2(\boldsymbol{\eta}_2) = \begin{bmatrix} 1 & \sin(\psi)\tan(\theta) & \cos(\varphi)\tan(\theta) \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi)/\cos(\theta) & \cos(\varphi)/\cos(\theta) \end{bmatrix} \tag{10}$$

4.2 Simplified Dynamics Equations

A simplified dynamic model is presented here to at least present the equations to the reader. Many simplifications are made, which is standard in some approaches where higher modelling fidelity is deemed only necessary after evaluating system performance. With respect to quadrotors it is often assumed that the blades of the rotors are considered to hinge directly from the hub, such that no hinge offset associated with rotor flapping is possible. Aerodynamic forces generated by relative wind and ground effects are ignored. Moreover, effects of the aerodynamic forces between rotor blades are not taken into account.

In order to obtain the dynamic equations, aerodynamic forces are separated into two groups. The first is composed of translational forces and the second is related to the rotational forces of motion.

The dynamics of the rotorcraft are given by

$$\begin{aligned} \boldsymbol{F} &= mg\boldsymbol{e}_D + \Sigma_{i=1}^{8} - \boldsymbol{F}_i, \\ \boldsymbol{F} &= m\ddot{\boldsymbol{r}}, \\ \boldsymbol{M} &= \Sigma_{i=1}^{8}(\boldsymbol{M}_i + \boldsymbol{r}_i \times (-\boldsymbol{F}_i)), \\ \boldsymbol{M} &= I_b\dot{\boldsymbol{\nu}}_2 + \Omega I_b \boldsymbol{\nu}_2, \end{aligned} \tag{11}$$

where ***F*** and ***M*** represent the total force and total moment, respectively; *m* represents the mass of the rotorcraft; $\boldsymbol{e}_D$ represents the down direction; and $\boldsymbol{r}_i$ denotes the centre of each pair of rotors in the body frame. The position vector of the center of gravity in the reference frame is denoted by ***r***. I_b represents the inertia of the rotorcraft; Ω represents the cross-product matrix for $\mathbf{v}_2$, while $\mathbf{v}_2$. represents the angular velocity in the body frame.

Since the rotation matrix for transforming the coordinates from the body frame to the reference frame is Ji, the equation governing the translation of the center of mass is

$$m\ddot{\boldsymbol{r}} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + \boldsymbol{J_1} \begin{bmatrix} 0 \\ 0 \\ \Sigma_{i=1}^{8} - F_i \end{bmatrix} \tag{12}$$

The equation governing the rotation of the center of mass is

$$I_b \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4 + F_6 - F_8) \\ L(F_1 - F_3 + F_5 - F_7) \\ -\Sigma_{i=1}^{4} M_i + \Sigma_{j=5}^{8} M_j \end{bmatrix} - \Omega I_b \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{13}$$

4.3 Simplified motor model

Each rotor has an angular speed ω_i and produces a vertical force F_i according to

$$F_i = k_F\,\omega_i^2 \tag{14}$$

The rotors also produce a moment according to

$$M_i = k_M\, \omega_i^2 \tag{15}$$

where k_F and k_M are coefficients of aerodynamic thrust and rotor load, respectively. Due to the independent action of the upper and lower motors, there are actually two sets of values: k_{Fu} and k_{Mu}, as well as k_{Fl} and k_{Ml}.

The rotor speed response is a first order system modelled as $k/(s + \tau)$, where τ is the time constant.

Although quadrotor vehicle dynamics are often assumed to be accurately modeled as linear for attitude and altitude control, this assumption is only reasonable at slow velocities. Even at moderate velocities, the impact of the aerodynamic effects resulting from variation in air speed is significant [4]. Proper characterization of the simplified motor model equations requires bench testing to provide appropriate values for coefficients, time constants, and time response. Proper characterization of aerodynamic effects requires the construction of a thrust test stand using a load cell to properly measure thrust, side force, and torque. This is a time consuming and labour intensive endeavour.

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5 Recommendations for mathematical model development

Accurate mathematical models of these rotorcraft are not readily available and the level of effort to create them from first principles and augmenting with real world data is a most time consuming effort. The challenge of this approach should be evident from Section 4 based on the fundamental physics required to understand and describe all of the forces and moments acting on the system. Simplifications were identified in Sections 4.2 and 4.3 in a first attempt to construct a mathematical model of the Draganflyer X8. This is a common approach in the robotics community where the time and resources are not available to invest in a proper model.

There are two recommended options that are available and should be pursued to overcome the challenges described above. In the subsections below, two ways ahead are proposed to produce a mathematical model that will allow for modern control methods to advance the performance of rUAVs.

5.1 System identification techniques

Development of a mathematical model for rotorcraft can also be achieved based on system identification similar to [5]. The paper describes the development of a parameterized model for a small-scale unmanned helicopter (Yamaha R-50 with 10 ft rotor diameter) and its identification using a frequency domain identification technique. The accuracy of the identified model is verified by comparing the model-predicted responses with the responses collected during flight experiments. Furthermore, the values of key identified parameters are compared with the values predicted by helicopter theory to show that the model has a physically meaningful parameterization.

5.2 Parameter Estimation Technology

In 1990, National Research Council Canada first applied its expertise in parameter estimation (PE) technology to develop simulator mathematical models for commercial clients. Since then, NRC has performed over 20 flight test data gathering programs for simulator model development for both fixed-wing and rotary-wing aircraft. NRC is able to perform parameter identification on the Draganflyer X8, and provide a simulation mathematical model in the desired state-space format for modern control system design and analysis [6].

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6 Conclusions

Modern control methods are well suited to provide improved rUAV performance over heuristic based controllers especially when subjected to external disturbances such as wind gusts. These techniques require an accurate mathematical model representation of the rUAV for control system design and analysis. A preliminary mathematical model of the Draganflyer X8 based on fundamental physics from first principles is presented in this paper. The formulation highlights the complexity of accurately representing the underlying dynamics of the system. Recommendations are made to pursue methods that are better able to capture rotorcraft dynamics and represent them in a full state-space mathematical model that allows access to modern control design methods. Specifically, DRDC Suffield experience with Parameter Estimation Technology would provide a high fidelity mathematical representation that fully describes the rUAV flight dynamics.

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In Applied Research Project 12pu, DRDC Suffield is investigating the use of highly manoeuvrable small rotorcraft unmanned aerial vehicles (rUAVs) to provide situational awareness to dismounted soldiers. The rUAV must provide useful information that contributes to improved situational awareness. It must do so while minimizing operational workload and allow the rUAV operator to continue with their primary tasks. Thus, operation of the rUAV must not compromise operator safety but provide battle-space awareness that provides a force multiplier to the dismounted soldier unit. A Draganflyer X8 was selected to carry an autonomy package consisting of a light power-efficient computer, LIDAR, and cameras for the purpose of developing autonomous navigation, obstacle avoidance and landing behaviours. The autonomous performance of the rUAV must increase to meet the project objective. This report presents the reasons for developing a mathematical model of the Draganflyer X8. The resulting mathematical can be used for control systems design and analysis. The mathematical model is used to frame stability and performance criteria to mathematically formulate the real-world control problem governed by the underlying physics and performance specifications. This mathematical representation of the real-world problem will then be used to select control strategies best suited for the Draganflyer X8 in achieving project performance objectives. This paper presents a mathematical model of the Draganflyer X8 from first principles highlighting the complexity involved in accurately representing system dynamics. Two recommendations are made that can be pursued to create an accurate mathematical model for control system design and analysis to achieve flight stability and performance objectives.

Dans le cadre du projet de recherche appliqué 12pu, RDDC Suffield étudie l’utilisation de petits véhicules aériens sans pilote à voilure tournante (rUAV) hautement manœuvrables dans le but de donner la connaissance de la situation à des soldats débarqués. Le rUAV doit fournir des renseignements utiles visant à améliorer la connaissance de la situation. Il doit le faire tout en réduisant la charge de travail opérationnelle et en permettant à l'opérateur du rUAV de poursuivre ses tâches principales. Par conséquent, l’emploi du rUAV ne doit pas mettre en péril la sécurité de l’opérateur, mais fournir une connaissance de l’espace de bataille qui constitue un multiplicateur de force pour l’unité débarquée. L’appareil Draganflyer X8 a été choisi pour porter un ensemble d’autonomie comprenant un ordinateur léger à faible consommation énergétique, un LIDAR et des caméras destinés à perfectionner les comportements de navigation autonome, d’évitement des obstacles et d’atterrissage. Le rendement en autonomie du rUAV doit être amélioré pour que l’appareil puisse réaliser l’objectif du projet. Le présent rapport explique ce qui motive l’élaboration d’un modèle mathématique du Draganflyer X8. Le modèle créé peut servir à concevoir et à analyser les systèmes de commande. Ce modèle mathématique permet également de définir des critères de stabilité et de rendement servant à formuler sous forme mathématique le problème de commande constaté dans le monde réel, qui est assujetti aux principes physiques sous-jacents ainsi qu’aux spécifications de rendement. La représentation mathématique du problème réel pourra ensuite être employée pour sélectionner les stratégies de commande les mieux adaptées à l’appareil Draganflyer X8 dans l’intérêt de l’atteinte des objectifs de rendement visés par le projet. Le rapport présente donc un modèle mathématique du Draganflyer X8 partant des principes fondamentaux, modèle qui fait ressortir la complexité d’une représentation fidèle de la dynamique du système. Il renferme aussi deux recommandations susceptibles de faciliter la création d’une modèle mathématique exact aux fins de la conception et de l’analyse de systèmes de commande destinés à assurer la stabilité du vol et la réalisation des objectifs de rendement.

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unmanned aerial vehicle; helicopter; model; stability; flight control algorithms