# Classification and anomaly detection algorithms for weak hyperspectral signal processing.

*Pierre Lahaie*

DRDC Valcartier, 2459 De la Bravoure Road, Quebec, Qc, Canada
Pierre.lahaie@drdc-rddc.gc.ca

## ABSTRACT

In applications involving weak light signal like hyperspectral or time distributed signals obtained in applications involving laser induced fluorescence spectral detection, fluorescence lifetime imaging, Raman Spectroscopy or hyperspectral imaging in low light environment, the photons arrive at such a rate that they can be counted or have to be intensified to obtain a usable signal. Detection and classification algorithms need to be designed and evaluated for weak hyperspectral signal processing. A new algorithm, Adaptive Shot Noise (ASN) based on the assumption that a signal respects the Poisson multivariate distribution has been developed using the method of the maximum likelihood. This algorithm demonstrates the capability to be used for detection and classification. Using Monte Carlo simulations its performances are compared with the Adaptive Coherence Estimator (ACE) classification and with an Integrated Signal Algorithm (ISA) and ACE for detection. This new algorithm provides a small increase in performance compared to ACE in very weak signal conditions for classification and in some conditions better performance over both ACE and ISA in detection. The algorithm behavior like ACE shows sensitivity to assumption on the spectral characteristics of the source for the detection, which is not the case for ISA.

***Index Terms—*** Detection, classification, algorithm, hyperspectral.

## 1. INTRODUCTION

When a spectral signal is very weak, photons can be counted, its natural distribution is a Poisson multivariate. This contrasts with the assumption that the signal is elliptically distributed and respects distributions that are multivariate Student or Gaussian [1,2,3,4,5,6]. The development of sensors able to count photons is raising the issue if detection and classification algorithms developed under the elliptically symmetric distributions are still good enough or if algorithms complying with other assumptions are required. Two kinds of sensors are interesting: photon counting and intensified detectors. Photon counting devices can be SPAD (Single photon avalanche photodiodes) or PM (photo multiplier tubes) arrays while intensified detectors can be ICCD (Intensified Charge Coupled Device), PM or avalanche diodes operated in analog mode. The detection and classification problems are discussed; a new algorithm ASN (Adaptive Shot Noise) is elaborated based on a simple signal model. Results are produced using Monte Carlo simulation to compare the performances of the developed algorithm with those of the Adaptive Coherent Estimator (ACE) [2].

## 2. THE SIGNAL MODEL

The signal received with time is a linear combination of a stationary background shown in figure 1 and of a contamination signal varying in intensity. The contamination is assumed to have a specific spectral shape and is member of a database shown in figure 2. Equation (1) gives the signal model where $r(\lambda)$ is the mean of the measurement with respect with the spectral index $\lambda$. $x(\lambda)$ is the mean of the background, $s(\lambda)$ is the spectrum of the signature and $a$ is the mean of its intensity (the signal strength).

$$r(\lambda) = x(\lambda) + as(\lambda) \qquad (1)$$

The variance (2) is the same as the mean because a Poisson distribution is assumed.

$$\sigma^2(\lambda) = x(\lambda) + as(\lambda) \qquad (2)$$

It is assumed that there is no correlation between the bands since photons are considered to be produced independently. These spectra are produced with 32 bands. In figures 1 and 2, the sum of all the functions is 1. The intensity in terms of photons are called the background strength $b$ and the signal strength $a$.

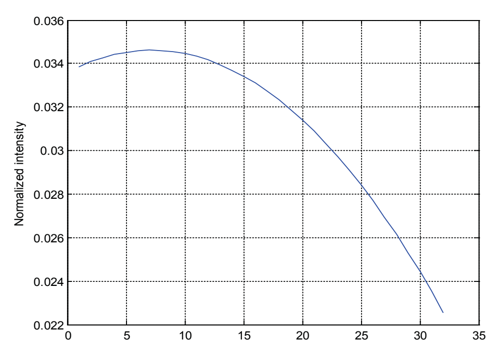


Figure 1: Mean of the background

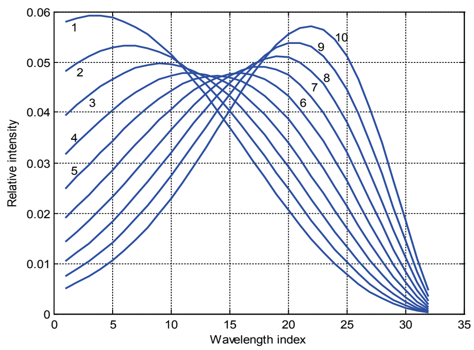

Figure 2: The signature database

The signal model for the photon counting sensor is simply a multivariate Poisson distribution for photon arrival. In the case of intensified sensors, each photon triggers a pulse of electrons which is modeled using an exponential distribution of counts for each detected photon [8].

## 3. EXISTING ALGORITHMS FOR DETECTION AND CLASSIFICATION

To tell if a signal has departed from the background level, a detection algorithm is required. It does not identify the nature of the source. Typically, a parameter related to the signal level is monitored. The mean and the variance of this parameter are used to compute a threshold corresponding to a given false alarm rate. The algorithm's performance is evaluated using the probability of detection against the probability of false alarm in specified operating conditions which is ROC curve (Receiver operating characteristic). Equation (3) and (4) respectively give the expressions for the intensity estimation of ACE and for the integrated signal algorithm [9].

$$a = \frac{\boldsymbol{s}\Sigma^{-1}(\boldsymbol{r}-\boldsymbol{\mu})}{\boldsymbol{s}\Sigma^{-1}\boldsymbol{s}} \tag{3}$$

$a$ is the estimation of the signal strength; $\boldsymbol{\mu}$ is the mean of the background; $\Sigma^{-1}$ is the inverse of a covariance matrix; $\boldsymbol{r}$ is a measurement under test and $\boldsymbol{s}$ is the signature of one of the member of the database. Expression (3) is valid for other algorithms such as ALS (Adaptive Least Square) and AMF (Adaptive matched filter) [9]. In (4) $w$ is the estimator for the intensity of the signal and $\mathbf{1}$ is a vector filled with one. It stands for ISA (Integrated Signal Algorithm).

$$w = \mathbf{1}.\boldsymbol{r} \tag{4}$$

The ACE classification algorithm is expressed by (5).

$$D_{ACE} = \left(\frac{\left(\boldsymbol{s}\Sigma^{-1}(\boldsymbol{r}-\boldsymbol{\mu})\right)^2}{(\boldsymbol{s}\Sigma^{-1}\boldsymbol{s})\left((\boldsymbol{r}-\boldsymbol{\mu})\Sigma^{-1}(\boldsymbol{r}-\boldsymbol{\mu})\right)}\right) \tag{5}$$

Since there is no correlation between bands the covariance matrix is replaced by the unity matrix.

## 4. NEW ALGORITHM FOR SHOT NOISE

ACE results from a particular consideration of the multivariate Student distribution. It is interpreted as the square of the cosine of the angle between the measurement and the signature vectors. The distribution for the shot noise is the Multivariate Poisson. The likelihood function is established with consideration that there is no correlation between the bands. The two assumptions are $H_0$: There is no contamination over the background, and therefore the mean for each channel is given by the following equation $\alpha_i = \langle m_i \rangle = x_i = \mu_i$ and $H_1$: There is a contamination that increases the number of photons. The mean of the number of photons to be measured in each channel is given by the following equation: $\alpha_i = \langle m_i \rangle = \mu_i + as_i$. In which, $m_i$ is the measurement in the $i^{\text{th}}$ band. The Poisson distributions corresponding to the two assumptions are for $H_0$ and $H_1$:

$$H_0: P_0(m_i) = \frac{\exp(-\mu_i)\mu_i^{m_i}}{m_i!} \tag{6}$$

$$H_1: P_1(m_i) = \frac{\exp\left(-(\mu_i + as_i)\right)(\mu_i + as_i)^{m_i}}{m_i!} \tag{7}$$

The likelihood ratio function is given by (8) considering that $\sum s_i = 1$.

$$R = \frac{\zeta(H1)}{\zeta(H0)} = \frac{\prod_i (\mu_i + as_i)^{m_i}}{\prod_i \mu_i^{m_i}} \exp(-a) \tag{8}$$

This function is the product of a monotonic increasing function (the polynomial) with a decreasing exponential. It therefore has a single maximum and the position of that maximum does not change if we take the logarithm.

$$\ln(R) = \sum_i m_i\left(\ln(\mu_i + as_i) - \ln(x_i)\right) - a \tag{9}$$

The derivative of (9) with respect to $a$ is:

$$\frac{\mathrm{d}[\ln(R)]}{da} = \sum_i \frac{m_i s_i}{\mu_i + as_i} - 1 = 0 \tag{10}$$

An iterative procedure is required to find the value of $a$ that satisfies (10). The value $a$ provides the parameter used in the detections algorithm. The classification algorithm uses (9) with the value $a$ inserted. The maximum value of (9) with respect to the signature index yields the closest signature to the signal. This algorithm is ASN.

## 5. MONTE CARLO SIMULATIONS

Due to the complexity of the problem, the algorithms for detection and classification are tested using Monte Carlo simulations. It is a standard procedure when there are models for the sources, the backgrounds and the sensors, but there is not enough real data or real data cannot be characterized thoroughly. In many case, the parameter that is difficult to characterize experimentally is $a$: the signal strength. The simulation process is the same method as what has been used in [9]. At first we select the average number of photons contained in the background and the average number of photons contained in the signature which is the signal strength. The average number of photons in each band is computed for a background only signal and then for a signal containing the signature with a given signal strength (the contamination). Random numbers respecting the Poisson distribution are generated for the background and for the contamination. To account for intensified signal, a random number respecting the exponential distribution with the selected gain is generated for each photon and they are added together. We therefore have two streams of data one for the background that is used for computing the probability of error and the contaminated signal used to estimate the probability of detection and the probability of correct classification and of classification error, i.e. the confusion matrix

## 6. RESULTS

### 6.1. Anomaly detection results

A set of data has been produced to compare the detection capability of each of the algorithms given by equation (3), (4) and (10). To produce the ROC curves, we used a background strength of 100 photons and variable signature strength. We show results in figure 3 and 4 for the case of 70 photons of signature strength to compare the performance of the detectors. Figures 3 and 4 give the probability of detection when the probability of false alarm is $10^{-4}$ for the photon sensor and for the intensified sensor. They display the results of the adaptive detectors for all the signatures and for the Integrated Signal detector. The adaptive detectors using the $5^{th}$ signature of figure 2 provide the best results. The ASN detector is almost always better than ACE by a small margin. The ISA provides a very good compromise in the case of the photon sensor and a good choice for the intensified sensor. When the signal due to the signature decreases, the performance of the detectors decreases. To produce the results of figure 3 and 4 we simulated $10^6$ signals for each of the signatures.

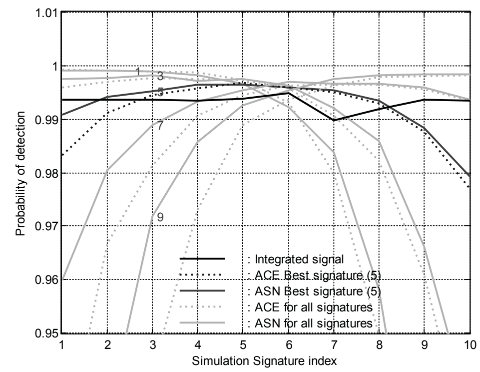


Figure 3: Probability of detection for the three detectors and for signatures (1, 3, 5, 7, 9) used in ACE and ASN detectors as function of the signature used in simulation used with photon counting sensor

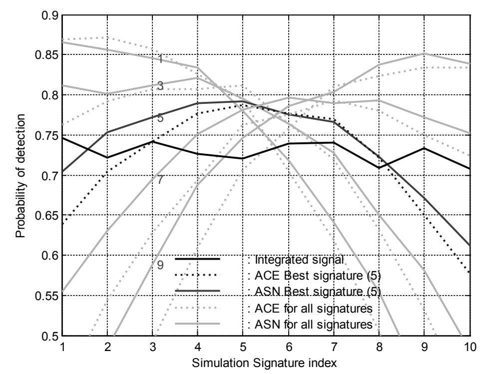


Figure 4 Probability of detection for the three detectors and for signatures (1, 3, 5, 7, 9) used in ACE and ASN detectors as function of the signature used in simulation used with Intensified sensor

### 6.2. Classification results

The classification results are shown in figures 5 to 8. They show the probability of error in classification as a function of signal level. The simulation size for each signal strength and signature is $10^5$. The results are provided for each of the signatures in a given situation. We can see that with low signal, the probability of error is smaller for the signatures that are on the edges of the database. This is due to the fact that there is no neighbor on one side to make an error. The plots in figure 5 are for the photon counter sensor and in figure 6 it is for the intensified sensor. The left side plots are the results of ACE and on the right side they are the results for ASN. For both sensors, ASN provides a better classification results than ACE, however the increase in performance is marginal. For the sensor comparison, the performance for both ACE and ASN are better for the photon sensor than for the intensified sensor. For the same probability of classification the intensified sensor seems to require two times more signal than the photon counter sensor.

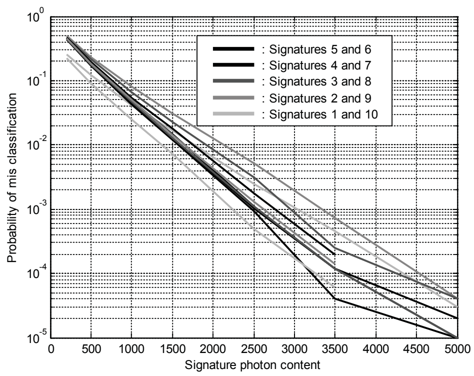


Figure 5: Classification results for ACE with photon counting sensor.

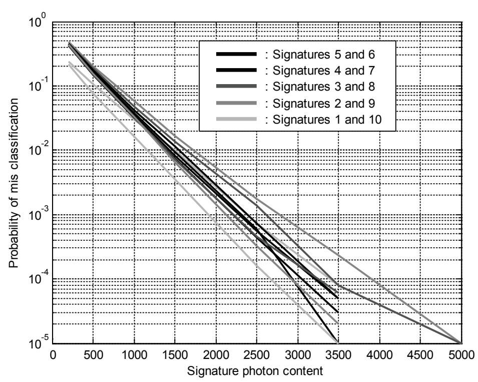


Figure 6: Classification results for ASN with photon counting sensor.

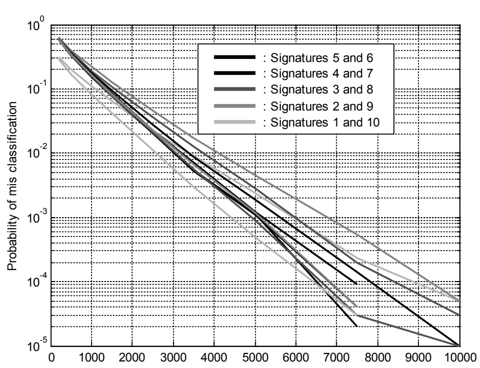


Figure 7: Classification results for ACE with intensified sensor.

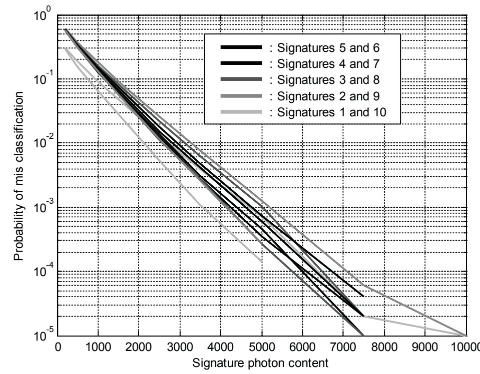


Figure 8: Classification results for ASN with intensified sensor.

## 7. CONCLUSION

In this paper, we propose a new algorithm for the classification of spectral signal composed of photons. One of the questions that we wanted to answer is whether or not a detector based on the maximum likelihood for Poisson multivariate distribution is better than conventional detectors used in hyperspectral detection and classification.
We developed a maximum likelihood detector called ASN (Adaptive Shot Noise) and we compared it with the Adaptive coherence estimator (ACE) for classification and with ACE and ISA for detection. The results show a small advantage for ASN compared to ACE. ACE in this study does not use a filter to reduce the impact of noise and especially noise from the background. This probably account partly for the differences between the two classifiers. ASN also shows an advantage compared to ACE with the two kinds of sensors, the photon counting sensor and the intensified sensor. In the case of anomaly detection the best detector is again ASN but specifically when the signature used in the detector is close to the signature of the source of the signal. When the signature of the source of the signal diverges from the signature used in the detector, ISA provides better performances than ASN and ACE. These conclusions are valid for the two types of sensors.
The detectors and classifiers studied in this paper will be particularly useful when the signal is very weak and where photon shot noise is the predominant noise source. These situations happen in spectral sensing of laser induced fluorescence or of Raman Effect. Application can also be found in hyperspectral sensing when the signal has a very low intensity or in fluorescence lifetime detection.

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