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Hyperspace Aperture Concept for Auroral Clutter Control in a Canadian Over-the-Horizon Radar

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Abstract

Under certain conditions, it is shown that adaptive arrays with up to six spatial dimensions (three on transmit and three on receive) can provide vast improvements in auroral-zone clutter suppression capability compared to one-dimensional adaptive receive arrays. The required conditions are that the array manifold vector and its associated clutter covariance matrix are factorable into the n-fold Kronecker product of the corresponding quantities for a one-dimensional linear array. When these conditions are satisfied, the overall system signal-to-clutter ratio (SCR) improvement scales with the nth power of the SCR improvement of a one-dimensional linear adaptive array. Some supporting experimental results are provided.

Résumé

Dans certaines conditions, il est possible de démontrer que l'utilisation de réseaux adaptatifs ayant jusqu'à six dimensions spatiales (trois à l'émission et trois à la réception) peut mener à des améliorations substantielles de la capacité d'élimination du fouillis en zone aurorale par rapport aux réseaux récepteurs adaptatifs à une dimension. Les conditions nécessaires sont que le vecteur de variété du réseau et sa matrice de covariance puissent être factorisés en un produit de Kronecker de puissance n des quantités correspondantes pour un réseau linéaire unidimensionnel. Lorsque ces conditions sont satisfaites, le rapport signal/fouillis global du système croît selon la n^e puissance de l'augmentation du rapport signal/fouillis d'un réseau adaptatif linéaire unidimensionnel. Des résultats expérimentaux sont donnés à lappui.

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Executive summary

Hyperspace Aperture Concept for Auroral Clutter Control in a Canadian Over-the-Horizon Radar

Ryan J. Riddolls; DRDC Ottawa TM 2012-159; Defence R&D Canada – Ottawa; December 2012.

Background: High Frequency Over-the-Horizon Radar (OTHR) provides a means to detect, locate, and track aircraft to a range of 4,000 km by using the bottom of the earth's ionosphere as a mirror to look beyond the earth's horizon. Previous efforts to operate OTHR in Canada were challenged by the problem of radar clutter caused by Bragg scatter from plasma irregularities in the auroral ionosphere. This radar clutter can mask radar echoes from aircraft targets. Sampled receive apertures and waveform-diverse transmit apertures provide adaptive beamforming opportunities that were not available for previous auroral OTHR efforts. While receive element sampling has been in common use for many years, adaptive transmit beamforming has only been studied more recently in the context of OTHR. Adaptive transmit beamforming is realized by the emission of distinguishable waveforms from different elements of a transmit array consisting of antennas spaced on the order of one-half of the radar wavelength. This is referred to as a Multiple-Input Multiple-Output (MIMO) radar architecture.

Results: Under certain conditions, it is shown that adaptive arrays with up to six spatial dimensions (three on transmit and three on receive) can provide vast improvements in auroral-zone clutter suppression capability compared to one-dimensional adaptive receive arrays. The required conditions are that the array manifold vector and its associated clutter covariance matrix are factorable into the n-fold Kronecker product of the corresponding quantities for a one-dimensional linear array. When these conditions are satisfied, the overall system signal-to-clutter ratio (SCR) improvement scales with the nth power of the SCR improvement of a one-dimensional linear adaptive array. Some supporting experimental results are provided.

Significance: A MIMO radar architecture can improve the SCR of the system in clutter-limited detection scenarios, which correspond to the OTHR auroral clutter case in Canada.

Future Work: Future work will show the operation of joint elevation-azimuth beamforming simultaneously on both transmit and receive. Furthermore, the hyperspace aperture concept can be extended to six dimensions by constructing three-dimensional arrays on transmit and receive that comprise the six-fold Kronecker product of linear transmit and receive subarrays.

Sommaire

Hyperspace Aperture Concept for Auroral Clutter Control in a Canadian Over-the-Horizon Radar

Ryan J. Riddolls; DRDC Ottawa TM 2012-159; R & D pour la défense Canada – Ottawa; décembre 2012.

Introduction : Le radar haute fréquence transhorizon fournit un moyen de détecter, de localiser et de poursuivre des aéronefs à des distances pouvant atteindre 4000 km en se servant de la base de l'ionosphère terrestre comme d'un miroir pour voir au-delà de l'horizon. Des efforts pour exploiter un radar transhorizon au Canada ont été déjoués par le problème du fouillis radar produit par la diffusion de Bragg sur les irrégularités plasmatiques de l'ionosphère aurorale. Ce fouillis peut cacher les échos radars d'aéronefs. Des ouvertures de réception à échantillonnage et des ouvertures d'émission à diversité de forme d'onde offrent des possibilités de formation de faisceau qui n'étaient pas disponibles aux précédents projets de radar transhorizon en zone aurorale. L'échantillonnage des éléments récepteurs est couramment utilisé depuis de nombreuses années, mais la formation de faisceau adaptative à l'émission n'a été étudiée que récemment dans le contexte du radar transhorizon. La formation de faisceau adaptative à l'émission est faite en émettant des formes dondes pouvant être distinguées à partir de différents éléments d'un réseau émetteur constitué d'antennes ayant un espacement de l'ordre de la moitié de la longueur d'onde du radar. Il s'agit de ce qu'on appelle une architecture radar entrée multiple sortie multiple (MIMO).

Résultats : Dans certaines conditions, il est possible de démontrer que l'utilisation de réseaux adaptatifs ayant jusqu'à six dimensions spatiales (trois à l'émission et trois à la réception) peut mener des améliorations substantielles de la capacité d'élimination du fouillis en zone aurorale par rapport aux réseaux récepteurs adaptatifs à une dimension. Les conditions nécessaires sont que le vecteur de variété du réseau et sa matrice de covariance puissent être factorisés en un produit de Kronecker de puissance n des quantités correspondantes pour un réseau linéaire unidimensionnel. Lorsque ces conditions sont satisfaites, le rapport signal/fouillis global du système croît selon la n^e puissance de l'augmentation du rapport signal/fouillis d'un réseau adaptatif linéaire unidimensionnel. Des résultats expérimentaux sont donnés à l'appui.

Portée : Une architecture radar MIMO peut améliorer le rapport signal/fouillis du système dans les scénarios où la détection est subordonnée au fouillis, ce qui est le cas pour le fouillis auroral du radar transhorizon à zone aurorale au Canada.

Recherches futures : Des travaux subséquents démontreront l'exploitation de la formation de faisceau conjointement en azimut et en site à la réception et à l'émission. Le concept d'ouverture hyperspatiale peut être étendu à six dimensions en construi-

sant à l'émission et à la réception un réseau tridimensionnel constitué du produit de Kronecker à la sixième puissance de sous-réseaux linéaires d'émission et de réception.

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1 Introduction

High Frequency Over-the-Horizon Radar (OTHR) provides a means to detect, locate, and track aircraft to a range of 4,000 km by using the bottom of the earth's ionosphere as a mirror to look beyond the earth's horizon [1]. Previous efforts to operate OTHR in Canada were challenged by the problem of radar clutter caused by Bragg scatter from plasma irregularities in the auroral ionosphere [2]. This radar clutter can mask radar echoes from aircraft targets. These irregularities move in convection patterns within the auroral zone. Because of the large-scale structure of the convection patterns, there will be a well-defined coupling between azimuth angle and Doppler shift, and between elevation angle and range [3]. Thus, echoes from a particular Doppler and range resolution cell will arrive from a well-defined azimuth and elevation angle combination. Adaptive beamforming can be employed to provide a null at this angular location, while maintaining the response to targets at other angles [3].

Sampled receive apertures and waveform-diverse transmit apertures provide adaptive beamforming opportunities that were not available for previous auroral OTHR efforts [2]. While receive element sampling has been in common use for many years, adaptive transmit beamforming has only been studied more recently in the context of OTHR [4]–[7]. Adaptive transmit beamforming is realized by the emission of distinguishable waveforms from different elements of a transmit array consisting of antennas spaced on the order of one-half of the radar wavelength [8]. This is referred to as a Multiple-Input Multiple-Output (MIMO) radar architecture. Such an architecture can improve the signal-to-clutter ratio (SCR) of the system in clutter-limited detection scenarios [9], which correspond to the OTHR auroral clutter case in Canada.

The effort described in this paper carries the MIMO concept further by considering transmit and receive arrays that are each up to three dimensions in structure. The combined transmit-receive array configuration can thus support beamforming in up to six spatial dimensions, and as such is referred to informally as a hyperspace aperture.

This paper considers specific hyperspace aperture configurations that suggest the possibility of a vast improvement in clutter suppression performance compared to one-dimensional adaptive receive apertures. The key step is the factoring of the hyperspace aperture manifold vector and clutter covariance matrix into the Kronecker product of the corresponding quantities for subarrays. It will be shown that if this factoring is possible, then the overall improvement in signal-to-clutter ratio (SCR) by the beamformer scales with the product of the subarray SCR improvements. With up to six spatial dimensions available, one can construct beamformers that provide SCR improvements scaling with the sixth power of the SCR improvement available from a one-dimensional linear adaptive array.

This paper is organized as follows. Section 2 describes the hyperspace aperture concept. Subsection 2.1 reviews the concept of array gain, and quantifies the improvement in SCR by an adaptive array. Subsection 2.2 discusses the matter of factoring array manifold vectors and covariance matrices to determine whether the overall transmit-receive beamformer can be set up with an array gain equal to the product of the gains of the subarray factors. Two cases are considered in Subsections 2.3 and 2.4. The first case pertains to the factoring of manifold vectors and covariance matrices in MIMO systems, where it is shown that a factorization into two subarrays, representing the radar transmit and receive antenna arrays, is always possible. The second case pertains to the case where the transmit and/or receive antenna array consists of a two- or three-dimensional array. It is shown that certain array geometries and clutter covariance structures allow the transmit and/or receive arrays to be factored into multiple subarray components.

Experiments which support the ideas of Section 2 are reported in Section 3. Subsection 3.1 provides a description of the setup, Subsection 3.2 examines joint transmit-receive beamforming, and Subsection 3.3 describes a two-dimensional array on receive. A conclusion is provided in Section 4.

2 Theory

In this section we discuss the theoretical ideas behind the hyperspace aperture concept. We begin with a discussion of array gain in Section 2.1. The remainder of the section is concerned with determining the conditions under which the array gain can be factored. One of the examples shows that choosing factorable conditions can lead to large improvements in array gain.

2.1 Array gain

We consider a transmitted radar pulse. The radar pulse scatters from a target and returns as a plane wave with complex amplitude ξ to the radar receiver, where it is mapped onto a spatial array of antenna elements by an array manifold vector denoted by $\mathbf{v}$. In addition to the target signal, clutter signals from the earth's auroral region are received at the array, and map onto the array elements in a manner to be determined. The array snapshot $\mathbf{u}$ is related to ξ by

$$\mathbf{u} = \mathbf{v}\xi + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is a random vector representing the clutter signal. Noise is ignored in this analysis as it is assumed that the observations are strongly clutter limited. The quantity ξ can be viewed as a non-random, but unknown quantity. A vector of beamformer weights $\mathbf{w}$ is applied to $\mathbf{u}$ in order to form an estimate of ξ:

$$\hat{\xi} = \mathbf{w}^H \mathbf{u}, \tag{2}$$

where H is the conjugate transpose. A reasonable choice for $\mathbf{w}$ is the Minimum Variance Distortionless Response (MVDR) beamformer given in [10]:

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{v}}{\mathbf{v}^H \mathbf{R}^{-1}\mathbf{v}}, \tag{3}$$

where $\mathbf{R}^{-1}$ is the inverse of the clutter covariance matrix:

$$\mathbf{R} = E(\mathbf{n}\mathbf{n}^H), \tag{4}$$

and E is the expectation. For simplicity of exposition in this paper, we will assume that the variance of the clutter is unity, so that the matrix $\mathbf{R}$ has ones on the diagonal.

Since the MVDR beamformer is distortionless, it does not affect the signal level ξ, and serves only to reduce the clutter level. After beamforming, it can be shown that the clutter has a variance differing from unity and given by

$$\sigma^2 = E[(\mathbf{w}^H\mathbf{n})(\mathbf{w}^H\mathbf{n})^H] = \mathbf{w}^H\mathbf{R}\mathbf{w} = \frac{1}{\mathbf{v}^H\mathbf{R}^{-1}\mathbf{v}}, \tag{5}$$

where we have used (3) and the fact that $(R^{-1})^H = R^{-1}$, which follows immediately from (4).

The SCR improvement of the MVDR beamformer, sometimes referred to as the array gain, is given as in [10] by

$$G = \frac{1}{\sigma^2} = \mathbf{v}^H \mathbf{R}^{-1} \mathbf{v}. \tag{6}$$

This expression is valid when $\mathbf{R}$ is normalized such that it has ones on the main diagonal (as previously noted), and $\mathbf{v}$ has the usual normalization such that $\mathbf{v}^H \mathbf{v}$ is equal to the number of sensors in the array.

2.2 Kronecker factorization

In this subsection, we examine the factorizations of the manifold vector $\mathbf{v}$ and the covariance matrix $\mathbf{R}$ that let us construct special array configurations for which the array gain G can be made particularly large.

Let us suppose that $\mathbf{v}$ and $\mathbf{R}$ can be factored into the N-fold Kronecker product of subarray manifold vectors and subarray covariance matrices:

$$\mathbf{v} = \mathbf{v}_1 \otimes \mathbf{v}_2 \otimes \ldots \otimes \mathbf{v}_N \equiv \bigotimes_{j=1}^{N} \mathbf{v}_j \tag{7}$$

$$\mathbf{R} = \mathbf{R}_1 \otimes \mathbf{R}_2 \otimes \ldots \otimes \mathbf{R}_N \equiv \bigotimes_{j=1}^{N} \mathbf{R}_j. \tag{8}$$

Here, the jth subarray manifold vector $\mathbf{v}_j$ is associated with the jth subarray clutter covariance matrix $\mathbf{R}_j$. The array gain can then be written as

$$G = \left(\bigotimes_{j=1}^{N} \mathbf{v}_j \right)^H \left(\bigotimes_{j=1}^{N} \mathbf{R}_j \right)^{-1} \left(\bigotimes_{j=1}^{N} \mathbf{v}_j \right). \tag{9}$$

Using the identities

$$\left(\bigotimes_{j=1}^{N} A_j \right)^{-1} = \bigotimes_{j=1}^{N} A_j^{-1} \tag{10}$$

$$\left(\bigotimes_{j=1}^{N} A_j \right)^{H} = \bigotimes_{j=1}^{N} A_j^{H} \tag{11}$$

$$\left(\bigotimes_{j=1}^{N} A_j \right) \left(\bigotimes_{j=1}^{N} B_j \right) = \left(\bigotimes_{j=1}^{N} A_j B_j \right), \tag{12}$$

it follows immediately that the array gain can be factored:

$$G = \bigotimes_{j=1}^{N} \left(\mathbf{v}_j^H \mathbf{R}_j^{-1} \mathbf{v}_j \right) = \prod_{j=1}^{N} G_j. \tag{13}$$

In other words, the array gain of the system is the product of the array gains of the subarrays. The structure given by (7) and (8) thus enables the possibility of multiplicative increases in array gain. In the following sections, we will see the conditions under which this property can be realized.

2.3 Factorability of MIMO systems

We consider a Multiple-Input Multiple-Output (MIMO) system, which features multiple distinguishable spatial phase centers on both the system transmit and receive. We show that the joint transmit-receive steering vector and joint transmit-receive clutter covariance matrix are factorable into Kronecker products.

To show the factorization, we consider distinguishable transmissions from M transmit elements, each received at N receive elements. Equation (1) can be easily modified by replacing the plane wave complex amplitude ξ with a summation of plane waves that are weighted by elements of a transmit array manifold vector:

$$\mathbf{u} = \mathbf{v}_r \sum_{j=1}^{M} v_{tj}\xi + \sum_{j=1}^{M} \mathbf{n}_j, \tag{14}$$

where $\mathbf{u}$ is a length-N observation vector, $\mathbf{v}_r$ is the receive array manifold vector, v_{tj} is the jth element of the transmit array manifold vector, and $\mathbf{n}_j$ is the clutter signal related to the emission from the jth transmit element.

The key feature that separates MIMO systems from non-MIMO systems is that the terms in the summations above are distinguishable. This means that the number of observations is not N but rather MN. The observation vector can therefore be rewritten as

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{MN} \end{bmatrix} = \begin{bmatrix} v_{t1}\mathbf{v}_r \\ v_{t2}\mathbf{v}_r \\ \vdots \\ v_{tM}\mathbf{v}_r \end{bmatrix} \xi + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_M \end{bmatrix} \tag{15}$$

$$=(\mathbf{v}_t \otimes \mathbf{v}_r)\xi + \mathbf{n}. \tag{16}$$

The observation vector $\mathbf{u}$ is in the same form as in (1) except that the manifold vector $\mathbf{v}$ has been factored into the product of the manifold vectors of the transmit array and the receive array. However, we also need to show that the clutter covariance factors into a similar Kronecker product. To do this, we compute the covariance

$$\mathbf{R} = E(\mathbf{n}\mathbf{n}^H) \tag{17}$$

$$= \begin{bmatrix} E(\mathbf{n}_1\mathbf{n}_1^H) & E(\mathbf{n}_1\mathbf{n}_2^H) & \dots & E(\mathbf{n}_1\mathbf{n}_M^H) \\ E(\mathbf{n}_2\mathbf{n}_1^H) & E(\mathbf{n}_2\mathbf{n}_2^H) & \dots & E(\mathbf{n}_1\mathbf{n}_M^H) \\ \vdots & \vdots & & \vdots \\ E(\mathbf{n}_M\mathbf{n}_1^H) & E(\mathbf{n}_M\mathbf{n}_2^H) & \dots & E(\mathbf{n}_M\mathbf{n}_M^H) \end{bmatrix}. \tag{18}$$

We adopt the notion of transmit and receive clutter covariance matrices. The receive clutter covariance matrix is expected to be most familiar to readers and it represents the covariance of clutter signals transmitted from a single transmit element and received at different receive elements. We denote the receive clutter covariance matrix as $\mathbf{R}_r$:

$$\mathbf{R}_r = E(\mathbf{n}_j\mathbf{n}_j^H), \tag{19}$$

where it is assumed that the transmit array is sufficiently small that $\mathbf{R}_r$ is not a function of the transmit element index j.

Conversely, the transmit clutter covariance matrix represents the covariance of signals transmitted by different transmit elements and received at a single receive element. If we define R_{tjk} as denoting the (j,k)-th element of the transmit clutter covariance matrix $\mathbf{R}_t$, then we find that

$$E(\mathbf{n}_j\mathbf{n}_k^H) = R_{tjk}\mathbf{R}_r. \tag{20}$$

It follows immediately that (18) can be written

$$\mathbf{R} = \begin{bmatrix} R_{t11}\mathbf{R}_r & R_{t12}\mathbf{R}_r & \dots & R_{t1M}\mathbf{R}_r \\ R_{t21}\mathbf{R}_r & R_{t22}\mathbf{R}_r & \dots & R_{t2M}\mathbf{R}_r \\ \vdots & \vdots & & \vdots \\ R_{tM1}\mathbf{R}_r & R_{tM2}\mathbf{R}_r & \dots & R_{tMM}\mathbf{R}_r \end{bmatrix} \tag{21}$$

$$= \mathbf{R}_t \otimes \mathbf{R}_r, \tag{22}$$

as desired.

From the above, we can see that the clutter covariance factors in a manner similar to that of the manifold vector. It follows that the array gain for the joint transmit-receive array can be written as the product of the transmit array gain (G_t) and receive array gain (G_r):

$$G = G_tG_r = \mathbf{v}_t^H\mathbf{R}_t^{-1}\mathbf{v}_t\mathbf{v}_r^H\mathbf{R}_r^{-1}\mathbf{v}_r. \tag{23}$$

For example, if an adaptive receive array with array gain $G_r = 20$ dB can be created, then employing a similar array on transmit will boost the overall system array gain to $G = 40$ dB. This result supports the intuitive idea that the system gain pattern is the product of the transmit and receive gain patterns, although in this case the gains are stochastic rather than deterministic.

2.4 Factorability of MISO/SIMO systems

We define Multiple-Input Single-Output (MISO) systems as having multiple distinguishable phase centers on transmit, but only one phase center on receive. In a similar manner, we can define Single-Input Multiple-Output (SIMO) systems as having a single phase center on transmit, but multiple distinguishable phase centers on receive.

In the MISO/SIMO case, we have distinguishable phase centers on either transmit or receive, but not both. While the previous subsection separated the overall system array gain into the product of transmit and receive array gains, in the MISO/SIMO case we are interested in determining if the individual transmit or receive gains can be further factored. The transmit or receive cases have identical formulation. Without loss of generality, let us thus consider a large receive array with MN elements and manifold vector $\mathbf{v}$, and attempt factoring into subarrays with M and N elements respectively. The elements of a manifold vector describe the complex amplitude of a plane wave at the array element locations. Let us denote the location of the jth element as $\mathbf{r}_j$. The manifold vector is given by

$$\mathbf{v} = \begin{bmatrix} e^{i\mathbf{k}^T\mathbf{r}_1} & e^{i\mathbf{k}^T\mathbf{r}_2} & \dots & e^{i\mathbf{k}^T\mathbf{r}_{MN}} \end{bmatrix}^T, \tag{24}$$

where i is the imaginary unit, and T is a matrix transpose. Suppose that $\mathbf{v}$ is factorable into the Kronecker product of the manifold vectors of two subarrays, denoted

$$\begin{aligned} \mathbf{v}_a &= \begin{bmatrix} e^{i\mathbf{k}^T\mathbf{r}_{a1}} & e^{i\mathbf{k}^T\mathbf{r}_{a2}} & \dots & e^{i\mathbf{k}^T\mathbf{r}_{aM}} \end{bmatrix}^T \\ \mathbf{v}_b &= \begin{bmatrix} e^{i\mathbf{k}^T\mathbf{r}_{b1}} & e^{i\mathbf{k}^T\mathbf{r}_{b2}} & \dots & e^{i\mathbf{k}^T\mathbf{r}_{bM}} \end{bmatrix}^T. \end{aligned} \tag{25}$$

We calculate the Kronecker product of the subarrays:

$$\mathbf{v}_a \otimes \mathbf{v}_b = \begin{bmatrix} \begin{bmatrix} e^{i\mathbf{k}^T(\mathbf{r}_{a1}+\mathbf{r}_{b1})} & e^{i\mathbf{k}^T(\mathbf{r}_{a1}+\mathbf{r}_{b2})} & \dots & e^{i\mathbf{k}^T(\mathbf{r}_{a1}+\mathbf{r}_{bN})} \end{bmatrix}^T \\ \begin{bmatrix} e^{i\mathbf{k}^T(\mathbf{r}_{a2}+\mathbf{r}_{b1})} & e^{i\mathbf{k}^T(\mathbf{r}_{a2}+\mathbf{r}_{b2})} & \dots & e^{i\mathbf{k}^T(\mathbf{r}_{a2}+\mathbf{r}_{bN})} \end{bmatrix}^T \\ \vdots \\ \begin{bmatrix} e^{i\mathbf{k}^T(\mathbf{r}_{aM}+\mathbf{r}_{b1})} & e^{i\mathbf{k}^T(\mathbf{r}_{aM}+\mathbf{r}_{b2})} & \dots & e^{i\mathbf{k}^T(\mathbf{r}_{aM}+\mathbf{r}_{bN})} \end{bmatrix}^T \end{bmatrix}. \tag{26}$$

The right side of (26) can be interpreted as the spatial convolution of the coordinates $\mathbf{r}_a$ and $\mathbf{r}_b$. Thus, the array manifold vector factors into the manifold vectors of two subarrays if the antenna array can be expressed as a spatial convolution of the two subarrays. This property reflects the fact that the exponential of a sum of two quantities is the product of the exponentials of each of the two quantities.

We now turn to the clutter covariance matrix to look for a similar factoring. If we define $R(\mathbf{r})$ as a three-dimensional clutter signal covariance function, then the associated sensor clutter covariance matrix can be written as

$$\mathbf{R} = \begin{bmatrix} 1 & R(\mathbf{r}_1 - \mathbf{r}_2) & \dots & R(\mathbf{r}_1 - \mathbf{r}_{MN}) \\ R(\mathbf{r}_2 - \mathbf{r}_1) & 1 & \dots & R(\mathbf{r}_2 - \mathbf{r}_{MN}) \\ \vdots & \vdots & & \vdots \\ R(\mathbf{r}_{MN} - \mathbf{r}_1) & R(\mathbf{r}_{MN} - \mathbf{r}_2) & \dots & 1 \end{bmatrix}. \tag{27}$$

Suppose that $\mathbf{R}$ is factorable into the Kronecker product of the clutter covariance matrices of two subarrays, denoted $\mathbf{R}_a$ and $\mathbf{R}_b$, with elements at locations $\mathbf{r}_a$ and $\mathbf{r}_b$. The entries of $\mathbf{R}$ must be equal to the covariance function R evaluated at locations corresponding to the spatial convolution of the coordinates $\mathbf{r}_a$ and $\mathbf{r}_b$:

$$R(\mathbf{r}_{a_k} - \mathbf{r}_{a_l} + \mathbf{r}_{b_m} - \mathbf{r}_{b_n}) = R(\mathbf{r}_{a_k} - \mathbf{r}_{a_l}) R(\mathbf{r}_{b_m} - \mathbf{r}_{b_n}), \tag{28}$$

for all integers k, l in the interval $(1, M)$ and m, n in the interval $(1, N)$.

Let us consider a simple example of how this relation can be satisfied. We consider a receiving array consisting of an M-by-N rectangular array of elements in the horizontal plane, with OTHR signals arriving approximately in the plane of the array. The array is in the xy plane, where x and y are horizontal coordinates, and d_x and d_y are the elemental spacings in the x and y directions, respectively. We assume that the clutter signals arrive from the $+y$ direction. The array manifold vector $\mathbf{v}$ can be factored into $\mathbf{v}_a$, the manifold vector of a horizontal linear array of M elements in the x direction, and $\mathbf{v}_b$, the manifold vector of a horizontal linear array of N elements in the y direction. Since the clutter signal is a propagating electromagnetic wave, let us say that the covariance function of the clutter is a complex exponential in the direction of propagation (y), and we allow for arbitrary covariance in the perpendicular (x) direction:

$$R(\mathbf{r}) \approx f(x) e^{i\kappa y}, \tag{29}$$

where $f(x)$ is some function equal to unity at $x = 0$ and between zero and unity magnitude for $x \neq 0$. Here, κ is the radar (and thus clutter) wavenumber. The above covariance function conveys the notion of a "wrinkled" wavefront, where the clutter signals vary as a complex exponential in the propagation direction, but have stochastic phase normal to the propagation direction. If d_x is the spacing between elements perpendicular to the clutter direction of arrival, and d_y is the spacing parallel to the clutter direction of arrival, then

$$\begin{aligned} R(\mathbf{r}_{a_k} - \mathbf{r}_{a_l} + \mathbf{r}_{b_m} - \mathbf{r}_{b_n}) &= f[(k-l)d_x] e^{i\kappa(m-n)d_z} \\ &= R(\mathbf{r}_{a_k} - \mathbf{r}_{a_l}) R(\mathbf{r}_{b_m} - \mathbf{r}_{b_n}), \end{aligned} \tag{30}$$

and thus the covariance matrix factors into the Kronecker product of the covariance matrices for the two horizontal subarrays. However, this factorization is not possible if the direction of arrival of the clutter deviates significantly from the $+y$ direction, in which case the array gain would degrade. This indicates that there are limitations on the clutter angular distributions that will allow the proposed scheme to work.

We now compare the array gain of factorable and non-factorable arrays to show the advantages offered by the Kronecker factoring. Consider a 2-by-2 array in the configuration of the previous example, i.e. a horizontal rectangular planar array in the xy plane. For simplicity of discussion, let us say that the target signal arrives from the $+x$ direction and the clutter signal arrives from the $+y$ direction, i.e. the competing components arrive along orthogonal coordinate axes. We also assume that the elements are laid out with half-wavelength spacing. If we factor the manifold vectors and covariance matrices, we find that

$$\begin{aligned} \mathbf{v}_a &= \begin{bmatrix} 1 & -1 \end{bmatrix}^T \quad \mathbf{v}_b = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \\ \mathbf{R}_a &= \begin{bmatrix} 1 & \rho_x \\ \rho_x^* & 1 \end{bmatrix} \quad \mathbf{R}_b = \begin{bmatrix} 1 & \rho_y \\ \rho_y^* & 1 \end{bmatrix}, \end{aligned} \tag{31}$$

where $\rho_x = f(d_x)$ and is assumed real, and $\rho_y \approx e^{i\kappa d_y}$, where κ is the radar wavenumber. The array gain of the 2-by-2 array is given from (13) and (31) by

$$\begin{aligned} G = G_a G_b &= (\mathbf{v}_a^H \mathbf{R}_a^{-1} \mathbf{v}_a)(\mathbf{v}_b^H \mathbf{R}_b^{-1} \mathbf{v}_b) \\ &\approx \left(\frac{2}{1-\rho_x}\right)\left(\frac{2-\rho_y-\rho_y^*}{1-|\rho_y|^2}\right) \approx \frac{2}{(1-\rho_x)(1-|\rho_y|)}. \end{aligned} \tag{32}$$

We now contrast this result with that of an L-shaped array, which is the same as the previous example but with one element removed. Without loss of generality we can choose to remove any of the four elements. The manifold vector and covariance matrix of an L-shaped array are not factorable, and they are

$$\mathbf{v} = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}^T \tag{33}$$

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_x & \rho_x\rho_y \\ \rho_x^* & 1 & \rho_y \\ \rho_x^*\rho_y^* & \rho_y^* & 1 \end{bmatrix}. \tag{34}$$

The 3×3 matrix inverse is analytically tractable, and we find that

$$G = \mathbf{v}^H \mathbf{R}^{-1} \mathbf{v} \approx \frac{2-\rho_y-\rho_y^*}{1-|\rho_y|^2} \approx \frac{1}{1-|\rho_y|}. \tag{35}$$

We can readily see that the loss of a single element has reduced the array gain from $G = G_a G_b$ in (32) to what is essentially $G = G_b$ in (35). The loss in array gain is a factor of $G_a \approx 2/(1-\rho_x)$, which is significant because $\rho_x \approx 1$. This result demonstrates the significant improvements in array gain that can be achieved by configuring the antenna arrays in a factorable manner.

3 Experiments

This section discusses experimental results that support the ideas developed in the previous section. The apparatus is explained in Subsection 3.1. Joint transmit-receive beamforming results are described in Subsection 3.2, and joint elevation-azimuth beamforming in Subsection 3.3.

3.1 Apparatus

An experimental OTHR is installed at Defence Research and Development Canada in Ottawa, Canada. Figure 1 shows the placement of antennas at the site. The OTHR features an array of eight Beverage antenna elements, laid out in a 4-by-2 configuration, of which four are used for transmit, and four for receive. These antennas were selected for the experiment because they do not require ground planes or towers. The antennas are 150 m long and suspended 6.5 m above the ground. The antennas generate about 1 dBi of gain at 5 MHz in the magnetic north direction, where one expects to receive strong auroral clutter echoes. Each of the four transmit

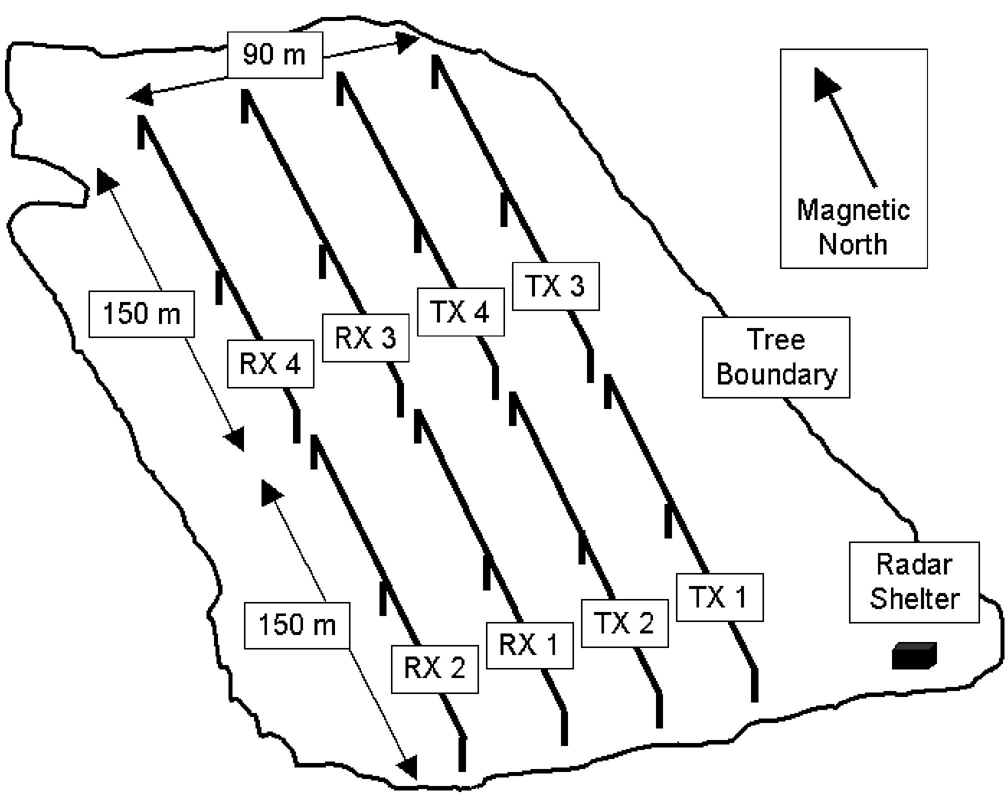

Figure 1: *Site layout. TX1 through TX4 are the transmit antennas and RX1 through RX4 are the receive antennas.*

antennas can be excited with independent waveforms at 4 kW peak power and up to 100% duty cycle, and each of the four receive antennas is connected to a digital radio receiver.

3.2 Joint transmit-receive beamforming

The objective of this experiment is to look at joint transmit-receive adaptive beamforming, to verify the theory of Subsection 2.3. In this case, antennas TX3, TX4, RX3, and RX4 in Figure 1 are not needed. To increase the antenna element gains to around 4 dBi, the four "rear" antennas (TX1, TX2, RX1, RX2) were connected to the four "front" antennas (TX3, TX4, RX3, RX4), to produce 4 Beverage antennas each 300 m long. The "extended" antennas TX1 and TX2 were then used for transmission, and the "extended" antennas RX1 and RX2 were used for reception. TX1 and TX2 were each excited with a 8-kW peak power, 7.5-ms long pulse every 25 ms, i.e. with a pulse repetition frequency (PRF) of 40 Hz. The carrier frequencies of the two transmit antenna signals were approximately 5 MHz, with an offset of 20 Hz (half the PRF) between them. This carrier frequency separation is resolvable during Doppler processing of the received signals [12]. This configuration effectively provides four distinguishable joint transmit-receive spatial channels, each with a Doppler bandwidth of 20 Hz, corresponding to $M = N = 2$ in the theory of Subsection 2.3.

The experimental data shown in Figure 2 were acquired on 12 Oct 2010 at 2352 UTC [7]. This Doppler spectrum was derived from 25.6 seconds of data, which at the effective PRF of 20 Hz yields a 512-point spectrum. The 64-point spectra shown in Figure 2 result from incoherent averaging over blocks of eight points to reduce the data variance. The black trace shows the baseline Doppler spectrum of the auroral echoes using a single transmit-receive channel (TX1-RX1). The blue trace shows the effect of adaptive beamforming on receive using the RX1 and RX2 signals. Although there were no target signals in the data, it was assumed for the purpose of comparison among beamformers that the target signal was in the magnetic west direction from the antennas so that signals from this direction are preserved. The spatial covariance matrix at each Doppler cell under test is estimated by taking the outer products of the signals in the four adjacent Doppler cells. In Figure 2 it is seen that the receive beamformer suppresses the clutter by about 15 dB. The green trace shows the effect of adaptive processing on transmit, using the signals due to the waveforms transmitted from TX1 and TX2. The transmit array is similar to the receive array and, as predicted in Subsection 2.3, suppresses the clutter by a similar amount. An exception is in the right-most third of the spectrum, where the suppression is degraded. Since the Doppler spectrum arises from a clockwise convection of plasma in the auroral zone in the evening sector (as viewed looking down on the earth) [3], Doppler-upshifted echoes arrive from directions to the northeast of the array, which are blocked by trees (as suggested by the depiction of the tree boundary in

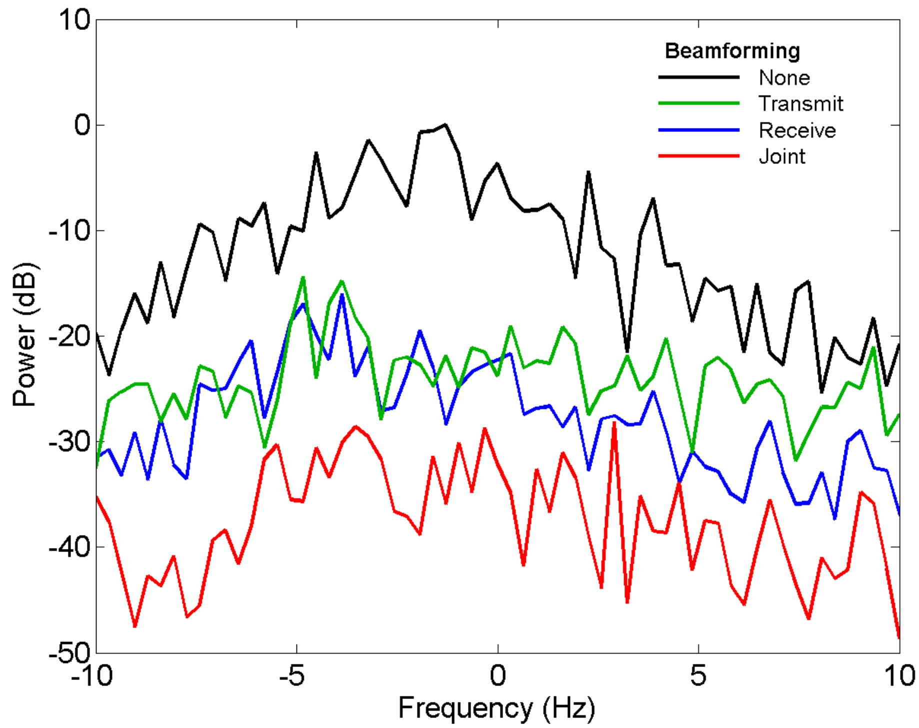

Figure 2: *Auroral clutter Doppler spectrum for different adaptive beamforming schemes. The black trace is single-channel data. The green trace is transmit-only beamforming. The blue trace is receive-only beamforming. The red trace is joint transmit-receive beamforming.*

Figure 1), leading to decorrelation of the transmit channel signals, and reduced array gain. The red trace shows joint transmit-receive adaptive beamforming. According to Subsection 2.3, this trace should display clutter suppression that combines the effect of the clutter suppression on transmit (green trace) and receive (blue trace). The experimental result shows about 30 dB of suppression, which supports the idea that the 15 dB of clutter suppression on transmit combines multiplicatively with the 15 dB of clutter suppression on receive.

3.3 Joint elevation-azimuth beamforming

The objective here is to examine joint elevation-azimuth beamforming, to demonstrate the results of Subsection 2.4. Since the transmit and receive antenna arrays have the same layout (see Figure 1) either one can be used to examine joint elevation-azimuth beamforming. The radar transmitter was under repair at the time of this experiment, so it was decided that signals on receive would be examined. Since both auroral clutter and signals of opportunity can satisfy the covariance requirements of

Subsection 2.4, results using the latter will be presented. In Subsection 2.4 it was shown that the clutter covariance of a horizontal planar array, with clutter incident on the array along one of the array axes, factors into a Kronecker product if the clutter covariance varies as a complex exponential in the direction of propagation, with arbitrary covariance in the perpendicular direction. To satisfy these assumptions, a total of 30 shortwave broadcast signals were examined in the 25-m band, and a signal at 12.1 MHz was chosen with direction of propagation within about 15 degrees of the magnetic north-south axis of the array, as determined by direction-finding techniques. The experimental data shown in Figure 3 were acquired on 18 Nov 2011 at 1930 UTC. The carrier of the radar broadcast was not locked to the radar receivers during the experiment. This limited the coherence time to about one second, resulting in the one-hertz resolution shown in Figure 3. The black trace shows the shortwave carrier signal. For adaptive beamforming, the shortwave carrier is taken as the clutter, and again we assume a target signal to have a direction of arrival from the magnetic west direction. The green trace shows the effect of adaptive processing in the elevation direction (using RX1 and RX3). We see that the carrier signal (near zero Hz) is suppressed by approximately 25 dB. The blue trace similarly

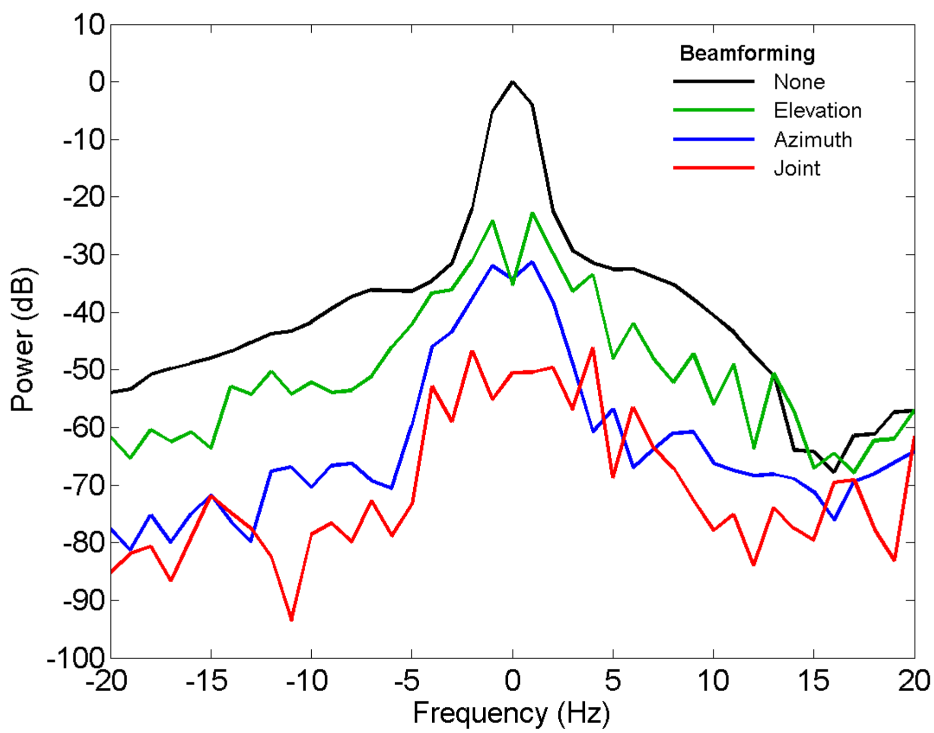

Figure 3: *Shortwave carrier signal. The black trace is single-channel data. The green trace is elevation beamforming. The blue trace is azimuth beamforming. The red trace is joint elevation-azimuth beamforming.*

shows adaptive beamforming in the azimuth direction (using RX1 and RX2), which causes a carrier suppression of about 30 dB. Joint elevation-azimuth beamforming is shown in the red trace, where the shortwave carrier is suppressed by about 50 dB, which represents the combined effect of the azimuth and elevation array gains. This is broadly consistent with the predictions of Subsection 2.4. The array gain of the joint beamformer away from the carrier is smaller, as the adaptive beamformer clutter suppression performance is bounded by the system noise floor.

4 Conclusion

The theoretical and experimental sections have presented examples of joint transmit-receive processing, and joint elevation-azimuth processing. Taken together, these represent the operation of joint four-dimensional spatial adaptive processing, which provide an example of a hyperspace radar aperture. Joint array gains have been shown to be products of the array gains of one-dimensional apertures, under the condition that the joint manifold vector and covariance matrix can be factored into the appropriate Kronecker products. Future work will show the operation of joint elevation-azimuth beamforming simultaneously on both transmit and receive. Furthermore, the hyperspace aperture concept can be extended to six dimensions by constructing three-dimensional arrays on transmit and receive that comprise the six-fold Kronecker product of linear transmit and receive subarrays.

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Under certain conditions, it is shown that adaptive arrays with up to six spatial dimensions (three on transmit and three on receive) can provide vast improvements in auroral-zone clutter suppression capability compared to one-dimensional adaptive receive arrays. The required conditions are that the array manifold vector and its associated clutter covariance matrix are factorable into the n-fold Kronecker product of the corresponding quantities for a one-dimensional linear array. When these conditions are satisfied, the overall system signal-to-clutter ratio (SCR) improvement scales with the nth power of the SCR improvement of a one-dimensional linear adaptive array. Some supporting experimental results are provided.

Dans certaines conditions, il est possible de démontrer que l'utilisation de réseaux adaptatifs ayant jusqu'à six dimensions spatiales (trois à l'émission et trois à la réception) peut mener à des améliorations substantielles de la capacité d'élimination du fouillis en zone aurorale par rapport aux réseaux récepteurs adaptatifs à une dimension. Les conditions nécessaires sont que le vecteur de variété du réseau et sa matrice de covariance puissent être factorisés en un produit de Kronecker de puissance n des quantités correspondantes pour un réseau linéaire unidimensionnel. Lorsque ces conditions sont satisfaites, le rapport signal/fouillis global du système croît selon la n^e puissance de l'augmentation du rapport signal/fouillis d'un réseau adaptatif linéaire unidimensionnel. Des résultats expérimentaux sont donnés à lappui.

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