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The three-dimensional spatial autocorrelation of ionospherically propagated wave packets at high latitudes

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Abstract

We analytically derive a three-dimensional spatial autocorrelation function for the phase of ionospherically propagated wave packets at near-vertical magnetic field dip angles. The correlation length in the direction normal to the plane of propagation is approximately equal to the outer scale size of plasma irregularities in the ionosphere. The correlation length in the horizontal direction in the plane of propagation is approximately equal to one-half the horizontal length of the ionospheric path transited by the wave packet. The vertical correlation length is shown to be equal to the horizontal correlation length multiplied by the tangent of the wave packet take-off angle.

Résumé

Nous développons analytiquement une fonction d'autocorrélation tridimensionnelle pour la phase de paquets d'ondes à propagation ionosphérique à des angles d'inclinaison du champ magnétique presque verticaux. La longueur de corrélation dans la direction normale au plan de propagation est environ égale à la taille des plus grandes irrégularités plasmatiques de l'ionosphère. La longueur de corrélation dans la direction horizontale du plan de propagation est environ égale à la moitié de la longueur horizontale du trajet ionosphérique suivi par le paquet d'ondes. On montre que la longueur verticale de corrélation est égale à la longueur de corrélation horizontale multipliée par la tangente de l'angle de rayonnement du paquet d'ondes.

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Executive summary

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Background: Correlation functions describe the degree to which measured quantities are consistent from one measurement to the next. In the case of measurements of the same quantity at different points in space, one speaks of the spatial autocorrelation function. Directional radio systems operate on the assumption of good correlation of the phase of an electric field at different points in space. The spatial autocorrelation of an electric field associated with an electromagnetic wave packet is modified by propagation through the ionosphere. This report theoretically derives a simple spatial autocorrelation function for ionospherically propagated wave packets.

Results: An analytic formula is derived based on a linear plasma density profile for the ionosphere and a power-law turbulence spectrum for ionospheric irregularities. The correlation length in the direction normal to the plane of propagation is approximately equal to the outer scale size of plasma irregularities in the ionosphere. The correlation length in the horizontal direction in the plane of propagation is approximately equal to one-half the horizontal length of the ionospheric path transited by the wave packet. The vertical correlation length is shown to be equal to the horizontal correlation length multiplied by the tangent of the wave packet take-off angle.

Significance: This work provides guidance for interpreting the results of radio experiments and for proposing future radio systems. In particular, autocorrelation functions provide insight on the degree to which ionospheric modes at various elevation and azimuth directions of arrival can be separated by analysis of the spatially dependent phase.

Future Work: It would be beneficial to look at the case of a general magnetic field dip angle, although it does not seem to be analytically tractable at this time. This analysis would support the interpretation of results obtained at lower latitudes, and illustrate the quality of the approximation of a vertical magnetic field dip angle made in the current work.

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The three-dimensional spatial autocorrelation of ionospherically propagated wave packets at high latitudes

Ryan J. Riddolls; DRDC Ottawa TM 2012-158; R & D pour la défense Canada – Ottawa; décembre 2012.

Contexte : Les fonctions de corrélation expriment le degré auquel les quantités mesurées sont cohérentes d'une mesure à l'autre. Dans le cas de mesures de la même quantité à différents points dans l'espace, on parle de fonction d'autocorrélation spatiale. Les systèmes radio directifs fonctionnent en présumant qu'il y a une bonne corrélation de la phase d'un champ électrique à différents points dans l'espace. L'autocorrélation spatiale d'un champ électrique associé à un paquet d'ondes électromagnétiques est modifiée par la propagation dans l'ionosphère. Le présent rapport présente le développement théorique d'une fonction d'autocorrélation spatiale simple pour les paquets d'ondes se propageant dans l'ionosphère.

Résultats : Une formule analytique est développée à partir d'un profil linéaire de densité de plasma dans l'ionosphère et d'un spectre de turbulence régi par une loi de puissance pour les irrégularités plasmatiques de l'ionosphère. La longueur de corrélation dans la direction normale au plan de propagation est environ égale à la taille des plus grandes irrégularités plasmatiques de l'ionosphère. La longueur de corrélation dans la direction horizontale du plan de propagation est environ égale à la moitié de la longueur horizontale du trajet ionosphérique suivi par le paquet d'ondes. On montre que la longueur verticale de corrélation est égale à la longueur de corrélation horizontale multipliée par la tangente de l'angle de rayonnement du paquet d'ondes.

Portée : Ces travaux fournissent des connaissances utiles à l'interprétation des résultats d'expériences radio et à l'établissement de propositions pour de futurs systèmes radio. En particulier, les fonctions d'autocorrélation donnent une idée du degré auquel les modes ionosphériques à diverses directions d'arrivée en azimut et en site peuvent être séparés par analyse des variations de la phase dans l'espace.

Recherches futures : Il serait utile d'étudier le cas d'un angle d'inclinaison du champ magnétique quelconque, même s'il semble impossible de résoudre ce problème au moyen d'une approche analytique actuellement. Cette analyse aiderait à l'interprétation de résultats obtenus à des latitudes plus basses et illustrerait la qualité de l'approximation d'un angle d'inclinaison vertical du champ magnétique faite dans les présents travaux.

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1 Introduction

The modelling of wave propagation through random media ifss a well-studied problem. A recent monograph on the subject [1] promulgates a three-level hierarchy for describing the methods that have been applied to the problem, which we briefly discuss here. The first and most basic method in the hierarchy is geometric optics, whereby the scattered wave field can be expressed in terms of a line integral of the refractive index fluctuations [2]–[5]. The line integral is taken along the zero-order ray trajectory, namely the ray that would occur in the absence of first-order refractive index fluctuations. Geometric optics provides good modelling of the resulting fluctuations of the phase and angle-of-arrival, but does not inherently contain diffractive effects. It is thus unable to accurately model fluctuations in amplitude or intensity. Geometric optics can be extended in an ad hoc manner to include diffractive effects by modelling the ionosphere as a series of phase screens interpolated by free space [6], [7].

The current work is concerned with phase, and thus a geometric optics approach is suitable. As a supporting example from the literature, if we take the plane-wave limit of Equation (30) in [8] (by setting the transmitter-phase screen distance to infinity), it is clear that the spatial and temporal correlation lengths are impacted, dominantly, by the phase variance. However, if there is interest in amplitude or intensity fluctuations, for the purpose of modelling fading for instance, one could consider the second method in the hierarchy of [1], referred to as the method of smooth perturbations [2] or the Rytov method [9]. In this method, the scattered wave field is expressed as an integral over a volume of refractive index fluctuations, which allows for the accounting of diffractive effects. A key feature of this method is the expression of the scattered wavefield as an exponential of a function to be determined. By allowing the function to be complex-valued, both amplitude and phase effects are captured in a straightforward manner.

For completeness, we mention the third method in the hierarchy, which is referred to as the method of strong fluctuations, where the scattered field is expressed as a functional integral with respect to the refractive index fluctuations [10]. As the name suggests, this method handles large-amplitude refractive index fluctuations where the other methods do not provide convergent solutions.

The current work aims to write down a simple three-dimensional autocorrelation function for ionospherically propagated wave packets. Section 2 describes the models of the ionosphere that are used. Section 3 describes properties of the ionospherically propagating wave packet. Section 4 computes the autocorrelation function and the result is summarized in Section 5.

2 Model of the ionospheric medium

In this section, we will present simple models for the coarse and fine structure of the ionosphere.

2.1 Coarse ionospheric structure

We consider an unmagnetized model of the ionosphere. We are interested in aspects of the ionospheric model that influence the propagation of wave packets through the medium. The key quantity is the medium refractive index [11]:

$$N^2 = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}, \tag{1}$$

where N is the refractive index, c is the speed of light, $k = |\mathbf{k}|$ is the wavenumber, ω is the frequency, and ω_p is the electron plasma frequency, defined as

$$\omega_p^2 = \frac{e^2 n}{\epsilon_0 m}, \tag{2}$$

where e is the charge on an electron, n is the density of plasma in electrons per unit volume, ϵ_0 is the permittivity of free space, and m is the mass of an electron.

In the ionosphere, n is a function of space. Let us consider a system of cartesian coordinates with xy in the plane of the earth and z vertical. The simplest space-dependent model of the ionosphere assumes that n is linearly related to altitude z, slowly varying compared to a wavelength, and independent of the horizontal coordinate [11]. Using (2), we can write a z-dependent plasma frequency:

$$\omega_p^2(z) = \omega^2 \frac{z}{z_0}. \tag{3}$$

Here, $z = 0$ is the bottom of the ionosphere. The model (3) is convenient for modelling the variation of refractive index in the ionosphere and its impact on the propagation of wave packets. Nominally, the range of applicability of the model is values of z such that $0 \leq z \leq z_0$. In other words, we assume that the plasma density in the ionosphere increases linearly up to the point $z = z_0$. However, it will be shown later that this assumption can be relaxed in cases where wave packets are injected into the ionosphere at low elevation angles to the horizon. In particular, it is shown that the linear plasma profile need only hold up to a height $z = z_0 \sin^2 \theta$, where θ is the launch angle of the wave packet with respect to the horizon.

2.2 Fine ionospheric structure

The linear plasma density profile described above normally contains fine structure in the form of plasma turbulence. We begin by writing the total plasma density n as the summation of a smooth linear profile n_0 and a small perturbation due to the turbulence n_1:

$$n = n_0 + n_1. \tag{4}$$

The turbulence n_1 can be characterized by its statistics. The relevant statistic is the spatial autocorrelation function:

$$R_{n_1}(\mathbf{R}) = \langle n_1(\mathbf{r}+\mathbf{R})n_1(\mathbf{r})\rangle\,, \tag{5}$$

where $\mathbf{r}$ is a point in space, and $\mathbf{R}$ is a spatial displacement, and the angle brackets represent the average of a statistical ensemble. For the sake of simplicity, we have assumed that n_1 is wide-sense stationary, such that R_{n_1} is only a function of the displacement $\mathbf{R}$. The spatial autocorrelation is related to the spatial spectrum, which is given by

$$S_{n_1}(\boldsymbol{\kappa}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dX\,dY\,dZ\,R_{n_1}(\mathbf{R})e^{-i\kappa_x X - i\kappa_y Y - i\kappa_z Z}, \tag{6}$$

where $\mathbf{R} = X\hat{\mathbf{x}} + Y\hat{\mathbf{y}} + Z\hat{\mathbf{z}}$ and $\boldsymbol{\kappa} = \kappa_x\hat{\mathbf{x}} + \kappa_y\hat{\mathbf{y}} + \kappa_z\hat{\mathbf{z}}$. The spectrum is normalized such that

$$\langle n_1^2\rangle = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\kappa_x d\kappa_y d\kappa_z\,S_{n_1}(\kappa_x,\kappa_y,\kappa_z), \tag{7}$$

where $\langle n_1^2\rangle$ is the mean-square density perturbation.

Various models exist for the turbulence spectrum. Most models reflect the dominant influence of the earth's magnetic field on the turbulence spectrum. A typical model is given by [9], [12]:

$$S_{n_1}(\boldsymbol{\kappa}) = \frac{8\pi\,\langle n_1^2\rangle}{\kappa_{0\perp}^2\kappa_{0\parallel}(1+\kappa_\perp^2/\kappa_{0\perp}^2+\kappa_\parallel^2/\kappa_{0\parallel}^2)^2}. \tag{8}$$

Here, $\kappa_\parallel$ is the component of $\boldsymbol{\kappa}$ along the magnetic field of the earth, $\kappa_\perp$ is the component of $\boldsymbol{\kappa}$ perpendicular to the magnetic field of the earth, and $\kappa_{0\perp}$ and $\kappa_{0\parallel}$ are characteristic scale lengths. For modelling high-latitude ionospheric turbulence, the magnetic field is near vertical. This lets us write the model in terms of cartesian coordinates:

$$S_{n_1}(\boldsymbol{\kappa}) = \frac{8\pi\,\langle n_1^2\rangle}{\kappa_{0\perp}^2\kappa_{0\parallel}(1+\kappa_x^2/\kappa_{0\perp}^2+\kappa_y^2/\kappa_{0\perp}^2+\kappa_z^2/\kappa_{0\parallel}^2)^2}. \tag{9}$$

The electrical conductivity along the magnetic field of the earth is large compared to the conductivity across the magnetic field. This tends to stretch the spatial turbulence structure along the magnetic field lines. Spatial stretching is equivalent to spectral compression, so we have $\kappa_{0\perp} \gg \kappa_{0\|}$. Typical numbers [12] are $\kappa_{0\perp} \approx 10^{-4}$ m^{-1} and $\kappa_{0\|} \approx 3{\times}10^{-8}$ m^{-1}.

This large anisotropy motivates a reduced-dimension spectral model that is easier to work with analytically for some applications. In view of the spectral compression, we re-write the phase spectrum such that there is an impulse in the κ_z direction [13], [14]. The procedure is

$$S_{n_1}(\boldsymbol{\kappa}) = \delta(\kappa_z) \int_{-\infty}^{\infty} \frac{d\kappa_z \, 8\pi \left\langle n_1^2 \right\rangle}{\kappa_{0\perp}^2 \kappa_{0\|} (1 + \kappa_x^2/\kappa_{0\perp}^2 + \kappa_y^2/\kappa_{0\perp}^2 + \kappa_z^2/\kappa_{0\|}^2)^2}. \tag{10}$$

The integration can be done by trigonometric substitution or residue theory, yielding

$$S_{n_1}(\boldsymbol{\kappa}) = \frac{4\pi^2 \left\langle n_1^2 \right\rangle \delta(\kappa_z)}{\kappa_{0\perp}^2 (1 + \kappa_x^2/\kappa_{0\perp}^2 + \kappa_y^2/\kappa_{0\perp}^2)^{3/2}}. \tag{11}$$

This reduced-dimension spectrum can be visualized as a horizontal disc-like structure.

3 Wave packet

In this section, we calculate the trajectory of the wave packet as determined by the coarse structure of the ionosphere. It will be shown later that it is not necessary to invoke the fine structure of the ionosphere for determining the wave packet trajectory.

3.1 Trajectory differential equation

Consider a wave packet that is launched from the ground at an elevation angle of θ with respect to horizontal. This wave packet travels in a straight line up to the ionosphere, and reaches the lower boundary of the ionosphere at an altitude of approximately 200 km. We denote this lower boundary as $z = 0$ in our system of coordinates. The unmagnetized ionosphere is rotationally symmetric around the z axis, so we can freely choose the azimuthal rotation of the xy plane. Without loss of generality, we rotate the xy plane such that the wave packet propagates in the xz plane. The components of the wavenumber are as follows

$$k_x = k\cos\theta \tag{12}$$

$$k_z = k\sin\theta. \tag{13}$$

Since the index of refraction, given by (1), is only a function of z, Snell's law states that the horizontal component of the wavenumber, denoted here as k_x, remains constant as the wave packet propagates through the ionosphere. Since $k = \omega/c$ at the point $z = 0$ where the wave packet enters the ionosphere, the horizontal component of the wavenumber is

$$k_x = \frac{\omega}{c}\cos\theta. \tag{14}$$

This value of k_x is maintained throughout the wave packet trajectory.

In order to ensure compliance with the dispersion relation (1), the vertical component of the wavenumber will adjust as follows :

$$k_z = \pm\sqrt{k^2 - k_x^2} = \pm\frac{\omega}{c}\sqrt{\sin^2\theta - \frac{z}{z_0}}. \tag{15}$$

Here, k_z takes positive values in the upward portion of the trajectory, and negative values in the downward portion of the trajectory.

We define the variable x as the horizontal coordinate of the wave packet location. The slope of the wave packet trajectory is given by:

$$\frac{dz}{dx} = \frac{k_z}{k_x}. \tag{16}$$

Inserting expressions for k_x and k_z yields the differential equation

$$\frac{dz}{dx} = \pm \left(\tan^2 \theta - \frac{z}{z_0} \sec^2 \theta \right)^{1/2}, \tag{17}$$

which we will proceed to solve in the next subsection.

3.2 Trajectory solution

To solve the first-order nonlinear differential equation (17), square both sides in order to clear the $\pm$ ambiguity:

$$\left(\frac{dz}{dx} \right)^2 = \tan^2 \theta - \frac{z}{z_0} \sec^2 \theta. \tag{18}$$

An inspection of this equation suggests a general form of solution given by

$$z = ax^2 + bx + c. \tag{19}$$

Inserting this trial solution and matching the coefficients of powers of x yields the system

$$a = -\frac{\sec^2 \theta}{4z_0} \tag{20}$$

$$c = z_0(\sin^2 \theta - b^2 \cos^2 \theta). \tag{21}$$

Evidently, b is a free parameter, so we choose $b = 0$. This means the solution for the trajectory can be written

$$z = -\frac{\sec^2 \theta}{4z_0} x^2 + z_0 \sin^2 \theta. \tag{22}$$

This is an inverted parabola. This solution is good for all x where z is non-negative, since the model of (3) assumes non-negative values of z. For values of x outside this range, the wave packet is below the ionosphere and the trajectory is a straight line.

Further insight on the trajectory in the ionosphere can be gained by writing (22) in the form:

$$\frac{z}{z_1} = 1 - \frac{x^2}{x_1^2}, \tag{23}$$

where we have taken the definitions

$$x_1 = z_0 \sin 2\theta \tag{24}$$

$$z_1 = z_0 \sin^2 \theta. \tag{25}$$

Here, the spatially dependent density perturbation n_1 is evaluated at a point $\mathbf{r}$ on the wave packet trajectory, and we adopt the notation $\mathbf{r}(x)$ to emphasize that the wave packet trajectory is parameterized by the independent variable x. Furthermore, we use $\lambda = 2\pi c/\omega$ to denote the wavelength and r_e to denote the classical electron radius, given by

$$r_e = \frac{e^2}{4\pi\epsilon_0 mc^2}. \tag{32}$$

This completes the calculation of the phase perturbation.

4.2 Two-dimensional phase autocorrelation

Let us consider the phase as calculated at two points on the ground, as shown in Figure 2. In this figure, the assumption is that plane waves enter the ionosphere on

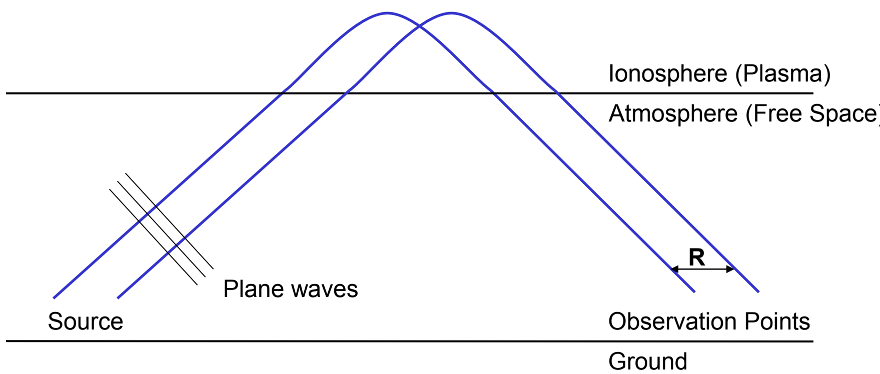

Figure 2: *Two trajectories through the ionosphere, with displacement* $\mathbf{R}$.

the left side of the figure, and after transiting the ionosphere, the rays are observed at two locations on the ground offset by a distance $\mathbf{R} = X\hat{\mathbf{x}} + Y\hat{\mathbf{y}}$. The autocorrelation of ϕ with respect to the displacement $\mathbf{R}$ is given by

$$R_{\phi_1}(\mathbf{R}) = (r_e\lambda \sec\theta)^2 \left\langle \int_{-x_1}^{x_1} dx\, n_1[\mathbf{r}(x) + \mathbf{R}] \int_{-x_1}^{x_1} dx'\, n_1[\mathbf{r}(x')] \right\rangle, \tag{33}$$

where the angle brackets represent an ensemble average. By using (5), we can write the phase autocorrelation as

$$R_{\phi_1}(\mathbf{R}) = (r_e\lambda \sec\theta)^2 \int_{-x_1}^{x_1} \int_{-x_1}^{x_1} dx\, dx'\, R_{n_1}\big[\mathbf{r}(x) - \mathbf{r}(x') + \mathbf{R}\big]. \tag{34}$$

The density autocorrelation R_{n_1} appearing on the right side is normally defined in terms of its Fourier transform, which we can write as

$$R_{\phi_1}(\mathbf{R}) = \frac{(r_e \lambda \sec\theta)^2}{(2\pi)^3} \int_{-x_1}^{x_1} \int_{-x_1}^{x_1} dx\, dx' \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\boldsymbol{\kappa}\, S_{n_1}(\boldsymbol{\kappa}) e^{i\boldsymbol{\kappa}\cdot[\mathbf{r}(x)-\mathbf{r}(x')+\mathbf{R}]}. \quad (35)$$

Next, we change the order of integration:

$$R_{\phi_1}(\mathbf{R}) = \frac{(r_e \lambda \sec\theta)^2}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\boldsymbol{\kappa}\, S_{n_1}(\boldsymbol{\kappa}) e^{i\boldsymbol{\kappa}\cdot\mathbf{R}} \int_{-x_1}^{x_1} \int_{-x_1}^{x_1} dx\, dx'\, e^{i\boldsymbol{\kappa}\cdot[\mathbf{r}(x)-\mathbf{r}(x')]}. \quad (36)$$

We substitute (11) for S_{n_1}. Since $S_{n_1} \sim \delta(\kappa_z)$, we have $\boldsymbol{\kappa}\cdot[\mathbf{r}(x)-\mathbf{r}(x')] = \kappa_x(x-x')$, and the rightmost double integral is

$$\int_{-x_1}^{x_1} \int_{-x_1}^{x_1} dx\, dx'\, e^{i\kappa_x(x-x')} = \frac{\sin^2(\kappa_x x_1)}{\kappa_x^2} = x_1^2 \operatorname{sinc}^2(\kappa_x x_1) = (z_0 \sin 2\theta)^2 \operatorname{sinc}^2(\kappa_x x_1). \quad (37)$$

The autocorrelation (36) can then be written

$$R_{\phi_1}(\mathbf{R}) = \frac{4(r_e \lambda z_0 \sin\theta)^2}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\kappa_x d\kappa_y\, 2\pi \left\langle n_1^2 \right\rangle \operatorname{sinc}^2(\kappa_x x_1) e^{i\kappa_x X + i\kappa_y Y}}{\kappa_{0\perp}^2 (1 + \kappa_x^2/\kappa_{0\perp}^2 + \kappa_y^2/\kappa_{0\perp}^2)^{3/2}}. \quad (38)$$

We recall the identity

$$\frac{2|x| K_1(a|x|)}{a} = \int_{-\infty}^{\infty} \frac{dk\, e^{ikx}}{(a^2+k^2)^{\frac{3}{2}}}, \quad (39)$$

where K_1 is the modified Bessel function of the second kind. By using (39) we find

$$R_{\phi_1}(\mathbf{R}) = \frac{8(r_e \lambda z_0 \sin\theta)^2 \left\langle n_1^2 \right\rangle}{2\pi} \int_{-\infty}^{\infty} d\kappa_x \operatorname{sinc}^2(\kappa_x x_1) \frac{\kappa_{0\perp} |Y| K_1\left[(\kappa_{0\perp}^2 + \kappa_x^2)^{1/2} |Y|\right]}{(\kappa_{0\perp}^2 + \kappa_x^2)^{1/2}} e^{i\kappa_x X}. \quad (40)$$

Under the condition $10^5\,\mathrm{m} \approx x_1 \gg \kappa_{0\perp}^{-1} \approx 10^4\,\mathrm{m}$, the factor $\operatorname{sinc}^2(\kappa_x x_1)$ acts as a Dirac delta function with respect to factors of $(\kappa_{0\perp}^2 + \kappa_x^2)^{1/2}$. Hence

$$R_{\phi_1}(\mathbf{R}) = 8(r_e \lambda z_0 \sin\theta)^2 \left\langle n_1^2 \right\rangle |Y| K_1(\kappa_{0\perp}|Y|) \frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa_x \operatorname{sinc}^2(\kappa_x x_1) e^{i\kappa_x X}. \quad (41)$$

We note the Fourier transform relationship

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} d\kappa_x \operatorname{sinc}^2(\kappa_x x_1) e^{i\kappa_x X} = \frac{1}{2x_1} \operatorname{tri}\left(\frac{X}{2x_1}\right), \quad (42)$$

where tri(x) $= (1-|x|)\mu(1-|x|)$ is the triangle function, and $\mu(x)$ is the unit step function. The final form of the correlation function is therefore

$$R_{\phi_1}(\mathbf{R}) = 2z_0 \tan\theta (r_e\lambda)^2 \left\langle n_1^2 \right\rangle \operatorname{tri}\left(\frac{X}{2x_1}\right) |Y| K_1(\kappa_{0\perp}|Y|). \quad (43)$$

This expression is good for two-dimensional displacements $\mathbf{R} = X\hat{\mathbf{x}} + Y\hat{\mathbf{y}}$. In the next subsection we extend this result to three-dimensional displacements.

4.3 Three-dimensional phase autocorrelation

We now consider three-dimensional displacements in the location of the phase observations. Figure 3 shows the impact of these displacements in the xz plane. We see

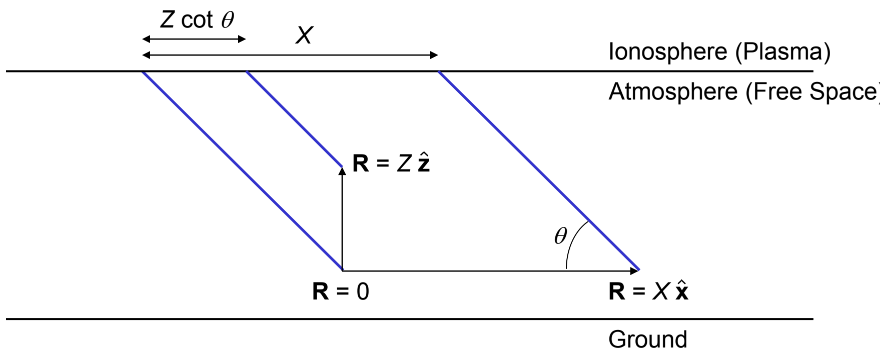

Figure 3: *Three-dimensional displacement on the ground, with relationship to two-dimensional displacement at base of ionosphere.*

that a displacement of $Z\hat{\mathbf{z}}$ moves the ionosphere exit location by the amount $Z\cot\theta\,\hat{\mathbf{x}}$. Therefore we can account for the horizontal and vertical observer displacements by making the replacement $X \to X + Z\cot\theta$ in (43). The three-dimensional correlation function is therefore

$$R_{\phi_1}(\mathbf{R}) = 2z_0 \tan\theta (r_e\lambda)^2 \left\langle n_1^2 \right\rangle \text{ tri}\left(\frac{X + Z\cot\theta}{2x_1}\right) |Y| K_1\left(\kappa_{0\perp}|Y|\right). \tag{44}$$

One can inspect this expression to determine the half-power correlation lengths in the three dimensions, which we denote using the subscript c. The functions $\mathrm{tri}(x)$ and $xK_1(x)$ are both unity at $x = 0$ and fall to one half at $x = 1/2$ and $x = 1$, respectively. Thus the half-power correlation lengths are

$$X_c = x_1 \tag{45}$$

$$Y_c = \kappa_{0\perp}^{-1} \tag{46}$$

$$Z_c = x_1 \tan\theta. \tag{47}$$

This completes the analysis of the three-dimensional correlation function.

5 Conclusion

A three-dimensional spatial autocorrelation function for ionospherically propagated wave packets at high latitudes has been provided. An analytic formula was derived based on a linear plasma density profile for the ionosphere and a power-law turbulence spectrum for ionospheric irregularities. The correlation length in the direction normal to the plane of propagation is approximately equal to the outer scale size of plasma irregularities in the ionosphere. The correlation length in the horizontal direction in the plane of propagation is approximately equal to one-half the horizontal length of the ionospheric path transited by the wave packet. The vertical correlation length is shown to be equal to the horizontal correlation length multiplied by the tangent of the wave packet take-off angle.

This work provides guidance for interpreting the results of experiments and for proposing future radio systems. In particular, autocorrelation functions provide insight on the degree to which ionospheric modes at various elevation and azimuth directions of arrival can be separated by analysis of the spatial dependent phase.

It would be beneficial to look at the case of general magnetic field dip angle, although it does not seem to be analytically tractable at this time. This analysis would support the interpretation of results obtained at lower latitudes, and illustrate the quality of the approximation of a vertical magnetic field made in the current study.

References

[1] Wheelon, A. D. (2001). Electromagnetic scintillation, Volume 1, geometric optics. New York: Cambridge University Press.

[2] Tatarskii, V. I. (1971). The effects of the turbulent atmosphere on wave propagation. Springfield, VA: National Technical Information Service.

[3] Rufenach, C. L. (1975). Ionospheric scintillation by a random phase screen: spectral approach, *Radio Sci.*, 10 (2), 155–165.

[4] Coleman, C. J. (1996). A model of HF sky wave radar clutter, *Radio Sci.*, 31 (4), 869–875, doi:10.1029/96RS00721.

[5] Ravan, M., Riddolls, R. J., Adve, R. S. (2012). Ionospheric and auroral clutter models for HF surface wave and over the horizon radar systems, *Radio Sci.*, 47, doi: 10.1029/2011RS004944.

[6] Dana, R., Wittwer, L. A. (1991). A general model for RF propagation through structured ionization, *Radio Sci.*, 26 (4), 1059–1068, doi:10.1029/91RS00263.

[7] Nickisch, L. J. (1992). Non-uniform motion and extended media effects on the mutual coherence function: an analytic solution for spaced frequency, position, and time, *Radio Sci.*, 27 (1), 9–22.

[8] Knepp, D. L. (1983). Analytic solution for the two-frequency mutual coherence function for spherical wave propagation, *Radio Sci.*, *18*(4), 535–549.

[9] Gherm, V. E., Zernov, N. N., Strangeways, H. J. (2005). HF propagation in a wideband ionospheric fluctuating reflection channel: physically based software simulator of the channel, *Radio Sci.*, 40 (1), 1–15, doi:10.1029/2004RS003093.

[10] Rytov, S. M., Kravtsov, Y. A., Tatarskii, V. I. (1989). Principles of statistical radiophysics, Volume 4, wave propagation through random media. Berlin, Germany: Springer.

[11] Budden, K. G. (1985). The propagation of radio waves: the theory of radio waves of low power in the ionosphere and magnetosphere. New York: Cambridge University Press.

[12] Kelley, M. C. (1989). The Earth's ionosphere. San Diego: Academic Press.

[13] Woodman, R. F., Basu, S. (1978). Comparison between in-situ spectral measurements of F-region irregularities and backscatter observations at 3m wavelength, *Geophys. Res. Lett.*, 5, 869–872, doi:10.1029/GL005i010p00869.

[14] Vallieres, X., Villain, J. P., Hanuise, C., Andre, R. (2004). Ionospheric propagation effects on spectral widths measured by Superdarn HF radars, *Ann. Geophys.*, 22, 2023–2031.

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We analytically derive a three-dimensional spatial autocorrelation function for the phase of ionospherically propagated wave packets at near-vertical magnetic field dip angles. The correlation length in the direction normal to the plane of propagation is approximately equal to the outer scale size of plasma irregularities in the ionosphere. The correlation length in the horizontal direction in the plane of propagation is approximately equal to one-half the horizontal length of the ionospheric path transited by the wave packet. The vertical correlation length is shown to be equal to the horizontal correlation length multiplied by the tangent of the wave packet take-off angle.

Nous développons analytiquement une fonction d'autocorrélation tridimensionnelle pour la phase de paquets d'ondes à propagation ionosphérique à des angles d'inclinaison du champ magnétique presque verticaux. La longueur de corrélation dans la direction normale au plan de propagation est environ égale à la taille des plus grandes irrégularités plasmatiques de l'ionosphère. La longueur de corrélation dans la direction horizontale du plan de propagation est environ égale à la moitié de la longueur horizontale du trajet ionosphérique suivi par le paquet d'ondes. On montre que la longueur verticale de corrélation est égale à la longueur de corrélation horizontale multipliée par la tangente de l'angle de rayonnement du paquet d'ondes.

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