An Approach to Sparing Analysis for a Finite Working Item Population

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DRDC CORA TM 2013-249
October 2012

Defence R&D Canada
Centre for Operational Research and Analysis

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Defence R&D Canada – CORA
Technical Memorandum
DRDC CORA TM 2013-249
October 2012

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Abstract

METRIC-based sparing models assume that equipment failures are generated by an infinite working item population. These models were first developed and applied in the US military which maintains large fleets of specific platforms obscuring the errors introduced by this simplification. However, in countries like Canada where fleet sizes tend to be small, the errors introduced by this assumption can be significant. In order to deal with small (or finite) fleet sizes, the T-METRIC model is proposed. The model is based on the use of truncated distributions for calculating the expected backorder. It is shown that the T-METRIC model can provide more accurate EBO values than the VARI-METRIC model, especially when there are a small number of working items.

Résumé

Les modèles d'analyse des pièces de rechange de la famille METRIC supposent que les pannes de matériel surviennent parmi une population infinie de pièces d'équipement. Ces modèles ont été élaborés et appliqués tout d'abord dans l'armée américaine, qui gère de vastes parcs de plateformes particulières, ce qui explique que cette simplification engendre des erreurs qui passent inaperçues. Toutefois, dans des pays comme le Canada, où les parcs d'équipements sont de taille plus modeste, les erreurs introduites par cette hypothèse peuvent être importantes. C'est pourquoi nous proposons le modèle T-METRIC pour la gestion des parcs de faible taille (ou de taille finie). Le modèle repose sur l'utilisation de distributions tronquées pour calculer le nombre estimé d'articles en souffrance. Nous montrons que le modèle T-METRIC peut donner des estimations plus justes que le modèle VARI-METRIC, surtout lorsque le nombre de pièces d'équipement disponibles est relativement faible.

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Executive summary

An Approach to Sparing Analysis for a Finite Working Item Population

Rui Zhang - NSERC Visiting Fellow
Yaw Asiedu ; DRDC CORA TM 2013-249; Defence R&D Canada – CORA; October 2012.

Background: Capital goods such as defense weapon systems, fail due to different processes such as corrosion, wear and tear, and fatigue. Due to the high replacement cost, these systems are repaired when they fail by removing and repairing the failed subcomponents. If a functioning spare is available, then it is installed on the system and the system is put back in service; otherwise, the replacement is delayed and the system becomes unavailable. Because of the impact system availability has on unit readiness and performance, it has always been an important area of research in military organizations. Most theoretical studies in this domain assume that the demand rate is invariant. That is, the number of working items is unlimited. This is the assumption used for computing the expected backorder in the VARI-METRIC model developed by Slay [1] and later improved by Sherbrooke [2] for the US military. However, this is not the case in practice and the application of the VARI-METRIC model may lead to erroneous results. This report develops an analytical model that is more applicable to situations where the number of working items is small; a situation that is prevalent in Canada's military.

Principal results: It was observed that results from simulations with small working item populations differed from results from the VARI-METRIC model and simulation results reported in the literature. This discrepancy, it was noted, was mainly due to the fact that the number of parts circulating in the simulated system was always upper bound. To rectify this, the truncated Negative Binomial and Poisson distributions were proposed for the calculation of the expected backorder in place of the standard Negative Binomial and Poisson distributions, used in the VARI-METRIC model. Simulated results show that with respect to the expected backorder and optimal stock policy, the T-METRIC model (using the truncated distributions) is either superior or similar to the VARI-METRIC model.

Future work: In order to deal with applications with small working item populations, two research directions deserving efforts from researchers are listed below: (1) efficient and flexible simulation tools a fixed number of working item population and (2) analytical approaches taking into account the number of working items in a population.

Sommaire

Une méthode d’analyse des pièces de rechange appliquée à une population finie d’articles en état de fonctionnement.

Rui Zhang - NSERC Visiting Fellow
Yaw Asiedu ; DRDC CORA TM 2013-249 ; R & D pour la défense Canada – CARO ; octobre 2012.

Contexte : Les biens d’équipement tels que les systèmes d’arme défensive tombent parfois en panne sous l’action de différents processus comme la corrosion, l’usure et la fatigue. En raison des coûts de remplacement élevés, on choisit de réparer les systèmes en panne en retirant de ceux-ci le ou les sous-composants défectueux pour les remettre en bon état. Si une pièce de rechange est disponible, elle est installée dans le système et celui-ci est remis en service ; dans le cas contraire, la réparation est retardée et le système n’est plus opérationnel. Compte tenu de l’incidence que peut avoir la disponibilité (ou non-disponibilité) des systèmes sur l’état de préparation et le rendement de l’unité, l’analyse des pièces de rechange a toujours été un sujet de recherche important au sein des organisations militaires. La plupart des études théoriques dans ce domaine supposent que le taux de demande est invariant, c’est-à-dire que le nombre d’articles en état de fonctionnement est illimité. C’est l’hypothèse qui est utilisée pour calculer le nombre estimé d’articles en souffrance avec le modèle VARI-METRIC, élaboré par Slay [1], puis amélioré par Sherbrooke [2] pour le compte de l’armée américaine. Cela dit, cette hypothèse n’est pas fondée en réalité et l’application du modèle VARI-METRIC peut donner de faux résultats. Ce rapport propose un modèle analytique qui est mieux adapté aux situations où le nombre d’articles en état de fonctionnement est relativement modeste, ce qui est le cas dans les Forces armées canadiennes.

Résultats : Nous observons que les résultats des simulations effectuées sur de petites populations d’articles en état de fonctionnement diffèrent de ceux du modèle VARI-METRIC et des résultats de simulations exposés dans la littérature. Il convient de souligner que cet écart s’explique principalement par le fait que le nombre de pièces qui circulent dans le système simulé est toujours assujetti à une limite supérieure. Pour corriger cette situation, nous proposons de calculer le nombre estimé d’articles en souffrance au moyen de versions tronquées de la distribution binomiale négative et de la distribution de Poisson plutôt que des versions classiques de ces distributions employées dans le modèle VARI-METRIC. Les résultats des simulations montrent que le modèle T-METRIC (qui utilise les distributions tronquées) est soit supérieur ou équivalent au modèle VARI-METRIC pour ce qui est du calcul du nombre estimé d’articles en souffrance dans le cadre d’une politique optimale de gestion du parc.

Recherches futures : Pour ce qui est de l’étude des petites populations d’articles en état de fonctionnement, nous proposons deux pistes de recherche qui méritent d’être explorées : 1) outils de simulation souples et efficaces au regard d’une population d’articles en état de fonctionnement à effectif fixe, et 2) méthodes analytiques qui tiennent compte du nombre d’articles en état de fonctionnement dans une population.

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1 Introduction

1.1 Background

Manufactured products will fail due to different processes such as corrosion, wear and tear, and fatigue. Products such as domestic electronics and appliances are generally discarded and replaced upon failure because they are inexpensive. However, capital goods such as defence weapon systems are repaired because the replacement costs tend to be very high. The repair often involves the removal of the failed component. If a functioning spare is available, then it is installed on the system and the system is put back in service; otherwise, the replacement is delayed and the system becomes unavailable. A defective component can either be discarded or repaired. If it is repairable, then a defective subcomponent is identified and replaced by a functioning one, or repaired. Replacement and repair is conducted either at the operating site or at another site within a maintenance/logistics support network.

In the military context, a prime equipment (PE) can be an aircraft, a tank, a submarine or a ship, etc. They contain tens or hundreds of units. These units are usually called line replaceable units (LRU), e.g. a truck's fuel pump. These LRUs are in turn made up of shop replaceable units (SRU), e.g. a fuel pressure sensor in the pump. Thus the PE can be thought of as a hierarchy of parts (LRUs and SRUs) at different indenture levels. In Figure 1, an example of PE with two indentures is depicted. The PE, located at Indenture 0, has two LRUs (Indenture 1); one LRU has two SRUs (Indenture 2) and the other has three SRUs (Indenture 2).

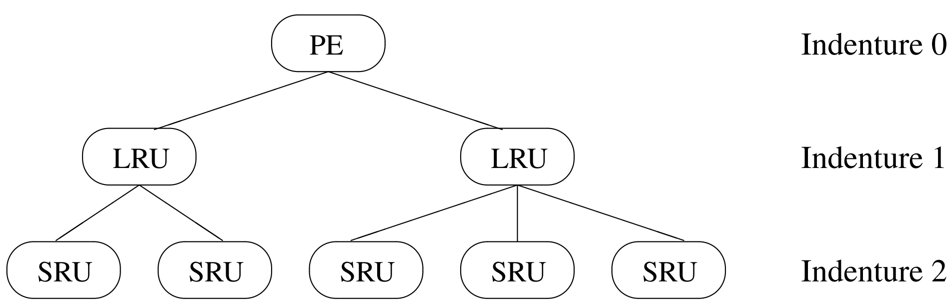

Figure 1: *A PE with a two indenture structure.*

In most military applications, operating sites where the weapon systems or PEs are located may be dispersed across a large geographical area. Carrying all the required maintenance equipment and stock of spare parts at each operating site may not be economically efficient. Repair equipment and spares are therefore also dispersed across multiple locations. A sample three-echelon support network is shown in Figure 2. It includes a depot, two intermediate sites and five bases. Generally, most of the costly repair facilities and spare

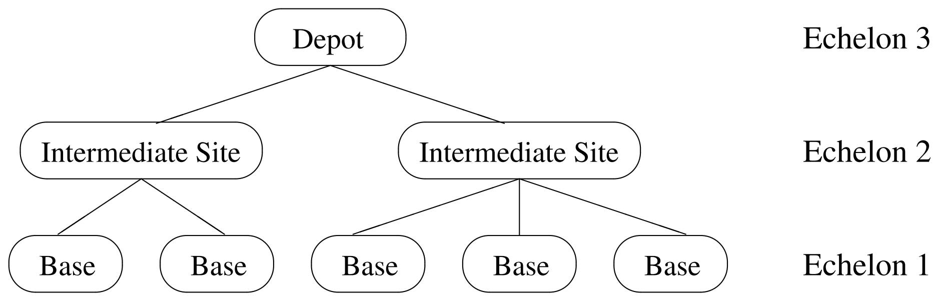

Figure 2: *A support network with three echelons.*

parts would be located at the depot, which is at the highest echelon level (Echelon 3) and supports all the operating sites or bases (Echelon 1) through two intermediate sites (Echelon 2). Note that this support system has a special tree structure. However, logistics support networks may be more complex with a graph structure. A multi-indenture PE supported by a multi-echelon network is collectively referred to as a multi-indenture, multi-echelon (MIME) system. In particular, Figures 1 and 2 describe a two-indenture, three-echelon system.

In practice, the major issue is how many spares to stock at the different locations in order to achieve an acceptable level of system availability. The expected backorder (EBO), which is the mean of the LRU backorder at the bases, is commonly used as a standard measure for this purpose. The major challenge in sparing analysis is how the EBO should be computed or approximated. This is often done using variants of the multi-echelon technique for recoverable item control (METRIC) model developed by Sherbrooke [3]. The VARI-METRIC model was developed by Slay [1] to account for the fact that the mean of the distribution of the number of units in the LRU repair and/or resupply pipeline is not equal to the variance for MIME systems as assumed in the METRIC model. Consequently, instead of approximating the LRU pipeline by a Poisson distribution, a Negative Binomial distribution is used.

1.2 Aim

As a widely accepted analytical model for MIME systems, the VARI-METRIC model can provide a quick approximation of the EBO. However, the VARI-METRIC model (and all METRIC-based models) assumes that the number of working items is unlimited. Given that the Canadian Forces often operate small fleets, e.g. the Royal Canadian Navy operates 4 submarines, 3 destroyers and 2 replenishment vessels, using the VARI-METRIC model for availability analysis may introduce large errors. This report develops a more robust analytical model, the T-METRIC model, with the aim to improve the accuracy of the EBO calculation for systems with small working item populations.

1.3 Scope

In developing the T-METRIC model, it is assumed that the Negative Binomial distribution is a good approximation of the LRU repair pipeline distribution and the use of other distributions is not investigated. Therefore, the central issue for this report is the modification of the Negative Binomial distribution to make it more applicable to problems with small working item populations.

1.4 Report organization

The remainder of this report is organized as follows. In Section 2, the basic assumptions and notations used in the METRIC-based models are introduced. This is followed by a more detailed look of the VARI-METRIC model. The details of the T-METRIC model are then discussed. In Section 3, sample problems are presented to show that the T-METRIC model performs better than the VARI-METRIC model with respect to EBO and optimal stock policy (OSP). Final remarks and discussions are provided in Section 4.

2 Modification of the VARI-METRIC model

2.1 Preliminaries

Analytical models for sparing analysis have been dominated by a series of METRIC-based models since the middle of the twentieth century. The METRIC model was originally developed by Sherbrooke [3] in 1968 for a single-indenture product supported by a two-echelon maintenance network. The multi-indenture, single-echelon (MISE) problem was addressed by Sherbrooke in [4]. The MOD-METRIC model was later developed by Muckstadt [5] for multi-indenture product structures and multi-echelon maintenance networks. An important limitation of the MOD-METRIC model is the assumption that the number of units in the pipeline at each operating site is Poisson distributed. This is often not true in practice. Consequently, the MOD-METRIC model can lead to substantial overestimation of the true backorder.

In [1], Slay derived an extension of the METRIC model called VARI-METRIC. The VARI-METRIC model was proven to perform substantially better than the METRIC or MOD-METRIC model. The substantial improvement achieved by the VARI-METRIC model is due to the fact that it accounts not only for the mean, but also for the variance of the number of components in repair. This is achieved by using a Negative Binomial rather than a Poisson distribution. For the cases studied in Graves [6], 11% of the stock levels computed using the METRIC model differ by at least one component from the optimal results while only 1% of the stock levels computed using the VARI-METRIC model differ from the optimal levels. In [2], Sherbrooke combined MOD-METRIC and VARI-METRIC to formulate a MIME version of the VARI-METRIC model. The author confirms through a number of examples, that in these cases, the VARI-METRIC estimate of EBO can be significantly larger than the MOD-METRIC EBO. Similar to [6], Sherbrooke also confirms that the VARI-METRIC estimate of the number of components in repair is very close to the actual result obtained using simulation.

The basis of METRIC-based models is Palm's Theorem [7], which is stated below.

Palm's Theorem Consider the $M/G/\infty$ queue with homogeneous Poisson input rate $\lambda > 0$ and a stationary service distribution F with mean $1/\mu$, then, as time t goes to ∞, the limiting distribution of the number of arrivals still in service is Poisson with mean λ/μ.

Due to the requirements of this theorem, METRIC-based models deploy two important assumptions: repair time is constant for all items and demand has a Poisson distribution with a constant mean. (Actually, the former assumption can be relaxed to independently and identically distributed repair times according to any distribution with a constant mean.) Accounting for the fact that in most real cases, repair capacity is limited, Diaz and Fu [8] modeled a single-indenture, two-echelon system with limited repair capacity at the depot, where a single repair shop with one or more repair channels is located. Sleptchenko et al.

[9] generalized the model of Diaz and Fu to MIME systems that may have multiple repair shops in the network.

The constant-mean-Poisson-demand assumption implies that the studied problem has an unlimited number of PEs providing an unlimited source of failures. In reality however, the number of PEs is limited and the failure rate changes with time. Recently, Nie and Wen [10] modeled a single-indenture, two-echelon system as a closed loop system and simulated it using Arena software. However, Nie and Wen did not make any statements to demonstrate or explain the significance of the closed loop simulation.

In addition to the two assumptions above, METRIC-based models also employ the following three assumptions:

1. an $(s-1,s)$ inventory policy is used for each item, since they have high costs and low demands (s is the number of spares of the item);
2. no items are condemned and discarded; and
3. no lateral transshipment is allowed between bases.

These three assumptions together with the two above are adopted in this report.

2.2 The VARI-METRIC model

Before introducing the T-METRIC model, the VARI-METRIC is first presented. The notations below describe a sample system with two indentures and one echelon (the base). The subscript $j = 1,2,...n$ refers to the n different SRUs at the same indenture level, which are components of the same LRU. The LRU is denoted by the reserved subscript $j = 0$.

- λ_0 is the demand rate of the LRU (number/day).
- R_0 is the repair time of the LRU (days).
- s_0 is the stock level of the LRU.
- λ_j is the demand rate of SRU-j (number/day).
- R_j is the repair time of SRU-j (days).
- s_j is the stock level of SRU-j.

Let $x_0(t)$ be the number of LRUs in the repair pipeline at a random time point t. Then, the mean and variance of $x_0(t)$ can be written as:

$$E[x_0] = \lambda_0 R_0 + \sum_{j=1}^{n} E[B(s_j)] \tag{1}$$

and

$$Var[x_0] = \lambda_0 R_0 + \sum_{j=1}^{n} Var[B(s_j)], \tag{2}$$

respectively, where $B(s_j)$ is the number of backorders of SRU-j given s_j spares, and $E[B(s_j)]$ and $Var[B(s_j)]$ are the mean and variance of $B(s_j)$. Let $x_j(t)$ be the number of SRU-j in the repair pipeline at time t. A backorder occurs if $x_j(t) > s_j$. Thus the mean and variance of the backorders can be calculated as:

$$E[B(s_j)] = \sum_{x=s_j+1}^{\infty} (x - s_j) P(x|\lambda_j R_j) \tag{3}$$

and

$$Var[B(s_j)] = \sum_{x=s_j+1}^{\infty} (x - E[B(s_j)])^2 P(x|\lambda_j R_j), \tag{4}$$

respectively, where $P(x|\lambda_j R_j)$ is the probability density function of the Poisson distribution with equal mean and variance $\lambda_j R_j$. Similar to the above calculations for the SRUs, the LRU EBO, $E[B(s_0)]$ can be calculated as:

$$E[B(s_0)] = \sum_{x=s_0+1}^{\infty} (x - s_0) P(x|E[x_0], Var[x_0]), \tag{5}$$

where $P(x|E[x_0], Var[x_0])$ is the probability density function of the Negative Binomial distribution with mean $E[x_0]$ and variance $Var[x_0]$.

One problem with the above calculations is that they assume infinite number of LRUs and SRUs. However, the fact that there are fixed numbers of spares and PEs, preconcludes this possibility. In Figure 3, the un-shaded bars are the probabilities of the number of LRUs in the repair pipeline in the simulation discussed in Section 1 and the shaded bars are the probabilities of the Negative Binomial distributions used by the VARI-METRIC model in computing EBO. It is clear that the simulated probabilities are different from what are used for the calculations. Note that there will not be more than five LRUs in the simulated system at any time point. Consequently, the summations in Equations (3) - (5) should not be taken to infinity but rather should be over a finite set of values. In the next section, the T-METRIC model is introduced to address these concerns.

2.3 The T-METRIC model

Suppose that a positive integer ω is the maximum number of parts possible in a repair pipeline, i.e., $x(t) \leq \omega$ for all t, where $x(t)$ is a random variable denoting the number of parts in the pipeline. In such a case, instead of assuming $x(t)$ is Poisson distributed

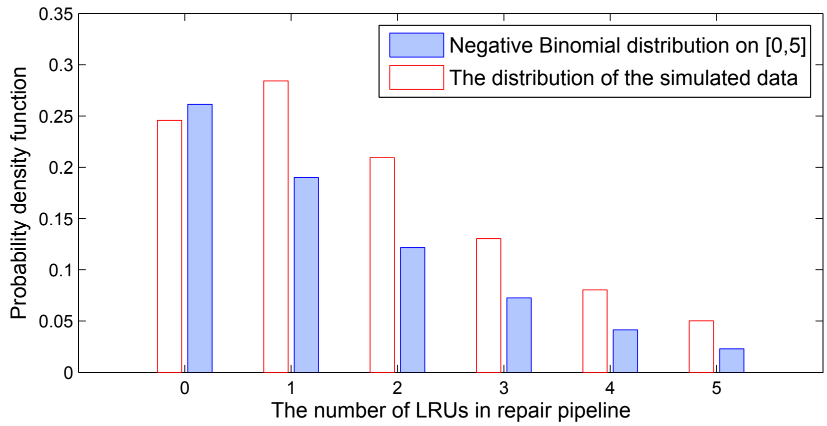

Figure 3: *Comparison of the probabilities of the number of LRUs in repair pipeline.*

with mean μ, it may be more appropriate to assume that $x(t)$ is distributed according to a truncated Poisson distribution:

$$P_{tr}(x|\omega,\mu) = \frac{P(x|\mu)}{\sum_{k=0}^{\omega} P(k|\mu)}, \tag{6}$$

where subscript "tr" is used to denote a truncated distribution and $x = 0, 1, 2, ..., \omega$. Similarly, a truncated Negative Binomial distribution can be defined as:

$$P_{tr}(x|\omega,\mu,\sigma) = \frac{P(x|\mu,\sigma)}{\sum_{k=0}^{\omega} P(k|\mu,\sigma)}. \tag{7}$$

Figure 4 compares the probability density function of the truncated and standard Negative Binomial distributions, where $\omega = 10$, $\mu = 9$ and $\sigma = 18$. The dashed curve denotes the standard distribution and the solid curve denotes the truncated distribution. The truncated distribution has a finite domain of $x \in [0, 10]$, while the standard distribution has positive probabilities in an infinite domain $x \geq 0$. Note that the figure depicts the curves only over the interval of $x \in [0, 15]$ for comparison. The differences can be clearly observed. Regions A and B are the areas enclosed by the two curves and the line, $x = 10$, to the left and right side of the line, respectively. The probabilities denoted as B are actually reallocated to the truncated distribution as the additional area denoted as A.

Based on the above observations, the proposed T-METRIC model replaces the standard distributions in the VARI-METRIC model with the truncated distributions in Equations (6) and (7); with the summation limits modified accordingly. As an illustration, assume that the logistics system supports only one type of PE, which has a single LRU, which in turn

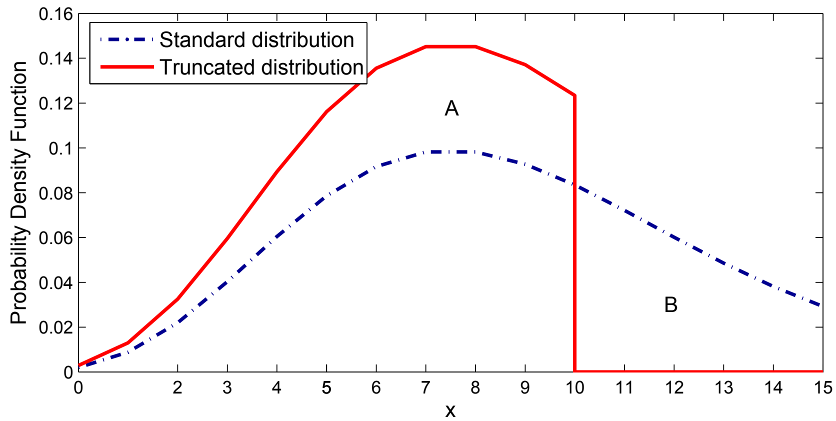

Figure 4: *Comparison of the standard and truncated Negative Binomial distributions.*

has n different SRUs. If there are γ PEs, where $\gamma > 0$, then the number of LRUs in the repair pipeline has an upper bound of $s_0 + \gamma$ and the number of SRU-j in the pipeline has an upper bound of $s_0 + s_j + \gamma$.

Using these new bounds, Equations (3) and (4) which are used to calculate the mean and variance of SRU-j backorder are modified to:

$$E[B(s_j)|\gamma] = \sum_{x=s_j+1}^{s_0+s_j+\gamma} (x - s_j)P_{tr}(x|\gamma, \lambda_j R_j) \quad (8)$$

and

$$Var[B(s_j)|\gamma] = \sum_{x=s_j+1}^{s_0+s_j+\gamma} (x - E[B(s_j)|\gamma])^2 P_{tr}(x|\gamma, \lambda_j R_j), \quad (9)$$

respectively. Similarly, Equations (1) and (2) for the mean and variance of the number of LRUs should be rewritten as:

$$E[x_0|\gamma] = \lambda_0 R_0 + \sum_{j=1}^{n} E[B(s_j)|\gamma] \quad (10)$$

and

$$Var[x_0|\gamma] = \lambda_0 R_0 + \sum_{j=1}^{n} Var[B(s_j)|\gamma], \quad (11)$$

respectively. Finally, Equation (5) for the EBO is represented in the T-METRIC model as:

$$E[B(s_0)|\gamma] = \sum_{x=s_0+1}^{s_0+\gamma} (x - s_0)P_{tr}(x|\gamma, E[x_0|\gamma], Var[x_0|\gamma]). \quad (12)$$

In Appendix A, an upper bound determination procedure (UBDP) is presented. Using the UBDP, an upper bound of a repair pipeline in any given system can be determined easily. Thus, the above calculations using truncated Poisson and Negative Binomial distributions can be easily extended to a system with more indentures and echelons.

3 Comparison of the T-METRIC and VARI-METRIC model

In this section, examples are presented to illustrate the benefits of using the T-METRIC model as opposed to the VARI-METRIC model. The comparisons are based on the EBO and OSP.

3.1 Comparison of the EBO

Two examples are presented here to compare the accuracy of the T-METRIC and VARI-METRIC model in calculating the EBO. Both problems are from Sherbrooke [2]

3.1.1 The MISE sample problem

The data for the two-indenture, one-echelon problem are shown in Table 1. In this example, the LRU has two SRUs: SRU-1 and SRU-2. The failure of the LRU is due to the failure of either SRU-1 or SRU-2 with equal probabilities of 0.5. The failure rate of LRU is two per day. The constant repair time of the LRU is half a day and for the SRUs, eight days.

Table 1: *The failure and repair data for the two-indenture, one-echelon sample problem (Source: Sherbrooke [2]).*

Part	Failure rate (per day)	Repair time (days)
LRU	2	0.5
SRU-1	$1 = 2 \times 0.5$	8.0
SRU-2	$1 = 2 \times 0.5$	8.0

This example was specifically selected by Sherbrooke to show that the VARI-METRIC model, which assumes a Negative Binomial distribution, is better than the MOD-METRIC model, which assumes a Poisson distribution, for the LRU repair pipeline. Sherbrooke showed that under a $4-10-10$ (4 LRUs, 10 SRU-1s and 10 SRU-2s) stock policy, the VARI-METRIC model can produce an EBO value (0.194) that is within $4\% = ((0.202 - 0.194) \div 0.202) \times 100\%$ of the $50,000$-year simulated EBO (0.202), while the MOD-METRIC model gives an EBO (0.056) with a higher error of 72.3% compared to 0.202.

As discussed in Section 2.3, the number of PEs together with the stock levels determines the maximum number of LRUs and SRUs in the system. Using the previous notations, $\gamma = 1$, $s_0 = 4$, $s_1 = 10$ and $s_2 = 10$ for this particular example. Thus, $\gamma + s_0 = 5$ is the upper bound of the LRU repair pipeline, and $\gamma + s_0 + s_1$ (or s_2) $= 15$ is the upper bound for the SRU-1 (or SRU-2) repair pipeline. Plugging these values into Equations (8) to (12), gives an EBO of 0.0372 for the T-METRIC model. These values are not close to any of the above results: 0.194, 0.056 and 0.202 for the VARI-METRIC model, the MOD-METRIC model and the simulation in [2], respectively.

Noting that the simulated EBO of 0.202 was obtained by assuming an infinite working item population, a new closed-loop simulation tool (CLS[1]) was developed. In a CLS simulation, the number of PEs is fixed and a PE cannot generate any demands unless it is working. Using the same number of PEs (i.e., $\gamma = 1$), an EBO of 0.0369 is determined using the CLS tool. This value is very close to the T-METRIC result of 0.0372. Table 2 shows the details for the various methods. The first row contains the EBOs under the $4-10-10$ stock policy and in the second row are the errors relative to the CLS EBO of 0.0369 (e.g. $0.8\% = ((0.0372-0.0369) \div 0.0369) \times 100\%$). Note that the T-METRIC model has a very small error of 0.8%, while the VARI-METRIC model has a high error of 425%.

Table 2: *Comparison of the EBOs for the* $4-10-10$ *stock policy in the two-indenture, one-echelon sample problem.*

Parameter	T-METRIC	VARI-METRIC	Sherbrooke [2]	The CLS tool
EBO	0.0372	0.194	0.202	0.0369
Relative error (%)	0.8	425	447	0

The T-METRIC and CLS model are also tested on the same example but under a different stock policy of $5-10-10$. Table 3 also contains the EBOs and the relative errors (compared to the CLS EBO of 0.0217). The error of the VARI-METRIC model is again very high (375%). Moreover, the gaps between the EBO from [2] and the EBO from the CLS are very high as well, i.e., 447% under the $4-10-10$ stock policy and 412% under the $5-10-10$ stock policy.

Table 3: *Comparison of the EBOs for the* $5-10-10$ *stock policy in the two-indenture, one-echelon sample problem.*

Parameter	T-METRIC	VARI-METRIC	Sherbrooke [2]	The CLS tool
EBO	0.0206	0.103	0.111	0.0217
Relative error (%)	−5.07	375	412	0

In order to explore these unusual gaps, the EBOs for the cases of $\gamma = 2, 3, \ldots, 15$ were also determined using the various approaches for the $4-10-10$ stock policy. Figure 5

[1] The CLS tool is an object-oriented, discrete-event simulation tool for MIME systems developed using Java. A brief description of the tool is provided in Appendix B.

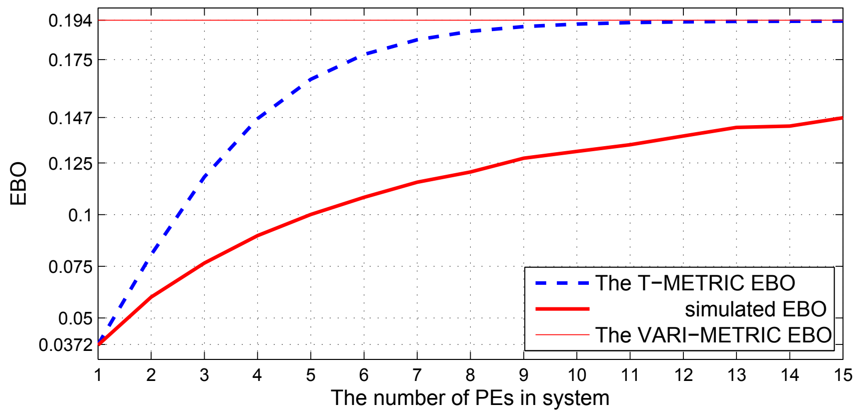

Figure 5: *Comparison of the EBOs of the cases with* $1-15$ *PEs for the two-indenture, one-echelon sample problem.*

shows the plot of the EBOs from the CLS (bold solid curve), the VARI-METRIC (upper horizontal dashed line) and the T-METRIC (bold dashed curve) model. As can be seen, the VARI-METRIC EBO values are independent of γ. The EBO from the T-METRIC model increases from 0.0372 to 0.194 (the VARI-METRIC value). In all of the tested cases, the CLS EBO stays below the T-METRIC EBO, following the same upward trend but increasing more slowly (up to only 0.147 with $\gamma = 15$). This figure validates that when the number of PE less than 15, it is better to use the T-METRIC model for EBO estimation.

Figure 6 depicts results from the CLS for higher values of γ. The EBO of 0.202 reported from the simulation in [2] was only replicated when the value of γ is around $1,024$, a very large number. Similarly, for the $5-10-10$ stock policy, $1,000$ working PEs gave a CLS EBO of 0.1107, which is close to the value of 0.111, reported by the simulation in [2]. These observations tend to support the statement by Graves in [6] "*(Assuming constant demand rate) seems reasonable when the expected number of shortage at a site (base) is small relative to the required number of working items (PEs) at that site.*". However, the required large number (i.e., $1,024$ for the $4-10-10$ stock policy and $1,000$ for the $5-10-10$ stock policy) is unrealistic in most real applications. This indicates the need for the T-METRIC model in the real world.

3.1.2 The MIME sample problem

This sample problem has two bases (Base-1 and Base-2) supported by a central depot (Depot). The PE has one LRU made up of two SRUs, SRU-1 and SRU-2. The failure and repair data are presented in Table 4. The daily LRU demand rates at Base-1 and Base-2 are 0.5 and 1, respectively. SRU-1 and SRU-2 have conditional failure probabilities of

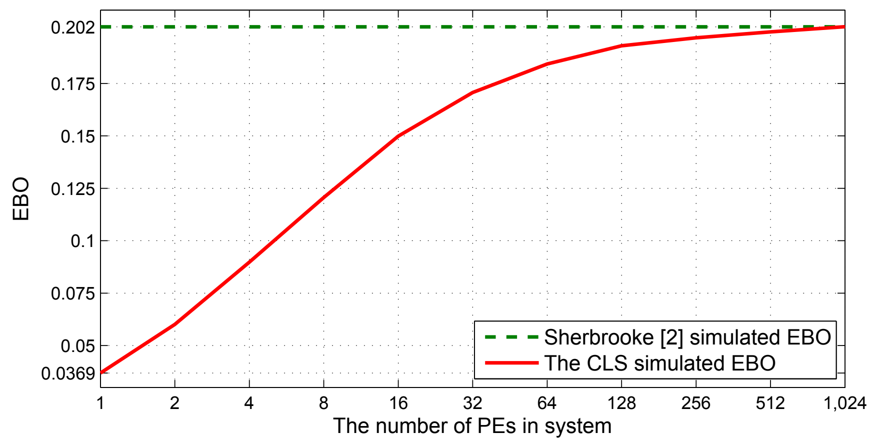

Figure 6: *Comparison of the EBOs of the cases with up to $1,024$ PEs for the two-indenture, one-echelon sample problem.*

$1/3$ and $2/3$, respectively. All parts are 50% base repairable. Note that the demand rates shown in Table 4 are the actual demand rates based on the base repairable percentages and conditional failure probabilities. For example, the LRU demand rate at Depot is $0.75 = (0.5+1)\times(1-0.5)$; the SRU-1 demand rate at Base-1 is $0.083 = 0.5\times0.5\times1/3$; and the SRU-1 demand rate at Depot is $0.375 = 0.75\times1/3+(0.083+0.167)\times(1-0.5)$. It takes 4 and 15 days to repair an LRU at the bases and depot, respectively. The SRU repair times are 8 and 30 days at the bases and depot, respectively. The resupply time from Depot to Base-1 or Base-2 is 15 days for all parts.

Table 4: *The failure and repair data for the two-indenture, two-echelon sample problem (Source: Sherbrooke [2]).*

Part	Demand rate (per day)			Repair time (days)		
	Base-1	Base-2	Depot	Base-1	Base-2	Depot
LRU	0.5	1	0.75	4	4	15
SRU-1	0.083	0.167	0.375	8	8	30
SRU-2	0.167	0.333	0.75	8	8	30

Table 5 shows the optimal stock policy determined by Sherbrooke [2] using the VARI-METRIC model and marginal analysis approach. For this stock policy, the VARI-METRIC model reports an EBO of 27.11, while the T-METRIC model only gives an EBO of 1.816. Using the CLS tool with one PE at Base-1 and two PEs at Base-2, the EBO was determined as 1.336. The number of PEs at Base-1 and Base-2 was selected in the simulations in this section is to reflect the LRU demand ratio, 0.5 to 1. Compared to the simulated EBO, the VARI-METRIC model over estimates the EBO by approximately 2,000%, while the T-METRIC's estimation error is only 35.9%. This demonstrates the advantage of T-METRIC model.

Table 5: *The optimal stock policy for the two-indenture, two-echelon sample problem determine in [2] using the VARI-METRIC model.*

Part	Stock		
	Base-1	Base-2	Depot
LRU	7	5	11
SRU-1	2	1	3
SRU-2	4	3	6

Figure 7 shows the plots of the CLS, T-METRIC and VARI-METRIC EBOs for the cases with the total number of PEs from 3 (i.e., one PE at Base-1 and two PEs at Base-2) to 12,288 (i.e., 4,096 PEs at Base-1 and 8,192 PEs at Base-2). The VARI-METRIC EBO of 27.11 is only replicated when the number of PEs is approximately equal to 12,288. Similar to the previous example, the T-METRIC model provides better estimates than the VARI-METRIC model.

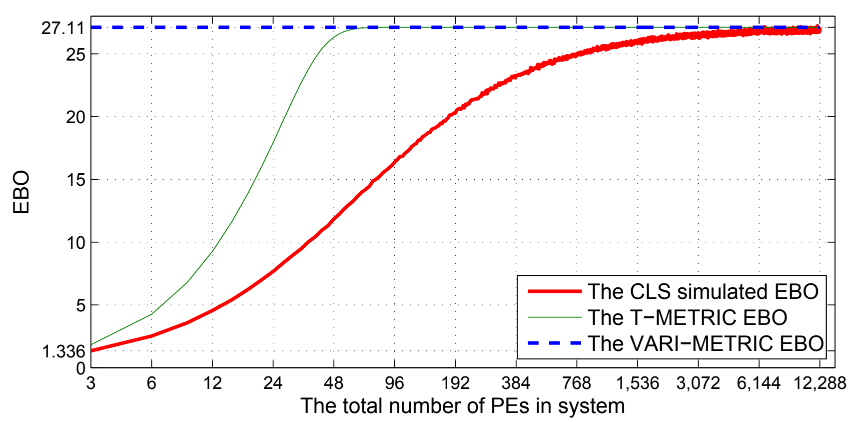

Figure 7: *Comparison of the EBOs of the cases with up to 12,288 PEs (i.e., 4,096 PEs at Base-1 and 8,192 PEs at Base-2), for the two-indenture, two-echelon sample problem.*

3.2 Comparison of the OSP

In the forgoing discussions, the stock policies were assumed and the EBOs compared. However, the main objective of sparing analysis is actually to determine the OSP for a system. Consider the same example as described in Table 1, but as an optimization problem to minimize the EBO subject to a limited stock budget. Suppose $\$20,000$ is available for procuring stocks of spares and it costs $\$2,000$ to stock an LRU, and $\$1,000$ for either SRU. The question to be considered is how many spares of each item should be procured and stored at each location to minimize the EBO. This issue was addressed for different number of PEs.

In Table 6, the first column lists the number of PEs for each test, i.e., $\gamma = 1,2,...,11$. Column "T-METRIC OSP" represents the OSPs found using the T-METRIC model. As can been seen, the 11 tests can be divided into three groups: tests with one to three PEs with an OSP of $3-7-7$ (3 LRUs, 7 SRU-1s and 7 SRU-2s); four to nine PEs with OSP $2-8-8$; and 10 and 11 PEs with OSP $1-9-9$. However, as expected the VARI-METRIC model reported the same OSP of $1-9-9$ for all of these 11 tests.

Table 6: *Comparison of the optimal stock policies for the two-indenture, one-echelon sample problem.*

# of PEs	T-METRIC OSP	The CLS EBO		Error (%)
		T-METRIC	VARI-METRIC	
1	3 - 7 - 7	0.1880	0.2385	26.9
2	3 - 7 - 7	0.3300	0.3795	15.0
3	3 - 7 - 7	0.4441	0.4862	9.5
4	2 - 8 - 8	0.5211	0.5736	10.1
5	2 - 8 - 8	0.5943	0.6491	9.2
6	2 - 8 - 8	0.6601	0.7125	7.9
7	2 - 8 - 8	0.7142	0.7634	6.9
8	2 - 8 - 8	0.7671	0.8102	5.6
9	2 - 8 - 8	0.8121	0.8536	5.1
10	1 - 9 - 9	0.8902	0.8902	0
11	1 - 9 - 9	0.9249	0.9249	0

The CLS EBOs based on the T-METRIC and VARI-METRIC OSP for each test are shown in the third and fourth columns, e.g. for a single-PE system, 0.1880 is the CLS EBO under the $3-7-7$ stock policy and 0.2385 is the CLS EBO under the $1-9-9$ stock policy. The last column compares these two CLS EBO results, i.e., for the single-PE test, $26.9\% = ((0.2385-0.1880) \div 0.1880) \times 100\%$. This measures the error in the EBO caused by the VARI-METRIC OSP relative to the T-METRIC OSP. This shows that the T-METRIC model would lead to an optimal sparing policy that ensures that the actual EBO is lower than the actual EBO based on the sparing from the VARI-METRIC model.

4 Conclusions

Many practical sparing decisions are made for systems with a few working items. However, the commonly used VARI-METRIC model assumes that the number of working items is unlimited, and that the repair pipelines may contain unlimited number of parts. This assumption introduces errors in the calculated EBO for some applications. The T-METRIC model was proposed to address these shortcomings.

The accuracy of the T-METRIC model was demonstrated by comparing it to the VARI-METRIC model for two sample problems. The first problem was a simple MISE system with two indentures and one echelon. When the system has fewer PEs (less than 15 in Figure 5), the error in the EBO value from the VARI-METRIC model is very large. For example, for the single-PE system, the VARI-METRIC model reported an EBO with an error of 425%, while the T-METRIC's EBO had only a small error of 0.8%.

In order to validate the T-METRIC model for MIME systems, a two-indenture, two-echelon sample problem was studied. Again, it was shown that the T-METRIC model provides a much more accurate estimation compared to the VARI-METRIC model, especially when the number of PEs is less than 48. For a system with a single PE, the T-METRIC EBO had an error of 35.9%, while the VARI-METRIC EBO had an error of almost 2,000%.

The two models were also compared by minimizing the EBO subject to a limited budget for procuring spares for the MISE sample problem. Similar to the EBO estimation, the T-METRIC model produced superioir spare policies when the number of PEs was small (less than 9). For instance, in the test with a single PE in the system, using the T-METRIC OSP produced a simulated EBO of 0.1880, while the simulated EBO, using the VARI-METRIC OSP was 0.2385. The results were similar for ten or more PEs.

All of the above comparisons show that for a system with a finite working item population, the T-METRIC model is more robust in spare parts logistics study and application. Although the T-METRIC model was tested only on the two-indenture, one-echelon and two-indenture, two-echelon systems, using the UBDP developed in Appendix A, the T-METRIC model can be easily applied to systems with arbitrary indenture and echelon structures.

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Annex A: The upper bound determination procedure

The UBDP is an algorithm for determining the upper bound of the number of parts (LRUs or SRUs) at each site (depot, base or intermediate site) in an arbitrary MIME system. Consider a system with L types of PEs, $E_l, E_2, ..., E_L$, which are made up of M types of parts, $T_1, T_2, ..., T_M$ and are supported by a network with N sites, $S_1, S_2, ..., S_N$. For instance, the sample system described by Figures A.1, A.2 and A.3 has eight sites ($N = 8$ in Figure A.1), two types of PEs ($L = 2$ in Figures A.2 and A.3) and eight types of parts ($M = 8$ in Figures A.2 and A.3). Note that E_1 (Figure A.2) and E_2 (Figure A.3) are each made up of only six parts.

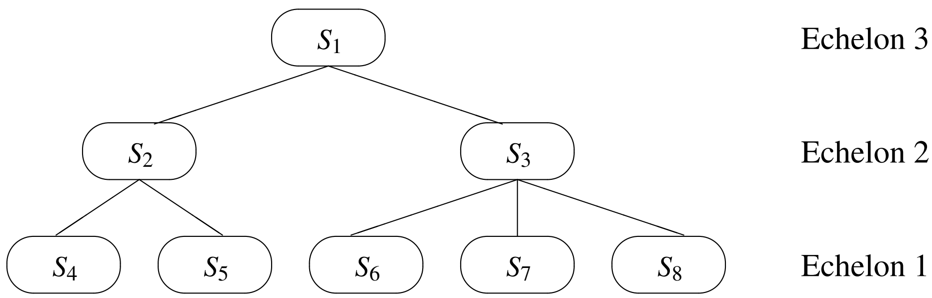

Figure A.1: *The support network in the sample system.*

Let Λ_β be the set of sites at Echelon β (e.g. $\Lambda_2 = \{S_2, S_3\}$). Let Φ_n be the set of sites at Echelon β directly supported by S_n. For example, in Figure A.1, S_4 and S_5 (at Echelon 1) are supported by S_2 (at Echelon 2), so $\Phi_2 = \{S_4, S_5\}$. In particular, $\Phi_n = \emptyset$, if $S_n \in \Lambda_1$.

Let Γ_α be the set of part types at Indenture α (e.g. $\Gamma_1 = \{T_1, T_2, T_3\}$). It is reasonable to assume that no part can be located at more than one indenture level, i.e., $\Gamma_\alpha \cap \Gamma_{\alpha'} = \emptyset$, if $\alpha \neq \alpha'$. For $T_m \in \Gamma_1$, let Ω_m be the set of PE types, which have an LRU of type T_m as a component. As can be seen in Figures A.2 and A.3, $\Omega_1 = \{E_1\}$, $\Omega_2 = \{E_1, E_2\}$ and $\Omega_3 = \{E_2\}$. A part at Indenture $\alpha > 1$ is defined as a parent of T_m, if T_m is a component of the part and T_m is at Indenture $\alpha + 1$. Let Ψ_m be the set of parents of T_m. For example, from Figure A.2, $\Psi_5 = \{T_2, T_3\}$.

Let $\theta_{ln} < \infty$ be the number of E_l located at base S_n, i.e., $S_n \in \Gamma_1$. In particular, $\theta_{ln} = 0$, if S_n is not a base. Let $s_{mn} < \infty$ be the number of spares with type T_m at S_n. Let $x_{mn}(t)$ be the number of T_m at S_n at a time point $t \geq 0$. Because θ_{ln} and $s_{mn} << \infty$, $x_{mn}(t)$ must have a

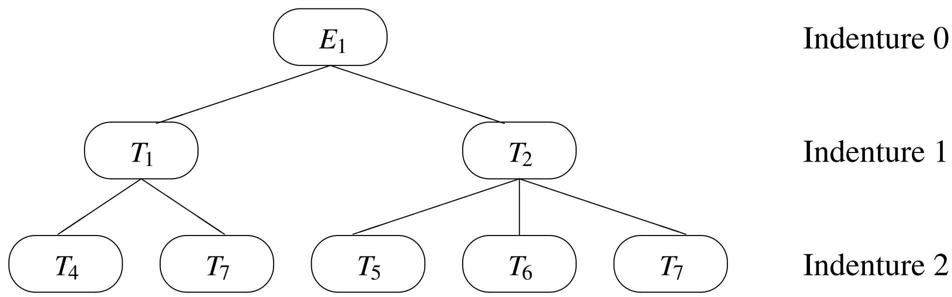

Figure A.2: *The indenture structure of E_1 in the sample system.*

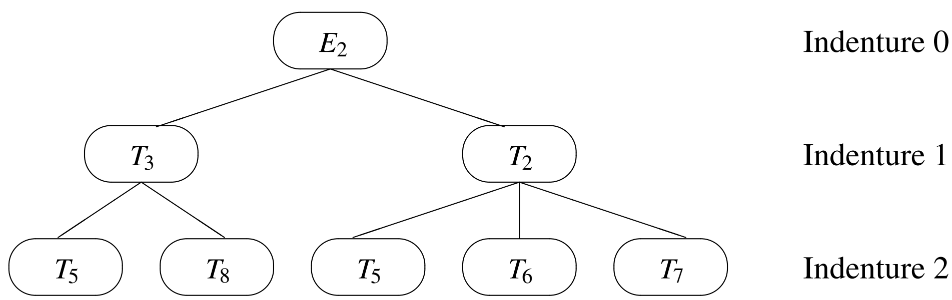

Figure A.3: *The indenture structure of E_2 in the sample system.*

non-negative upper bound ω_{mn}, i.e., $\omega_{mn} = \sup\{x_{mn}(t), t \geq 0\}$. It can be calculated as:

$$\omega_{mn} = \begin{cases} s_{mn} + \sum_{E_l \in \Omega_m} \theta_{ln} + \sum_{S_k \in \Phi_n} \omega_{mk}, \text{ if } T_m \in \Gamma_1, \\ s_{mn} + \sum_{T_j \in \Psi_m} \omega_{jn} + \sum_{S_k \in \Phi_n} \omega_{mk}, \text{ if } T_m \in \Gamma_\alpha \text{ and } \alpha > 1, \end{cases} \quad \text{(A.1)}$$

where in both cases, the first term is the number of spares of T_m at S_n and the last term represents the sum of the upper bounds for the parts of type T_m at the sites supported by S_n. In the case of $T_m \in \Gamma_1$, the second term denotes the sum of the number of PEs which have an LRU of type T_m as a component. In the other case, the second term denotes the sum of the corresponding upper bounds of the parents of T_m.

In practice, all of the PEs are installed at the operating bases, i.e., S_4 to S_8 in Figure A.1, and the PEs will not be moved to any other locations. Therefore, the UBDP starts computing the upper bounds from the LRUs (the parts at Indenture 1), and proceeds to the highest indenture level. At each indenture level, the UBDP starts from the operating bases (the sites at Echelon 1), and moves up to the highest echelon level. In the UBDP algorithm below, δ denotes the largest indenture level of all the PEs in the system (e.g. $\delta = 2$ from Figures A.2 and A.3) and λ denotes the largest echelon level of the logistics support system (e.g. $\lambda = 3$ in Figure A.1).

The UBDP

For $\alpha = 1$ to δ

  For each T_m in Γ_α, do the followings.

    For $\beta = 1$ to λ

      For each S_n in Λ_β

        Compute ω_{mn} using Equation (A.1).

      End

    End

  End

End

It should be remarked that for a system with M types of parts and N sites, the UBDP can determine all the upper bounds, ω_{mn} in $O(MN)$ time.

Note that the UBDP does not take into account either demand rates or failure probabilities. That is, the upper bounds are independent of these two factors. However, in reality, demand rates together with failure probabilities determine the repair and flow of parts in a logistics support network, and hence the repair pipelines. The UBDP considers the worst case scenario which may have a small probability of occurring in practice. For instance, in Equation (A.1), $\sum_{T_j \in \Psi_m} \omega_{jn}$ assumes that a part type T_m may be responsible for every failure of its parent T_j. Assume that in Figure A.3, T_5 and T_8 are each responsible for 50% of the failures of T_3. These are steady state probabilities. However, it is possible, although highly unlikely, that a sequence of T_3 failures over a period of time may be due to say only T_5 which will be in excess of the expected number of failures attributable to T_5.

In the remainder of this section, the UBDP is illustrated using the data in Tables A.1 and A.2. Since the PEs are located only at the bases, Table A.1 lists the θ_{1n} and θ_{2n} for $n = 4, 5, \ldots, 8$. Table A.2 contains information on the spares. For instance, the number of spares

Table A.1: *The number of E_1 and E_2 in the sample system.*

PE	S_4	S_5	S_6	S_7	S_8
E_1	3	4	5	3	5
E_2	2	4	5	0	1

of T_2 at S_1 is $s_{21} = 6$. Since the UBDP is being illustrated for T_2 and T_6, all the unrelated information are not disclosed here.

Given that T_2 is at indenture level 1 with $\Omega_2 = \{E_1, E_2\}$ and T_6 is at indenture level 2 with $\Psi_2 = \{T_2\}$. The UBDP starts the calculation with T_2 at the bases, S_4 to S_8. For

Table A.2: *The number of spares of T_2 and T_6 in the sample system.*

Part	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
T_2	6	4	5	6	0	2	1	10
T_6	3	3	5	3	8	7	9	6

example, at S_4, using Equation (A.1) gives $\omega_{24} = s_{24} + (\theta_{14} + \theta_{24}) = 6 + (3 + 2) = 11$. The UBDP then moves to the next level, Echelon 2. Since S_2 has $\Phi_2 = \{S_4, S_5\}$, $\omega_{22} = s_{22} + (\omega_{24} + \omega_{25}) = 4 + (11 + 8) = 23$. Similarly, $\omega_{23} = 5 + (12 + 4 + 6) = 37$, since $\Phi_3 = \{S_6, S_7, S_8\}$. Using these two values (ω_{22} and ω_{23}) gives the upper bound for T_2 at S_1 (Echelon 3) as $\omega_{21} = 6 + (23 + 37) = 66$, since $\Phi_1 = \{S_2, S_3\}$. The detailed results can be seen in the first row of Table A.3.

Table A.3: *The upper bounds of T_2 and T_6 in the sample system.*

Part	Echelon 1					Echelon 2		Echelon 3
	S_4	S_5	S_6	S_7	S_8	S_2	S_3	S_1
T_2	11	8	12	4	16	23	37	66
T_6	14	16	19	13	22	56	96	221

For T_6, again the UBDP starts from the bases. Since $\Psi_6 = T_2$, the upper bound at S_4 is $\omega_{64} = s_{64} + \omega_{24} = 3 + 11 = 14$. Similar calculations can also be done for the other bases. For the higher echelon levels, $\omega_{62} = 3 + (14 + 16) + 23 = 56$, $\omega_{63} = 5 + (13 + 19 + 22) + 37 = 96$ and $\omega_{61} = 3 + (56 + 96) + 66 = 221$. The last row of Table A.3 shows all the upper bounds for T_6.

Annex B: The closed-loop simulation tool

The CLS tool is an object-oriented, discrete-event simulation tool developed in Java for MIME systems. Compared to open-system simulation tools, the CLS tool has one important feature, i.e, a failed PE cannot generate any demand while under repair.

Figure B.1 shows a sample two-indenture, three-echelon system. The dashed lines represent the flow of failed parts and the solid lines represent the flow of functioning parts. Once a PE fails, a failed LRU is identified. The failed LRU can be repaired locally at the base or sent to the depot for repair. Failure of an LRU is always traceable to an SRU. Consequently, the repair of an LRU requires the repair or replacement of an SRU. SRU repairs due to LRU repairs at the base can be done either at the base or depot. Once a part is repaired, the part is stocked locally. Requests for functioning parts are satisfied immediately using local stock (if spares are available) or as backorder (if no spare are available locally). Note that the depot cannot get any support from the base for part repair or resupply, but it can get support from external third-party suppliers.

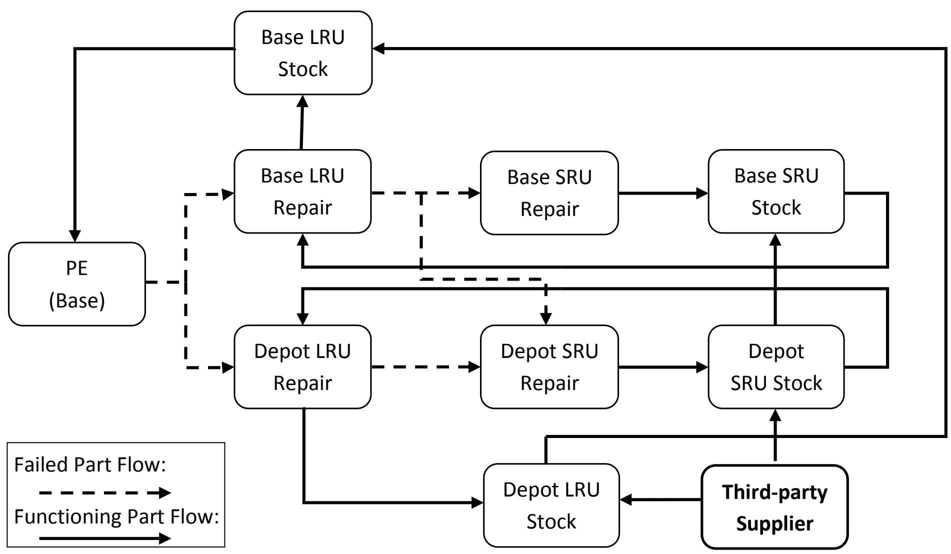

Figure B.1: *A sample of the repair and resupply support processes simulated by the CLS tool (Source: Muckstadt [11]).*

Such a system can be easily represented and simulated using the CLS tool. Objects are created for PEs, parts (LRUs and SRUs) and locations (operating bases, intermediate repair sites, central depot and third-party suppliers). A PE object has information such as its operating base, the installed LRUs and failure rate. A part object has information such as its sub-components (if it has any), repair rate and repair time. A location object has information such as repair equipment and parts inventory at the location, the locations supporting it and those it supports. Given these capabilities, the CLS tool can build any indenture and echelon structures of a real-world system in a simulation. In addition, for the convenience of users, input data can be read from an Excel file and output results can be written to an Excel file. All of these make the CLS tool a very flexible and powerful tool for studying MIME systems.

List of acronyms

CLS	Closed-Loop Simulation
EBO	Expected Backorder
LRU	Line Repairable Unit
METRIC	Multi-Echelon Technique for Recoverable Item Control
MIME	Multi-Indenture, Multi-Echelon
MISE	Multi-Indenture, Single-Echelon
OSP	Optimal Stock Policy
PE	Prime Equipment
SRU	Shop Repairable Unit
UBDP	Upper Bound Determination Procedure

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METRIC-based sparing models assume that equipment failures are generated by an infinite working item population. These models were first developed and applied in the US military which maintains large fleets of specific platforms obscuring the errors introduced by this simplification. However, in countries like Canada where fleet sizes tend to be small, the errors introduced by this assumption can be significant. In order to deal with small (or finite) fleet sizes, the T-METRIC model is proposed. The model is based on the use of truncated distributions for calculating the expected backorder. It is shown that the T-METRIC model can provide more accurate EBO values than the VARI-METRIC model, especially when there are a small number of working items.

Les modèles d'analyse des pièces de rechange de la famille METRIC supposent que les pannes de matériel surviennent parmi une population infinie de pièces d'équipement. Ces modèles ont été élaborés et appliqués tout d'abord dans l'armée américaine, qui gère de vastes parcs de plateformes particulières, ce qui explique que cette simplification engendre des erreurs qui passent inaperçues. Toutefois, dans des pays comme le Canada, où les parcs d'équipements sont de taille plus modeste, les erreurs introduites par cette hypo-thèse peuvent être importantes. C'est pourquoi nous proposons le modèle T-METRIC pour la gestion des parcs de faible taille (ou de taille finie). Le modèle repose sur l'utilisation de distributions tronquées pour calculer le nombre estimé d'articles en souffrance. Nous montrons que le modèle T-METRIC peut donner des estimations plus justes que le mo-dèle VARI-METRIC, surtout lorsque le nombre de pièces d'équipement disponibles est relativement faible.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Closed-Loop Simulation
Expected Backorder
Finite Working Item Populations
Multiple Echelons
Miltiple Indentures
Optimal Stock Policy
Sparing Analysis
T-METRIC
Truncated Distribution
VARI-METRIC