Defence Research and Development Canada | Recherche et développement pour la défense Canada

DRDC | RDDC

Future Small Arms Research WBE 1.2.4: Relative Localization for Network of Soldiers

Yifeng Zhou
Communications Research Centre

Franklin Wong
DRDC – Valcartier Research Centre

Defence Research and Development Canada

Scientific Report
DRDC-RDDC-2016-R019
February 2016

Canada

Future Small Arms Research WBE 1.2.4: Relative Localization for Network of Soldiers

Yifeng Zhou
Communications Research Centre

Franklin Wong
DRDC – Valcartier Research Centre

Defence Research and Development Canada
Scientific Report
DRDC-RDDC-2016-R019
February 2016

Abstract

In this Scientific Report, we investigate the development of relative localization techniques based on internode distance measurements for small wireless networks where global navigation satellite systems are not available. Internode distance measurement errors are assumed to be small compared to their corresponding internode distances. High precision ranging can be achieved by using technologies such as ultra-wide band (UWB) ranging. A number of approaches are proposed, which include the Linear Least Squares (LLS) approach, the Maximum Likelihood Estimation (MLE) approach, the Map Registration Approach (MAP), the Multidimensional Scaling (MDS) approach and the enhanced MDS approaches. Computer simulations are used to demonstrate and compare the performances and effectiveness of the proposed approaches. Both fully and partially connected networks are simulated for studying the relative localization performance. Finally, conclusions are drawn on the results for relative localization in small networks.

Significance for defence and security

The work reported here is part of the Department of National Defence's Future Small Arms Research (FSAR) program that is mandated to assess technologies that can provide the dismounted soldier with enhanced situational awareness. The aim of this research is to develop advanced relative localization techniques for networked soldiers, *i.e.*, to provide the net-enabled soldier with the ability to accurately determine his/her location relative to the rest of his/her section or platoon. Knowledge of the relative locations of the soldiers enables the calculation of target locations with the aid of additional sensors, and the potential to handoff target location among section or platoon members.

Localization is a pre-requisite for many military operations. Although GPS can be used for the purpose of localization, it requires line-of-sight (LOS) conditions to satellites. Also, it does not work reliabily in urban and indoor environments. In addition, GPS is subject to jamming. Therefore, many alternative localization and navigation techniques have been developed in recent years. These include image/ladar/Doppler/Directional Range aiding of inertial, beacon-based navigation, and navigation using signals of opportunity. However, each of these techniques has its own advantages and limitations. It is noted that, in the area of wireless sensor networks, localization is considered one of the most important features. As a result, numerous techniques have been developed for localization in wireless sensor network applications. Although these techniques are developed under different contexts, their concepts and approaches are applicable and can be extended to the problem of localization for net-enabled soldiers. In this study, we focus on relative localization techniques. By relative localization, we mean the localization process only provides relative coordinates that are defined without reference to an external coordinate system. On the other hand, global coordinates are defined in the form of specific geographic coordinates such as latitude and longitude. Relative coordinates can be derived from corresponding global coordinates. For the problem of target location hand-off, the knowledge of the relative locations of the soldiers is sufficient. In this study, we develop relative localization techniques for small networks of dismounted soldiers. The use of internode distance measurements (distance between soldiers) using the ultra-wide band (UWB) ranging technology is proposed for localization. UWB radios employ very short pulse waveforms with energy spread over a wide swath of the frequency spectrum. Due to the inherently fine temporal resolution of UWB, arriving multi-path components can be sharply timed at a receiver to provide accurate time of arrival estimates, and thus the internode distance measurements. A number of approaches are proposed and discussed including the Linear Least Squares (LLS) approach, the Maximum Likelihood Estimation (MLE) approach, the Map Registration Approach (MAP), the Multidimensional Scaling (MDS) approach and the enhanced MDS approaches.

Computer simulations are used to demonstrate and compare the performances and effectiveness of the proposed approaches. Fully and partially connected networks are simulated for studying the localization performance. The simulation results show that, among the approaches discussed, MLE and MAP are the most viable solutions to relative localization for networks of small size. MLE is able to provide the superior localization performance in both fully and partially connected network scenarios. Although the global convergence of MLE cannot be guaranteed in theory, our simulation results showed that, when LLS was used as the initial estimates, MLE has converged to the desired optimal estimates almost every time. For partially connected networks with low connectivity, however, MLE may suffer from the problem of not having sufficient numbers of anchors in iterating across the entire network. The performance of MAP is close to that of MLE in fully connected networks and partially connected networks with moderate and high connectivity levels, and deteriorates as the network connectivity decreases. Unlike MLE, MAP is always able to provide a localization solution in spite of the network connectivity level.

Résumé

Dans ce rapport scientifique, nous étudions le développement des techniques relatives de localisation basé sur des mesures internodales de distance pour les petits réseaux sans fil où les systèmes de navigation par satellite mondiale ne sont pas disponibles. Les erreurs de mesure de distance d'internode sont supposées être faibles par rapport à leur distances internodales correspondant. La mesure de distance en haute précision peut être réalisée en utilisant des technologies telles que ultra-large band (UWB). Un certain nombre d'approches est proposé, qui comprend l'approche de linéaire des moindres carrés (LLS), l'approche d'estimation du maximum de vraisemblance (MLE), l'approche d'enregistrement Plan (MAP), l'approche de multidimensionnelle Scaling (MDS) et les approches MDS amliorées. Les simulations informatiques sont utilisées pour démontrer et comparer les performances et l'efficacité des approches proposées. Deux réseaux, un qui est entièrement connecté et l'autre qui est partiellement connecté, sont simulés pour étudier la performance de la localisation relative. Des conclusions sont tirées, basées sur les résultats, sur la localisation relative dans des petits réseaux.

Importance pour la défense et la sécurité

Le travail présenté ici fait partie du programme 'Future Small Arms Research (FSAR)' du Ministère de la défense nationale qui est mandaté afin d'évaluer les technologies qui peuvent fournir aux soldats débarqués une sensibilisation accrue de la situation. Le but de cette recherche est de développer une technique avancée de localisation par rapport aux soldats en réseau, *i.e.*, pour fournir au soldat réseauté la possibilité de déterminer avec précision sa / son emplacement par rapport à sa section ou son peloton. La connaissance des emplacements relatifs des soldats permet le calcul des emplacements des cibles à l'aide de capteurs supplémentaires, et le potentiel de transférer l'emplacement de la cible entre les membres de la section ou du peloton.

La localisation est une exigence préalable pour de nombreuses opérations militaires. Bien qu'un GPS puisse être utilisé pour la localisation, il faut avoir une ligne de vue aux satellites. Cette technique, pourtant, ne fonctionne pas en milieu urbain et à l'intérieur des bâtiments. En outre, le GPS est soumis à du brouillage. Beaucoup de techniques de localisation et des techniques de navigation alternatives ont été développées au cours des dernières années, qui comprennent l'image / ladar / Doppler / Directional Range qui aident la navigation inertie par balises, et la navigation à l'aide des signaux d'opportunité. Cependant, chacune de ces techniques a ses propres avantages et leurs limites. Il est à noter que, dans le domaine de capteur sans fil, la localisation est considérée comme l'une des caractéristiques les plus importantes, et de nombreuses techniques ont été mises au point pour améliorer la performance de la localisation dans les applications de réseaux de capteurs sans fil. Bien que ces techniques soient développées en vertu de contextes différents, leurs concepts et approches sont applicables et peuvent être étendus au problème de la localisation des soldats réseautés. Dans cette étude, nous nous concentrons sur la localisation relative. Par localisation relative, on entend que le processus de localisation ne fournit que les coordonnées relatives qui sont définies sans référence à un système de coordonnée externe. D'autre part, les coordonnées globales sont définies sous la forme des coordonnées géographiques spécifiques tels que latitude et longitude. Les coordonnées relatives peut être dérivées à partir des coordonnées globales correspondantes. Pour le problème de transfert de l'emplacement de cible, la connaissance des emplacements relatifs des soldats est suffisante. Dans cette étude, nous développons des techniques de localisation relative pour les petits réseaux de soldats débarqués. L'utilisation des mesures de distance internodale (distance entre entre les soldats) utilisant la bande ultra-large (UWB) est proposée pour la localisation. Les radios UWB emploient des très courtes formes d'onde de pouls avec la propagation de l'énergie sur une large la bande du spectre de fréquence. En raison de la résolution temporelle inhérente de l'UWB, les signaux de multi-trajet arrivant peuvent être chronométrés précisément à un récepteur pour fournir les estimations d'arrivée exactes, et donc les mesures de distance internodale. Un certain nombre d'approche est proposé et discuté y compris l'approche linéaire des moindres carrés (LLS), l'approche d'estimation du maximum de vraisemblance (MLE), l'approche d'enregistrement Plan (MAP), l'approche de multidimensional scaling (MDS) et les approches MDS amliorés.

Les simulations par ordinateur sont utilisées pour démontrer et comparer les performances et l'efficacité des approches proposées. Les réseaux entièrement et partiellement connectés sont simulés pour l'étude de performance de la localisation. Les résultats des simulations montrent que, parmi les approches discutées, les MLE et MAP sont les solutions les plus viables à la localisation relative pour les réseaux de petite taille. MLE est en mesure de fournir une performance supérieure dans les deux scénarios de réseau entièrement et partiellement connectés. Bien que la convergence globale du MLE ne peut pas être garantie en théorie, nos résultats ont démontré que, lorsque LLS est utilisé comme estimation initiale, MLE a convergé vers les estimations optimales souhaitées presque chaque fois. Pour les réseaux partiellement connectés avec faible connectivité, MLE peut souffrir d'un problème de pénurie de nombre de point d'ancrage dans l'itération à travers l'ensemble du réseau. La performance de la MAP est proche de celle du MLE dans les réseaux entièrement connectés et partiellement connectés avec des niveaux de connectivité modérés et élevés, et se détériore avec la diminution de la connectivité du réseau. Contrairement au MLE, MAP est toujours en mesure de fournir une solution de localisation malgré le niveau de connectivité du réseau.

Table of contents

List of figures

1 Introduction

The work reported here is part of the Department of National Defence's Future Small Arms Research (FSAR) program that is mandated to assess technologies that can provide the dismounted soldier with an integrated suite of equipment for enhanced situational awareness. The aim of this research is to assess the viability of advanced relative localization techniques for networked soldiers, *i.e.*, to provide the net-enabled soldier with the ability to accurately determine his/her location relative to his/her section or platoon. Knowledge of the relative locations enables the calculation of target locations with the aid of additional sensors and the potential to handoff target locations among section or platoon members.

Localization refers to the process of estimating the locations of objects based on various types of measurements and the use of a number of anchors. Anchors are simply objects that know their coordinates *a priori*. In some applications, anchors are also capable of transmitting beacons signals for ranging purpose. Localization is a pre-requisite for many military operations where location information must be known *a priori* in order to monitor the environment, gather data measurements, track objects to make right decisions. Although GPS can be used for providing coordinates, it requires line-of-sight (LOS) conditions to satellites. Also, it does not work reliably in urban and indoor environments. In addition, GPS is subject to jamming. Therefore, many alternative localization and navigation techniques have been developed in recent years, which include image/ladar/Doppler/Directional Range aiding of inertial, beacon-based navigation, and navigation using signals of opportunity. These techniques have their own advantages and limitations. It is interesting to note that, in the area of wireless sensor networks, localization is considered to be one of the most important features. Numerous localization techniques have been developed for localization of sensor nodes in wireless sensor networks. Although these techniques are developed under different contexts and scenarios, their concepts and approaches are applicable and can be extended to the problem of localization for net-enabled soldiers.

In this study, we focus on relative localization techniques. Relative localization provides relative coordinates that are defined without reference to an external coordinate system. On the other hand, global localization provides coordinates that are defined in the form of specific geographic coordinates such as latitude and longitude. Relative coordinates can be derived from corresponding global coordinates. For a set of nodes, relative coordinates of the nodes provide their topological configuration. Relative coordinates are not unique, and they are arbitrary rigid transformations of their global coordinates. Relative coordinates can be transformed into global coordinates using anchor nodes with known global coordinates. For the problem of target location hand-off, relative localization provides one of the data needed to determine the location of a target. The other data required for handoff are distance to and azimuth of the target.

In the last two decades, many localization techniques have been developed for wireless sensor network applications [1][2]. In this study, we examine a number of relative localization techniques for small networks of dismounted soldiers based on the use of internode distance measurements (distance between soldiers in this case). The ultra-wide band (UWB) ranging technology is proposed for obtaining the internode distance measurements. UWB radios employ very short pulse waveforms with energy spread over a wide swath of the frequency spectrum. Due to the inherently fine temporal resolution of UWB, arriving multi-path components can be sharply timed at a receiver to provide accurate time of arrival estimates, and thus the internode distance measurements. Five relative localization approaches are formulated and discussed, which are the Linear Least Squares (LLS) approach, the Maximum Likelihood Estimation (MLE) approach, the Map Registration approach (MAP), the Multidimensional Scaling (MDS) approach and the enhanced MDS approaches. The LLS and MLE methods are based on the multilateration technique, which is seen to be one of the most popular localization techniques [1][3]. Multilateration is a simple localization technique that is based on distance measurements from multiple anchor nodes to the sensor node to be localized. The estimated location is determined by the minimization of the sum of squared distances between a hypothesized sensor location to all the anchor locations. MDS and MAP are based on the approaches in [4], [5] and [6], respectively. They use inter-node distance measurements to provide relative coordinates. MDS requires the full knowledge of the Euclidean distance matrix of the nodes, which is usually not available in practice due to the limited ranging capability. Unavailable distance measurements need to be approximated, which may introduce large localization errors. The MAP is a more elaborated approach that is proposed to counter this difficulty by dividing the network into many small sub-groups with adjacent groups sharing common nodes, constructing local maps for all sub-groups, and merging them into a global map. The MAP is able to alleviate the problems due to the shortest path distances for remote sensor nodes.

The rest of the report is organized as follows. In Section 2, the LLS and MLE methods are formulated and discussed. The procedures for determining the anchors are discussed in detail in this section. In Section 3, the MDS and MAP are discussed and formulated in the context of relative localization. In Section 4, the performance of various approaches are evaluated using computer simulations for different application scenarios. Both fully and partially connected networks are simulated for studying the relative localization performance. Finally, conclusions are drawn on the proper solutions for relative localization of small wireless networks.

2 Linear least squares and maximum likelihood estimation approaches

In general, the Linear Least Squares (LLS) and Mximum Likelihood Estimation (MLE) methods have two steps. In the first step, three node locations are estimated and are used as anchors. In the second step, the location of the rest of the nodes are estimated iteratively using LLS and MLE. The first step is common to LLS and MLE. They differ only in the second step. In the following, we discuss the two steps in detail.

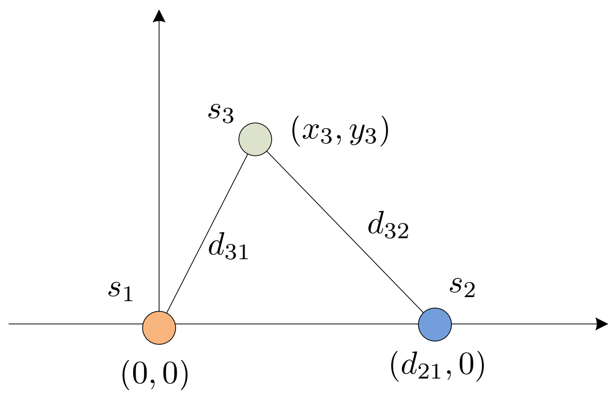

Figure 1: *Geometry of the first three anchor nodes.*

2.1 Initial anchor selection

We arbitrarily select a node, denoted by s_1, as the first anchor and define it as the origin of the coordinate system. Then, the coordinates of s_1 are given by $(0,0)$. We select a neighbour node of s_1, denoted by s_2, as the second anchor, and define the line connecting s_1 and s_2 as the x-axis. The coordinates of s_2 are given by $(d_{12}, 0)$, where d_{12} is the measured distance between s_1 and s_2. We then select a node that is a neighbour to both s_1 and s_2 with measured distances d_{31} and d_{32}, respectively. The geometry of the first three anchor nodes are shown in Fig. 1. Denote the coordinates of s_3 by (x_3, y_3). The estimation of the coordinates of s_3 can be considered in two scenarios. In the first scenario, the distance measurements d_{21}, d_{31} and d_{32} satisfy the triangle inequality relationship

$$d_{31} + d_{32} > d_{21}, \;\; d_{31} + d_{21} > d_{32}, \;\; d_{32} + d_{21} > d_{31}. \tag{1}$$

As shown in Fig. 2 (left), the coordinates of s_3 can be obtained as the intersections of two circles with centers at s_1 and s_2 and radii of d_{31} and d_{32}, respectively. Mathematically, the coordinates can be computed using the following equations [7]

$$x_3^2 + y_3^2 = d_{31}^2, \quad (x_3 - d_{21})^2 + y_3^2 = d_{32}^2, \tag{2}$$

with solution given by

$$x_3 = \frac{d_{21}^2 + d_{31}^2 - d_{32}^2}{2d_{21}}, \quad y_3 = \pm\sqrt{d_{31}^2 - x_3^2}. \tag{3}$$

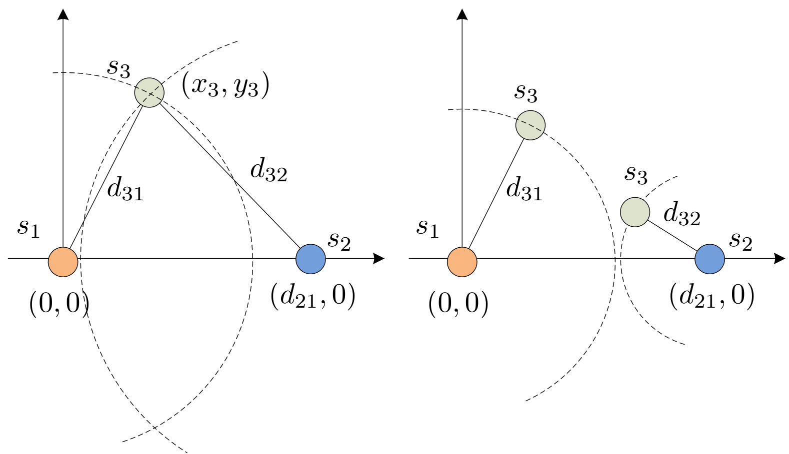

Figure 2: *Intersection geometry of anchor nodes.*

In (3), the positive root is selected for y_3. Note the selection of the positive root is arbitrary and will not affect the performance of relative localization. However, when the triangle inequality is satisfied due to distance measurement noise, the two circles will not intersect, as shown in right part of Fig. 2. In this case, the triangle inequality does not hold, *i.e.*, $d_{31} + d_{32} < d_{21}$, and y_3 obtained by (3) will be an imaginary number.

When the triangle inequality is not satisfied, we propose to estimate the coordinates of s_3 from the following nonlinear least squares problem

$$\min_{x_3,y_3}(\sqrt{x_3^2+y_3^2}-d_{31})^2+(\sqrt{(x_3-d_{21})^2+y_3^2}-d_{32})^2. \tag{4}$$

Note that (4) is a nonlinear optimization problem and an analytical solution does not exist. Numerical techniques are required to solve for minimizing x_3 and y_3. There are many numerical methods that can be applied including the gradient decent methods and Newton's method [8]. In this study, the Nelder-Mead simplex search method [9], which is a commonly used nonlinear optimization technique, is used due to its availability in Matlab. The Nelder-Mead method approximates a local minimum. It provides a local minimum solution depending on the initial point that starts the iteration. When the initial point is sufficiently close to the actual solution, the Nelder-Mead method is able to provide the global minimum; otherwise the solution is a local minimum. In this study, the initial point is selected based on the condition of the first three distance measurements. If

$$d_{31}+d_{32}<d_{21} \ \text{ or } \ d_{21}+d_{32}<d_{31}, \tag{5}$$

the initial point is chosen to be $(d_{31},0)$; otherwise, the initial point is given by $(-d_{31},0)$.

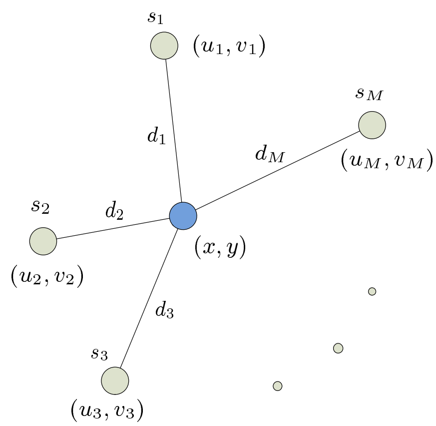

Figure 3: *Geometry of anchor nodes and the node to be localized.*

While not guaranteed, the algorithm was found to converge to the desired optimal estimates almost every time in the simulation.

2.2 Calculation methods

The second step is to estimate the locations of the rest of the nodes based on the initial anchors. This step iterates the all network, and the coordinates of one node will be determined. Once the location of the new node is determined, the node becomes an anchor and will be used in the next round of computing. In this step, two variants of the multilateration technique are used, referred to as the maximum likelihood and linear least squares methods. Multilateration is a simple technique for estimating the location of a point based on its distance measurements to multiple anchor nodes. Fig. 3 shows the geometric configuration of anchors and the node to be localized. In the figure, M anchors are used, and their coordinates and measured distances to the node to be localized are denoted by $\{u_m, v_m, d_m\}$, for $m = 1, 2, \ldots, M$, respectively. The multilateration approach consists of estimating the coordinates (x, y) given $\{u_m, v_m, d_m; m = 1, 2, \ldots, M\}$. In general, multilateration minimizes the following sum of squared errors between the measured distances and hypothetical ones based on the unknown sensor node location

$$\min_{x,y} \sum_{m} \left[\sqrt{(x-u_m)^2 + (y-v_m)^2} - d_m \right]^2, \tag{6}$$

which is a nonlinear least squares problem. Under the assumption that $\{d_m; m = 1, 2, \ldots, M\}$ contain additive measurement errors that are independent, identically distributed (*i.i.d.*) Gaussian processes with zero means, the nonlinear least squares problem (6) can be shown to be equivalent to the maximum likelihood estimator [10]. We refer to the formulation (6) as the Maximum Likelihood Estimator (MLE). Since (6) is a nonlinear minimization problem, a closed-form solution does not exist, and numerical techniques are required. As mentioned before, numerical optimization techniques are subject to convergence difficulties and always suffer from the local minimum problem. The *lsqnonlin* in Matlab was applied to numerically solved the minimization problem. The *lsqnonlin* function uses the trust-region-reflective algorithm, which is a subspace trust-region method based on the interior-reflective Newton method described in [11][12].

The maximum likelihood approach can be considered as a formulation in the distance domain. In practice, multilateration is often formulated as the following minimization problem in the squared distance domain to simplify the solution

$$\min_{x,y} \sum_{m} \left[(x-u_m)^2 + (y-v_m)^2 - d_m\right]^2, \tag{7}$$

or equivalently,

$$\min_{x,y} \sum_{m} \left[r_x^2 - 2u_m x - 2v_m y + r_m^2 - d_m^2\right]^2, \tag{8}$$

where $r_x^2 = x^2 + y^2$ and $r_m^2 = u_m^2 + v_m^2$. It can be shown that (8) is equivalent to the least squares solution of the following equations

$$B\underline{z} - \frac{r_x^2}{2} \cdot \underline{\mathbf{1}} = \underline{\eta}, \tag{9}$$

where $\underline{z} = [x, y]^T$, $\underline{\mathbf{1}}$ denotes an all one vector of length M, and

$$\underline{\eta} = -\frac{1}{2} \begin{bmatrix} d_1^2 - r_1^2 \\ d_2^2 - r_2^2 \\ \vdots \\ d_M^2 - r_M^2 \end{bmatrix} \quad B = \begin{bmatrix} u_1 & v_1 \\ u_2 & v_2 \\ \vdots & \vdots \\ u_M & v_M \end{bmatrix} \tag{10}$$

Note that (9) contains the term r_x^2 that is nonlinear in $\underline{x}$, and a direct solution is not available. However, the nonlinear term r_x^2 can be eliminated from the equation by the use of projection operations. Define $P_{\mathbf{1}}^{\perp}$ as the orthogonal projection onto the unitary subspace of $\underline{\mathbf{1}}$. The orthogonal projection $P_{\mathbf{1}}^{\perp}$ is given by [13]

$$P_{\mathbf{1}}^{\perp} = \mathbf{I}_M - \frac{1}{M}\underline{\mathbf{1}} \cdot \underline{\mathbf{1}}^T \tag{11}$$

where $\mathbf{I}_M$ denotes the identity matrix of size M. Multiplying both sides of (9) by $P_{\mathbf{1}}^{\perp}$ yields,

$$A\underline{z} = \underline{b} \tag{12}$$

where $A = P_{\mathbf{1}}^{\perp} B$ and $\underline{b} = P_{\mathbf{1}}^{\perp} \underline{\eta}$. It can be seen that the projection leads to an equation that is linear in $\underline{z}$ and has a closed-form least squares (LS) solution given by [13]

$$\underline{z}_{ls} = (A^T A)^{-1} A^T \underline{b} = (B^T P_{\mathbf{1}}^{\perp} B)^{-1} B^T P_{\mathbf{1}}^{\perp} \underline{\eta} \tag{13}$$

In this report, the solution (13) is referred to as the linear least squares method. The elimination of the nonlinear terms r_x^2 is different from other approaches discussed in the literature such as the atomic multilateration approach [1]. In [1], the elimination is done by selecting one of the equations in (9), and subtracting it from the rest of the equations. The approach of orthogonal projection is equivalent to subtracting the average of all equations from each equation in (9). Intuitively, it is expected to be more robust although a rigorous analysis is still required. It should also be noted that equation (12) is not equivalent to (9). They are equivalent only if all vector in (9) lie in the $(M-1)$-dimensional unitary space of $\underline{\mathbf{1}}$. In other words, the relationship of the vector components in (9) are ignored in (12).

3 MDS and MAP for relative localization

The Multidimensional Scaling (MDS) and MAP are also considered for relative localization in this report.

3.1 The MDS approach

The MDS localization method [4][14] is based on internode distance measurements and the application of the popular Multidimensional Scaling (MDS) technique in statistics. The MDS technique has its origins in psychometrics and psychophysics. It is a data analysis technique that can be used to represent a set of data as a configuration of points in some Euclidean spaces based on their similarity measures. The distances of the resulting configuration of points resemble the original similarities. There are many types of MDS techniques, including metric MDS and nonmetric MDS, replicated MDS, weighted MDS, deterministic and probabilistic MDS [15][16]. The classical MDS method is more attractive than the others because it has analytical solutions that can be obtained via eigendecomposition of a transform of the Euclidean distance matrix. MDS is closely related to principal component analysis and factor analysis. The technique has found many applications such as cluster analysis, machine learning and computational chemistry[16]. MDS was first proposed for solving the problem of sensor localization by Shang *et al.* [4][14], where either connectivity information or distance measurements between neighbor nodes were used for localization. In [17], the authors proposed an iterative MDS algorithm that uses a multivariate optimization for sensor location estimation. The iterative MDS is similar to the least squares refinement step in [14]. The iterative MDS approach is less tractable than the classical MDS solution because it involves complex computations and suffers from global convergence problems. In general, the MDS technique is relatively resilient to distance errors due to the over-determined nature of the solution. However, MDS requires full knowledge of the Euclidean distance matrix of the sensor nodes, which is usually not available in practice due to the limited transmission range of beacons or ranging modules on each sensor node. A commonly used approach is to approximate the distances between nodes that are separated further than the transmission range by their shortest path distances. The shortest path distances can be computed using shortest path algorithms such as Dijkstra's (single source shortest path problem) [18] or Floyd's (all pairs shortest path problem) [19]. The approximation of the Euclidean distance matrix introduces sensor localization errors, especially when the shortest paths do not correspond well with the Euclidean distance in sparse networks or networks of irregular topology. The MDS localization algorithm can be summarized as follows.

1. All sensor nodes measure their distances to neighbor nodes and send them to a central node during the stage of sensor deployment. Once all the distance measurements are collected from each individual node at the central node, an Euclidean distance matrix D for all sensors in the network is constructed, where the ijth element denotes the distance between the ith and jth sensor nodes. Unavailable distances (or missing components in the Euclidean distance matrix) are approximated by their corresponding shortest path distances using either Dijkstra's or Floyd's algorithm.

2. Transformation of the distance matrix D into its corresponding Gram matrix form by assuming that the geometric center of the sensor coordinates is at the origin. The Gram matrix E can be computed by [20]

$$E = -\frac{1}{2} L \cdot D \cdot L^T \quad \text{where} \quad L = I - \frac{1}{N} \underline{\mathbf{1}} \cdot \underline{\mathbf{1}}^T \tag{14}$$

   and D denotes the squared distance matrix.

3. Realization of eigendecomposition on the Gram matrix, retaining the first two columns of the eigenvectors, which correspond to the two largest eigenvalues (in the case of sensor localization on a plane). The coordinates of the sensor nodes are then given by the eigenvectors weighted by the square roots of their corresponding eigenvalues. The resulting sensor locations are relative coordinates and are independant of arbitrary translation and rotation or reflection of the origin.

MDS produces relative coordinates of the sensor nodes. If global coordinates of the nodes are required, the resulting relative coordinates can be transformed given a sufficient number of anchor nodes. A minimum of three anchor nodes will be required to construct an affine transformation to convert the relative coordinates to global ones. The affine transformation may include one or a combination of scaling, rotation/reflection and translation [4][14][17]. The optimal transformation is found by minimizing the discrepancy between the global and relative coordinates of the anchor nodes after transformation. A commonly used discrepancy error is the sum of the squared distances between the two sets of coordinates of the anchor nodes. The computed optimum transformation is applied to the relative positions of the rest of the sensor nodes to obtain their global coordinates.

3.2 The MAP approach

The MAP approach refers to the map registration approach proposed by Zhou *et al.* [5][21]. It is known that the MDS approach requires that the Euclidean distance matrix for all nodes be known, which may not be always available in practice due to the limited ranging distance of the nodes. When two nodes are out of their transmission range, the distance between them cannot be directly obtained, and needs to be estimated. In MDS, the unavailable internode distances are typically approximated by their shortest path distances. A shortest path distance corresponds well to the corresponding Euclidean distance in a network of regular topology or a densely distributed network of nodes. However, in a sparse network or a network of nodes of irregular topology, a shortest path distance may not match its Euclidean distance and the use of the approximated distance matrix will result in degraded localization performance [4][5]. A more elaborate approach is to divide the network into many small sub-groups of nodes, where adjacent groups share common nodes. A commonly used approach for forming a sub-group is to include a node and its one- or two-hop neighbors. Hop neighbors are nodes that are within the maximum ranging distance of a given node, or the nodes to which the distances can be directly measured. For each sub-group of nodes, a local map with relative coordinates of the nodes, is built using some localization techniques (*e.g.*, MDS). The local maps are then merged into a global map based on the common nodes shared by adjacent sub-group of nodes. The approaches that are based on local map merging can alleviate the problems caused by using the shortest path distances for remote sensor nodes, which usually do not match their Euclidean distances well in an irregular network. In addition, dividing a network into a set of smaller sub-networks allows the use of sophisticated localization algorithms which are too computationally expensive for a large network.

In [4], an incremental greedy algorithm was proposed for merging the local maps in a sequential manner. One local map is randomly selected as the core map, which is grown by merging the local maps one by one. Each time, a local map that has the maximal number of common nodes with the core map is selected and merged with the core map. A rigid body transformation of the local map is effected to minimize the conformational difference between the common node locations in the core map with those in the local map. The incremental greedy algorithm is locally optimal since it only explores the commonalities of the shared nodes in two maps. In practice, the common nodes are often shared by more than two local maps. In some cases, adjacent local maps may not have a sufficient number of common nodes. The sequential merging process can also lead to error propagation, and perhaps, unacceptable errors as the network grows. The MAP was introduced to counter the problems of the sequential approach. In MAP, instead of using a sequential pairwise approach for merging local maps, the construction of the global map is considered at a global level. For each local map, an affine transformation is defined to transform the local map to a global map. The set of optimal affine transformations are determined simultaneously by considering all available nodes that are shared by various local maps. The discrepancy is represented by the sum of the squared distances of all nodes to their respective geometric centers in the global map. The MAP algorithm is described as follows.

Since the proposed local map registration algorithm minimizes the overall discrepancies of the locations of all sensor nodes, it is able to counter the problems associated with approaches based on pairwise map merging, and achieve the global optimal performance. The problem of finding the optimal rigid transformation for two maps based on common nodes has closed-form solutions. They are similar to the techniques developed in the areas of computer vision and photogrammetry for finding the optimal transformation that aligns two sets of data points [22]. The approach by Arun *et al.* [23] is shown to have provable optimality and the advantage of computational efficiency over other methods. Arun's approach minimizes the squares error between two sets of matched points under rotation and translation (scaling is not considered), and the optimal transformation is obtained using a singular value decomposition (SVD). The problem of finding a set of optimal transforms, however, is not trivial due to the highly nonlinear optimization criterion involved, and analytic solutions are not known to exist. In [5], a gradient projection algorithm is developed for finding the optimal transforms for transforming local maps to a global map. The algorithm is developed based on a general idea by Jennrich in [24][25] and is particularly suitable to the constrained optimization problem of coordinate transformation. The algorithm is iterative. It has faster convergence and is computationally more efficient than many general numerical optimization techniques [8][26] for nonlinear programming.

Assume that a network consists of N nodes. For each node, the local map is assumed to contain its neighbor nodes within k-hops. Define a neighbor vector $\underline{c}_i$ of length N for the ith node. The nth component of $\underline{c}_i$ is given by 1 or 0 depending on whether the nth node is a neighbor node or not. Define a neighbor matrix $C = [\underline{c}_1, \underline{c}_2, \ldots, \underline{c}_N]$. For the ith local map, define an orthogonal matrix $U_i \in \mathcal{R}^{2\times 2}$ and a row vector $T_i \in \mathcal{R}^{1\times 2}$ to represent rotation/reflection (or a combination) and translation, respectively. Define $U \in \mathcal{R}^{2N\times 2}$ and $T \in \mathcal{R}^{N\times 2}$ as

$$U = [U_1; U_2; \ldots; U_N] \text{ and } T = [T_1; T_2; \ldots; T_N] \tag{15}$$

Let $\mathbf{z}_{ij} \in \mathcal{R}^{1\times 2}$ denote the local coordinates of the ith sensor node in the jth local map.

If the ith sensor node is not in the jth local map, then $\mathbf{z}_{ij} = \mathbf{0}$. Define a data matrix $Z_{ij} \in \mathcal{R}^{N\times 2}$, where the jth row of Z_{ij} is $\mathbf{z}_{ij}$. If the ith node is not in the jth local map, then, Z_{ij} is an all-zero matrix. Let $C_i = diag(\underline{c}_i)$ be a diagonal matrix of $N \times N$, where *diag* puts the elements of $\underline{c}_i$ on its diagonal. For the ith local map, we construct a data matrix X_i

$$X_i = [Z_{i1}, Z_{i2} \ldots, Z_{iN}]. \tag{16}$$

Let Y_i denote an affine transform of X_i given by

$$Y_i = X_i U + C_i T. \tag{17}$$

All Y_i are in a same coordinate system that is referred to as the *global coordinate system*. The global coordinates of the sensor nodes form the *global map*. In MAP, the optimal U is obtained from the following optimization problem [5]

$$\min_U tr\{U^T \Sigma U\}, \tag{18}$$

subject to the constraint that U_i is an orthogonal matrix for $i = 1, 2, \ldots, N$. Denote $\mathcal{M}$ as the manifold that consists of all $U = [U_1, U_2, \ldots, U_N]$ where each U_i is an orthogonal matrix of 2×2. Then, the constraint implies that the optimal U is in the manifold $\mathcal{M}$. In (18), tr denotes the trace of a square matrix, and

$$\Sigma = \sum_i X_i^T P_i^{\perp} X_i - A_s^T B_s^{-1} A_s \tag{19}$$

$$B_s = \sum_i \tilde{C}_i^T P_i^{\perp} \tilde{C}_i, \;\; A_s = \sum_i \tilde{C}_i^T P_i^{\perp} X_i \tag{20}$$

$$P_i^{\perp} = I - \frac{1}{N_i} \underline{c}_i \underline{c}_i^T, \tag{21}$$

Also, $\tilde{C}_i$ is C_i with its first column removed. The translation matrix T is related to the optimal U by

$$T = [\mathbf{0}; -B_s^{-1} A_s U]. \tag{22}$$

Note that an analytic solution does not exist for the constrained optimization problem (18), and numerical techniques are required. In this study, an iterative algorithm is developed for finding the optimal U, which is based on the gradient projection (GP) technique by Jennrich [24][25]. At each step, the GP algorithm finds a next descent point in the direction of the negative gradient of the criterion function at a current point. In general, the next point will not be in the desired manifold $\mathcal{M}$. To deal with this issue, the GP algorithm projects the point onto $\mathcal{M}$ and obtains the next desired estimate there. The algorithm terminates until some pre-defined stoping rules are satisfied. The detailed discussion of the GP algorithm can be found in [21]. In spite of the iterative nature of the algorithm, it has faster convergence and is computationally more efficient than many general numerical optimization techniques for nonlinear programming [8][26]. Although, each iteration of the algorithm requires the projection of a set of matrices onto $\mathcal{M}$, the implementation is relatively simple because they only involve singular value decomposition (SVD) of 2×2 matrices, which has closed-form solutions.

Since the MAP algorithm minimizes an overall discrepancies of the locations of all sensor nodes, it is able to counter the problems associated with approaches based on pairwise map merging such as the incremental greedy algorithm in [4]. The pairwise map merging approach is considered locally optimal since it only explores the commonalities of the shared sensor nodes in two maps in each step. In practice, the common sensor nodes are often shared by more than two local maps. Furthermore, the sequential merging process can also lead to error propagation and perhaps unacceptable errors as the network grows. The MAP algorithm provides a globally optimal performance since it constructs the global map at a global level, *i.e.*, the set of optimal affine transformations for all local maps are obtained simultaneously by considering all available nodes that are shared by various local maps.

The proposed localization approach does not rely on the knowledge of the mobile beacon locations, which is important for applications where access to GPS satellites is not available. However, it is necessary to point out that the localization results by MAP are relative sensor locations, *i.e.*, the estimated sensor locations are given in an arbitrary coordinate system. If global coordinates of the sensor nodes are desired, then, a number of anchors are needed to determine a rigid transformation of the relative coordinates into global coordinates.

3.3 Enhancing MDS performance

In the simulation study, various approaches are evaluated including LLS, MLE, MDS, MAP and the enhanced MDS approaches (MDS-LLS, MDS-MLE and MDS-MAP). By enhanced approached, we mean the MDS approach in which the unavailable internode distance measurements are replaced by estimates from LLS, ML, and MAP. These enhanced MDS approaches are referred to as MDS-LLS, MDS-MLE and MDS-MAP, in this report, respectively.

4 Simulations and performance analysis

In this section, we use computer simulations to demonstrate the effectiveness and performance of the proposed relative localization techniques including LLS, MLE, MDS, MAP and the enhanced MDS approaches (LLS-MDS, MLE-MDS and MAP-MDS). MLE uses the LLS solution based on previous MLE estimates as the initial estimate in each iteration after the initial node selection process. For the MDS approach, Dijkstra's algorithm [18] is used to compute the shortest paths to approximate the unavailable inter-node distances. For MAP, a local map is constructed for each node, which consists of all direct neighbor nodes within its maximum ranging distance.

The root mean square errors (RMSE) of the location estimates are used as a performance metric. In this study, since the estimates are relative location of the nodes, their RMSEs cannot be directly computed. Relative coordinates of a node are known to be displaced from the global coordinates system by an arbitrary rigid transformation (rotation, reflection, translation). In order to compute meaningful RMSE for relative location estimates, we apply a rigid transformation to align the relative location estimates of a node to best conform to its ground truth node location. The rigid transformation consists of an orthogonal matrix that represents a rotation or a reflection, and a translation vector. The best rigid transform minimizes the discrepancy between the estimated locations and the ground truth locations of the nodes. The discrepancy error is the sum of the squared distances between the two sets of locations. The problem of finding the optimal orthogonal matrix and translation is solved by using *extended orthogonal Procrustes analysis* [27] except that an unknown scale factor is not considered. After estimated locations are aligned with the ground truth locations of the nodes, the sum of the squared distances between the two sets of coordinates is computed, and an averaged RMSE error is computed to measure the accuracy of relative location estimates.

4.1 Scenario simulations

In the simulations, the nodes are assumed to be uniformly distributed in a square area of 100 meters by 100 meters. All nodes are assumed to have a common maximum ranging distance that can be configured. The maximum ranging distance determines whether the internode distance measurement between a pair of nodes is available or not. If a pair of nodes are located within the maximum ranging distance, then the internode distance measurement is available; otherwise, the internode distance is considered unavailable and need to be estimated for some of the approaches (*e.g.*, MDS). In the simulations, the network is assumed to be connected, *i.e.*, each of the nodes of the network is connected to each other either one-hop or *via* multiple hops in terms of internode ranging. The connectivity of a network is guaranteed by adjusting the maximum ranging distance. When unconnected nodes are detected, we increase the maximum ranging distance until the network is connected. The algebraic connectivity of a network is used to check whether the network is connected or not. The algebraic connectivity is computed as the second smallest eigenvalue of the Laplacian matrix of the network [28]. This eigenvalue reflects how well the nodes in a network are connected. The algebraic connectivity is non-zero if and only if all nodes of a network are connected. In the simulations, the algebraic connectivity of a network is computed and compared with a pre-defined threshold. If the algebraic connectivity was smaller than the threshold, the maximum range for distance measurement was increased by pre-define value. The process repeats until the algebraic connectivity was equal to or larger than the threshold. In the simulations, another step is also implemented to ensure that each node has at least three neighbor nodes that are within the maximum ranging distance. This is considered the minimum requirement for the implementation of the multilateration approaches (*i.e.*, MLE and LLS).

The distance measurement errors are assumed to be additive and uniformly distributed. Denote $\tilde{d}_{ij}$ and d_{ij} as the actual and measured distances between the ith and jth nodes, respectively. Then, we have $d_{ij} = \tilde{d}_{ij} + \varepsilon_{ij}$, where ε_{ij} is simulated to be uniformly distributed in $\tilde{d}_{ij}[-\sigma, \sigma]$ and σ is in the range $[0, 1]$. The distance measurement error is set proportional to the actual distance. With a fixed σ, the longer the distance between two nodes, the larger the mean distance measurement error. In the following, we discuss the localization performance of various approaches using two scenarios: fully and partially connected networks.

4.2 Fully connected networks

By a fully connected network, we mean that the maximum ranging distance for all nodes in the network is sufficiently large such that each node is able to measure its distances to all other nodes in the network. For a fully connected network, distance measurements between all pairs of nodes are available. Figs. 4 and 5 show the variation of RMSE for the MLE, LLS and MDS estimates versus the ranging error parameter σ for fully connected networks with 5 and 10 nodes, respectively. The ranging error parameter σ is written in the form of percentage. In the simulations, σ varies from 0 to 0.01 (or 1%), and for each value of σ, 1000 tests are repeated to obtain the averaged RMSE results. The averaged internode distances for the networks of 5 and 10 nodes are calculated as 52.39 and 52.25 meters, respectively. When $\sigma = 0.01$, it would translate into an averaged internode distance measurement error range of about ± 0.5 meter. For each test, all nodes are randomly redeployed and their random internode distance measurement errors re-generated. Thus, the RMSE results are averaged over both node distribution and random distance measurement errors. For a fully connected network, since all pairs of nodes are within the maximum ranging distance, the Euclidean distance matrix is completely available, and MAP and the enhanced MDS approaches become equivalent to MDS. Thus, only LLS, MLE and MDS are evaluated for fully connected networks. As discussed before, the LLS solutions are sensitive to geometric distribution of the nodes. In order to isolate the impact of node distribution on localization, a condition number of 400 is used to avoid scenarios that would

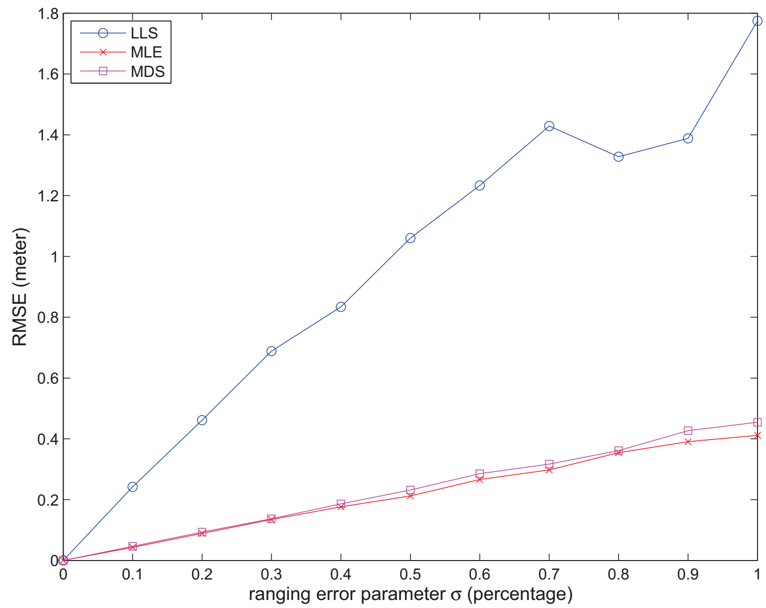

Figure 4: *Variation of RMSE as a function of σ in a fully connected network of $N = 5$ nodes for LLS, MLE and MDS approaches.*

Table 1: *Maximum ranging distance, averaged internode distance and connectivity level*

	Fig. 6	Fig. 7	Fig. 8
Maximum ranging distance (m)	80	100	120
Averaged internode distance (m)	45.73	51.11	52.21
Connectivity level	7.72	8.77	8.99

result in ill-conditioned data matrix. In Figs. 4 and 5, it can be observed that the RMSE of the LLS, MLE and MDS estimates increase as σ increases. The MLE and MDS estimates have similar performance, and both outperform the LLS estimates significantly, especially as σ increases.

4.3 Partially connected networks

In a partially connected network, a node may not be able to measure the distances to all other nodes in the network due to limited maximum ranging distance, *i.e.*, some internode distances are not available. For the MDS approaches, this means that the unavailable internode distances will need to be estimated using their corresponding shortest path distances. The shortest path distances are approximates of the Euclidean distances. Partially connected networks with 10 nodes are simulated. Three scenarios are simulated with the

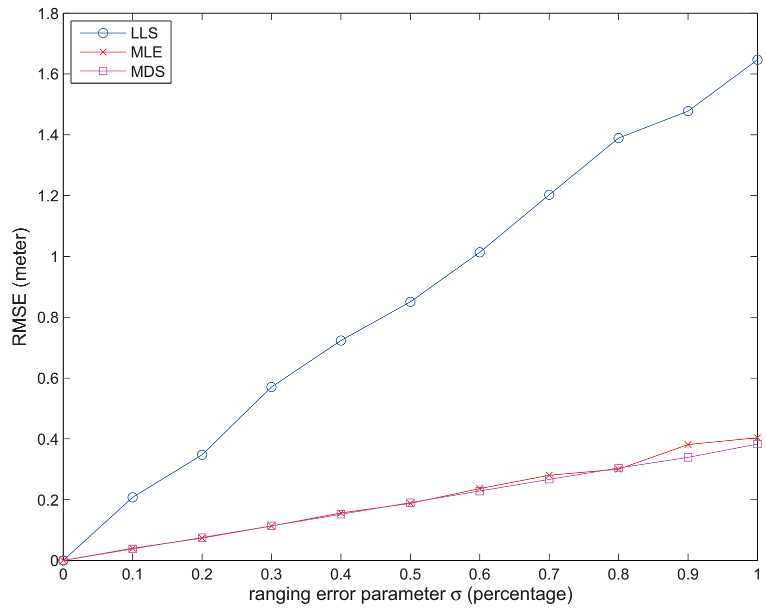

Figure 5: *Variation of RMSE as a function of σ in a fully connected network of $N = 10$ nodes for LLS, MLE and MDS approaches.*

maximum ranging distances being set to 80, 100 and 120 meters, respectively. Figs. 6 to 8 show the variations of RMSE as a function of the ranging error parameter σ for LLS, MLE, MDS, MAP and the enhanced MDS approaches in those three scenarios. The maximum ranging distance of a node determines the number of nodes that it can measure distances to, which can be characterized by the concept of connectivity levels. In this study, we compute the connectivity level as the averaged number of nodes that a node can measure distance to. Connectivity level increases as the maximum ranging distance is increased. Table 1 lists the maximum ranging distances, the averaged internode distances and the connectivity levels in Figs. 6, 7 and 8. In Fig. 6, the connectivity level is calculated as 7.72, which means that each node is able to its distance measurements to an average of about 77% of the nodes in the network. In the simulations, σ varies from 0 to 0.01 (or 1% in terms of percentage). For each value of σ, 1000 tests are repeated to obtain the averaged results. In each test, nodes are re-deployed and random ranging errors are reproduced. Similarly, a condition number of 500 is used to avoid the ill-conditioned data matrix for LLS. In Figs. 6, 7 and 8, the top figures show the variations of RMSE as a function of σ for the LLS, MLE, MDS and MAP estimates. In all three scenarios, all approaches in the simulation study show similar performance patterns. The RMSE of the LLS and MLE estimates increase as σ increases, while the RMSE of the MDS and MAP estimates are relatively constant over the tested range of σ. The accuracy of the MDS and MAP estimates is dominated by the connectivity level of the network rather than the assumed relatively small ranging errors. In all three scenarios, MLE performs the best. The performance of MDS and MAP improves as the network connectivity improves, as can be observed from Fig. 6 to 8. As shown in Fig.8, MDS and MAP perform as well as MLE when the connectivity level is 8.99 except for small values of σ. LLS outperforms MDS and MAP for small values of σ, and is outperformed by MDS and MAP as σ increases. The demarcation point for LLS moves down as the network connectivity increases as observed from Figs.6 to 8. For example, the point is at about 0.8 and 0.3, respectively, in Figs.6 and Fig.7. In Figs. 6, 7 and 8, the bottom figures show the variations of RMSE as a function of σ for the enhanced MDS approaches (*i.e.*, MLE-MDS, LLS-MDS and MAP-MDS). Unlike the regular MDS approach that uses the shortest path distances to approximate the unavailable internode distances, MLE-MDS, LLS-MDS and MAP-MDS use the MLE, LLS and MAP estimates, respectively, to compute and replace the unavailable internode distances. In general, the enhanced MDS approaches improve over the regular MDS approach. In addition, MLE-MDS improves over MLE while LLS-MDS is able to provide improvement over LLS for most values of σ. On the other hand, MAP-MDS and MAP have similar performance. It is observed that MDS and MAP produce large RMSE when the network has low connectivity levels, and improve as the connectivity improves. Although the RMSE for MLE and LLS are less affected by the network connectivity level, MLE and LLS may run into problems in the iteration process due to the issue of insufficient number of anchors for localization in the case of low network connectivity levels. Finally, note that, in contrast to the case of fully connected networks, in a partially connected network, MDS and MAP have non-zero RMSE even when $\sigma = 0$ (*i.e.*, no ranging errors). This error is due to the approximation errors of the unavailable internode distances.

5 Conclusions

In this report, novel localization approaches, including MLE, LLS, MDS, MAP and the enhanced MDS approaches, have been proposed for estimating the relative locations of a set of nodes based on their internode distance measurements. The performance of these proposed were discussed and analyzed using computer simulations. Small networks were considered, *i.e.*, networks that contain about 5 to 10 nodes. The accuracy of internode distance measurement was assumed to be relatively high. In the simulations, distance measurement errors were simulated to be additive and uniformly distributed with minimum and maximum boundaries that are proportional to the actual distances. Two types of networks were simulated to study the performance of the proposed localization approaches: fully and partially connected networks. Nodes were simulated to be randomly distributed in a square of 100 meters by 100 meters.

For fully connected networks, it was observed that the RMSE for MLE, LLS and MDS increase as the distance measurement error parameter σ increases. MLE and MDS showed similar localization performance, and outperformed LLS significantly. These three approaches showed similar performance for the 5 and 10 node cases.

For partially connected networks, the RMSE for the LLS and MLE estimates increase as

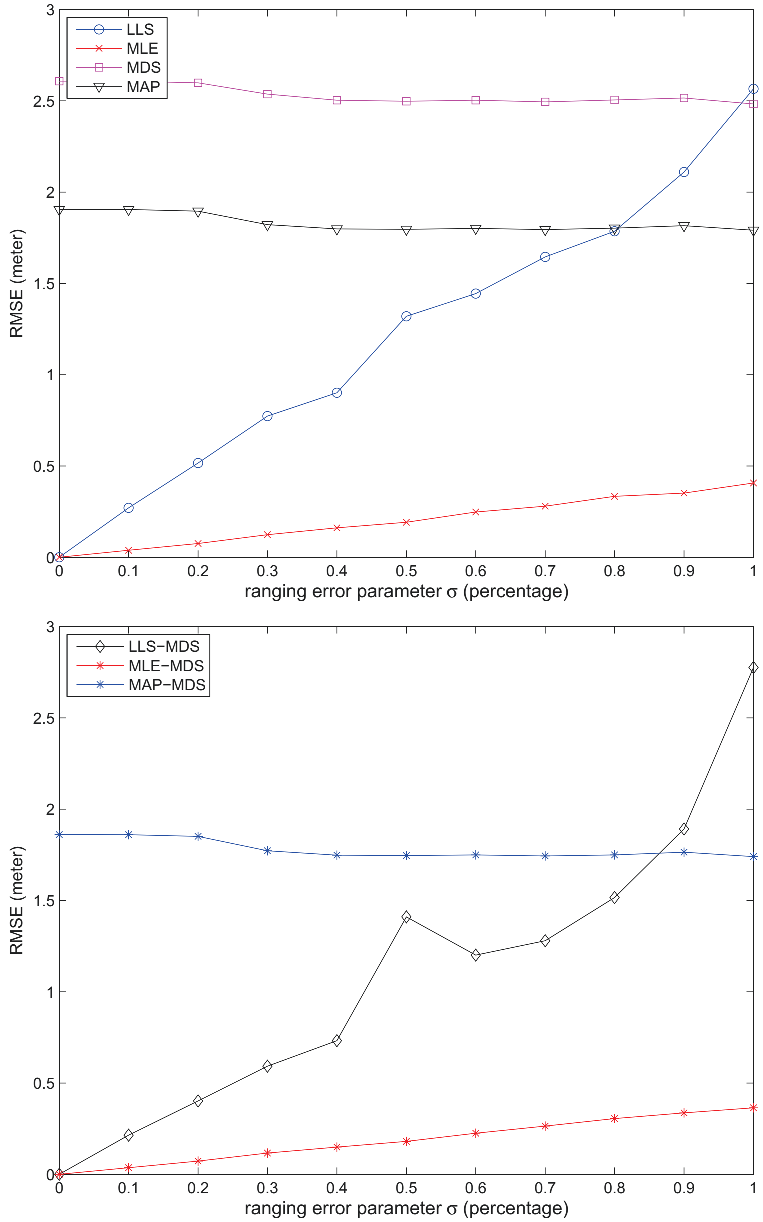

Figure 6: *Variation of RMSE as a function of* σ *for LLS, MLE, MDS, MAP and the enhanced MDS approaches in a partially connected network with a maximum ranging distance of* 80 *meters.*

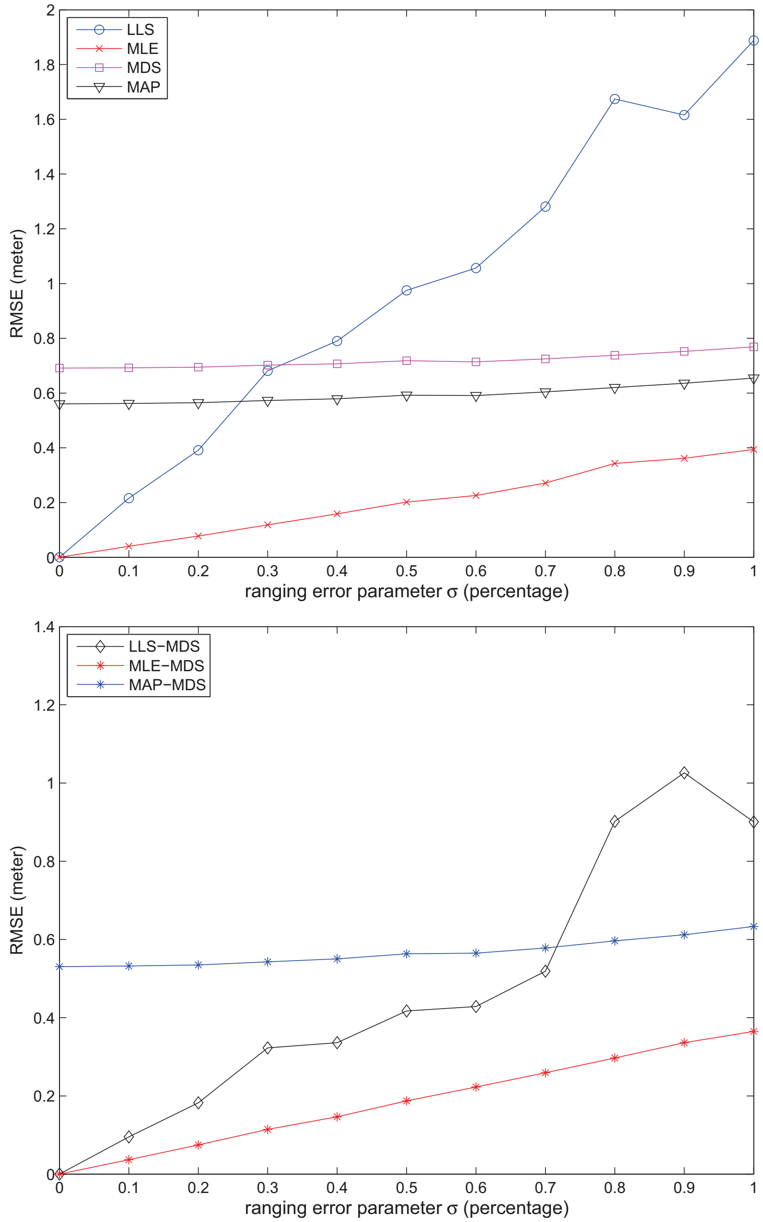

Figure 7: *Variation of RMSE as a function of σ for LLS, MLE, MDS, MAP and the enhanced MDS approaches in a partially connected network with a maximum ranging distance of 100 meters.*

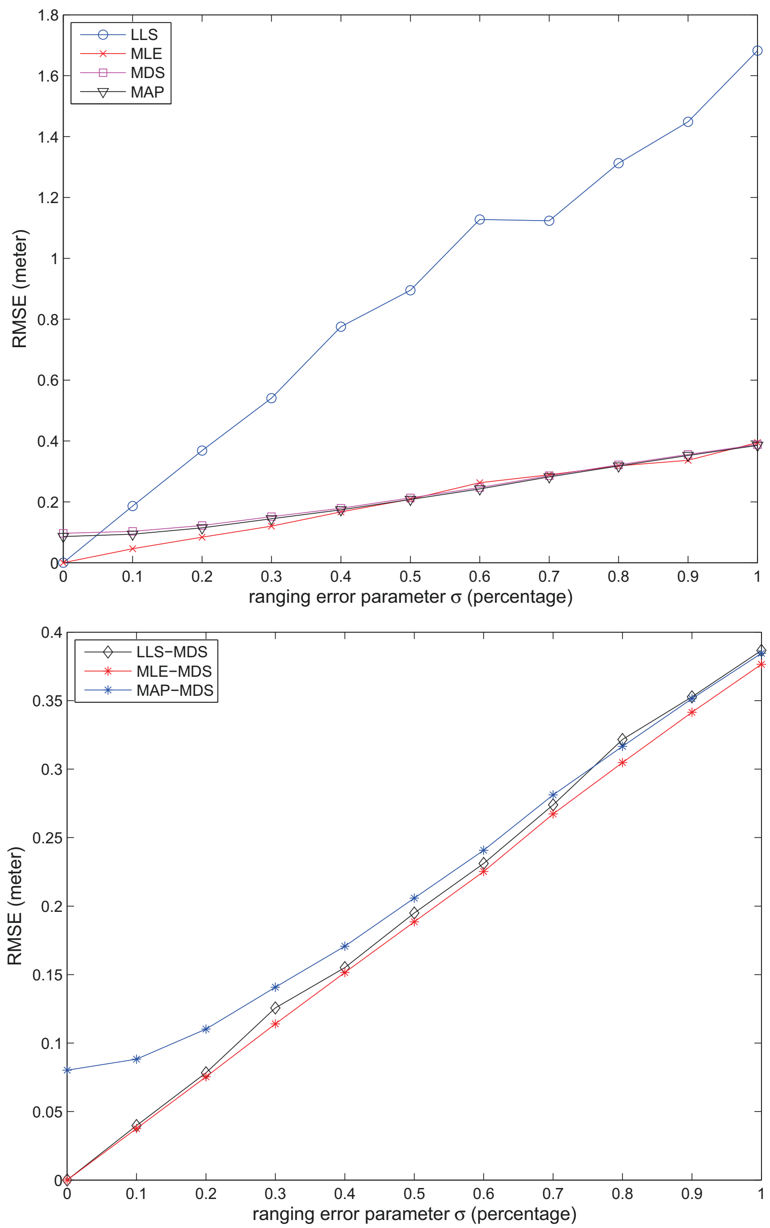

Figure 8: *Variation of RMSE as a function of* σ *for LLS, MLE, MDS, MAP and the enhanced MDS approaches in a partially connected network with a maximum ranging distance of 120 meters.*

σ increases while the RMSE for the MDS and MAP estimates are relatively constant over the tested range of σ. In this case, the observations indicated that the accuracy of the MDS and MAP estimates is dominated by the connectivity level of the networks rather than by the distance measurement errors. MLE performed the best. The performance of MDS and MAP improved as network connectivity level increases. It was observed that as the network connectivity increases, the RMSE for the MDS and MAP estimates approached those of the MLE estimates. LLS outperformed MDS and MAP when connectivity level and σ were both small, and was outperformed by MDS and MAP when the networks had increased connectivity. In general, the enhanced MDS approaches improves over the regular MDS approach. Both MLE-MDS and LLS-MDS improve over MLE and LLS, respectively. MAP-MDS and MAP had similar performance.

In conclusion, we consider MLE and MAP, among all proposed approaches, the viable solutions to relative localization for small wireless networks. MLE is able to provide the superior localization performance in both fully and partially connected network scenarios. Since MLE involves a nonlinear optimization problem, its global minimum convergence depends on the selection of the initial estimates and cannot be guaranteed. However, our simulation results showed that, when LLS was used to provide the initial estimates, the MLE algorithm has converged to the desired optimal estimates almost every time. Moreover, since small networks were considered in this study, significant impact of error propagation on the performance of MLE was not observed, which usually occurs to iterative algorithm such as MLE. For partially connected networks with low connectivity, however, MLE may suffer from the problem of not having sufficient numbers of anchors in iterating across the entire network. On the other hand, the performance of MAP is close to that of MLE in fully connected networks. It has similar trends as MDS in partially connected works with moderate and high connectivity levels. Compared to MLE, MAP has the advantage of always being able to provide a localization solution in spite of the network connectivity level, although the localization accuracy may be low as in the case of networks with low connectivity levels. Finally, it should be pointed out that MAP has additional advantages when compared to MDS. These advantages have not been discussed since they are beyond the scope of this report. More details can be found in [5][21].

This page intentionally left blank.

References

[1] A. Savvides, C. C. Han, and M. B. Srivastava, "Dynamic fine-grained localization in ad hoc networks of sensors," *Proc. the 7th Annual ACM/IEEE International Conference on Mobile Computing and Networking (MobiCom'01)*, pp. 166–179, Rome, Italy, July 2001.

[2] G. Mao, B. Fidan and B. D. O. Anderson, "Wireless sensor network localization techniques," *Computer Networks*, vol. 51, issue 10, pp. 2529–2553, July 2007.

[3] A. Savvides, H. Park, and M. B. Srivastava, "The bits and flops of the *N*-hop multilateration primitive for node localization problems," *Proc. the First ACM International Workshop on Wireless Sensor Networks and Applications*, pp. 112–121, Atlanta, Georgia, USA, September 2002.

[4] Y. Shang and W. Ruml, "Improved MDS-based localization," *Proc. IEEE INFOCOM 2004, The 23rd Annu. Joint Conf. of the IEEE Computer and Communications Societies*, Hong Kong, China, March 2004.

[5] Y. Zhou and L. Lamont, "An optimal local map registration technique for wireless sensor network localization problems," *Proc. 11th International Conference on Information Fusion (FUSION08)*, Cologne, Germany, June 30–July 03, 2008.

[6] Y. Zhou and L. Lamont, "A mobile beacon based localization approach for wireless sensor network applications," *Proc. the Fifth International Conference on Sensor Technologies and Applications (SENSORCOMM)*, Nice, France, August 2011.

[7] C. Savarese, J. M. Rabaey, and J. Beutel, "Location in distributed ad-hoc wireless sensor networks," *Proc. 2001 IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 4, pp. 2037–2040, Salt Lake City, UT, May 2001.

[8] J. E. Dennisand and R. B. Schnabel, *Numerical methods for unconstrained optimization and nonlinear equations*, Englewood Cliffs, NJ: Prrntice-Hall, 1983.

[9] J. C. Lagarias, J. A. Reeds, M. H. Wright, and P. E. Wright, "Convergence properties of the Nelder-Mead Simplex Method in low dimensions," *SIAM Journal of Optimization*, vol. 9, no. 1, pp. 112–147, 1998.

[10] J. M. Mendel, *Lessons in Digital Estimation Theory*, Prentice Hall, Englewood Cliffs, NJ, 1987.

[11] T. F. Coleman and Y. Li, "On the convergence of reflective Newton methods for large-scale nonlinear minimization subject to bounds," *Mathematical Programming*, vol. 67, no. 2, pp. 189–224, 1994.

[12] T. F. Coleman and Y. Li, "An interior, trust region approach for nonlinear minimization subject to bounds," *SIAM Journal on Optimization*, vol. 6, pp. 418–445, 1996.

[13] G. H. Golub and C. F. Van Loan, *Matrix Computation*, 3*rd Ed.*, The Johns Hopkins University Press, London, 1996.

[14] Y. Shang, W. Ruml, Y. Zhang, and M. Fromherz, "Localization from connectivity in sensor networks," *IEEE Trans. Parallel Distri. Sys.*, vol. 15, no. 11, pp. 961–974, November 2004.

[15] F. W. Young and R. M. Hamer, *Multidimensional Scaling: History, Theory and Applications*, Erlbaum, New York, 1987.

[16] I. Borg and P. Groenen, *Modern Multidimensional Scaling, Theory and Applications*, New York: Springer-Verlag, 1997.

[17] X. Ji and H. Zha, "Sensor positioning in wireless ad hoc networks using multidimensional scaling," *Proc. IEEE* 23*rd Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM* 2004*)*, vol. 4, pp. 2652–2661, Hong Kong, China, March 2004.

[18] E. W. Dijkstra, "A note on two problems in connection with graphs," *Numerische Mathematik*, 1, pp. 269–271, 1959.

[19] S. Warshall, "A theorem on Boolean matrices," *Journal of the ACM*, vol. 9, no. 1, pp. 11–12, January 1962.

[20] J. Dattorro, *Convex Optimization and Euclidean Distance Geometry*, Meboo Publishing, 2005.

[21] Y. Zhou and L. Lamont, "Optimal local map registration technique for wireless sensor network localization problems," *in Advances in Wireless Sensors and Sensors Networks*, *ed.* S. C. Mukhopadhyay and H. Leung, *Springer Lecture Notes in Electrical Engineering*, 2010.

[22] D. W. Eggert, A. Lorusso, and R. B. Fisher, "Estimating 3-D rigid body transformations: a comparison of four major algorithms," *Machine Vision and Applications*, vol. 9, pp. 272–290, 1997.

[23] K. S. Arun, T. S. Huang, and S. D. Blostein, "Least-squares fitting of two 3-d points sets," *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 9, no. 5, pp. 698–700, September 1987.

[24] R. I. Jennrich, "A simple general procedure for orthogonal rotation," *Psychometrika*, vol. 66, no. 2, pp. 289–306, June 2001.

[25] R. I. Jennrich, “A simple general method for oblique rotation,” *Psychometrika*, vol. 67, no. 1, pp. 7–19, March 2002.

[26] D. P. Bertsekas, *Nonlinear programming*, Athena Scientific, 2*nd ed.*, 1999.

[27] J. C. Gower and G. B. Dijksterhuis, *Procrustes Problems*, Oxford University Press, 2004.

[28] F. R. K. Chung, *Spectra Graph Theory*, American Mathematical Society, 1997.

This page intentionally left blank.

DOCUMENT CONTROL DATA

(Security markings for the title, abstract and indexing annotation must be entered when the document is Classified or Protected.)

1. ORIGINATOR (The name and address of the organization preparing the document. Organizations for whom the document was prepared, e.g. Centre sponsoring a contractor's report, or tasking agency, are entered in section 8.) DRDC – Valcartier Research Centre 2459 de la Bravoure Road, Québec QC G3J 1X5, Canada	2a. SECURITY MARKING (Overall security marking of the document, including supplemental markings if applicable.) UNCLASSIFIED 2b. CONTROLLED GOODS (NON-CONTROLLED GOODS) DMC A REVIEW: GCEC DECEMBER 2014

3. TITLE (The complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U) in parentheses after the title.)

Future Small Arms Research WBE 1.2.4: Relative Localization for Network of Soldiers

4. AUTHORS (Last name, followed by initials – ranks, titles, etc. not to be used.)

Zhou, Y.; Wong, F.

5. DATE OF PUBLICATION (Month and year of publication of document.)	6a. NO. OF PAGES (Total containing information. Include Annexes, Appendices, etc.)	6b. NO. OF REFS (Total cited in document.)
February 2016	36	28

7. DESCRIPTIVE NOTES (The category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

Scientific Report

8. SPONSORING ACTIVITY (The name of the department project office or laboratory sponsoring the research and development – include address.)

DRDC – Valcartier Research Centre
2459 de la Bravoure Road, Québec QC G3J 1X5, Canada

9a. PROJECT OR GRANT NO. (If appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant.)	9b. CONTRACT NO. (If appropriate, the applicable number under which the document was written.)
02aa	

10a. ORIGINATOR'S DOCUMENT NUMBER (The official document number by which the document is identified by the originating activity. This number must be unique to this document.)	10b. OTHER DOCUMENT NO(s). (Any other numbers which may be assigned this document either by the originator or by the sponsor.)
DRDC-RDDC-2016-R019	

11. DOCUMENT AVAILABILITY (Any limitations on further dissemination of the document, other than those imposed by security classification.)

Unlimited

12. DOCUMENT ANNOUNCEMENT (Any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in (11)) is possible, a wider announcement audience may be selected.)

Unlimited

13. ABSTRACT (A brief and factual summary of the document. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual.)

In this Scientific Report, we investigate the development of relative localization techniques based on internode distance measurements for small wireless networks where global navigation satellite systems are not available. Internode distance measurement errors are assumed to be small compared to their corresponding internode distances. High precision ranging can be achieved by using technologies such as ultra-wide band (UWB) ranging. A number of approaches are proposed, which include the Linear Least Squares (LLS) approach, the Maximum Likelihood Estimation (MLE) approach, the Map Registration Approach (MAP), the Multidimensional Scaling (MDS) approach and the enhanced MDS approaches. Computer simulations are used to demonstrate and compare the performances and effectiveness of the proposed approaches. Both fully and partially connected networks are simulated for studying the relative localization performance. Finally, conclusions are drawn on the results for relative localization in small networks.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

DRDC Scientific Report; MANET, wireless sensor networks, localization, GPS, MDS, MAP, Analysis, least squares, maximum likelihood, global coordinate, relative coordinate, ultrasound ranging, mobile beacon