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Letter

Recovery Probability Analysis for Sparse Signals Via OMP

It is known that use of a random measurement (sensing) matrix usually results in good recovery performance via orthogonal matching pursuit. This paper provides the probability of ensuring the recovery of sparse signals using orthogonal matching pursuit for the case where all entries of the measurement matrix are independently selected from a Gaussian distribution. The analysis relies on the mutual-coherence property of the sensing matrix.

I. INTRODUCTION

Compressed sensing (CS), also known as compressive sampling, has been successfully applied to a large number of applications, such as image processing [1], decentralized detection using wireless sensor networks [2], direction-of-arrival estimation [3], and radar [4]. It focuses on how to recover a signal vector $\mathbf{x}$ of size $L \times 1$ from a measurement vector $\mathbf{r}$ of size $M \times 1$ with $M \ll L$. The basic mathematical model is

$$\mathbf{r} = \mathbf{\Phi x}, \tag{1}$$

where $\mathbf{\Phi}$ is an $M \times L$ measurement matrix that is independent of signal $\mathbf{x}$. Since $\mathbf{\Phi}$ has many fewer rows than columns, recovering $\mathbf{x}$ from $\mathbf{r}$ is an undetermined problem that has infinite solutions. Therefore, CS exploits the sparsity of $\mathbf{x}$ to make the solution unique and ensure that the mapping between $\mathbf{r}$ and $\mathbf{x}$ is one-to-one [5]. In a typical CS system, it is assumed that

$$\mathbf{x} = \mathbf{\Psi s}, \tag{2}$$

where $\mathbf{s}$ is a sparse vector of size $N \times 1$ with K nonzero elements. This type of signal $\mathbf{x}$ given by (2) is said to be K-sparse. Also, $\mathbf{\Psi}$ denotes an $L \times N$ matrix which is called the basis matrix of an appropriate transform domain. Substituting (2) into (1), $\mathbf{r}$ can be rewritten as

$$\mathbf{r} = \mathbf{\Theta s}, \tag{3}$$

where $\mathbf{\Theta} = \mathbf{\Phi\Psi}$ is an $M \times N$ sensing matrix.

In principle, reconstructing the sparse signal $\mathbf{s}$ from compressed measurements $\mathbf{r}$ is NP-hard [6]. Nonetheless, many suboptimal algorithms (approximations) have been proposed in the literature to solve this kind of problem, such as ℓ_1/ℓ_2-based optimization techniques [7], basis pursuit [8], and orthogonal matching pursuit (OMP) [9]. Among these approaches, OMP is favorable for the sparse signal recovery, due to its interesting attractions. The main advantages of this greedy algorithm are its simplicity and speed [10]. In addition to its fast implementation, OMP is also empirically competitive in terms of recovery performance [11].

Theoretical analysis of OMP to date has been carried out mainly based on two properties of the sensing matrix $\mathbf{\Theta}$: the restricted-isometry property and mutual coherence (MC). A sensing matrix $\mathbf{\Theta}$ satisfies the restricted-isometry property of order K if there exists a constant δ such that

$$(1-\delta)\,\|\mathbf{s}\|_2^2 \leq \|\mathbf{\Theta s}\|_2^2 \leq (1+\delta)\,\|\mathbf{s}\|_2^2 \tag{4}$$

holds for any K-sparse vector $\mathbf{s}$ [11]. In particular, the minimum of all constants δ satisfying (4) is referred to as the isometry constant δ_K of sensing matrix $\mathbf{\Theta}$ [8]. As stated in [12], checking whether a sensing matrix $\mathbf{\Theta}$ satisfies (4) involves high computational complexity. On the other hand, the analysis based on MC can be described as follows: When sensing matrix $\mathbf{\Theta}$ has normalized columns, any K-sparse signal $\mathbf{s}$ can be exactly reconstructed from the measurements $\mathbf{r} = \mathbf{\Theta s}$ via OMP as long as [9]

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\mathbf{\Theta})}\right), \tag{5}$$

where $\mu(\mathbf{\Theta})$ represents the MC of $\mathbf{\Theta}$ [5]. In this work, we consider the MC-based approach because of its advantages from an implementation point of view.

It is known that the use of a random measurement matrix $\mathbf{\Phi}$ (typically using Gaussian distributions) usually results in good recovery performance [13]. Note that when $\mathbf{\Phi}$ is random, the sensing matrix $\mathbf{\Theta}$ becomes random as well. Therefore, it is desirable to consider the random nature of $\mathbf{\Theta}$ in OMP performance evaluation, which requires a probabilistic approach. To the best of our knowledge, despite the considerable attention that has been paid to OMP, such a probabilistic analysis has not been reported in the literature. This motivated our work.

The goal of this paper is to provide the probability of ensuring the perfect recovery of K-sparse signals using OMP. In particular, for the first time in the literature, we introduce and derive the probability of perfect recovery as a useful measure to evaluate the performance of OMP assuming that $\mathbf{\Phi}$ (and thus $\mathbf{\Theta}$) is random. It turns out that the direct derivation of the exact probability of perfect recovery is very difficult. So we will apply a bounding technique to approximate the MC of $\mathbf{\Theta}$, which makes the probability analysis tractable. The accuracy of the performed analysis is well demonstrated by our extensive numerical experiments.

NOTATION Throughout this paper, bold uppercase symbols denote matrices and bold lowercase symbols denote vectors. Superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ stand for the complex conjugate, transpose, and Hermitian, respectively. Also, we use $E[\cdot]$ to denote the statistical expectation, and $\mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma})$ represents a real-valued multidimensional Gaussian vector with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$. Also, $\|\mathbf{a}\|_2$ stands for the Euclidean norm of the vector $\mathbf{a}$. Furthermore, $\mathbf{0}$ represents a zero matrix, $\mathbf{I}_N$ is an $N \times N$ identity matrix, and diag$\{\mathbf{a}\}$ stands for a diagonal matrix with the elements of the vector $\mathbf{a}$ as its diagonal entries. We use a_i and $A_{i,j}$ to denote, respectively, the ith element of the vector $\mathbf{a}$ and element (i, j) of the matrix $\mathbf{A}$. We use $\mathbf{A} := \mathbf{B}$ to denote that $\mathbf{A}$, by definition, equals $\mathbf{B}$, and $\mathbf{A} =: \mathbf{B}$ to denote that $\mathbf{B}$, by definition, equals $\mathbf{A}$.

II. PROBABILITY OF PERFECT RECOVERY

We begin this section with the definition of MC. Then we determine the probability of perfect recovery based on the MC property of sensing matrix $\mathbf{\Theta}$. Since direct derivation of the exact probability does not appear to be possible, we will apply a bounding technique to approximate the MC of $\mathbf{\Theta}$. Throughout this section, we draw a random measurement matrix $\mathbf{\Phi}$ whose entries are independent and identically distributed Gaussian random variables (RVs).

MC is one of the most fundamental quantities in the context of CS. The MC of sensing matrix $\mathbf{\Theta} = [\,\boldsymbol{\theta}_1 \cdots \boldsymbol{\theta}_N\,]$ is defined as [5]

$$\mu(\mathbf{\Theta}) := \max_{1 \leq i \neq j \leq N} \frac{\left|\boldsymbol{\theta}_i^{\mathrm{H}} \boldsymbol{\theta}_j\right|}{\|\boldsymbol{\theta}_i\|_2 \left\|\boldsymbol{\theta}_j\right\|_2}, \tag{6}$$

which represents the worst-case coherence between any two columns $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_j$ of sensing matrix $\mathbf{\Theta}$. Suppose that $\mathbf{\Theta}$ has normalized columns, i.e., $\|\boldsymbol{\theta}_1\|_2 = \cdots = \|\boldsymbol{\theta}_N\|_2 = 1$. Also, let $\mathbf{G} = \mathbf{\Theta}^{\mathrm{H}}\mathbf{\Theta}$ denote the Gram matrix of $\mathbf{\Theta}$. Then (6) can be rewritten as

$$\mu(\mathbf{\Theta}) = \max_{i<j} \left\{\left|G_{i,j}\right|\right\}, \tag{7}$$

where one can show that

$$G_{i,j} = \sum_p \sum_q \Phi_{p,q}^* \Psi_{q,i}^* \sum_l \Phi_{p,l} \Psi_{l,j}. \tag{8}$$

Note that the Gram matrix $\mathbf{G}$ has a unit diagonal—i.e., $G_{i,j} = 1$—for $1 \leq i \leq N$, and $|G_{i,j}| = |G_{j,i}|$ for $1 \leq i \neq j \leq N$.

It has been shown in [9] that when sensing matrix $\mathbf{\Theta}$ has normalized columns and

$$K < \frac{1}{2}\left(1 + \frac{1}{\mu(\mathbf{\Theta})}\right),$$

OMP will exactly recover any K-sparse signal $\mathbf{s}$ generated from $\mathbf{r} = \mathbf{\Theta s}$. This motivates us to define the probability of perfect recovery P_{MC} as

$$P_{\mathrm{MC}} := \Pr(\mu(\mathbf{\Theta}) < \tau) = \Pr\left(\max_{i<j}\left\{\left|G_{i,j}\right|\right\} < \tau\right), \tag{9}$$

where $\tau := 1/2K - 1$. The problem is to find the cumulative distribution function of RV $\max_{i<j}\{|G_{i,j}|\}$. It turns out that this is a very difficult problem. To make the problem tractable, we will first determine an upper and a lower bound of the RV $\max_{i<j}\{|G_{i,j}|\}$. To this end, we arrange all upper-right off-diagonal elements of matrix $\mathbf{G}$

in a column and define the vector $\mathbf{g}$ as

$$\mathbf{g} := \left[G_{1,2} \cdots G_{1,N} \cdots G_{(N-2),(N-1)} \; G_{(N-2),N} \; G_{(N-1),N} \right]^{\mathrm{T}} =: \left[g_1 \cdots g_\eta \right], \quad (10)$$

where $\eta := N(N-1)/2$. All entries of measurement matrix $\boldsymbol{\Phi}$ are independently selected from a Gaussian distribution of zero mean and variance σ_Φ^2. Therefore, each element of vector $\mathbf{g}$ is a summation of many independent and identically distributed RVs. The multidimensional central limit theorem states that when $N \gg 1$, the distribution of such a random vector can be well approximated by a multivariate normal distribution with mean $\boldsymbol{\mu}_g$ and covariance matrix $\boldsymbol{\Sigma}_g$, i.e., $\mathbf{g} \sim \mathcal{N}(\boldsymbol{\mu}_g, \boldsymbol{\Sigma}_g)$. It is possible to derive the exact and closed-form expressions for the mean $\boldsymbol{\mu}_g$ and covariance matrix $\boldsymbol{\Sigma}_g$.

For any matrix $\mathbf{A}$ of size $a \times b$, the following inequality holds [14]:

$$\frac{1}{\sqrt{ab}} \|\mathbf{A}\|_2 \le \|\mathbf{A}\|_{\max} \le \|\mathbf{A}\|_2, \quad (11)$$

where $\|\mathbf{A}\|_{\max} = \max |A_{i,j}|$, for $i = 1, \ldots, a$ and $j = 1, \ldots, b$. This enables us to bound the MC $\mu(\boldsymbol{\Theta})$ as

$$\frac{1}{\eta} \|\mathbf{g}\|_2^2 \le \max_{i<j} \left\{ \left| G_{i,j} \right|^2 \right\} \le \|\mathbf{g}\|_2^2 . \quad (12)$$

LEMMA 1 Let $\lambda_1, \ldots, \lambda_r$ be nonzero eigenvalues of $\boldsymbol{\Sigma}_g$. Also, suppose that $\mathbf{u}_n$ is the eigenvector of $\boldsymbol{\Sigma}_g$ corresponding to λ_n for $n = 1, \ldots, \eta$. Then (12) can be equivalently written as

$$\frac{1}{\eta} \|\mathbf{h}\|_2^2 \le \max_{i<j} \left\{ \left| G_{i,j} \right|^2 \right\} \le \|\mathbf{h}\|_2^2 , \quad (13)$$

where elements of vector $\mathbf{h}$ are independent Gaussian RVs with mean $\mathbf{u}_n^{\mathrm{H}} \boldsymbol{\mu}_g$ and variance $\lambda_n : h_n \sim \mathcal{N}(\mathbf{u}_n^{\mathrm{H}} \boldsymbol{\mu}_g, \lambda_n)$, $n = 1, \ldots, \eta$.

PROOF See Appendix A.

We observe that $\max_{i<j}\{|G_{i,j}|^2\}$ is lower bounded by $(1/\eta)\|\mathbf{h}\|_2^2$ and upper bounded by $\|\mathbf{h}\|_2^2$. Thus, we approximate it as $\max_{i<j}\{|G_{i,j}|^2\} \approx \zeta \|\mathbf{h}\|_2^2$, where $(1/\eta) \le \zeta \le 1$. Note that the ζ value can be numerically determined to ensure that this approximation is accurate.

It is well-known that the knowledge of the moment-generating function (MGF) provides an easy way to characterize the distribution of the sum of independent random variables. Thus, in the following lemma, we derive the MGF of $\|\mathbf{h}\|_2^2 = \sum_n |h_n|^2$.

LEMMA 2 The MGF of $\|\mathbf{h}\|_2^2$ is given by

$$\mathcal{M}_{\|\mathbf{h}\|_2^2}(s) := E\left[e^{s\|\mathbf{h}\|_2^2} \right] = \prod_{n=1}^{\eta} \left[\frac{1}{1 - s\lambda_n} \exp\left(\frac{s\lambda_n \left| \mathbf{u}_n^{\mathrm{H}} \boldsymbol{\mu}_g \right|^2}{1 - s\lambda_n} \right) \right]. \quad (14)$$

PROOF See Appendix B.

Having been well equipped, we find the probability of perfect recovery in the following lemma.

LEMMA 3 The probability of perfect recovery corresponding with the MC approach is given by

$$P_{\mathrm{MC}} \approx \frac{\mathbf{j}}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{s} \mathcal{M}_{\|\mathbf{h}\|_2^2}(s) \left[\exp\left(-\mathbf{j} \frac{\tau^2 s}{\zeta^2} \right) - 1 \right] ds. \quad (15)$$

PROOF See Appendix C.

III. NUMERICAL RESULTS

In this section, we evaluate the recovery performance of OMP via the probability of perfect recovery. To this end, we present two different experiments. The first experiment studies the performance of a simple CS-based single-input, single-output (SISO) radar in terms of the probability of perfect recovery. The second one uses such probability to investigate the performance of a complex collocated CS-based multiple-input, multiple-output (MIMO) radar. It is worth mentioning that since the number of targets in a radar scene is often limited, sparse modeling and CS theory are applicable to the field of radar. Throughout all simulations, the carrier frequency is considered to be 1 GHz. The results that we show are obtained by averaging the corresponding quantity over 10^4 independent simulation runs, where both sensing matrix and target locations are randomly generated at each test instance. The application of CS to collocated MIMO radars was recently investigated in [15], where the CS-based MIMO radar system was implemented on a small-scale network consisting of M_{t} transmit and N_{r} receive elements, each equipped with single antenna. According to [15], all the receive elements forward their compressively obtained measurements to a fusion center, which combines them appropriately. Exploiting the sparsity of radar signals in various spaces, the fusion center formulates the problem of target parameters' estimation as that of the recovery of the sparse vector $\mathbf{s}$ from measurements $\mathbf{r} = \boldsymbol{\Theta}\mathbf{s}$.

We begin with the SISO case. Fig. 1 examines the probability of perfect recovery P_{MC} versus $\tau = 1/(2K-1)$ for different σ_Φ^2 values. For simulation purposes, we consider a grid of size 66×45. Also, it is assumed that the targets fall on the grid points. From Fig. 1, it is observed that increasing σ_Φ^2 improves the probability of perfect recovery. Furthermore, Fig. 1 reveals that the simulation results achieve almost the same P_{MC} as analytical results for different values of σ_Φ^2.

Next, we study the performance of a collocated CS-based MIMO radar system. The simulation parameters of the second experiment have been chosen to be the same as those used in the previous experiment. The only difference is the grid size, which is increased from 66×45 (for SISO) to 105×91 (for MIMO). The probability of

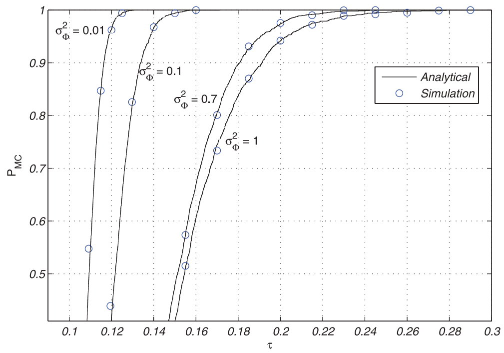

Fig. 1. Probability of perfect recovery P_{MC} versus $\tau = 1/(2K-1)$ for SISO radar.

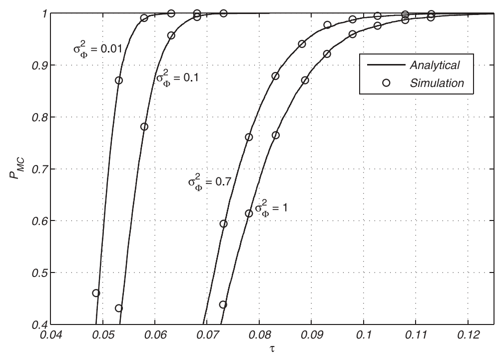

Fig. 2. Probability of perfect recovery P_{MC} versus $\tau = 1/(2K-1)$ for 10×10 MIMO radar.

perfect recovery P_{MC} versus $\tau = 1/(2K-1)$ for a 10×10 MIMO radar system is depicted in Fig. 2. It can be seen from the figure that the simulation and analytical results produce almost the same P_{MC} for different σ_Φ^2 values. Note that in both experiments, the curve-fitting parameter $\zeta = 0.368$ leads to the best match between analytical and simulation results.

IV. CONCLUSIONS

In this letter, assuming that all entries of the measurement matrix are independently selected from a Gaussian distribution, we studied the notion of the probability for OMP performance evaluation. In particular, we calculated the probability of perfect recovery as a useful measure to study the recovery performance of sparse signals using OMP. We defined the probability of perfect recovery based on the MC property of the sensing matrix. Since direct calculation of such probability is difficult, we applied a bounding technique to approximate the MC. Numerical experiments confirmed the accuracy of our performed analysis.

APPENDIX A. PROOF OF LEMMA 1

Let $\lambda_1, \ldots, \lambda_r$ be non zero eigenvalues of $\mathbf{\Sigma}_g$. Also, suppose that $\mathbf{u}_n$ is the eigenvector of $\mathbf{\Sigma}_g$ corresponding to λ_n for $n = 1, \ldots, \eta$. The eigenvalue decomposition of $\mathbf{\Sigma}_g$ is formed as

$$\mathbf{\Sigma}_g = \begin{bmatrix} \mathbf{U} & \mathbf{U}_0 \end{bmatrix} \begin{bmatrix} \mathbf{\Delta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{\mathrm{H}} \\ \mathbf{U}_0^{\mathrm{H}} \end{bmatrix}, \tag{16}$$

where $\mathbf{\Delta} = \mathrm{diag}\{\lambda_1, \ldots \lambda_r\}$ is a positive-definite diagonal matrix of size $r \times r$, $\mathbf{U} := [\,\mathbf{u}_1 \cdots \mathbf{u}_r\,]$ is an $\eta \times r$ matrix, $\mathbf{U}_0 := [\,\mathbf{u}_{r+1} \cdots \mathbf{u}_\eta\,]$ is a matrix of dimension $\eta \times (\eta - r)$, and

$$\begin{bmatrix} \mathbf{U} & \mathbf{U}_0 \end{bmatrix} \begin{bmatrix} \mathbf{U}^{\mathrm{H}} \\ \mathbf{U}_0^{\mathrm{H}} \end{bmatrix} = \mathbf{I}. \tag{17}$$

Then

$$\begin{bmatrix} \mathbf{U}^{\mathrm{H}} \\ \mathbf{U}_0^{\mathrm{H}} \end{bmatrix} \mathbf{\Sigma} \begin{bmatrix} \mathbf{U} & \mathbf{U}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{\Delta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{18}$$

Let us define $\mathbf{h}$ as

$$\mathbf{h} = \begin{bmatrix} \mathbf{U}^{\mathrm{H}} \\ \mathbf{U}_0^{\mathrm{H}} \end{bmatrix} \mathbf{g}. \tag{19}$$

One can show that $\mathbf{h} \sim \mathcal{N}(\boldsymbol{\mu}_h, \mathbf{\Sigma}_h)$, where

$$\boldsymbol{\mu}_h := \begin{bmatrix} \mathbf{u}_1^{\mathrm{H}} \boldsymbol{\mu}_g \\ \vdots \\ \mathbf{u}_\eta^{\mathrm{H}} \boldsymbol{\mu}_g \end{bmatrix} \quad \text{and} \quad \mathbf{\Sigma}_h := \begin{bmatrix} \mathbf{\Delta} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{20}$$

Since $\mathbf{\Delta}$ is a diagonal matrix, all elements of $\mathbf{h}$ are independent Gaussian RVs—i.e., $h_n \sim \mathcal{N}(\mu_n, \lambda_n)$, where $\mu_n := \mathbf{u}_n^{\mathrm{H}} \boldsymbol{\mu}_g$ for $n = 1, \ldots, \eta$. Based on (17) and (19), we have $\|\mathbf{h}\|_2^2 = \mathbf{h}^{\mathrm{H}}\mathbf{h} = \|\mathbf{g}\|_2^2$.

This completes the proof.

APPENDIX B. PROOF OF LEMMA 2

As stated in Lemma 1, all elements of vector $\mathbf{h}$ are independent Gaussian RVs—i.e. $h_n \sim \mathcal{N}(\mu_n, \lambda_n)$, where $\mu_n = \mathbf{u}_n^{\mathrm{H}} \boldsymbol{\mu}_g$ for $n = 1, \ldots, \eta$. Therefore, $|h_n|$ are independent Ricean RVs with a probability distribution function given by

$$f_{|h_n|}(x_n) = \frac{2x_n}{\lambda_n} \exp\left(-\frac{x_n^2 + |\mu_n|^2}{\lambda_n}\right) I_0\left(\frac{2\,|\mu_n|\,x_n}{\lambda_n}\right), \tag{21}$$

$n = 1, \ldots, \eta$, where $I_0(\cdot)$ denotes the modified Bessel function of the first kind of order 0. Thus, the MGF of $\|\mathbf{h}\|_2^2 = \sum_{n=1}^{\eta} |h_n|^2$ can be calculated as

$$E\left[e^{s\|\mathbf{h}\|_2^2}\right] = E\left[\exp\left(s\sum_{n=1}^{\eta}|h_n|^2\right)\right] = E\left[\prod_{n=1}^{\eta}\exp\left(s|h_n|^2\right)\right]$$
$$= \int_0^\infty \cdots \int_0^\infty dx_1 \cdots dx_\eta \prod_{n=1}^{\eta}\exp\left(sx_n^2\right)\frac{2x_n}{\lambda_n}\exp\left(-\frac{x_n^2+|\mu_n|^2}{\lambda_n}\right) I_0\left(\frac{2|\mu_n|x_n}{\lambda_n}\right)$$
$$= \prod_{n=1}^{\eta}\left[\int_0^\infty \frac{2x_n}{\lambda_n}\exp\left(-\frac{(1-s\lambda_n)x_n^2+|\mu_n|^2}{\lambda_n}\right) I_0\left(\frac{2|\mu_n|x_n}{\lambda_n}\right)dx_n\right]$$
$$= \prod_{n=1}^{\eta}\left[\frac{1}{1-s\lambda_n}\int_0^\infty \frac{2y_n}{\lambda_n}\exp\left(-\frac{y_n^2+|\mu_n|^2}{\lambda_n}\right) I_0\left(\frac{2y_n}{\lambda_n}\frac{|\mu_n|}{\sqrt{1-s\lambda_n}}\right)dy_n\right]$$
$$= \prod_{n=1}^{\eta}\left[\frac{1}{1-s\lambda_n}\exp\left(\frac{|\mu_n|^2 s\lambda_n}{1-s\lambda_n}\right)\right]. \quad (22)$$

This completes the proof.

APPENDIX C. PROOF OF LEMMA 3

Let

$$Y := \frac{1}{\eta}\sqrt{\|\mathbf{h}\|_2^2} = \frac{1}{\eta}\sqrt{\sum\nolimits_n |h_n|}^2.$$

Then one can show that $f_Y(y) = 2\eta^2 y f_{\|\mathbf{h}\|_2^2}(\eta^2 y^2)$. Recalling that

$$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathcal{M}_X(s)\exp(-\mathbf{j}xs)\,ds,$$

we have

$$f_Y(y) = \frac{\eta^2 y}{\pi}\int_{-\infty}^{\infty} M_{\|\mathbf{h}\|_2^2}(s)\exp\left(-\mathbf{j}\eta^2 y^2 s\right)ds. \quad (23)$$

After replacing $1/\eta$ with a curve-fitting parameter such as ζ, the probability of perfect recovery is given by

$$P_{\mathrm{MC}} \approx \frac{1}{\pi\zeta^2}\int_0^\tau\int_{-\infty}^{+\infty} y\mathcal{M}_{\|\mathbf{h}\|_2^2}(s)\exp\left(-\mathbf{j}\frac{y^2 s}{\zeta^2}\right)ds\,dy$$
$$= \frac{1}{\pi\zeta^2}\int_{-\infty}^{+\infty}\int_0^\tau y\mathcal{M}_{\|\mathbf{h}\|_2^2}(s)\exp\left(-\mathbf{j}\frac{y^2 s}{\zeta^2}\right)dy\,ds$$
$$= \frac{\mathbf{j}}{2\pi}\int_{-\infty}^{+\infty}\frac{1}{s}\mathcal{M}_{\|\mathbf{h}\|_2^2}(s)\left[\exp\left(-\mathbf{j}\frac{\tau^2 s}{\zeta^2}\right)-1\right]ds. \quad (24)$$

This completes the proof.

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