Global Network Connectivity Assessment via Local Data Exchange for Underwater Acoustic Sensor Networks

Fiscal Year 2014-2015

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Contract Report # AMBUSH.2.1

PWGSC Contract Number: W7707-145674
Contract Scientific Authority: Stephane Blouin, Defence Scientist

Contract Report
DRDC-RDDC-2015-C325
March 2015

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Abstract

In this report, the problem of connectivity assessment for an underwater random sensor network is investigated. The weighted vertex connectivity is introduced as a metric to evaluate the connectivity of the weighted expected graph of a random sensor network where the elements of the weight matrix characterize the operational probability of their corresponding communication links. The weighted vertex connectivity measure extends the notion of vertex connectivity for random graphs by taking into account the joint effects of the path reliability and the network robustness to node failure. The approximate weighted vertex connectivity measure is defined subsequently as a lower bound on the introduced connectivity metric which can be found by applying a series of a polynomial-time shortest path algorithm. The performance of the proposed algorithms is validated by simulation and experimental results.

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1 Introduction

An underwater acoustic sensor network consists of a number of fixed or mobile sensors, deployed in the underwater environment which are capable of sending/receiving data using acoustic communication channels [1], [2]. A typical objective for such networks is to perform data aggregation for applications as diverse as underwater exploration, ocean sampling, and disaster prevention [3], [4], [5]. Unlike the communication channels used in terrestrial sensor networks, there are a large number of factors which influence the communication between underwater nodes such as multi-path propagation, temperature fluctuation, scattering and reverberation, variation of sound speed profile, and underwater currents [6], [7], [8]. Moreover, all mentioned factors vary over time and space in an unpredictable manner resulting in highly temporal and spatially variable acoustic channels, and making random graphs a good candidate to model the underwater acoustic sensor networks [9], [10].

Based on the results given in [11], [12], [13], various tasks such as consensus, distributed estimation, and target localization over a random sensor network are achieved cooperatively as long as the expected communication graph of the network remains connected. Moreover, the performance and convergence rate of the cooperative algorithms running over a random network depend heavily on the connectivity degree of the expected graph of the network [14], [15].

Vertex connectivity is introduced in [16], [17] as a measure of connectivity to evaluate the robustness of a sensor network to node failure. Based on the vertex connectivity notion, a fault-tolerant topology control procedure is proposed in [16] to minimize the power consumption of the network while maintaining a certain degree of connectivity over the entire network. In [17], efficient algorithms are introduced to improve the vertex connectivity of a heterogenous wireless sensor network using relay node placement. Various polynomial-time algorithms have been provided in the literature to measure the vertex connectivity degree of a graph [18], [19].

The general idea behind these algorithms is that the degree of vertex connectivity for any pair of nonadjacent nodes in a graph can be formulated as a problem of finding multiple vertex-disjoint paths between those nodes [20]. By establishing multiple disjoint paths between a source and a destination in an optimal manner, the network characteristics such as energy conservation, load balancing, and robustness to failure can be improved. Different procedures are proposed in the literature to obtain a set of disjoint paths between two nodes satisfying certain properties. An algorithm is proposed in [21] to find a predetermined number of vertex-disjoint paths from a source node to a destination node with minimum total weight in a directed graph. In [22], the authors propose a procedure to find a number of parallel vertex-disjoint paths which can be used to transmit data with maximum reliability.

In this report, a novel metric of connectivity is developed to evaluate the connectivity of the expected communication graph of a random sensor network. The notion of weighted vertex connectivity is introduced as an extension of the vertex connectivity notion reflecting the combined effects of the reliability of the paths and the network robustness to node failure on the connectivity of the expected communication graph [23]. An approximation of the weighted vertex connectivity measure is subsequently proposed which provides a lower bound on the original metric and can be computed by applying a polynomial-time shortest path algorithm sequentially. The proposed connectivity measure and its approximation are computed in a simulation environment, and are verified by experiments.

The remainder of the report is organized as follows. In Section 2, some relevant preliminaries are given and the problem is formulated. The weighted vertex connectivity degree is proposed in Section 3, and an approximation of the weighted vertex connectivity metric along with a time-efficient algorithm to obtain this measure are presented in Section 4. An illustrative example to demonstrate the required steps for finding the weighted vertex connectivity and approximate weighted vertex connectivity metrics is given in Section 5. The simulation and experimental results are presented in Sections 6 and 7, respectively. Finally, the concluding remarks are offered in Section 8.

2 Preliminaries and Problem Formulation

Throughout this report, the set of positive and nonnegative real numbers are denoted by $\mathbb{R}_{>0}$ and $\mathbb{R}_{\geq 0}$, respectively. Also, $\mathbb{N}_n := \{1,2,\ldots,n\}$, $|\Phi|$ is the cardinality of a finite set Φ, and ϕ^i represents the i-th element of the set Φ. Moreover, the power set of a finite set Φ, denoted by $\mathscr{P}(\Phi)$, is the set of all subsets of Φ.

Definition 1 *Let $G = (V,E)$ denote a random directed graph (digraph) composed of a set of nodes V and a set of edges E. Let also the probability matrix $P = [p_{ij}]$ represent the existence probability of all directed edges in G, where $p_{ij} \in [0,1]$ is the probability of the existence of the edge $(j,i) \in E$. Define $A = [a_{ij}]$ as the adjacency matrix of G, where a_{ij} is a binary random variable such that:*

$$a_{ij} = \begin{cases} 1, & \textit{with probability } p_{ij}, \\ 0, & \textit{with probability } 1 - p_{ij}, \end{cases} \tag{1}$$

and $(j,i) \in E$ if and only if $a_{ij} = 1$.

Definition 2 *Define $\hat{G} = (\hat{V},\hat{E})$ as the expected graph of the random digraph $G = (V,E)$, where the set of nodes and edges of $\hat{G}$ are denoted by $\hat{V}$ and $\hat{E}$, respectively. Furthermore, the weighted adjacency matrix of $\hat{G}$ is represented by $\hat{A} = [\hat{a}_{ij}]$, where $\hat{a}_{ij} = p_{ij}$ for every pair of distinct nodes $i, j \in \hat{V}$. Moreover, $(j,i) \in \hat{E}$ if and only if $p_{ij} \neq 0$.*

Let the *communication graph* of a network composed of n sensors be specified by a random digraph $G=(V,E)$ with the probability matrix $P=[p_{ij}]$, where its node and edge sets are defined as:

$$V=\{1,2,\ldots,n\}, \tag{2a}$$
$$E=\{(i,j)\in V\times V \mid a_{ji}=1\}. \tag{2b}$$

Let also $\hat{G}=(\hat{V},\hat{E})$ be the *expected communication graph* of the network, where $\hat{V}=V$ and:

$$\hat{E}=\{(i,j)\in \hat{V}\times \hat{V} \mid p_{ji}\neq 0\}. \tag{3}$$

Due to the importance of the connectivity of the expected communication graph in achieving desired cooperative objective over a random sensor network, the main objective of this report is to introduce appropriate global measures for the connectivity of an expected communication graph, and develop efficient algorithms to evaluate them.

3 Weighted Vertex Connectivity Measure

Consider a group of sensors represented by the node set of a random digraph $G=(V,E)$, where every directed edge in E is characterized by a binary random variable. Let also the binary random variables describing the probabilistic nature of all edges be independent. The concept of *vertex connectivity* (VC) is introduced in [20] as a measure of global connectivity of digraphs. The VC degree of G is defined as the minimum number of nodes that should be removed in order for G to lose strong connectivity. Let $G'=(V',E')$ represent a deterministic digraph with node set V' and edge set E'. Then the VC degree of G', denoted by $\kappa(G')$, is given by:

$$\kappa(G')=\min_{i,j\in V',\, i\neq j}\kappa_{i,j}(G'), \tag{4}$$

where,

$$\kappa_{i,j}(G')=\begin{cases} N_{i,j}(G'), & \text{if } (i,j)\notin E',\\ |V'|-1, & \text{if } (i,j)\in E', \end{cases} \tag{5}$$

and $N_{i,j}(G')$ denotes the maximum number of vertex-disjoint paths connecting node i to node j in G'. The main idea behind the algorithms provided in the literature to compute the VC degree of a graph is that the minimum number of nodes whose removal disconnects any pair of nonadjacent nodes is equal to the maximum number of mutually vertex-disjoint directed paths between them (see Menger's Theorem [20]). However, this measure does not account for the probability matrix of random networks and merely demonstrates the robustness of the network to node failure. This calls for a more accurate measure of connectivity to capture the probabilistic nature of the communication links. The *weighted vertex connectivity* (WVC) measure is introduced here to extend the notion of the VC degree to the more general case of weighted digraphs, where the elements of the weight matrix denote the operational probability of their corresponding communication links.

This non-negative measure is positive for a strongly connected digraph, and a larger value of this measure represents "stronger" connectivity, taking into account the combined effects of the operational probability of the paths and the network robustness to node failure.

To clarify this new concept, the multiplicative weight of a path is subsequently defined, based on the mutual independence of the binary random variables used for describing the probabilistic nature of the edges of the network.

Definition 3 *Let $\Pi_{i,j}$ denote the set of all directed paths from node i to node j whose lengths are greater than one in the expected communication graph $\hat{G} = (\hat{V}, \hat{E})$ with probability matrix $P = [p_{ij}]$. Let also $\pi^k_{i,j} \in \Pi_{i,j}$ represent the k-th element of $\Pi_{i,j}$ defined as $\pi^k_{i,j} = \{v^k_0, v^k_1, \ldots, v^k_{m_k-1}, v^k_{m_k}\}$, which denotes a directed path of length $m_k > 1$ from node i to node j such that $v^k_0 = i$, $v^k_{m_k} = j$, and $(v^k_{l-1}, v^k_l) \in \hat{E}$ for all $l \in \mathbb{N}_{m_k}$. Then, the multiplicative weight of path $\pi^k_{i,j}$, denoted by $W(\pi^k_{i,j})$, is defined as follows:*

$$W(\pi^k_{i,j}) = \prod_{l=1}^{m_k} p_{v^k_l v^k_{l-1}}. \tag{6}$$

Since each element of P represents the probability of the existence of its corresponding edge in $\hat{G}$ and all edges are characterized by a set of mutually independent binary random variables, the multiplicative weight can be interpreted as the operational probability of a given directed path.

Definition 4 *Consider $\pi^s_{i,j}$ and $\pi^t_{i,j}$ as two distinct directed paths from node i to node j in $\hat{G}$ which are described by the node sets $\pi^s_{i,j} = \{v^s_0, v^s_1, \ldots, v^s_{m_s-1}, v^s_{m_s}\}$ and $\pi^t_{i,j} = \{v^t_0, v^t_1, \ldots, v^t_{m_t-1}, v^t_{m_t}\}$, respectively, with $v^s_0 = v^t_0 = i$ and $v^s_{m_s} = v^t_{m_t} = j$ for $s, t \in \mathbb{N}_{|\Pi_{i,j}|}$. Let also m_s and m_t denote the lengths of two directed paths $\pi^s_{i,j}$ and $\pi^t_{i,j}$, respectively, such that $m_s > 1$ and $m_t > 1$. Then, $\pi^s_{i,j}$ and $\pi^t_{i,j}$ are* vertex-disjoint *paths if $(\pi^s_{i,j} \setminus \{v^s_0, v^s_{m_s}\}) \cap (\pi^t_{i,j} \setminus \{v^t_0, v^t_{m_t}\}) = \emptyset$.*

The notion of the local WVC measure for any pair of distinct nodes $i, j \in \hat{V}$ in the expected communication graph $\hat{G} = (\hat{V}, \hat{E})$ is now introduced. This measure, denoted by $\hat{\kappa}_{i,j}(\hat{G})$, is defined as the maximum of the summation of the multiplicative weights of the vertex-disjoint paths from node i to node j in $\hat{G}$. In other words, $\hat{\kappa}_{i,j}(\hat{G})$ represents the maximum of the summation of the operational probability of vertex-disjoint paths connecting node i to node j in $\hat{G}$. Consider $\mathscr{P}(\Pi_{i,j})$ as the power set of $\Pi_{i,j}$, and let $\hat{\mathscr{P}}(\Pi_{i,j}) \subseteq \mathscr{P}(\Pi_{i,j})$ contain all nonempty subsets of $\Pi_{i,j}$ composed of a set of mutually vertex-disjoint paths from i to j in $\hat{G}$. Then:

$$\hat{\kappa}_{i,j}(\hat{G}) = \begin{cases} \sum_{k=1}^{|\hat{\Pi}_{i,j}|} W(\hat{\pi}_{i,j}^{k}), & \text{if } (i,j) \notin \hat{E}, \\ \max\left((|\hat{V}|-1)p_{ji}, p_{ji} + \sum_{k=1}^{|\hat{\Pi}_{i,j}|} W(\hat{\pi}_{i,j}^{k})\right), & \text{if } (i,j) \in \hat{E}, \end{cases} \tag{7}$$

where

$$\hat{\Pi}_{i,j} = \underset{\Pi \in \hat{\mathscr{P}}(\Pi_{i,j})}{\operatorname{argmax}} \sum_{k=1}^{|\Pi|} W(\pi^{k}), \tag{8}$$

and $\Pi = \{\pi^k \mid k \in \mathbb{N}_{|\Pi|}\}$ denotes a path set (composed of $|\Pi|$ elements). The WVC metric of $\hat{G}$, denoted by $\hat{\kappa}(\hat{G})$, is defined below as the global connectivity measure of the expected communication graph $\hat{G}$:

$$\hat{\kappa}(\hat{G}) = \min_{i,j \in \hat{V}, i \neq j} \hat{\kappa}_{i,j}(\hat{G}). \tag{9}$$

Remark 1 *The proposed algorithm to find the WVC measure of a strongly connected weighted digraph is composed of two steps. As the first step, a modified iterative deepening depth-first search (IDDFS) procedure is used to find the set of all directed paths between any pair of distinct nodes in a digraph. In the second step, the problem of finding the local WVC metric is formulated as a maximum weight clique (MWC) problem, where the existing algorithms can be used to solve it [24].*

4 Approximate Weighted Vertex Connectivity Measure

The number of all possible paths between any pair of distinct nodes increases exponentially with the network size. On the other hand, the MWC problem is NP-hard. Thus, it is desired to find an approximation of the WVC metric which can be obtained using a polynomial-time algorithm. To this end, the most reliable path in $\hat{G}$ is defined next.

Definition 5 *Given an expected communication graph $\hat{G}$ with probability matrix P, let $\Pi_{i,j}$ denote the set of all directed paths from node i to node j with length greater than one. The* most reliable path *directed from i to j in* $\hat{G}$*, denoted by* $\pi^r_{i,j}$*, is defined as a path in* $\Pi_{i,j}$ *with the largest multiplicative weight. The path* $\pi^r_{i,j}$ *is, in fact, a path from i to j with the highest probability of existence whose length is greater than one.*

The notion of local *approximate weighted vertex connectivity* (AWVC) measure for any pair of distinct nodes $i, j \in \hat{V}$, denoted by $\overline{\kappa}_{i,j}(\hat{G})$, is defined as follows:

$$\overline{\kappa}_{i,j}(\hat{G}) = \begin{cases} \sum_{k=1}^{l_{ij}} W(\pi^{r,k}_{i,j}), & \text{if } (i,j) \notin \hat{E}, \\ \max\left((|\hat{V}|-1)p_{ji}, p_{ji} + \sum_{k=1}^{l_{ij}} W(\pi^{r,k}_{i,j})\right), & \text{if } (i,j) \in \hat{E}, \end{cases} \tag{10}$$

where $W(\pi^{r,k}_{i,j})$ represents the multiplicative weight of the k-th most reliable path from node i to node j after removing the internal nodes and their corresponding adjacent edges of $k-1$ previously found most reliable paths $\pi^{r,l}_{i,j}$, $l \in \mathbb{N}_{k-1}$, from $\hat{G}$. Moreover, l_{ij} is the maximum number of the vertex-disjoint most reliable paths directed from i to j with length greater than one such that after deletion of their internal nodes along with the corresponding adjacent edges no path will remain from i to j.

Then $\overline{\kappa}(\hat{G})$ is defined as the global AWVC metric of digraph $\hat{G}$, which is related to the previous local AWVC measure as follows:

$$\overline{\kappa}(\hat{G}) = \min_{i,j \in \hat{V}, i \neq j} \overline{\kappa}_{i,j}(\hat{G}). \tag{11}$$

The result of the next proposition will be used to obtain the set of most reliable paths from node i to node j with maximum multiplicative weight, denoted by $\pi^{r,k}_{i,j}$ for any $k \in \mathbb{N}_{l_{ij}}$, using the standard shortest path algorithms.

Proposition 1 *Consider a pair of distinct nodes* $u, v \in \hat{V}$ *in the expected communication graph* $\hat{G}$*. The problem of finding the most reliable path from u to v in* $\hat{G}$ *with the weight matrix* $P = [p_{ij}]$ *is equivalent to finding the shortest path connecting u to v in* $\hat{G}$ *with the modified weight matrix* $\overline{P} = [\overline{p}_{ij}]$*, where* $\overline{p}_{ij} = -\ln(p_{ij})$ *for all* $i, j \in \hat{V}$*,* $i \neq j$*.*

In order to develop a polynomial-time algorithm to obtain the global AWVC measure $\overline{\kappa}(\hat{G})$ as a lower bound on the computationally-expensive WVC metric $\hat{\kappa}(\hat{G})$, Dijkstra's algorithm is used for any pair of distinct nodes $i, j \in \hat{V}$ in a number of steps by considering the modified weight matrix $\overline{P}$. Let l_{ij} represent the number of steps which should be taken to find the local measure $\overline{\kappa}_{i,j}(\hat{G})$. In the k-th step ($k \in \mathbb{N}_{l_{ij}}$), the most reliable path $\pi^{r,k}_{i,j}$ with length greater than one is identified and its multiplicative weight $W(\pi^{r,k}_{i,j})$ (obtained based on the original weight matrix P) is added to $\overline{N}_{i,j}(\hat{G})$, which is initially set to zero.

Then, all internal nodes of the k-th most reliable path $\pi_{i,j}^{r,k}$ along with the edges adjacent to them are removed from $\hat{G}_{ij}$, and the next step starts by applying a new Dijkstra's algorithm to the modified graph $\hat{G}_{ij}$. In the last step (i.e., when $k = l_{ij}$), only one directed path exists from i to j in the modified $\hat{G}_{ij}$. Then the local AWVC degree $\overline{\kappa}_{i,j}(\hat{G})$ is determined according to equation (10), and by comparing the computed local AWVC degrees for all pairs of distinct nodes in $\hat{G}$, the minimum value denoted by $\overline{\kappa}(\hat{G})$ is found.

5 A Computational Example of the WVC and AWVC Metrics

An illustrative example is given here to demonstrate the required steps for finding the proposed WVC and AWVC measures for a random network composed of six nodes. Let Fig. 1 depict the expected graph $\hat{G}$ of the network. Note that the existence probability of each link appears as a weight on its corresponding edge in $\hat{G}$, which yields the following probability matrix P:

$$P = \begin{bmatrix} 0 & 0 & 0.7 & 0 & 0 & 0.7 \\ 0.7 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0.95 \\ 0.9 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & 0.6 & 0 \end{bmatrix}. \tag{12}$$

Since at least two vertex-disjoint paths exist between any pair of distinct nonadjacent nodes

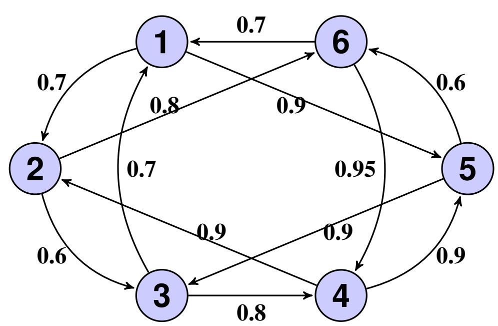

Figure 1: *The expected graph $\hat{G}$ of the considered example.*

in $\hat{G}$, it then follows that $\kappa(\hat{G}) = 2$. For this example, the minimum local WVC measure corresponds to the directed paths from node 5 to node 2, i.e., $\hat{\kappa}(\hat{G}) = \hat{\kappa}_{5,2}(\hat{G})$. In order to compute $\hat{\kappa}_{5,2}(\hat{G})$, it is first required to find the set $\Pi_{5,2}$ containing all distinct paths from node 5 to node 2 with length greater than one in $\hat{G}$. This results in four different paths $\pi_{5,2}^{k}$, $k \in \mathbb{N}_4$, as follows:

$$\begin{aligned} \pi_{5,2}^{1} &= \{5,3,1,2\}, \quad \pi_{5,2}^{2} = \{5,3,4,2\}, \\ \pi_{5,2}^{3} &= \{5,6,1,2\}, \quad \pi_{5,2}^{4} = \{5,6,4,2\}. \end{aligned} \tag{13}$$

Moreover, the path set $\hat{\mathscr{P}}(\Pi_{5,2})$ is given by:

$$\hat{\mathscr{P}}(\Pi_{5,2}) = \{\pi_{5,2}^1, \pi_{5,2}^2, \pi_{5,2}^3, \pi_{5,2}^4, \{\pi_{5,2}^1, \pi_{5,2}^4\}, \{\pi_{5,2}^2, \pi_{5,2}^3\}\}, \tag{14}$$

every element of which is composed of a set of vertex-disjoint paths belonging to $\Pi_{5,2}$. By solving the combinatorial optimization problem (8), one arrives at $\hat{\Pi}_{5,2} = \{\pi_{5,2}^1, \pi_{5,2}^4\}$, which results in $\hat{\kappa}_{5,2}(\hat{G}) = W(\pi_{5,2}^1) + W(\pi_{5,2}^4) = 0.954$ or $\hat{\kappa}(\hat{G}) = 0.954$.

In order to find the AWVC metric, note that for this example $\overline{\kappa}(\hat{G}) = \overline{\kappa}_{5,2}(\hat{G})$, i.e., the minimum local AWVC measure is given by $\overline{\kappa}_{5,2}(\hat{G})$. By applying Dijsktra's algorithm to $\hat{G}$ with the modified weight matrix $\overline{P}$, the first most reliable path from node 5 to node 2 is obtained as $\pi_{5,2}^{r,1} = \pi_{5,2}^2$. After removing the internal nodes of $\pi_{5,2}^{r,1}$ from $\hat{G}$ along with the edges adjacent to them and applying Dijkstra's algorithm for the second time, one obtains $\pi_{5,2}^{r,2} = \pi_{5,2}^3$. Since no path exists from 5 to 2 in $\hat{G}$ after removing the internal nodes of $\pi_{5,2}^{r,2}$, it can be concluded that $l_{52} = 2$ and $\overline{\kappa}_{5,2}(\hat{G}) = W(\pi_{5,2}^2) + W(\pi_{5,2}^3) = 0.942$, or $\overline{\kappa}(\hat{G}) = 0.942$. This example also confirms that $\overline{\kappa}(\hat{G}) \leq \hat{\kappa}(\hat{G})$.

6 Simulation Results

Consider a network of six underwater acoustic sensors which broadcast their data periodically as described in [10], [25]. Assume that the existence probability of the communication links of the network is given by a time-varying probability matrix $P(k)$ as below:

$$P(k) = \begin{bmatrix} 0 & 0 & 0.7 & 0 & 0 & p_{16}(k) \\ p_{21}(k) & 0 & 0 & 0.9 & 0 & 0 \\ 0 & p_{32}(k) & 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0.95 \\ 0.9 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0.8 & 0 & 0 & p_{65}(k) & 0 \end{bmatrix}, \tag{15}$$

where $k \in \mathbb{N}$ denotes the k-th broadcast cycle and $p_{16}(k) = 0.7 + 0.25\sin(0.01k)$, $p_{21}(k) = 0.7 + 0.2\sin(0.005k)$, $p_{32}(k) = 0.6 + 0.1\sin(0.01k)$, $p_{65}(k) = 0.6 + 0.1\sin(0.005k)$. Let the length of each broadcast cycle be $T = 6$sec, and the length of each broadcast interval be $\Delta = 1$sec. Let also the broadcast order of the i-th sensor be specified by $\delta_i = i - 1$ for $i \in \mathbb{N}_6$, and choose $\alpha = 0.025$ as the learning rate. The entries of the probability matrix $P(k)$ along with the topology of the expected communication graph $\hat{G}$ are estimated in a distributed manner by each sensor based on the network estimation algorithms proposed in [10], [25].

In Figs. 2 and 3, the proposed WVC and AWVC measures of the expected communication graph $\hat{G}$ versus the number of broadcast cycles are shown from the viewpoint of sensors 1 and 3, respectively.

The topology of $\hat{G}$ along with the time-varying probability matrix $P(k)$ is first estimated by each sensor in a distributed fashion and then the global WVC and AWVC metrics are obtained for the estimated probability matrix $P(k)$ in Figs. 2 and 3 from the viewpoint of sensors 1 and 3.

The *bottleneck pair* (i^*, j^*) is defined as an ordered pair of distinct nodes $i^*, j^* \in \hat{V}$ which has the smallest local connectivity degree $\hat{\kappa}_{i^*,j^*}(\hat{G})$ (or $\overline{\kappa}_{i^*,j^*}(\hat{G})$) among all pairs of distinct nodes in the expected communication graph $\hat{G}$ of a random network. The index of the bottleneck pair (i^*, j^*) is defined as an integer $\mathscr{I}(i^*, j^*) \in \mathbb{N}_{|\hat{V}|^2-|\hat{V}|}$ such that:

$$\mathscr{I}(i^*, j^*) = \begin{cases} (|\hat{V}|-1)(i^*-1)+j^*, & \text{if } j^* < i^*, \\ (|\hat{V}|-1)(i^*-1)+j^*-1, & \text{if } j^* \geq i^*. \end{cases} \tag{16}$$

The index of the bottleneck pair for the WVC and AWVC measures versus the number of broadcast cycles is depicted in Figs. 4 and 5 from the viewpoint of sensors 1 and 3, respectively. It can be implied from Figs. 4 and 5 that $(3,6)$ with index $\mathscr{I}(3,6) = 15$ is the bottleneck pair of the network most of the time from the beginning of the simulation up to $t = 1800$sec (first 300 cycles). Moreover, the most dominant bottleneck pair between $t = 1800$sec and $t = 3780$sec is $(2,1)$ with index $\mathscr{I}(2,1) = 6$, while $(5,2)$ with index $\mathscr{I}(5,2) = 22$ is the most often bottleneck pair of the random UASN for $t > 3780$sec.

7 Experimental Results

The experimental results were obtained during a sea trail conducted in Bedford Basin (N.S., Canada) in July and August of 2014 over a 20-day period. Four nodes were deployed in the Bedford Basin at various locations shown in Fig. 6, where the coordinates of the node locations along with the sea-floor depth at the mooring location of the nodes is given in Table 1. Moreover, the approximate distance separating each node from the others is shown in Table 2 which ranges from 640m to 1.64km.

Table 1: *Node locations and the basin depth at mooring locations.*

	[Latitude, Longitude]	Depth (m)	Location
Node 1	[44.6905 -63.6489]	50	Southwest
Node 2	[44.6999 -63.6550]	52	Northwest
Node 3	[44.7020 -63.6474]	52	Northeast
Node 4	[44.6983 -63.6343]	57	Southeast

A view of a node employed in this experiment is demonstrated in Fig. 7. Each node was equipped with a Global Positioning System, a battery pack, an acoustic modem, two acoustic releases, a mooring, ropes and cables. It also had an acoustic modem set at a depth of

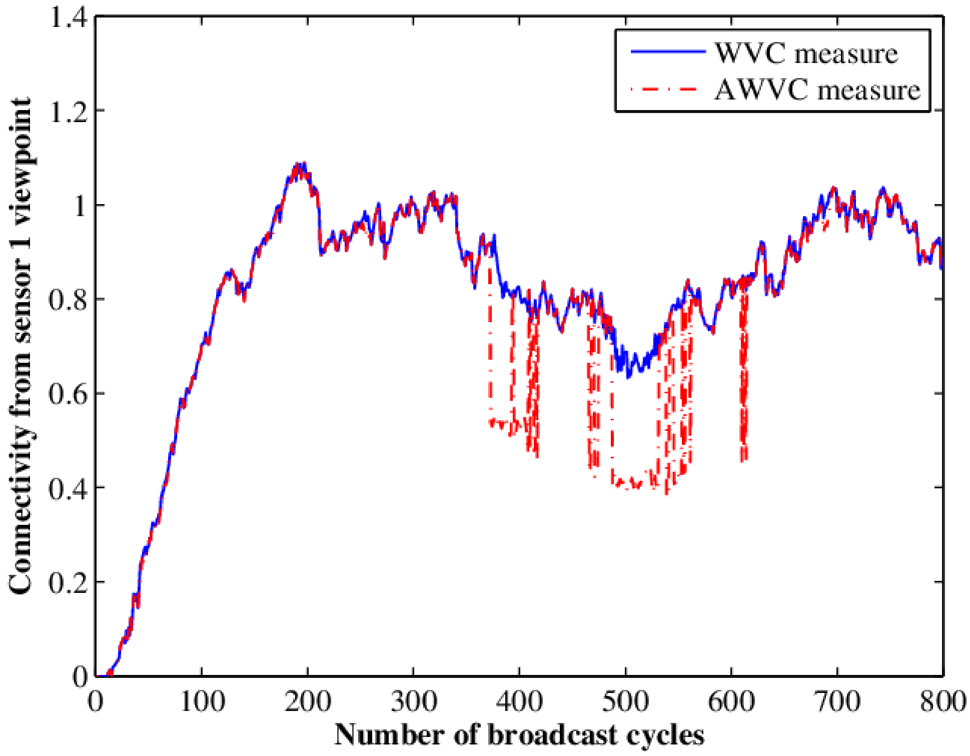

Figure 2: *WVC measure $\hat{\kappa}(\hat{G})$ and AWVC measure $\overline{\kappa}(\hat{G})$ from the viewpoint of sensor 1.*

Table 2: *Node-to-node direct separation distances.*

Node 1 to 2	Node 1 to 3	Node 1 to 4
1.15km	1.29km	1.45km
Node 2 to 3	Node 2 to 4	Node 3 to 4
0.64km	1.64km	1.11km

5m below surface to ensure they would reside in the mixed layer, where acoustic communication is usually difficult to establish. During the trial, various parameters of the acoustic modems including power level, bit rate, and message length were altered to investigate their effects on the operational probability of the underwater acoustic communication channels.

Four different power levels offered by the commercial acoustic modems are labeled as P_1, P_2, P_3 and P_4, respectively, where $P_1 < P_2 < P_3 < P_4$. Three options considered for the bit rate of the modems were b_1, b_2 and b_3, where $b_1 < b_2 < b_3$. Furthermore, two different message lengths investigated in the experiments were l_1 and l_2, where $l_1 < l_2$.

All experiments were operated remotely and automatically from DRDC's main building through an antenna and a repeater installed on the platform located at DRDC's calibration

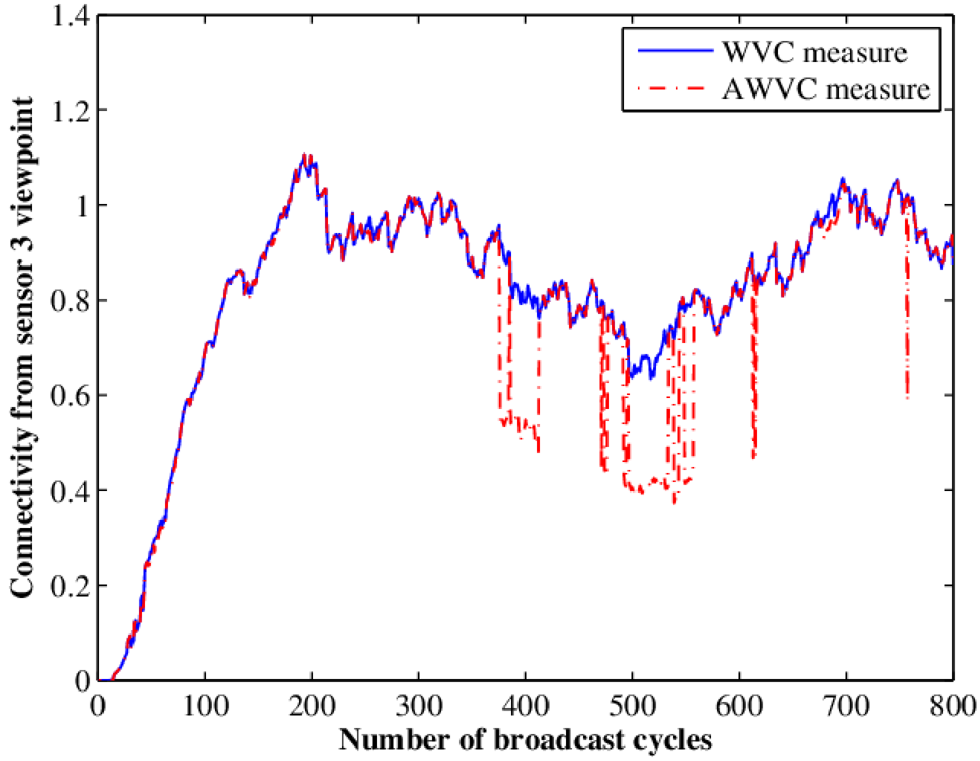

Figure 3: *WVC measure $\hat{\kappa}(\hat{G})$ and AWVC measure $\overline{\kappa}(\hat{G})$ from the viewpoint of sensor 3.*

barge position. The probing signals were modulated messages communicated frequently. In-house control and routing procedures were changing the modem settings and querying the other modems to retrieve received messages and the modem internal data.

The existence probability of all underwater acoustic communication channels for a particular combination of the power level, bit rate and message length for all nodes was then obtained resulting in the expected communication graph $\hat{G}$ depicted in Fig. 8. For the considered configuration of the underwater nodes, the probability matrix P was obtained as:

$$P = \begin{bmatrix} 0 & 0.36 & 0.35 & 0.47 \\ 0.26 & 0 & 0.58 & 0.11 \\ 0.31 & 0.85 & 0 & 0.31 \\ 0.08 & 0 & 0.09 & 0 \end{bmatrix}. \quad (17)$$

Based on the proposed WVC measure, $\hat{\kappa}(\hat{G}) = 0.105$ for this scenario, where $(2,4)$ represents the bottleneck pair in the network. Using the AWVC metric, on the other hand, it can be shown that $\overline{\kappa}(\hat{G})$ is also equal to 0.105, and $(2,4)$ is, again, the bottleneck pair, which means that these two connectivity measures are the same in this case. This shows that the network is not well-connected, and more experiments with different configurations are planned in order to determine a configuration with a sufficiently high connectivity.

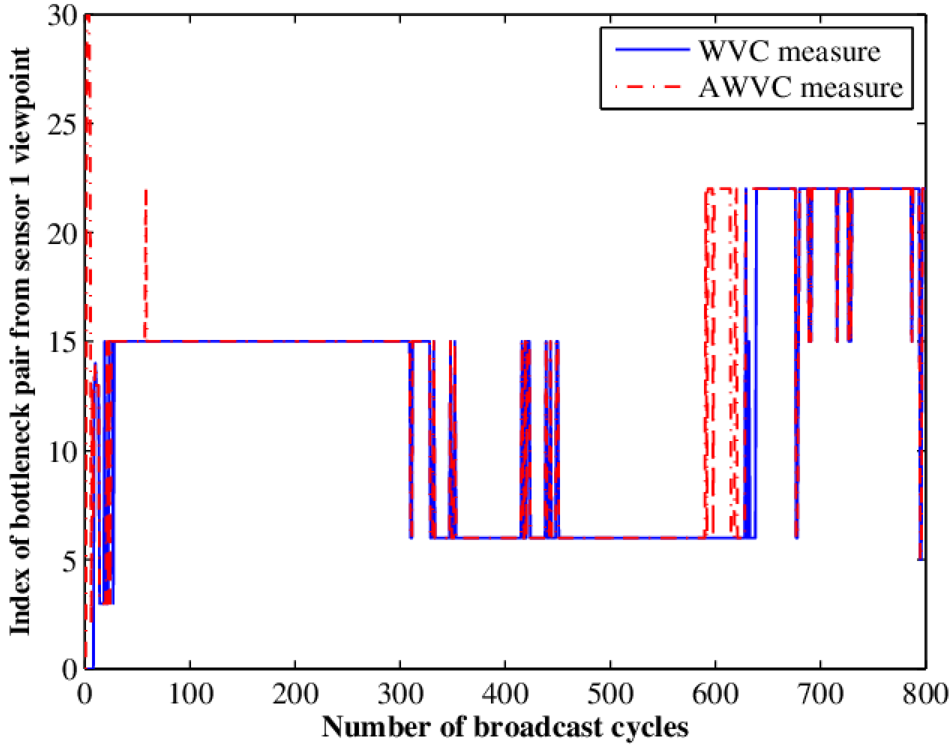

Figure 4: *Index of the bottleneck pair with the WVC and AWVC measures from the viewpoint of sensor 1.*

8 Conclusions

Connectivity assessment of an underwater sensor network using distributed algorithms is investigated in this report. The weighted vertex connectivity is defined as a novel measure to evaluate the connectivity of a weighted digraph representing the expected communication graph of the sensor network. The elements of the weight matrix denote the operational probability of their corresponding communication links in the network. This measure is, in fact, an extension of the vertex connectivity metric introduced in the literature. It describes the combined effects of the path reliability and network robustness to node failure on connectivity of the expected communication graph, reflecting the performance of the cooperative algorithms in random sensor network. An approximation of the proposed connectivity metric is subsequently presented which provides a lower bound on the weighted vertex connectivity and can be computed by applying a series of polynomial-time shortest path algorithms. The efficacy of the proposed measures and the corresponding algorithms is confirmed by simulation and experimental results.

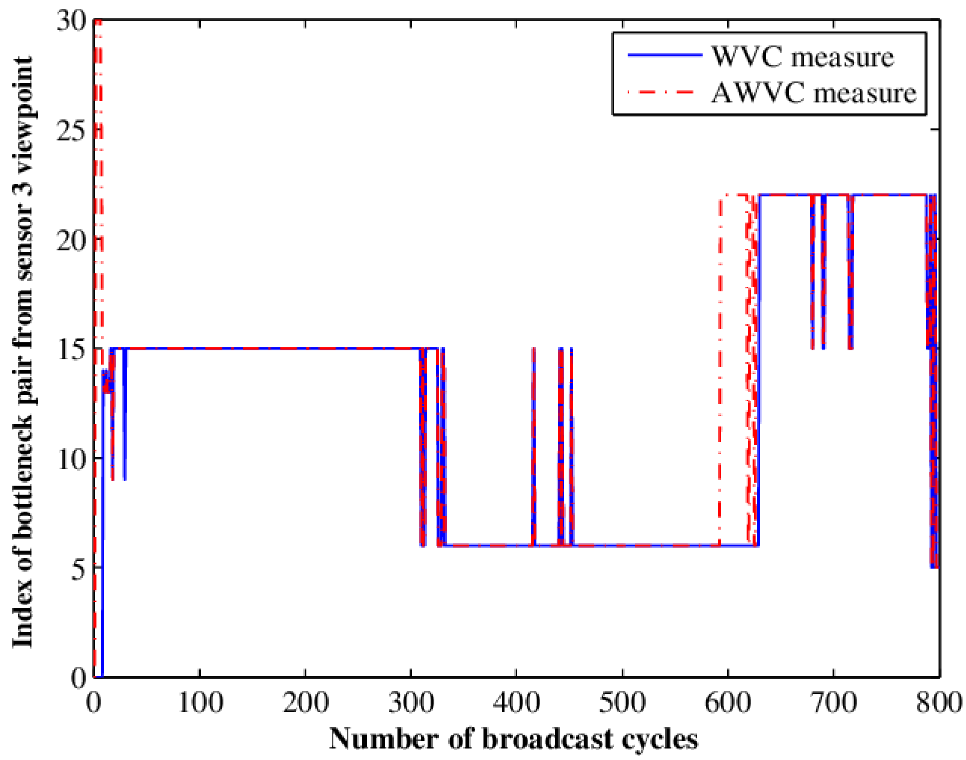

Figure 5: *Index of the bottleneck pair with the WVC and AWVC measures from the viewpoint of sensor 3.*

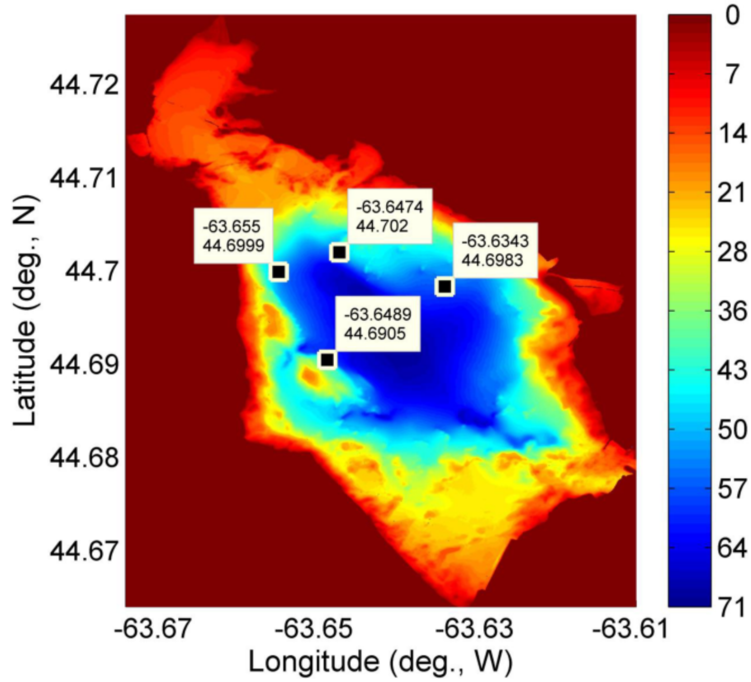

Figure 6: *Location of deployed nodes in Bedford Basin.*

Figure 7: *View of a node used in the experiment.*

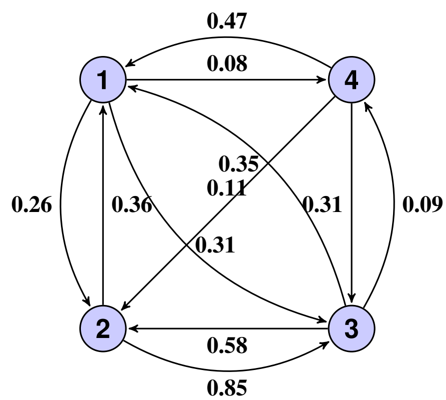

Figure 8: *The expected graph $\hat{G}$ for the considered experiment.*

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