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Passive Geolocation Using TDOA Method from UAVs and Ship/Land-Based Platforms for Maritime and Littoral Area Surveillance

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Defence R&D Canada – Ottawa
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Abstract

Passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. Several location methods have been investigated in the past. This report presents passive geolocation of radar emitter using the time difference of arrival (TDOA) location method from Unmanned Aerial Vehicles (UAVs) working in conjunction with ship/land-based platforms to fulfill maritime and littoral area surveillance. Instead of using multiple stationary sensors, multiple TDOA measurements are obtained over time intervals from UAV-based sensors along UAV flight trajectories. A simple least-squares (LS) solution is derived based on TDOA measurements and known sensor locations to solve three-dimensional (3-D) location estimation. The proposed location method takes into account of both TDOA measurement noise and bias. Therefore, the location estimation can be achieved more accurately when biased measurements are present. The advantages and effectiveness of the proposed method are demonstrated using computer simulation.

Résumé

Le problème de la géolocalisation des émetteurs radar a fait l’objet de nombreuses études au cours des dernières décennies. Plusieurs méthodes de localisation ont été étudiées dans le passé. Cet article présente une technique de géolocalisation passive des émetteurs radar, fondée sur la méthode de la différence entre les temps d’arrivée (TDOA) obtenus avec des véhicules aériens sans pilote (UAV) travaillant en collaboration avec des plates-formes terrestres ou embarquées sur un navire pour exercer une surveillance des zones maritimes et du littoral. Plutôt que de faire appel à plusieurs capteurs stationnaires, on calcule, à des intervalles de temps déterminés, plusieurs mesures de la différence entre les temps d’arrivée (TDOA) obtenues à partir de capteurs embarqués sur un UAV et prises le long de la trajectoire de vol de ce dernier. Par une application simple de la méthode des moindres carrés, on détermine une solution au problème de la localisation d’un objet dans l’espace tridimensionnel (3-D), à partir des mesures de différence entre les temps d’arrivée, et des positions connues du capteur. La méthode de géolocalisation proposée prend en compte le bruit et l’erreur systématique sur la mesure de la différence entre les temps d’arrivée. Par conséquent, on obtient une estimation plus précise de la position avec des mesures comportant une erreur systématique. Une simulation sur ordinateur fait ressortir les avantages et l’efficacité de la méthode proposée.

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Executive summary

Passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. This report addresses a proposed three-dimensional (3-D) geolocation of radar emitters from multiplatform-based sensors to fulfill electronic support measures (ESM) roles for maritime and littoral surveillance.

Several location techniques such as time difference of arrival (TDOA), frequency difference of arrival (FDOA), angle of arrival (AOA), etc., have been investigated in the past. The TDOA method, one of the precise location techniques, is selected in this report for the derivation of multiplatform geolocation scheme for radar emitter localization. The TDOA method is based on measuring the difference of time arrivals of emitted signals/pulses at a pair of sensors. The TDOA measured at each pair of sensors defines a hyperbolic curve with the sensors as foci. Three TDOA measurements from three pairs of sensors define three hyperbolic curves. If the TDOA measurements were assumed to be accurate, the three hyperbolic curves would intersect at the actual emitter location for 3-D localization. However, in most of cases, the TDOA measurements are noisy and biased.

To deal with noisy measurements, the proposed scheme takes into account of both TDOA measurement noise and bias by modeling the noise and bias terms in the TDOA model. To reduce the number of sensors required by the TDOA location method, this report presents the use of UAVs as moving sensor platforms in conjunction with ship/land-based platforms to obtain multiple TDOA measurements. In addition, the use of UAVs as sensor platforms can increase ESM systems capability and detection range/coverage for better geolocation of radar emitters.

The theoretical performance of the proposed location scheme is obtained using computer simulation with additive noise and bias. It is confirmed that more accurate location estimates are achieved because the proposed location scheme allows estimation and subsequent removal of measurement bias. However, this study did not take into consideration of some of the real-life problems caused by multipath, time of arrival ambiguity between sensors, asynchronous illumination of the sensors by an emitter and UAV trajectory selection.

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Sommaire

La localisation géographique passive des émetteurs radar demeure un grave problème pour le soutien électronique (SE) de la surveillance maritime et littorale. Le présent rapport porte sur une technique proposée de localisation géographique en trois dimensions (3D) des émetteurs radar à partir de capteurs situés sur plusieurs plates-formes aux fins des mesures de soutien électronique (MSE) de la surveillance maritime et littorale.

Plusieurs techniques de localisation, comme la différence de temps d'arrivée (TDOA), la différence de fréquences d'arrivée (FDOA) et l'angle d'arrivée (AOA), ont fait l'objet d'un examen par le passé. La technique TDOA, l'une des plus précises, est sélectionnée dans le présent rapport pour la dérivation d'un plan de localisation géographique à partir de plusieurs plates-formes en vue de la localisation d'émetteurs radar. Elle se fonde sur la mesure de la différence du temps d'arrivée à une paire de capteurs d'impulsions/de signaux émis. La TDOA mesurée à chaque paire de capteurs définit une courbe hyperbolique, les capteurs étant les points de convergence. Trois mesures de la TDOA prises à trois paires de capteurs définissent trois courbes hyperboliques. Si les mesures de la TDOA étaient présumées être précises, il y aurait intersection entre les trois courbes hyperboliques à l'emplacement réel des émetteurs en vue de la localisation 3D. Toutefois, dans la plupart des cas, les mesures de la TDOA sont bruitées et biaisées.

Comme solution, le plan proposé tient compte à la fois du bruit et du biais des mesures de la TDOA par leur intégration au modèle de la TDOA. Afin de réduire le nombre de capteurs requis par la technique de localisation de la TDOA, le présent rapport présente l'utilisation de véhicules aériens télépilotés (VAT) comme plates-formes mobiles à capteurs utilisées de façon combinée avec des plates-formes embarquées/au sol pour l'obtention de plusieurs mesures de la TDOA. En outre, l'utilisation de VAT comme plates-formes à capteurs permet d'accroître la mobilité des systèmes MSE et la couverture/portée de détection et, du même coup, la localisation géographique des émetteurs radar.

La performance théorique du plan proposé de localisation est obtenue au moyen d'une simulation informatique qui prévoit un biais et un bruit additifs. Il est confirmé que des estimations plus précises de l'emplacement sont obtenues parce que le plan proposé de localisation permet une estimation et le retrait subséquent du biais des mesures. Toutefois, la présente étude n'a pas tenu compte de certains des problèmes réels dus à l'ambiguïté causée par la propagation par trajets multiples ou les écarts de temps d'arrivée aux capteurs, à l'illumination asynchrone des capteurs par un émetteur ou à la sélection de la trajectoire des VAT.

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1. Introduction

Unmanned Aerial Vehicles (UAVs) today are gaining wide acceptance as valuable reconnaissance, surveillance, targeting and intelligence gathering tools to support various missions [1]. The UAVs can also be used in combat situations such as strike, military defence, etc. However, one common use for a UAV today is to find a target emitter and determine its location. This report presents the use of UAVs as sensor platforms for passive geolocation of radar emitters to fulfill electronic support measures (ESM) roles for electronic warfare (EW) applications.

Passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. Several location methods such as time difference of arrival (TDOA) [1], frequency difference of arrival (FDOA) [1,3], angle of arrival (AOA) [4], etc. have been investigated in the past. Two precise location techniques are typically used for emitter localization: TDOA and FDOA [5]. In this report, the TDOA location method is presented for the derivation of three-dimensional (3-D) geolocation of radar emitters. An extensive work based on the TDOA technique has been done in the past to solve location problems. Fang [6] derived an exact close-form location solution when the number of TDOA measurements is equal to the number of unknowns, i.e., the position coordinates of an emitter. This solution, however, cannot make use of extra measurements when they become available to improve position accuracy. Schau and Robinson [7] derived a location solution for the more general situation with extra TDOA measurements from extra sensors. Although a closed-form solution has been developed, the estimation may not be accurate when noisy (random and/or non-random) measurements present. Smith and Abel [8] later extended the method in [7] into a linear Least-Squares solution using a sensor array to deal with random noise but not to handle non-random) measurement errors. Manolakis [9] presented an explicit solution to a 3-D position problem based on three range measurements from three stationary stations. However, the method did not take range measurement uncertainty into account in the location estimation. Chan and Ho [10] proposed an explicit approach using TDOAs measured at multiple sensor locations. This method is an approximate realization of the maximum-likelihood estimator for the case when the TDOA estimation errors are small and assumed to be independent zero-mean stationary Gaussian random noise. To handle non-random errors, Poirot and McWilliams [11] derived a location solution based on bearing measurements, which allows estimate and removal of non-random measurement errors.

This report presents geolocation of radar emitters using the TDOA method from UAV-based sensors working in conjunction with ship/land-based sensors for maritime and littoral area surveillance. In the past, many mathematical models based on TDOA measurements have been developed for geolocation of radar emitters. However, most of them are based on an assumption that TDOA measurement noises are random with zero-mean. In this report, a mathematical model including both random measurement noise and bias is presented in order to handle both random and non-random noise properly. Instead of using multiple stationary sensors, multiple TDOA measurements are obtained over time intervals from UAV-based sensors along UAV flight trajectories. A simple least-squares (LS) solution is derived from TDOA measurements in combination with known sensor locations to solve three-dimensional (3-D) location estimation. The proposed location method takes account of both TDOA measurement noise and bias, and it allows estimation and removal of measurement errors that are not random. Therefore, the location estimation can be achieved more accurately when biased measurements are present. Simulation results are included to present the effectiveness of the proposed method.

2. The TDOA Method

The TDOA based location technique, in general, requires three or more spatially separated sensors (long baseline) with coordinated measurements. Fig. 1 illustrates the geometry of TDOA location technique using multiple stationary sensors for localization of a target emitter. Each sensor measures the time interval or time of arrival (TOA) for signals propagating from the emitter to a sensor. The TOA is dependent on the emitter-sensor geometry and medium characteristics. In the case of a constant velocity medium as assumed in the following study, the TOA is function of the emitter-sensor ranges [5].

The signal travels at the speed of light. Therefore, if we know when the signal left the transmitter and arrived at the receiver, we will know the distance or path length between the transmitter and the receiver. However, under the situations of targeting and intelligence gathering for the purpose of electronic warfare, we have no way of knowing when the signal left the transmitter. However, we can measure when the signal is arrived. By determining the differences between the arrival times or the TDOAs at each pair of sensors, we will be able to estimate the location of the emitter's position.

Let t_m be the TOA for the signal radiating from the emitter to arrive at sensor m, and d_m be the distance from the emitter to sensor m. The TDOA between sensors m and n is then defined by $\Delta t_{m,n}$ as follows

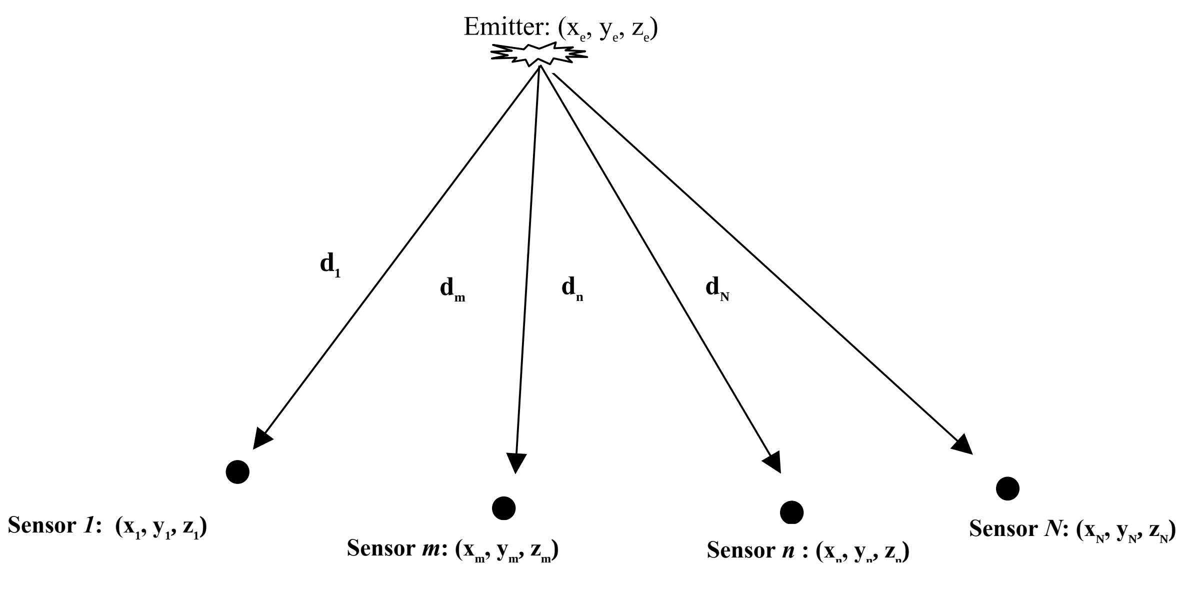

Figure 1. *Location based on TDOA method using multiple stationary sensors*

$$\Delta t_{m,n} = t_m - t_n = \frac{1}{c}(d_m - d_n),$$
$$m, n = 1,2,\ldots, N(m \neq n), \quad (1)$$

where c is the speed of light, N is the number of sensors, and d_m is the distance defined by

$$d_m = \sqrt{(x_e - x_m)^2 + (y_e - y_m)^2 + (z_e - x_m)^2}, \quad (2)$$

where (x_e, y_e, z_e) and (x_m, y_m, z_m) are the position coordinates of the emitter and sensor m, respectively. Eq.1 can be expressed in terms of range difference as

$$c\Delta t_{m,n} = \sqrt{(x_e - x_m)^2 + (y_e - y_m)^2 + (z_e - z_m)^2}$$
$$- \sqrt{(x_e - x_n)^2 + (y_e - y_n)^2 + (z_e - z_n)^2}. \quad (3)$$

In order to solve three unknowns (i.e., x_e, y_e and z_e), a minimum of three TDOA measurements/equations from four stationary sensors is required.

The TDOA location technique is also called hyperbolic location method. For each pair of sensors, the TDOAs define the possible emitter's locations along a hyperbolic curve with the sensors as foci. With three stationary sensors, an intersection of hyperbolic curves corresponds to a possible emitter location for two-dimensional (2-D) position-localization, while with four or more than four stationary sensors, hyperbolic curves intersect at an emitter location for 3-D position-localization.

3. Geolocation Using TDOA Method from UAVs and Ship/Land-Based Sensors

As stated above, one common use for UAVs is to find a target emitter and determine its location. A UAV can be operated alone or operated in conjunction with existing systems/platforms (such as ship, land, manned aircraft or space based stations) to increase the capability of the existing systems/platforms and detection range/coverage. Figs. 2 and 3 show the possible scenarios in the use of UAVs as sensor platforms working in conjunction with ship/land-based sensors for passive geolocation of radar emitters. Fig. 2 illustrates passive geolocation using TDOA measurements from a UAV sensor and a ship/land-based sensor, while Fig. 3 shows two UAV sensors operating in conjunction with a ship/land-based sensor to locate a target.

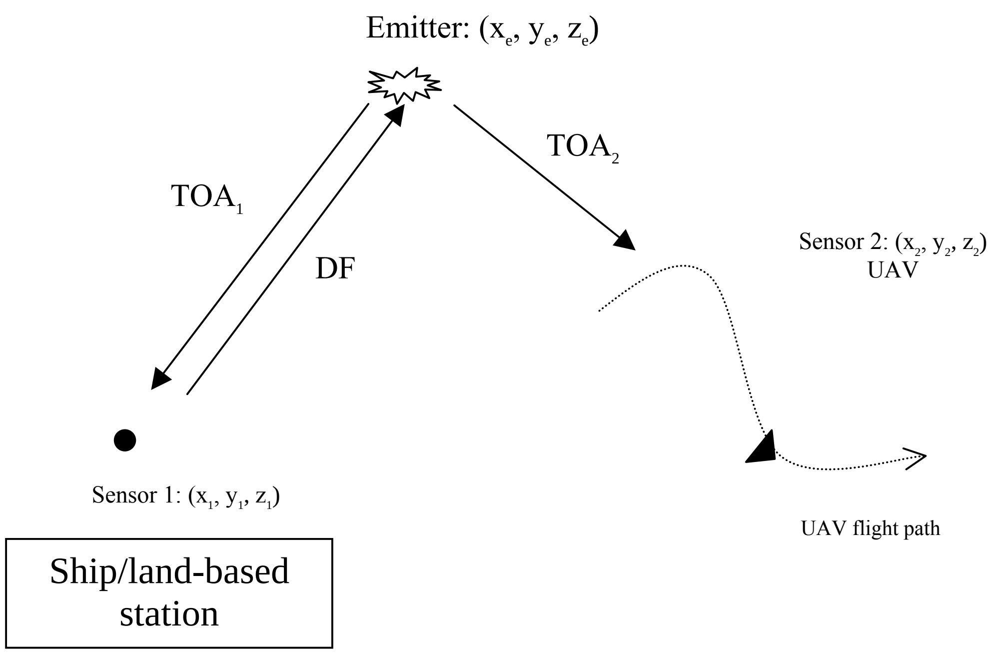

Figure 2. *Location using TDOAs from a UAV and a ship/land-based platform*

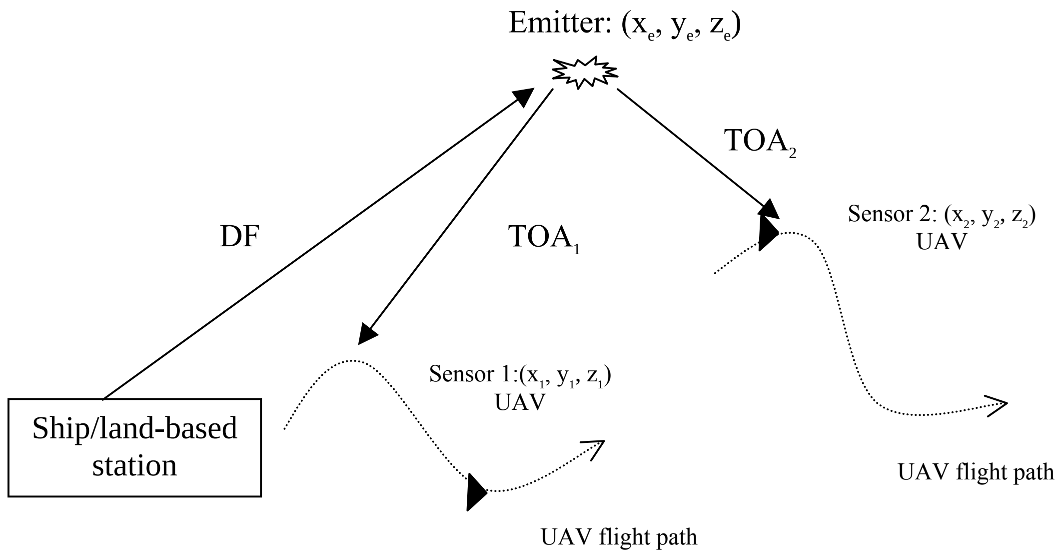

Figure 3. *Location using TDOAs from two UAVs and a ship/land-based platform*

To intercept radiated signals, time-synchronized ESM sensors are utilized to obtain TDOA measurements. TDOA measurements are collected at various sensor locations along UAV pre-determined fight paths. Sensor information (such as emitter information and UAV navigation data) can be processed on-board or transferred to the ship/land-based station for analysis and location determination. A direction finding (DF) system is assumed to be available at the ship/land-based station to provide an initial location of the emitter to allow UAV to get close to the target

3.1 Formulation of the problem

For simplicity, Figs. 2 and 3 are combined as given in Fig. 4 to illustrate the geometry between the emitter and the sensors. The spatial coordinates of sensor *1* (on a fixed/moving platform) and sensor *2* (on a moving platform) at time interval *i* are defined by $(x_{i,1},y_{i,1},z_{i,1})$ and $(x_{i,2},y_{i,2},z_{i,2})$, respectively. Multiple TDOA measurements are collected from various sensor positions over time intervals along the UAV flight paths. Multiplying the TDOAs given in Eq.1 by the speed of light gives us the range difference as follows

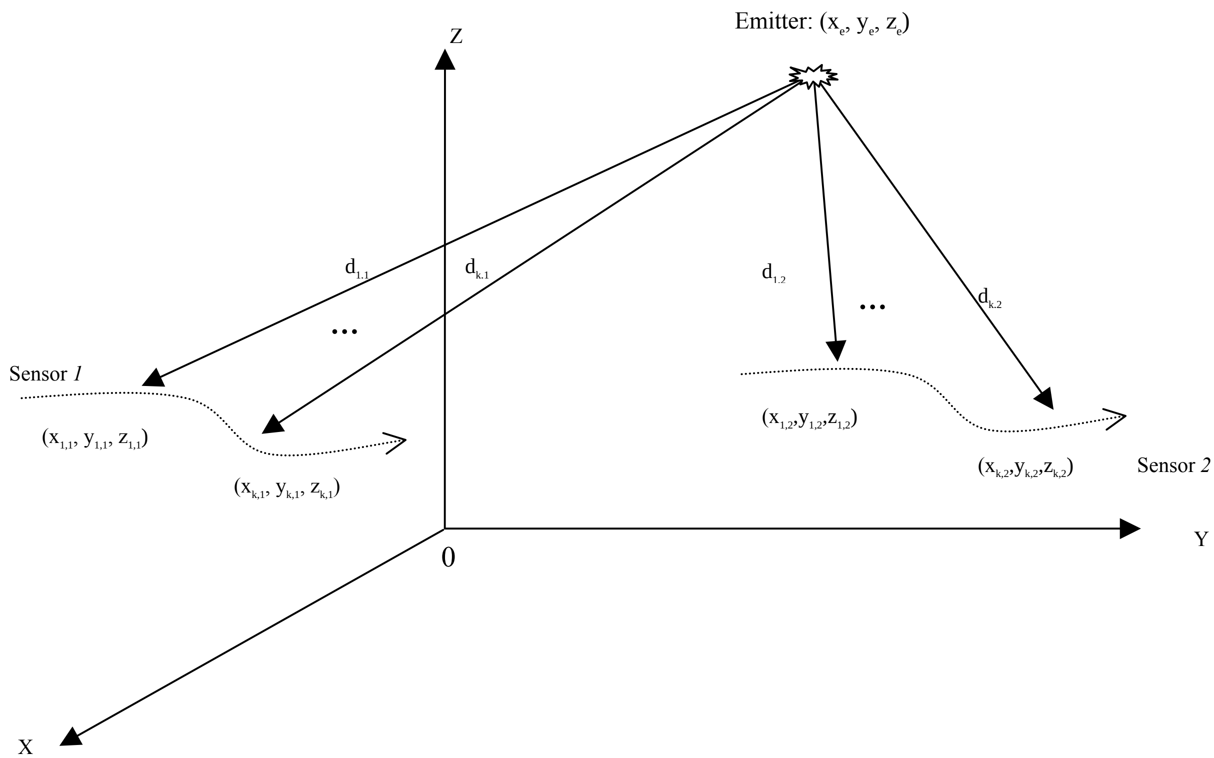

Figure 4. *Geometry between an emitter and two sensors*

$$c\Delta t_i = d_{i,1} - d_{i,2}, \tag{4}$$

where i represents the i^{th} time interval, c the speed of light, Δt_i the time difference of arrival between sensor *1* and sensor *2* at the i^{th} time interval, and $d_{i,1}$ - $d_{i,2}$ the range difference between sensor *1* and sensor *2* at the i^{th} time interval. The above equation can be expanded as

$$
\begin{aligned}
c\Delta t_i = & \sqrt{(x_e - x_{i,1})^2 + (y_e - y_{i,1})^2 + (x_e - z_{i,1})^2} \\
& - \sqrt{(x_e - x_{i,2})^2 + (y_e - y_{i,2})^2 + (x_e - z_{i,2})^2}, \\
& i = 1,2,\ldots k,
\end{aligned}
\tag{5}
$$

where k is the number of measurements along a UAV flight path. To solve for the three unknowns x_e y_e and z_e, a minimum of three TDOA measurements are required, which allows Eq.5 to be expand into three equations. The location solution derived from Eq.5 is based on the geometry between the sensors and the emitter without consideration of TDOA measurement noise and bias. However, with noisy measurements, the emitter location derived from the above geometry will not intersect in a single point. Thus, a statistical model is needed to obtain the desired position of an emitter.

3.2 Statistical model

Considering TDOA measurement noise, Eq.5 can be expressed as the range difference $c\Delta t_i$ plus the additive noise n_i as

$$
\begin{gathered}
y_i = c\Delta t_i + n_i = h_i(x_e, y_e, z_e, x_{i,1}, y_{i,1}, z_{i,1}, x_{i,2}, y_{i,2}, z_{i,2}) + n_i, \\
i = 1,2,\ldots k,
\end{gathered}
\tag{6}
$$

where

$$
\begin{aligned}
h_i(x_e, y_e, z_e, x_{i,1}, y_{i,1}, z_{i,1}, x_{i,2}, y_{i,2}, z_{i,2}) = & \sqrt{(x_e - x_{i,1})^2 + (y_e - y_{i,1})^2 + (z_e - z_{i,1})^2} \\
& - \sqrt{(x_e - x_{i,2})^2 + (y_e - y_{i,2})^2 + (z_e - z_{i,2})^2}.
\end{aligned}
\tag{7}
$$

The position coordinates of the emitter are defined by (x_e, y_e, z_e). The position coordinates of the two sensors at the i^{th} time interval are given by ($x_{i,1}$, $y_{i,1}$, $z_{i,1}$) and ($x_{i,2}$, $y_{i,2}$, $z_{i,2}$), respectively.

3.3 A least-squares approach for passive geolocation

Since the function $h_i(x_e,y_e,z_e,x_{i,1},y_{i,1},z_{i,1},x_{i,2}, y_{i,2},z_{i,2})$ in Eq.7 is a nonlinear function of positions of the emitter and the sensors, the function is linearized by a Taylor series expansion about an initial estimate of emitter location (x_{e0},y_{e0},z_{e0}). If retaining only the first-order terms, Eq.6 at the i^{th} time interval can be represented by

$$y_i - h_i(x_{e0}, y_{e0}, z_{e0}, x_{i,1}, y_{i,1}, z_{i,1}, x_{i,2}, y_{i,2}, z_{i,2}) = \frac{\partial h_i}{\partial x_e} \Delta x_e + \frac{\partial h_i}{\partial y_e} \Delta y_e + \frac{\partial h_i}{\partial z_e} \Delta z_e + n_i, \quad (8)$$

where

$$\begin{aligned} \Delta x_e &= x_e - x_{e0}, \\ \Delta y_e &= y_e - y_{e0}, \\ \Delta z_e &= z_e - z_{e0.} \end{aligned}$$

For k measurements of TDOA data, Eq.8 can be expressed as the linear model in a matrix form as

$$\underset{k\times 1}{Y} = \underset{k\times 3}{A}\,\underset{3\times 1}{X} + \underset{k\times 1}{N}. \quad (9)$$

where

$$\underset{k\times 1}{Y} = \begin{bmatrix} y_1 - h_1(x_{e0}, y_{e0}, z_{e0}, x_{1,1}, y_{1,1}, z_{1,1}, x_{1,2}, y_{1,2}, z_{1,2}) \\ y_2 - h_2(x_{e0}, y_{e0}, z_{e0}, x_{2,1}, y_{2,1}, z_{2,1}, x_{2,2}, y_{2,2}, z_{2,2}) \\ \vdots \\ y_k - h_k(x_{e0}, y_{e0}, z_{e0}, x_{k,1}, y_{k,1}, z_{k,1}, x_{k,2}, y_{k,2}, z_{k,2}) \end{bmatrix}, \quad \underset{k\times 3}{A} = \begin{bmatrix} \frac{\partial h_1}{\partial x_e} & \frac{\partial h_1}{\partial y_e} & \frac{\partial h_1}{\partial z_e} \\ \frac{\partial h_2}{\partial x_e} & \frac{\partial h_2}{\partial y_e} & \frac{\partial h_2}{\partial z_e} \\ \vdots & & \\ \frac{\partial h_k}{\partial x_e} & \frac{\partial h_k}{\partial y_e} & \frac{\partial h_k}{\partial z_e} \end{bmatrix},$$

$$\underset{3\times1}{X} = \begin{bmatrix} \Delta x_e \\ \Delta y_e \\ \Delta z_e \end{bmatrix}.$$

It is desired to estimate the emitter location that best fits TDOA measurements. In particular, to find the $\hat{X}$ that minimizes the sum of squares of difference between the measurements and the estimated functions is a natural choice for a goodness-of-fit criterion [4] as

$$\min \left\| Y - A\hat{X} \right\|^2, \tag{10}$$

where

$$\hat{X} = \left[\Delta\hat{x}_e \quad \Delta\hat{y}_e \quad \Delta\hat{z}_e \right]^T.$$

The formal least-squares solution for $\hat{X}$ is obtained by minimizing Eq.10 and given by

$$\hat{X} = (A^T A)^{-1} A^T Y, \tag{11}$$

and the estimated location of the emitter is given by

$$\begin{bmatrix} \hat{x}_e \\ \hat{y}_e \\ \hat{z}_e \end{bmatrix} = \begin{bmatrix} x_{e0} \\ y_{e0} \\ z_{e0} \end{bmatrix} + \begin{bmatrix} \Delta\hat{x}_e \\ \Delta\hat{y}_e \\ \Delta\hat{z}_e \end{bmatrix}. \tag{12}$$

If it is desired to weigh the measurements according to *a priori* confidence in each measurement, then the weighted least-squares solution is [10]

$$\hat{X} = (A^T W A)^{-1} A^T W Y, \tag{13}$$

where W is a simply diagonal and positive definite matrix. As can be seen from Eq.9, the k measurements of TDOAs are used to solve three unknowns x_e,y_e, and z_e. The redundancy of the complete set of TDOAs is used to increase noise immunity.

Note: The proposed method is an iterative scheme starting with a rough initial estimate and improving the estimate at each interval by minimizing the local linear-sum-squared errors. A direction finding (DF) system may be used at the ship/land-based station to provide an initial coarse estimate of emitter position to allow UAVs to get close to a target to perform geolocation of a radar emitter.

3.4 Measurement noise and bias

The estimates obtained using Eq.11 are the best statistical estimation if the noise term n_i in Eq.8 is normal distributed random noise with zero mean [9]. However, in most cases, n_i is a combination of random noise and bias (non-random noise). Therefore, the location estimates based on an assumption of zero-mean random noise will not be accurate. To represent the measurement noise properly, we consider n_i to be [9]

$$n_i = \text{bias} + \text{random noise}\,. \tag{14}$$

If considering measurement bias, for example, as a constant a_1, then we have

$$n_i = a_1 + e_i\,, \tag{15}$$

i.e.

$$\begin{aligned} & y_i - h_i(x_{e0}, y_{e0}, z_{e0}, x_{i,1}, y_{i,1}, z_{i,1}, x_{i,2}, y_{i,2}, z_{i,2}) \\ & = \frac{\partial h_i}{\partial x_e}\Delta x_e + \frac{\partial h_i}{\partial y_e}\Delta y_e + \frac{\partial h_i}{\partial z_e}\Delta z_e + a_1 + e_i\,, \end{aligned} \tag{16}$$

where e_i is the zero-mean random part of n_i and a_1 is a constant error. The associated estimates $\hat{X}$ and the matrix A for Eq.16 would be

$$\underset{4\times1}{\hat{X}} = \begin{bmatrix} \Delta\widehat{x}_e \\ \Delta\widehat{y}_e \\ \Delta\widehat{z}_e \\ \widehat{a}_1 \end{bmatrix}, \qquad \underset{k\times4}{A} = \begin{bmatrix} \frac{\partial h_1}{\partial x_e} & \frac{\partial h_1}{\partial y_e} & \frac{\partial h_1}{\partial z_e} & 1 \\ \frac{\partial h_2}{\partial x_e} & \frac{\partial h_2}{\partial y_e} & \frac{\partial h_2}{\partial z_e} & 1 \\ \vdots & & & \\ \frac{\partial h_k}{\partial x_e} & \frac{\partial h_k}{\partial y_e} & \frac{\partial h_k}{\partial z_e} & 1 \end{bmatrix}. \tag{17}$$

If considering measurement bias as a combination of constant and linear time-varying components, then we have

$$n_i = a_1 + a_2 t_i + e_i \,, \tag{18}$$

i.e.

$$\begin{aligned} & y_i - h_i(x_{e0}, y_{e0}, z_{e0}, x_{i,1}, y_{i,1}, z_{i,1}, x_{i,2}, y_{i,2}, z_{i,2}) \\ & = \frac{\partial h_i}{\partial x_e}\Delta x_e + \frac{\partial h_i}{\partial y_e}\Delta y_e + \frac{\partial h_i}{\partial z_e}\Delta z_e + a_1 + a_2 t_i + e_i \,, \end{aligned} \tag{19}$$

where t_i is time of observation. The estimates $\hat{X}$ and the associated matrix A for Eq.19 would be

$$\underset{5\times1}{\hat{X}} = \begin{bmatrix} \Delta\widehat{x}_e \\ \Delta\widehat{y}_e \\ \Delta\widehat{z}_e \\ \widehat{a}_1 \\ \widehat{a}_2 \end{bmatrix}, \qquad \underset{k\times5}{A} = \begin{bmatrix} \frac{\partial h_1}{\partial x_e} & \frac{\partial h_1}{\partial y_e} & \frac{\partial h_1}{\partial z_e} & 1 & \mathrm{t}_1 \\ \frac{\partial h_2}{\partial x_e} & \frac{\partial h_2}{\partial y_e} & \frac{\partial h_2}{\partial z_e} & 1 & \mathrm{t}_2 \\ \vdots & & & & \\ \frac{\partial h_k}{\partial x_e} & \frac{\partial h_k}{\partial y_e} & \frac{\partial h_k}{\partial z_e} & 1 & \mathrm{t}_\mathrm{k} \end{bmatrix}. \tag{20}$$

Although higher order measurement errors may exist, for simplicity, only the two cases of measurement bias (i.e., constant and linear time-varying errors) are considered here. Obviously, choosing the correct model is important for location estimation. A well-known theorem from linear model theory is provided in [11] to identify measurement bias.

4. Numerical Simulation

The objective of numerical simulation is to demonstrate the effectiveness of the proposed method. The unit of distance used in the following simulation is in kilometers (km). In Figs. 5, 6 and 7, we assume that the emitter's true position is at (x_e, y_e, z_e)=(9.84, 12.08, 0.00) and the initial position of the emitter for location estimation is at (x_{e0}, y_{e0}, z_{e0}) = (19.84, 22.08, 10.00), i.e., about 17.32 km away from its true position. In order to illustrate the accuracy of 3-D location estimation, the location estimation error is used and defined by

$$\text{Location Estimation Error} = \sqrt{(\hat{x}_{e,i} - x_e)^2 + (\hat{y}_{e,i} - y_e)^2 + (\hat{z}_{e,i} - z_e)^2}\ , \qquad (21)$$

where $(x_e,\ y_e,\ z_e)$ is the emitter's true position and $(\hat{x}_{e,i}, \hat{y}_{e,i}, \hat{z}_{e,i})$ is the location estimates at the i^{th} time interval.

In the following, two possible scenarios for passive geolocation of a radar emitter are given to demonstrate the effectiveness of the proposed method.

4.1 Geolocation from a ship/land-based sensor accompanied by a UAV-based sensor

Figs. 5 and 6 show location estimation using noisy TDOA measurements from two sensors with sensor *1* on a UAV and sensor *2* on a ship/land-based platform. In the figures, the crosses represent the position of the moving sensor at different time intervals along the UAV fight path, the square represents the sensor on the ship/land-based station, and the circle represents the true position of the emitter. For simplicity, we take the position of the ship/land-based station to be the origin in the figures. Based on the scenarios given in Figs. 5 and 6, the distance between the emitter and sensor *1* varies from 20.58 km to 21.52 km, while the distance between the emitter and sensor *2* is 15.58 km. This gives range difference variation from 5.00 km to 5.94 km, corresponding to TDOA variation from 16.66 to 19.81 microseconds (μsec).

Fig. 5 demonstrates the effectiveness of the proposed method in handling zero-mean random measurement noise, in which noisy TDOAs were produced by adding Gaussian random noise with standard deviation of 33.0 nanoseconds (nsec). The associated location estimation errors over estimation intervals are given in Fig. 6. As can be seen, the proposed estimation method converges to the true location after first few estimation intervals although the initial location of the emitter is assumed to be 17.32 km away from its true position. The location estimate at the first estimation interval in Fig.5 was computed using first 15 measurements and the subsequent estimates incorporated each new measurement with one old measurement removed from the data set.

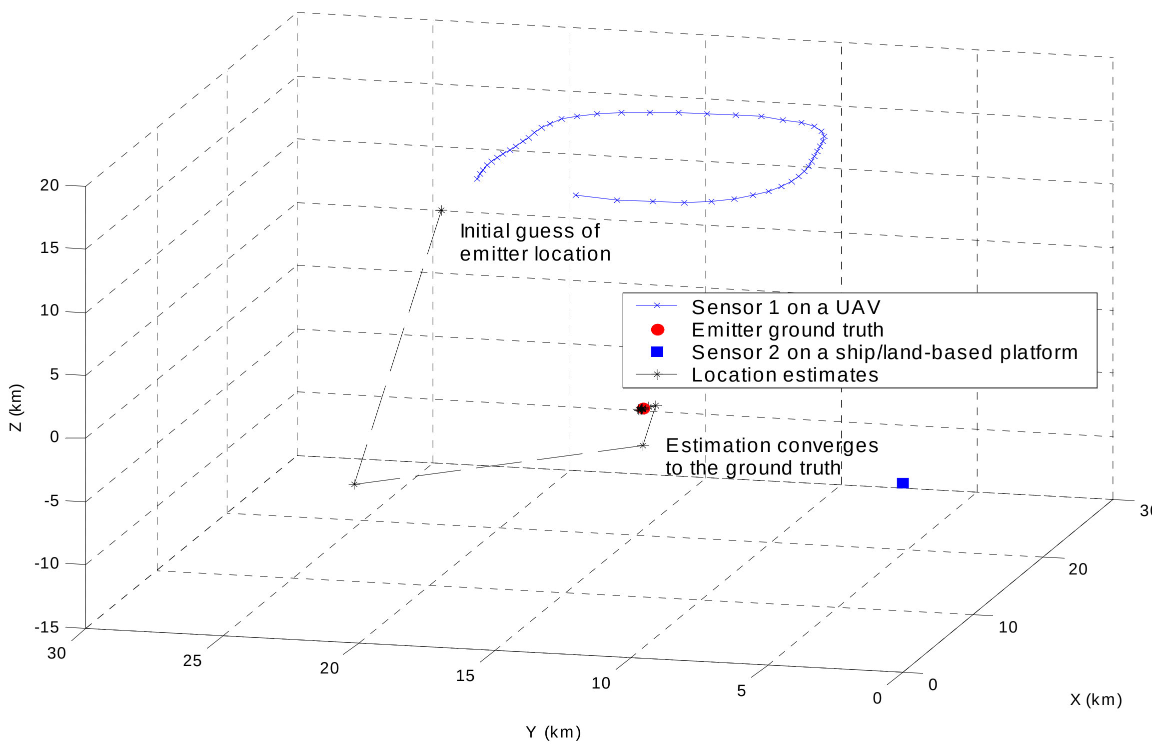

Figure 5. *Location estimation against the ground truth with random TDOA measurement noise*

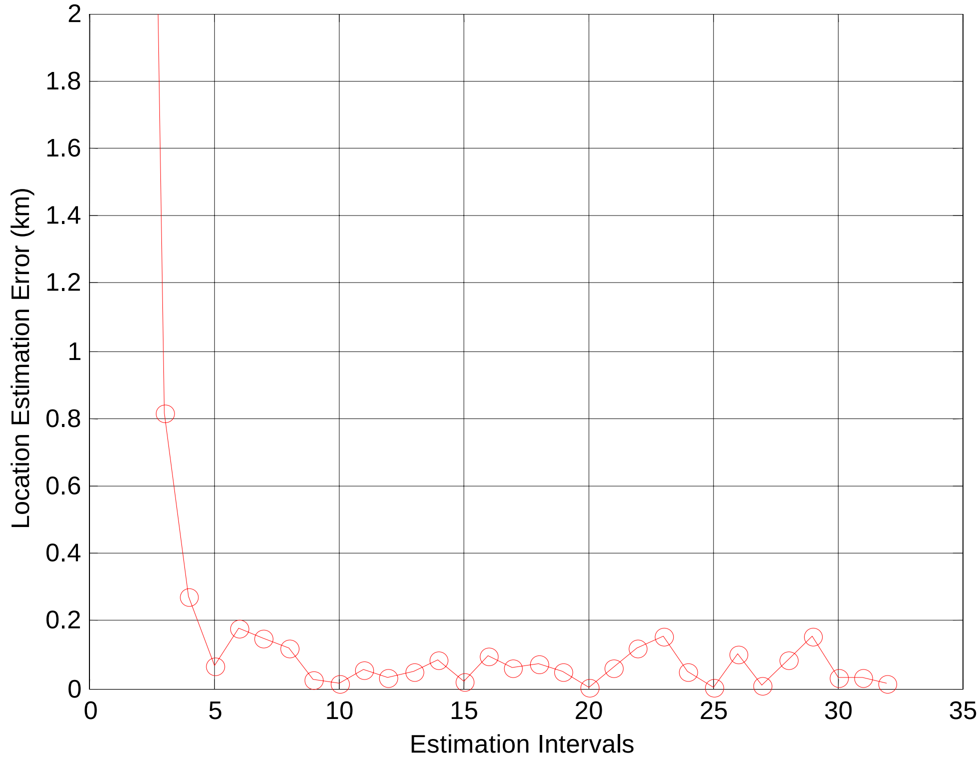

Figure 6. *Location estimation errors with random TDOA measurement noise*

Figs 7 and 8 illustrate the effectiveness of the proposed method in handling both random and non-random measurement noises, where the noisy TDOAs were produced by adding the measurement bias of 7.0 μsec and white Gaussian noise with its standard deviation of 33.0 nsec. As can be seen from Fig.7, the proposed method converges to the true location although TDOA measurement bias is present in the measurements. This is because the proposed method allows the estimation of the measurement bias and has it removed while computing the location of the emitter. The estimated bias in this simulation was 7.06 μsec while the true measurement bias was 7.0 μsec. The associated estimation errors are illustrated by the circles in Fig.8. To demonstrate the effectiveness of the proposed method in handling non-random bias, Fig.8 also shows location estimation errors given by the squares when there is no bias estimation and removal. As can be seen in Fig.8, the estimation errors without bias estimation and removal are larger and are about 2.0 km. The location estimates given in Fig.7 and 8 were computed using 40 TDOA measurements.

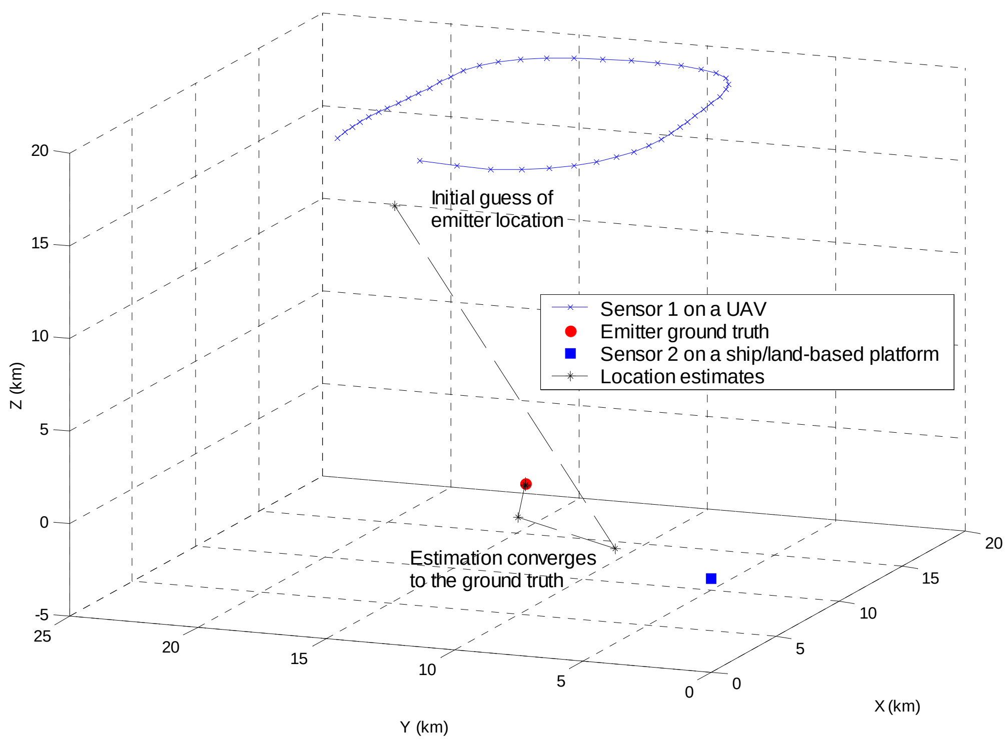

Figure 7. *Location estimations against the ground truth with random and non-random TDOA measurements noises*

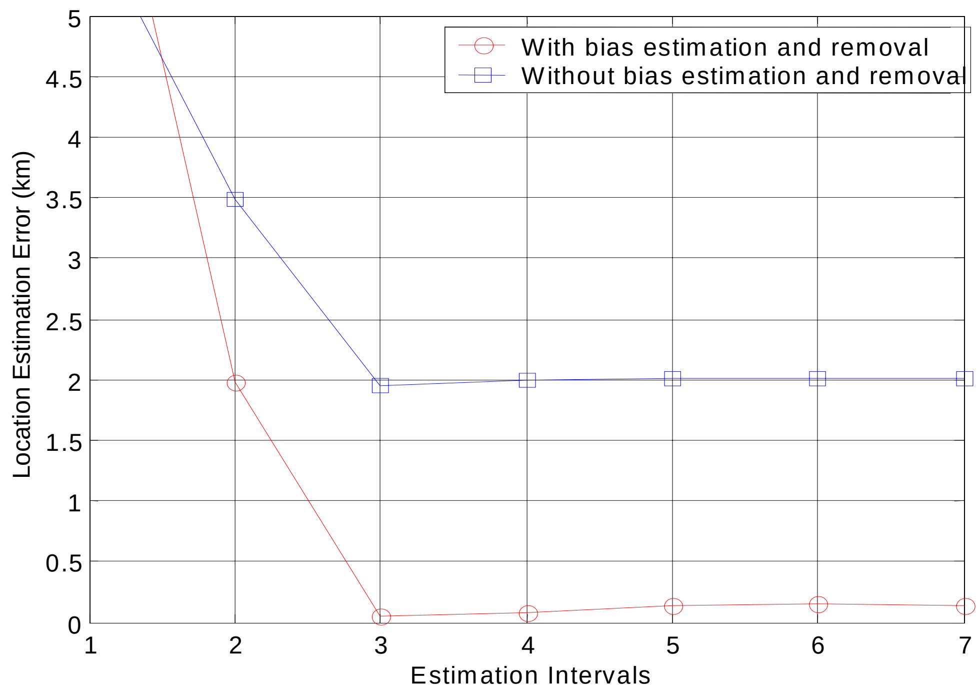

Figure 8. *Location estimation errors with random and non-random TDOA measurement noises*

It has been found in the numerical simulation that to obtain the same estimation accuracy, the estimation of both emitter location and measurement bias requires more TDOA measurements than does the estimation of location alone. This seems reasonable because more parameters are being estimated. The requirement for additional data is certainly justified when accurate location estimation is desired. It has also been found that an increase in number of TDOA measurements will improve location estimation accuracy when noisy measurements are presented.

4.2 Geolocation from a ship/land-based sensor accompanied by two UAV-based sensors

Figs.9 and 10 show location estimation from two UAV sensors and a ship/land-based sensor. According to the scenario given in Fig. 9, the distance between the emitter and sensor *1* varies from 30.14 km to 30.99 km, while the distance between the emitter and sensor *2* varies from 20.33 km to 22.44 km. The associated range difference varies from 7.73 km to 10.65 km, corresponding to TDOA variations from 25.76 μsec to 35.49 μsec.

To illustrate the effectiveness of the proposed method in handling both random and non-random measurement noises, the noisy measurements were produced by adding a constant TDOA bias of 7.0 μsec and white Gaussian noise to the TDOA data. As can be seen in Fig. 9, the proposed method begins to converge to the emitter ground truth at the fifth estimation intervals even though a measurement bias of 7.0 μsec was introduced. To achieve the location accuracy given in Fig.10, 40 TDOA measurements were used in the simulation. As stated previously, more TDOA measurements are required to handle a measurement bias if accurate location estimation is desired.

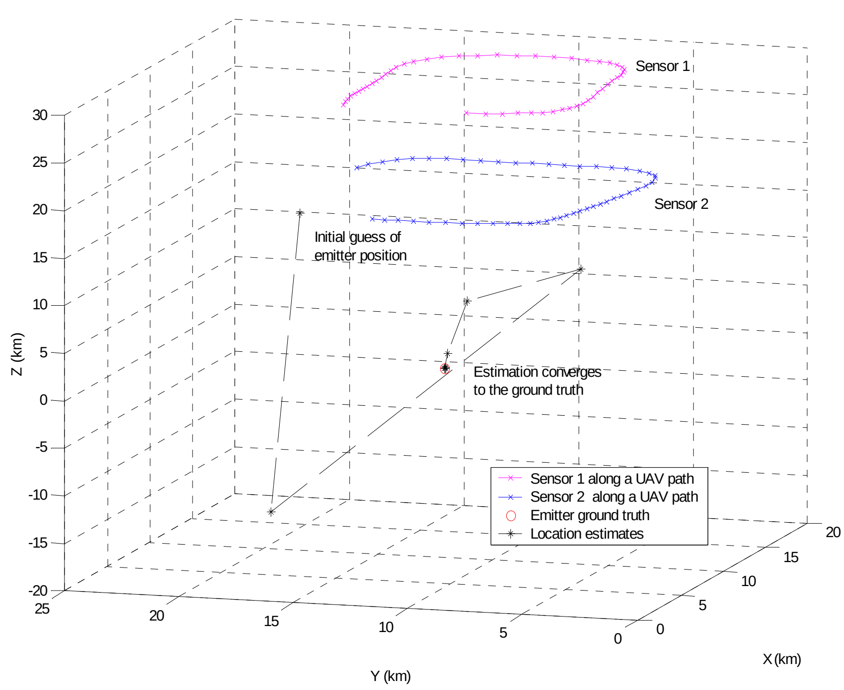

Figure 9. *Location estimation against the ground truth with random and non-random TDOA measurements noises*

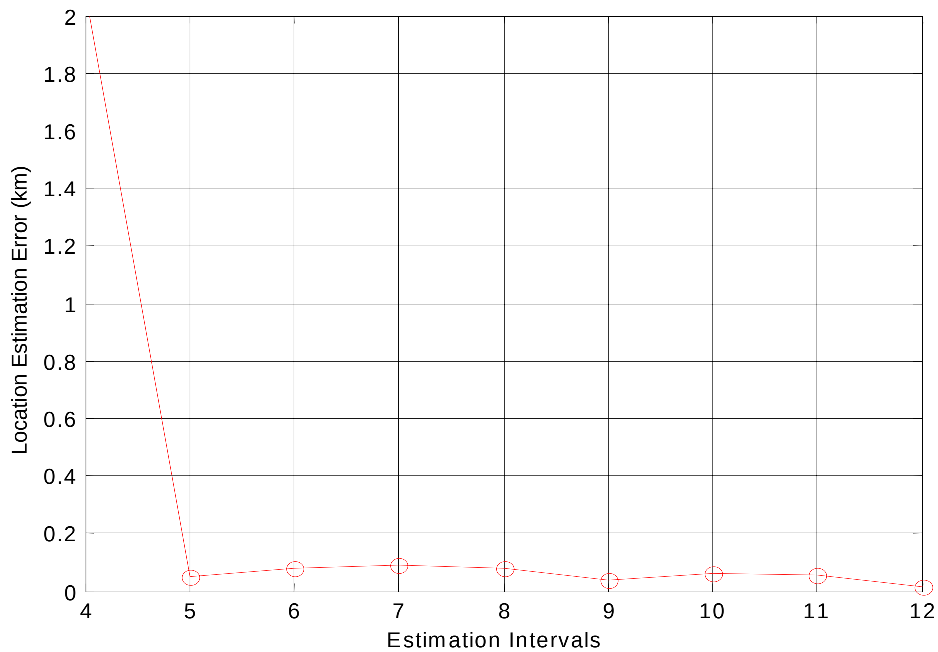

Figure 10. *Location estimation errors with random and non-random measurement noises*

5. Summary and Conclusion

This report has addressed passive geolocation using TDOA method from UAVs working in conjunction with a ship/land-based platform. Implicit in the development is the assumption that a mechanism for measuring the TDOAs on radar pulses exists. In this approach, multiple TDOA measurements are collected from UAV sensors over time intervals along UAV flight trajectories. A mathematical model including random and non-random TDOA measurement errors is derived to achieve an accurate formulation of a position-location problem when noisy measurements are present. A simple least-squares solution based on the model is given to solve 3-D location estimation using TDOA measurements. The simulation results have shown that accurate location estimates are achieved even though a measurement bias presents in the TDOA measurements. The proposed method is an iterative scheme starting with a rough initial estimate and improving the estimate at each interval by minimizing the local linear-sum-squared errors. With reasonable initial guesses, the method does converge in most cases although the absolute convergence is not proven.

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Passive geolocation of radar emitters remains an important problem in electronic support (ES) for maritime and littoral surveillance. Several location methods have been investigated in the past. This report presents passive geolocation of radar emitter using the time difference of arrival (TDOA) location method from Unmanned Aerial Vehicles (UAVs) working in conjunction with ship/land-based platforms to fulfill maritime and littoral area surveillance. Instead of using multiple stationary sensors, multiple TDOA measurements are obtained over time intervals from UAV-based sensors along UAV flight trajectories. A simple Least-Squares (LS) solution is derived based on TDOA measurements and known sensor locations to solve three-dimensional (3-D) location estimation. The proposed location method takes into account of both TDOA measurement noise and bias. Therefore, the location estimation can be achieved more accurately when biased measurements are present. The advantages and effectiveness of the proposed method are demonstrated using computer simulation.

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