COMPRESSIVE SENSING FOR RADAR SIGNALS:

PART III: MIMO RADARS

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Contract Report
DRDC-RDDC-2015-C209
March 2015

I. Compressed Sensing General Theory

Compressed sensing (CS) is a novel sensing/sampling paradigm that goes against the common wisdom in data acquisition. Based on the CS theory, one can recover certain signals from far fewer samples or measurements than traditional methods (like Shannon theorem) use. To make this possible, CS relies on two principles: sparsity, which pertains to the signals of interest, and incoherence, which pertains to the sensing modality.

Sparsity expresses the idea that the information of a signal may be much smaller than suggested by its length. In many application such as digital images and video cameras the Nyquist rate is so high that too many samples result and it is a serious challenge to store or transmit these samples. Also, in applications which deals with high-bandwidth signals, it is necessary to build a high-rate A/D converter which is very expensive if it is not impossible. CS addresses this issue by reducing the number of measurements considerably.

Incoherence extends the duality between time and frequency and expresses the idea that signals having a sparse representation must be spread out in the domain in which they are acquired.

The important point is that one can design efficient sensing protocols that capture the useful information content embedded in a sparse signal and condense it into a small amount of data. These protocols are nonadaptive and simply require correlating the signal with a small number of fixed waveforms that are incoherent with the sparsifying basis.

In [1], the authors consider the model problem of reconstructing an object from incomplete ferequency measurements. They show that a discrete object f can be recovered exactly from observations on small set of frequencies provided that f is sparse.

A. The Sensing Problem

In sensing, information about a signal f is obtained by linear functionals recording the values

$$y_k =< f, \phi_k >, \quad k = 1, \ldots, m. \tag{1}$$

That is, we simply correlate the object we wish to acquire with the ϕ_k's. If the sensing waveforms are Dirac delta functions (spikes), for example, then y is a vector of sampled values of f in the time or space domain.

B. Sparsity and Incoherence in Compressive Sampling

We expand the signal vector $f \in \mathbb{R}^n$ in an orthonormal basis as

$$f = \mathbf{\Psi}\mathbf{x}, \tag{2}$$

where $\mathbf{\Psi}$ is the $n \times n$ matrix of orthonormal vectors and $\mathbf{x}$ is the vector of coefficients. Sparsity implies that when f has a sparse expansion, one can discard the small coefficients without perceptual loss. Let us use the basis $\mathbf{\Phi}$ for sensing f. Note that $\mathbf{\Psi}$ is used to represent f. The coherence between sensing basis and representation basis is defined as

$$\mu(\mathbf{\Phi}, \mathbf{\Psi}) = \sqrt{n}. \max_{1 \leq k,j \leq n} |\langle \varphi_k, \psi \rangle|. \tag{3}$$

the coherence measures the largest correlation between any two elements of $\mathbf{\Phi}$ and $\mathbf{\Psi}$. Also we have $\mu(\mathbf{\Phi}, \mathbf{\Psi}) \in [1, \sqrt{n}]$. Compressive sampling is mainly concerned with low coherence pairs and random matrices are widely incoherent with any fixed basis $\mathbf{\Psi}$. To this end, we can select an orthobasis $\mathbf{\Phi}$ in which the elemnets are i.i.d. Gaussian random variables.

The reconstruction of f is obtained via using linear programming (l_1-minimization). So it's necessary that f is sufficiently sparse. It means that the number of measurments, m, must satisfy a condition to get exact recovery. If

$$m \geq C.\mu^2(\mathbf{\Phi}, \mathbf{\Psi}).S. \log n, \tag{4}$$

for some positive constant C, the solution is exact with overwhelming probability. In (3), we can see the role of coherence coefficeint. The smaller the coherence, the fewer samples are needed. The proof and more detailed information have been presented in [2].

C. Robust Signal Recovery

Another key point in CS is $restricted\ isometry\ property$ (RIP). For each integer $S = 1, 2, ...,$ define the isometry constant δ_S of the sensing matrix A ($A = \mathbf{\Phi\Psi}$) as the smallest number such that

$$(1 - \delta_S)\|\mathbf{x}\|_{l_2}^2 \leq \|A\mathbf{x}\|_{l_2}^2 \leq (1 + \delta_S)\|\mathbf{x}\|_{l_2}^2, \tag{5}$$

holds for all S-sparse vectors $\mathbf{x}$. A obeys the RIP of order S if δ_S is not close to one. When this property holds, S-sparse vectors cannot be in the null space of A and then the following linear program gives an accurate reconstruction

$$\min_{\tilde{\mathbf{x}} \in \mathbb{R}^n} \|\tilde{\mathbf{x}}\|_{l_1} \text{ subject to } A\tilde{\mathbf{x}} = y (= A\mathbf{x}). \tag{6}$$

Assume that $\delta_{2S} < \sqrt{2} - 1$. Then the solution $\mathbf{x}^*$ to (5) obeys

$$\|\mathbf{x}^* - \mathbf{x}\|_{l_2} \leq C_0.\|\mathbf{x} - \mathbf{x}_S\|_{l_1}/\sqrt{S},$$

$$\|\mathbf{x}^* - \mathbf{x}\|_{l_1} \leq C_0.\|\mathbf{x} - \mathbf{x}_S\|_{l_1}, \tag{7}$$

for some constant C_0, where $\mathbf{x}_S$ is the vector $\mathbf{x}$ with all but the largest S components set to 0. If $\mathbf{x}$ is S sparse, then $\mathbf{x} = \mathbf{x}_S$ and the recovery is exact. Also if $\mathbf{x}$ is not S-sparse, then (6) asserts that the quality of the recovered signal is as good as if one knew ahead of time the location of the S largest values of $\mathbf{x}$ and decided to measure those directly.

It should be noted that the real-world signals are corrupted by noise, so we have

$$y = A\mathbf{x} + \mathbf{z}, \tag{8}$$

where z denotes the error. In this case l_1-minimization with relaxed constraints for reconstruction

$$\min_{\tilde{\mathbf{x}} \in \mathbb{R}^n} \|\tilde{\mathbf{x}}\|_{l_1} \text{ subject to } \|A\tilde{\mathbf{x}} - y\|_{l_2} \leq \epsilon, \tag{9}$$

where ϵ bounds the amount of noise in the data. For noisy data, we can rewrite (6) as

$$\|\mathbf{x}^* - \mathbf{x}\|_{l_2} \leq C_0.\|\mathbf{x} - \mathbf{x}_S\|_{l_1}/\sqrt{S} + C_1.\epsilon, \tag{10}$$

for some constants C_0 and C_1. There is an interesting point in (6) and (9). Both of them hold for general form of vectors (spars and non-sparse). Hence, in noisy environment, the solution $\mathbf{x}^*$ consists of the coefficients of the signal which corrupted by noise, i.e. still $\mathbf{x}^*$ at locations where the signal is of small value, is zero. So the noise do not affect selecting S largest components of the signal. Note that noise is generally non-sparse.

This establishes Compressive Sensing as a practical and robust sensing mechanism. It works with all kinds of not necessarily sparse signals, and it handles noise gracefully. The proof and more detailed information have been presented in [3].

II. Compressive Sensing Applications In Communications and Networks

A. Sparse Channel Estimation

CS has been used in communications domain for sparse channel estimation. Adoption of multiple-antenna in communication system design and operation at large bandwidths, enables sparse representation of channels in appropriate bases. Conventional technique of training based estimation using least-square (LS) methods may not be an optimal choice. Various recent studies have employed CS for sparse channel estimation. Compressed channel estimation (CCS) gives much better reconstruction using its non-linear reconstruction algorithm as opposed to linear reconstruction of LS-based estimators. Also, the estimation of underwater acoustic channels, which are inherently sparse, through CS technique yields results better than the conventional least square estimator.

B. Spectrum Sensing In Cognitive Radios

CS based technique is used for speedy and accurate spectrum sensing in cognitive radio technology based standards and systems. IEEE 802.22 is the first standard to use the concept of cognitive radio, providing an air interface for wireless communication in the TV band. Fast Fourier sampling (FFS) - an algorithm based on CS - is used to detect wireless signals as proposed in [Y. Tachwali et. al., $The\ feasibility\ of\ a\ fast\ fourier\ sampling\ technique\ for\ wireless\ microphone\ detection\ in\ IEEE\ 802.22\ air\ interface$, in Proc. IEEE INFOCOM, pp. 15, 2010].

Also, wideband spectrum sensing scheme using distributed CS is proposed for cognitive radio networks in [Y. Wang et. al., $Distributed\ compressive\ wideband\ spectrum\ sensing$, IEEE Proc. Inf. Theory Appl., pp. 14, 2009].

C. Ultra-Wideband (UWB) Systems

In the emerging technology of UWB communication, CS plays a key role by reducing the high data-rate of ADC at receivers [P. Zhang, et. al., $A\ compressed\ sensing\ based\ ultra-wideband\ communication\ system$, in Proc. IEEE ICC, pp. 15, 2009]. CS, as used in pulse-based UWB communication utilizes time sparsity of the signal through a filter-based CS approach applied on continuous time signals.

D. Wireless Sensor Networks

CS finds its applications in data gathering for large WSNs, consisting of thousands of sensors deployed for tasks like infrastructure or environment monitoring. This approach of using compressive data gathering helps in overcoming the challenges of high communication costs and uneven energy consumption by sending m weighted sums of all sensor readings to a sink which recovers data from these measurements. Although, this increases the number of signals sent by the initial m sensors, but the overall reduction in transmissions and energy consumption is significant since $m \ll n$ (where n is the total number of sensors in large-scale WSN). This also results in load balancing which in turn enhances lifetime of the network.

E. Distributed Compression in Wireless Sensor Networks (WSNs)

Distributed source coding (DSC) is a compression technique in WSNs in which one signal is transmitted fully and rest of the signals are compressed based on their spatial correlation with main signal. DSC performs poorly when sudden changes occur in sensor readings, as these changes reflect in correlation parameters and main signal fails to provide requisite base information for correct recovery of side signals. Only spatial correlation is exploited in DSC, while under no-event conditions, sensor readings usually have a high temporal correlation as well.

F. Radar Systems

CS is used in radar systems to achieve better target resolution than classical techniques. It is demonstrated in [4] that when the target scene is sparse in range-Doppler plane, the compressed sensing can be very effective. In [6], the idea of applying CS in the radar systems has been extended to the MIMO radar.

III. MIMO RADAR - DIVERSITY MEANS SUPERIORITY

A. Introduction

MIMO radar is an emerging technology that is attracting the attention of researchers because of its superior capabilities compared with a standard phased-array radar. A MIMO radar system can transmit via its antennas multiple probing signals that may be chosen quite freely. For collocated transmit and receive antennas, the MIMO radar paradigm has been shown to offer higher resolution, higher sensitivity to detecting slowly moving targets, better parameter identifiability (maximum number of targets that can be uniquely identified by the radar), and direct applicability of adaptive array techniques.

B. Problem Formulation

Consider a MIMO radar system with M_t transmit antennas and M_r receive antennas. Let $x_m(n)$ denote the discrete-time baseband signal transmitted by the mth transmit antenna. Also, let θ denote the location parameter(s) of a generic target. Then, under the assumption that the transmitted probing signals are narrowband and that the propagation is nondispersive, the baseband signal at the target location can be described by the expression

$$\sum_{m=1}^{M_t} e^{-j2\pi f_0 \tau_m(\theta)} x_m(n) \triangleq \mathbf{a}^*(\theta)\mathbf{X}(n),\ n = 1, \ldots, N \tag{11}$$

where f_0 is the carrier frequency of the radar, $\tau_m(\theta)$ is the time needed by the signal emitted via the mth transmit antenna to arrive at the target, $(.)^*$ denotes the conjugate transpose, N denotes the number of samples of each transmitted signal pulse

$$\mathbf{X}(n) = [x_1(n)\ x_2(n)\ \ldots\ x_{M_t}(n)]^T,\ n = 1, \ldots, N \tag{12}$$

and

$$\mathbf{a}(\theta) = [e^{j2\pi f_0 \tau_1(\theta)}\ e^{j2\pi f_0 \tau_2(\theta)}\ \ldots\ e^{j2\pi f_0 \tau_{M_t}(\theta)}]^T \tag{13}$$

where $(.)^T$ denotes the transpose.

Let $y_m(n)$ denote the signal received by the mth receive antenna. Let

$$\mathbf{Y}(n) = [y_1(n)\ y_2(n)\ \ldots\ y_{M_r}(n)]^T, n = 1, \ldots, N \tag{14}$$

and let

$$\mathbf{b}(\theta) = [e^{j2\pi f_0 \tilde{\tau}_1(\theta)}\ e^{j2\pi f_0 \tilde{\tau}_2(\theta)}\ \ldots\ e^{j2\pi f_0 \tilde{\tau}_{M_t}(\theta)}]^T \tag{15}$$

where $\tilde{\tau}_m(\theta)$ is the time needed by the signal reflected by the target located at θ to arrive at the mth receive antenna. Then, under the simplifying assumption of point targets, the received data vector can be described by

$$\mathbf{Y}(n) = \sum_{k=1}^{K} \beta_k \mathbf{b}^c(\theta_k)\mathbf{a}^*(\theta_k)\mathbf{X}(n) + \epsilon(n),\ n = 1,\ \ldots\ , N \tag{16}$$

where K is the number of targets that reflect the signals back to the radar receiver, β_k are complex amplitudes proportional to the radar cross sections (RCSs) of those targets, θ_k are the target location parameters, $\epsilon(n)$ denotes the interference-pulse-noise term, and $(.)^c$ denotes the complex conjugate. The unknown parameters to be estimated are β_k and θ_k for $k = 1,\ \ldots,\ K$. Also, determining the maximum number of targets that can be uniquely identified is of interest.

C. Parameter Identifiability

Parameter identifiability is to establish the uniqueness of the solution to the parameter estimation problem as either the signal to interference plus noise ratio (SINR) or the snapshot number N goes to infinity.

Based on the analysis provided in [5], for K_{max}, the maximum number of targets that can be uniquely identified, we have

$$K_{max} \in \left[\frac{2(M_t + M_r) - 5}{3}, \ \frac{2M_t M_r}{3} \right) \tag{17}$$

For a phased-array radar (which uses M_r receiving antennas, and for which we can basically assume that $M_t = 1$), we have

$$K_{max} = \left\lceil \frac{2M_r - 3}{3} \right\rceil \tag{18}$$

Hence, the maximum number of targets that can be uniquely identified by a MIMO radar can be up to M_t times that of its phased-array counterpart.

D. Nonparametric adaptive techniques for parameter estimation

As a significant advantage of a MIMO radar system, it is possible to use adaptive localization and detection techniques directly which leads to have much better resolution and much better interference rejection capability than their data-independent counterparts.

Let

$$\tilde{\mathbf{A}} = [\beta_1^* \mathbf{a}(\theta_1) \ \beta_2^* \mathbf{a}(\theta_2) \ \ldots \ \beta_K^* \mathbf{a}(\theta_K)] \tag{19}$$

Then the sample covarinace matrix of the target reflected waveform is $\tilde{\mathbf{A}}^* \hat{\mathbf{R}}_{xx} \tilde{\mathbf{A}}$, where

$$\hat{\mathbf{R}}_{xx} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{X}(n) \mathbf{X}^*(n) \tag{20}$$

is the sample covariance matrix of the transmitted waveforms. When orthogonal waveforms are used for MIMO probing and $N \geq M_t$, then $\hat{\mathbf{R}}_{xx}$ is a scaled identity matrix. Then $\tilde{\mathbf{A}}^* \hat{\mathbf{R}}_{xx} \tilde{\mathbf{A}}$ has full rank. In other words, the target reflected waveforms are not completely correlated with each other, if the columns of $\tilde{\mathbf{A}}$ are linearly independent of each other, which requires that $K \leq M_t$. The fact that the target reflected waveforms are noncoherent allows the direct application of many adaptive techniques for target localizaton.

The expression of the signal matrix at the output of the receiving array is

$$\mathbf{Y} = \mathbf{b}^c(\theta)\beta(\theta)\mathbf{a}^*(\theta)\mathbf{X} + \mathbf{Z} \tag{21}$$

where $\mathbf{X} = [\mathbf{x}(1)\ \mathbf{x}(2)\ \ldots \mathbf{x}(N)]$, the columns of $\mathbf{Y} \in \mathfrak{C}^{M_r \times N}$ are the received data samples $\mathbf{y},\ n = 1,\ \ldots,\ N$, and $\beta(\theta) \in \mathfrak{C}$ denotes the complex amplitude of the reflected signal from θ, which is proportional to the radar cross section (RCS) of the focal point θ. The matrix $\mathbf{Z} \in \mathfrak{C}^{M_r \times N}$ denotes the residual term, which includes the noise, interference and jamming. The problem is to estimate $\beta(\theta)$ for each θ of interest from the observed data matrix $\mathbf{Y}$.

E. Absence of array calibration errors

A simple way to estimate $\beta(\theta)$ is via the LS method. However, as any other data-independent beamforming-type method, the LS method completely fails to work in the presence of strong interfernece and jamming. We present below some methods which have much higher resolution and much better interference rejection capability than does the LS method.

1) Capon: The Capon estimator consists of two main steps: (1) the Capon beamforming step and (2) a LS estimation step. The Capon beamformer can be formulated as follows

$$\min_{\mathbf{w}} \mathbf{w}^*\hat{\mathbf{R}}_{yy}\mathbf{w} \quad \text{subject to} \quad \mathbf{w}^*\mathbf{b}^c(\theta) = 1 \tag{22}$$

where $\mathbf{w} \in \mathfrak{C}^{M_r \times 1}$ is the weight vector used to acheive noise,interference, and jamming suppression while keeping the desired signal undistorted and $\hat{\mathbf{R}}_{yy}$ is the sample covariance matrix of the observed data samples.

The solution to (12) is as follows

$$\hat{\mathbf{w}}_{Capon} = \frac{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^c(\theta)}{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^c(\theta)} \tag{23}$$

and the Capon estimate of $\beta(\theta)$ is

$$\hat{\beta}_{Capon}(\theta) = \frac{\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{Y}\mathbf{X}^*\mathbf{a}(\theta)}{N[\mathbf{b}^T(\theta)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{b}^c(\theta)][\mathbf{a}^*(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)]} \tag{24}$$

2) APES: The amplitude and phase estimation (APES) approach is a non parametric spectral analysis method with superior estimation accuracy. We apply this method to MIMO radar system to acheive better amplitude estimation accuracy. We can formulate the APES method as

$$\min_{\mathbf{w},\beta} \|\mathbf{w}^*\mathbf{Y} - \beta(\theta)\mathbf{a}^*(\theta)\mathbf{X}\|^2 \quad \text{subject to} \quad \mathbf{w}^*\mathbf{b}^c(\theta) = 1 \tag{25}$$

The APES beamformer weight vector is

$$\hat{\mathbf{w}}_{APES} = \frac{\hat{\mathbf{Q}}^{-1}\mathbf{b}^c(\theta)}{\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{b}^c(\theta)} \tag{26}$$

and the APES estimate of $\beta(\theta)$ is obtained as

$$\hat{\beta}_{APES}(\theta) = \frac{\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{Y}\mathbf{X}^*\mathbf{a}(\theta)}{N[\mathbf{b}^T(\theta)\hat{\mathbf{Q}}^{-1}\mathbf{b}^c(\theta)][\mathbf{a}^*(\theta)\hat{\mathbf{R}}_{xx}\mathbf{a}(\theta)]} \tag{27}$$

F. Presence of array calibration errors

The previous data-dependent methods assume that the transmitting and receiving arrays are perfectly calibrated. However, in practice, array calibration errors are often inevitable. The presence of array calibration errors and the related small data sample number problem can significantly degrade the performance of the data-dependent beamforming methods discussed so far.

1) RCB: We consider the application of the robust Capon beamformer (RCB) to a MIMO radar system that suffers from calibration errors. RCB allows $\mathbf{b}(\theta)$ to lie in an uncertainty set. Without loss of generality, we assume that $\mathbf{b}(\theta)$ belongs to an uncertainty sphere

$$\|\mathbf{b}(\theta) - \bar{\mathbf{b}}(\theta)\|^2 \leq \epsilon_r \tag{28}$$

where both $\bar{\mathbf{b}}(\theta)$, the nominal receiving array steering vector, and ϵ_r are given.

The RCB method is based on the following covariance fitting formulation

$$\max_{\sigma^2(\theta),\mathbf{b}(\theta)} \sigma^2(\theta) \quad \text{subject to} \quad \hat{\mathbf{R}}_{yy} - \sigma^2(\theta)\mathbf{b}^c(\theta)\mathbf{b}^T(\theta) \geq 0$$
$$\|\mathbf{b}(\theta) - \bar{\mathbf{b}}(\theta)\|^2 \leq \epsilon_r \tag{29}$$

where $\sigma^2(\theta)$ denotes the power of the signal of interest and $\mathbf{P} \geq 0$ means that the matrix $\mathbf{P}$ is positive semidefinite.

By using the Lagrange multiplier methodology, the solution to (19) is found to be

$$\hat{\mathbf{b}}(\theta) = \bar{\mathbf{b}}(\theta) - [\mathbf{I} + \lambda(\theta)\hat{\mathbf{R}}_{yy}^c]^{-1}\bar{\mathbf{b}}(\theta) \tag{30}$$

where $\mathbf{I}$ denotes the identity matrix. The lagrange multiplier $\lambda(\theta) \geq 0$ in (20) is obtained as the solution to the constraint equation

$$\|[\mathbf{I} + \lambda(\theta)\hat{\mathbf{R}}_{yy}^c]^{-1}\bar{\mathbf{b}}(\theta)\|^2 = \epsilon_r \tag{31}$$

which can be solved efficiently by using Newton method since the left side of (21) is a monotically decreasing function of $\lambda(\theta)$.

G. Parametric techniques for parameter estimation

We consider parametric techniques for target parameter estimation as well as target number detection for MIMO radar with collocated antennas. We employ maximum likelihood (ML) method for the estimation of target parameters and a Baysian information criterion (BIC) is used for target number detection. The received data matrix can be rewritten as

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{b}^c(\theta_k)\beta_k\mathbf{s}^T(\theta_k) + \tilde{\mathbf{Z}}, \quad \text{with} \quad \mathbf{s}^T(\theta_k) = \mathbf{a}^*(\theta_k)\mathbf{X} \tag{32}$$

where the column of $\tilde{\mathbf{Z}}$ comprise independently and identically distributed circularly symmetric complex Gaussian random vectors with zero mean and unknown covariance matrix.

1) ML and BIC: Consider first the ML estimation of the target parameters β_k and θ_k for $k = 1, \ldots, K$ by assuming that the target number K is known a priori. Based on the analysis in [5], we have the following concentrated negative log-LF of the unknown parameters

$$f_2(\theta_k, \beta_k) = N \ \ln\left|\left[\mathbf{Y} - \sum_{k=1}^{K} \mathbf{b}^c(\theta_k)\beta_k \mathbf{s}^T(\theta_k)\right] \times \left[\mathbf{Y} - \sum_{k=1}^{K} \mathbf{b}^c(\theta_k)\beta_k \mathbf{s}^T(\theta_k)\right]^*\right| \tag{33}$$

The minimization of this cost function must be conducted with respect to $3K$ unknown real-valued variables $\theta_k \quad \text{for} \quad k = 1, \ldots, K$ and the real and imaginery parts of $\beta_k \quad \text{for} \quad k = 1, \ldots, K$. This optimization problem does not appear to admit a closed-form solution and [5] peresent two cyclic optimization algorithms as ACO and ECO for target parameter estimation.

The estimates of the target parameters β_k and $\theta_k \quad \text{for} \quad k = 1, \ldots, K$ are associated with the assumed target number K. Once we got the estimates of β_k and $\theta_k \quad \text{for} \quad k = 1, \ldots, K$ for various values of K, the target number K can be detected by minimizing the following BIC cost function

$$\text{BIC}(K) = 2f_2(\hat{\theta}_k, \hat{\beta}_k \quad \text{for} \quad k = 1, \ldots, K) + 3K \ \ln N \tag{34}$$

where $\hat{\theta}_k$ and $\hat{\beta}_k$ are the estimates of θ_k and β_k, respectively.

IV. MIMO Radar: Concepts, Performance Enhancements, and Applications

A. Introduction

The notion of MIMO radar is simply that there are multiple radiating and receiving sites. The collected informaion is then processed together. Various possible signaling techniques are used for MIMO radar. The transmit antennas radiate signals, which may or may not be correlated, and the receive antenna attempt to disentangle these signals.

There is a continuum of MIMO radar systems concepts; however, there are two basic regimes of operation considered in the current literature. In the first regime, the transmit array elements (and receive array elements) are broadly spaced, providing independent scattering responses for each antenna pairing, sometimes referred to as $statistical\ MIMO\ radar$. In the second regime, the transmit array elements (and recieve array elements) are closely spaced so that the target is in the far field of the transmit-receive array, sometimes referred to as $coherent\ MIMO\ radar$.

There are a variety of potential advantages to using MIMO radar. For given design choices, some of these advantages can be traded for others: improved target detection performance, improved angle estimation accuracy, and decreased minimum detectible velocity. For he first type of MIMO radar in which the individual transmit and receive antennas are seprated widely, the diversity provided by the multiplicity in trasmit and receive angles can be exploited to improve the statistics of the detection performance. For the second type of MIMO radar in which the antennas are spaced relatively closely, angle estimation performance can be improved. The estimation performance improvement can be dramatic when optimized sparse arrays are used.

B. MIMO Channel

Between the transmitter and receiver is the channel. In some sense, the role of radar processing is to estimate and interpret this channel. Without loss of generality, a baseband sampled signal can be considered. The $n_R \times n_S$ received data matrix $\mathbf{Z}$ is given by

$$\mathbf{Z} = \sum_{\delta} \mathbf{H}_{\delta} \mathbf{S}_{\delta} + \mathbf{N} \tag{35}$$

where n_R and n_S are respectively the number of receiving antennas and the number of samples in block. $\mathbf{N}$ contains the sum of noise and external interference. The summation in (1) is over delay δ, which correspond to different range cells.

If the illuminated region contained a single sample scatter in the far field at delay δ, then the channel matrices at all delays would be zero with the exception of $\mathbf{H}_{\delta}$, which would have the structure

$$(\mathbf{H}_{\delta})_{n,m} \propto e^{ik\mathbf{u}.(\mathbf{y}_n + \mathbf{x}_m)} \tag{36}$$

where k is the wavenumber ($k = 2\pi/\lambda$), $\mathbf{u}$ is the pointing vector from transmitter to scatter, and $\mathbf{x}_m$, $\mathbf{y}_n$ are 3-vectros of physical locations for the transmitter and receiver phase centers, respectively. The argument of the exponential reflects differential pathlengths between transmitter and receiver phase centers, given a far-field target in direction $\mathbf{u}$.

C. MIMO Virtual Array: Resolution and Sidelobes

Considering the argument of the exponential in Eq. (2), it can be seen that the MIMO radar apperas to have phase centers located at the virtual locations $\{\mathbf{x}_m + \mathbf{y}_n\}$. Equivalently, we say that the MIMO virtual array centers can be constructed by convolving the locations of the real transmitter and receiver locations.

With a single-point-scattering target, the channel matrix can be written in the form

$$\mathbf{a}(\mathbf{u}) \equiv vec(\mathbf{H}) = \mathbf{a}_R(\mathbf{u}) \otimes \mathbf{a}_T(\mathbf{u}) \tag{37}$$

based on the conventional response vectors

$$(\mathbf{a}_T)_m \equiv e^{ik\mathbf{u}.\mathbf{x}_m} \tag{38}$$

and

$$(\mathbf{a}_R)_n \equiv e^{ik\mathbf{u}.\mathbf{y}_n} \tag{39}$$

D. MIMO radar in clutter-free environments

To investigate the performance of MIMO radar in clutter-free environments, we introduce a Cramer-Rao bound on angle estimation performance. The bound allows arbitrary inter-waveform correlation. The accuracy of angle-of-arrival estimates for an illuminated scatterer depends on the signals transmitted by the radar. For our analysis, the salient characteristic of the transmitted signals is their covariance matrix. The same-time auto- and cross-correlations of the transmitted waveforms are contained within the transmitter's correlation matrix $\mathbf{C}$:

$$\mathbf{C} \equiv \frac{1}{n_S}\left(\mathbf{S}_\delta \mathbf{S}_\delta^\dagger\right)^T \tag{40}$$

Information about the external universe is contained within the recorded data matrix $\mathbf{Z}$. However, given a single point scatter in a range cell at delay δ and the ideal different-time cross-correlations assumed above, no information about the environment is lost if the data are correlated with respect to the transmitted signals at the matching delay. This correlation can be used to construct estimators of the channel at a given delay. For example, consider

$$\hat{\mathbf{H}}_\delta = \frac{1}{n_S}\mathbf{Z}\mathbf{S}_\delta^\dagger = \mathbf{H}\mathbf{C}^T + \frac{1}{n_S}\mathbf{N}\mathbf{S}_\delta^\dagger \tag{41}$$

According to the analysis provided in [5], for the Cramer-Rao estimation bound we have

$$\text{var}\{\theta\} = \frac{n_T}{2|b|^2 p k^2 \cos^2(\theta)(n_R\sigma_T^2 + n_T\sigma_R^2)} \tag{42}$$

where b is the attenuation contained in the channel matrix and p is the total noise normalized transmit power. Also, σ_T and σ_R are the standard deviations of antenna distance along the linear array.

V. MIMO radar detection

A significant benefit of MIMO radar over SIMO radar can be illustrated using a simple model of detection for a narrowband signal. We consider a large-deviation model for detection performance. In general terms, the Kullback-Leibler (KL) divergence is used to characterize the probability of detection P_d. KL divergence is expressed by

$$I(p_2, p_1) = \int \log\left(\frac{p_2(x)}{p_1(x)}\right) p_2(x)dx \tag{43}$$

for two probability densities, $p_1(x)$ and $p_2(x)$, associated with two hypotheses, θ_1 and θ_2. Here, the signal-present hypothesis is denoted θ_1. For MIMO radar, we can evaluate the KL divergence by using a Gaussian signal model. Specifically, let

$$\mathbf{Z} = \mathbf{H}\mathbf{S} + \mathbf{N} \tag{44}$$

express the $n_R \times n_S$ matrix $\mathbf{Z}$ of observations in terms of the $n_R \times n_T$ channel matrix $\mathbf{H}$, the $n_T \times n_S$ waveform matrix $\mathbf{S}$, and the $n_R \times n_S$ noise matrix $\mathbf{N}$. Both $\mathbf{H}$ and $\mathbf{N}$ are chosen to be complex, circular, Gaussian random variates. Their covariances are given by

$$\begin{aligned} \text{cov}(\mathbf{H}) &= \mathbf{R} \otimes \mathbf{T}^* \\ \text{cov}(\mathbf{N}) &= \mathbf{I}_{n_R} \otimes I_{n_S} \end{aligned} \tag{45}$$

Based on the analysis in [5], for he Kullback-Leibler divergence we have

$$\begin{aligned} \log|\mathbf{R} \otimes (\mathbf{S}^\dagger \mathbf{T} \mathbf{S})^* + \mathbf{I}_{n_S n_R}| + \text{tr}(\mathbf{R} \otimes (\mathbf{S}^\dagger \mathbf{T} \mathbf{S})^* + \mathbf{I}_{n_S n_R})^{(-1)} - n_S n_R \\ = \log|\mathbf{R} \otimes ((\mathbf{S}\mathbf{S}^\dagger)^{1/2} \mathbf{T} (\mathbf{S}\mathbf{S}^\dagger)^{1/2})^* + \mathbf{I}_{n_R n_T}| \\ + \text{tr}(\mathbf{R} \otimes ((\mathbf{S}\mathbf{S}^\dagger)^{1/2} \mathbf{T} (\mathbf{S}\mathbf{S}^\dagger)^{1/2})^* + \mathbf{I}_{n_R n_T})^{-1} - n_R n_T \end{aligned} \tag{46}$$

as long as $n_T \leq n_S$.

We are interested in the expected value of Eq. (4) over an ensemble of waveform matrices $\mathbf{S}$. This ensemble is a surrogate for the search performed by the radar. We model this search by assuming that each $\mathbf{S}$ has the singular value decomposition $\mathbf{S} = \mathbf{U}\mathbf{\Lambda}\mathbf{W}^\dagger$, with unitaries $\mathbf{U}$, $\mathbf{W}$, and quasidiagonal rectangular matrix $\mathbf{\Lambda}$ with nonnegetaive diagonal entries. We assume that the unitaries $\mathbf{U}$ and $\mathbf{W}$ are random and uniformly distributed over all unitaries of the appropriate dimensions. The expectation over $\mathbf{U}$ will yield a function of $\mathbf{D} = \mathbf{\Lambda}\mathbf{\Lambda}^\dagger$ that can be optimized. Since Eq. (4) is nonconvex, we approximate the expectation of Eq. (4) in two regimes: that of high integrated SNRs, where the log term dominates, and that of low integerated SNR, where the KL divergence has a quadratic character. Both approximate KL divergences are convex, but in different senses.

A. High SNR

Regarding [5], we have

$$\text{E[KL log term]} \leq s \sum_k \log\left(1 + \frac{\beta r_k d}{s}\right) \tag{47}$$

with integral s, $1 \leq s \leq n_T$. In (5), $\beta = \text{tr}(\mathbf{T})/n_T$, r_k denotes the eigenvalues of $\mathbf{R}$ and d is the summation of diagonal entries of $\mathbf{D}$. The bound is maximized when $s = n_T$ and hence $\mathbf{D} = (d/n_T)\mathbf{I}n_T$. For this choice of $\mathbf{D}$, Eq. (5) becomes an equality that is achieved at all values of $\mathbf{U}$. In other words, uncorrelated waveforms of equal power provide a uniform upper bund on the approximate, average KL divergence in the high-SNR regime.

B. Weak-Signal Regime

The weak-signal approximation to the KL divergence is

$$\text{E[K.L.quad. approx.]} = \frac{1}{2}\text{tr}\mathbf{R}^2.(\alpha \text{tr}(\mathbf{D}^2) + \beta(\text{tr}(\mathbf{D}))^2) \tag{48}$$

for unknown scalars α and β where $\alpha > 0$. The approximation is maximized when $\mathbf{D}$ is rank 1.

C. Optimal Beamforming without Search

In the special case in which there is a single target, the KL divergence is

$$\log[1 + \|\mathbf{a}_R\|^2(\mathbf{a}_T^\dagger(\mathbf{S}\mathbf{S}^\dagger)^*\mathbf{a}_T)] - \frac{\|\mathbf{a}_R\|^2(\mathbf{a}_T^\dagger(\mathbf{S}\mathbf{S}^\dagger)^*\mathbf{a}_T)}{1 + \|\mathbf{a}_R\|^2(\mathbf{a}_T^\dagger(\mathbf{S}\mathbf{S}^\dagger)^*\mathbf{a}_T)} \tag{49}$$

which is maximized by maximizing $\mathbf{a}_T^\dagger(\mathbf{S}\mathbf{S}^\dagger)^*\mathbf{a}_T$ over $\mathbf{S}$ given a constraint on $\text{tr}(\mathbf{S}\mathbf{S}^\dagger)$. The maximum is achieved when $\mathbf{S} \propto (\mathbf{a}_T)^*\mathbf{s}^\dagger$ for some $n_S \times 1$ vector $\mathbf{s}$, which amounts to forming a beam on the target.

D. Nonfading Targets

The improvement in detection suggested by the KL divergence disappears when the target exhibits no fading. In this case, we have

$$\text{E[KL divergence]} = \frac{\text{tr}(\mathbf{S}\mathbf{S}^\dagger)}{n_T}\text{tr}(\mathbf{H}^\dagger\mathbf{\Gamma}^{-1}\mathbf{H}) \tag{50}$$

The result states that the total energy returned is the same no matter what waveforms are employed; only that energy matters for detection.

VI. MIMO Radar with Moving Targets in Clutter

A. GMTI Radars

Ground moving-target indicator (GMTI) radar offers a potentially important application for MIMO radar techniques. Sensors illuminating the ground suffer strong clutter returns that degrade detection performance. Moving targets generate responses that differ from those of stationary ground clutter returns. Adaptive filtering in Doppler and angle can mitigate the effects of ground clutter. The resulting performance is often characterized in terms of an effective SNR loss incurred as a result of the adaptive filtering. By improving the seprability of targets and clutter, MIMO techniques can significantly reduce SNR loss, given equivalent physical apertures.

B. Signal Model

Consider, at a frequency of w radians per second, the response of a target at position $\mathbf{r}$ to a signal transmitted from a phase center at $\mathbf{x}_T$ and received at a phase center at $\mathbf{y}_R$. The target is assumed to be moving with velocity vector $\mathbf{v}_t$, while the sensor platform moves with velocity vector $\mathbf{v}_p$. The function $\text{dist}(\mathbf{r}, \mathbf{q})$ expresses the Euclidean distance between $\mathbf{r}$ and $\mathbf{q}$, with $\mathbf{q}$ representing a position on the sensor platform. The distance function can be linearized about $(\mathbf{r}_0, \mathbf{q}_0)$ by the approximation

$$\text{dist}(\mathbf{r}, \mathbf{q}) \approx \text{dist}(\mathbf{r}_0, \mathbf{q}_0) + [(\mathbf{r} - \mathbf{r}_0) - (\mathbf{q} - \mathbf{q}_0)].\mathbf{u}(\mathbf{r}_0, \mathbf{q}_0) \tag{51}$$

where $\mathbf{u}$ is a unit vector pointing from the sensor to the target. With this notation, the response vector at time t can be represented by the far-field approximation

$$e^{-2i(w/c)\text{dist}(\mathbf{r}_0,\mathbf{q}_0)}.e^{i(w/c)[2(\mathbf{v}_p-\mathbf{v}_t)t+\mathbf{y}_R+\mathbf{x}_T].\mathbf{u}(\mathbf{r}_0,\mathbf{q}_0)} \tag{52}$$

where the exponent expresses phase change along the path fom $\mathbf{x}_T$ to $\mathbf{y}_R$. The speed of light is denoted by c. The leftmost vector in Eq. (2) is common to all reponse vectors and is ignored in the following. The target response is represented only at the center frequency. Only narrowband models are considered.

Define, for a target at a fixed direction $\mathbf{u}$ and Doppler $\delta \equiv \mathbf{v}_t.\mathbf{u}/\|\mathbf{v}_p\|$, the response vector $\mathbf{a}(u_x, u_y, \delta)$ with components [5]

$$a_{TRt}(u_x, u_y, \delta) = e^{i(w/c)[(2t\mathbf{v}_p + \mathbf{y}_R + \mathbf{x}_T).u - 2t\|\mathbf{v}_p\|\delta]} \tag{53}$$

The vector $\mathbf{a}(u_x, u_y, \delta)$ has the tensor product structure

$$\mathbf{a}(u_x, u_y, \delta) = \mathbf{a}_T(u_x, u_y) \otimes \mathbf{a}_R(u_x, u_y) \otimes \mathbf{a}_D(u_x, u_y, \delta) \tag{54}$$

with

$$(\mathbf{a}_D)_t(u_x, u_y, \delta) \equiv e^{i(w/c)2(\mathbf{v}_p.\mathbf{u} - \|\mathbf{v}_p\|\delta)t} \tag{55}$$

C. SNR Loss

The performance of GMTI radar is often characterized in terms of adapted SNR. Ground clutter returns interfere with the target return and impair. Since the clutter exhibits correlation in angle and Doppler, it can be suppressed when the target is separated from the clutter ridge. Adaptive filtering in angle and Doppler is used to achieve the best possible SNR by adaping the filtering to the observed clutter statistics. Also, it is common to measure the performance of GMTI radars in terms of SNR loss, which is expressed by the ratio of adapted SNR to the SNR achieved without any clutter returns.

Given a target response vector $\mathbf{a}$, a weight vector $\mathbf{w}$, and a total noise-plus-clutter covariance $\mathcal{R}$, the adapted SNR can be expressed as follows [5]

$$\frac{|\mathbf{w}^\dagger \mathbf{a}|^2}{\mathbf{w}^\dagger \mathcal{R} \mathbf{w}} \tag{56}$$

When the noise-plus-clutter covariance represents additive white noise with identity covariance, the best adapted SNR becomes $|\mathbf{a}|^2$. The addition of clutter degrades SNR. The ratio of the largest adapted SNR with noise and clutter to the SNR with noise alone is expressed by [5]

$$\max_{\mathbf{w}} \frac{|\mathbf{w}^\dagger \mathbf{a}|^2}{|\mathbf{a}|^2 \mathbf{w}^\dagger \mathcal{R} \mathbf{w}} = \frac{\mathbf{a}^\dagger \mathcal{R}^{-1} \mathbf{a}}{|\mathbf{a}|^2} \tag{57}$$

Regarding [5], SNR loss is expressed by

$$\frac{\mathbf{a}^\dagger \mathcal{R}^{-1} \mathbf{a}}{|\mathbf{a}|^2} \approx \sin^2\left(\frac{\pi b(\epsilon)}{2}\right) \tag{58}$$

As a function of SNR loss, the target Doppler becomes approximately [5]

$$\mathbf{v}_t.\mathbf{u} \approx \frac{2}{\pi}\text{arcsin}(\sqrt{\text{SNR}_{\text{loss}}}).\|\mathbf{v}_p\|\sqrt{\frac{M_{22}}{|\mathbf{M}|} + \rho^{-1}} \tag{59}$$

This approximation is tight for losses greater than 3 dB [5].

VII. Generalized MIMO Radar Ambiguity Functions

A. Introduction

Two of the primary functions of a radar are to detect targets and estimate parameters of a model used to describe those targets. Early radars could distinguish one unambiguous parameter, $range$. MIMO radar can enable the unambiguous observation of additional target parameters. Modern radar systems are designed to be highly accurate for their intended purpose. Designers and engineers need to know the level of resolution to expect from a particular system configuration. Some of the tools used to characterize performance are statistical parameter estimation bounds and ambiguity functions. Typically, parameter estimation bounds such as the Cramer-Rao (CR) bound depend on the ambiguity function. The classic ambiguity function was introduced by

Woodward and is used to characterize the local and global resolution properties of time delay and Doppler for narrowband waveforms.

B. Background

The earliest radar systems were designed to make simple measurements. As systems became more complex and precise, the inevitable issues of accuracy and resolution arose. Early researchers introduced a function called the $ambuigity\ function$ that captures some of the inherent resolution properties of a radar system. The ambuigity function is generally identified with Woodward because of his pioneering work. Woodward was interested in characterizing how well one could identify the target parameters of time delay (range) and Doppler (range rate) based on the transmission of a known waveform $s(t)$. Based on Woodward work, the $radar\ ambuigity\ function$ is as follows [5]

$$\chi(\Delta\tau, \Delta f_v) = \left| \int s(t)s^*(t - \Delta\tau)e^{-j2\pi\Delta f_v t}dt \right|^2 \tag{60}$$

Among the more well-known properties of this function is the fact that there is an inherent ambuigity or duality between resolution in time and resolution in frequency. For a given time-bandwidth product, targets can not be resolved perfectly in time and frequency simultaneously.

It was recognized that Woodward's ambuigiy function needed modification to handle larger bandwidth signals, long duration, and targets with high velocity. Some researchers use parametric models that more accurately reflect the actual physical phenomena involved with moving targets and reflecting signals. As a traveling wavefield reflects off a moving target, the field either expands or compresses in time as a result of the movement of the target. When a narrowband waveform is transmitted, this compressive effect is ignored for the waveform's complex envelope and considered only for the carrier. The condition that must be met for this compressive effect to be ignored is based on the time-bandwidth product TB, target velocity v, and the propagation speed of traveling waves c:

$$\frac{2vBT}{c} \ll 1 \tag{61}$$

The ambuigity function derived for conditions that violate (2) is

$$\chi(\tau, f_v) = \left| \sqrt{\lambda} \int s(t) s^*(\lambda(t-\tau)) e^{-j2\pi f_v t} dt \right|^2 \tag{62}$$

The term $\lambda = 1 + f_v/f_c$ specially accounts the streching/compressing in time of the reflected signal.

C. MIMO signal model

A general signal model for MIMO radar is introduced to clarify interaction between the transmitted signals, the target, and the noise. A MIMO radar system consists of N_T transmit sensors and N_R receive sensors. A series of independent signals is transmitted from each transmit sensor in a coherent fashion. The propagation of a signal fom a transmit sensor to a receive sensor consists of propagation through a channel with three components: a forward-propagating channel to the target, a reflecting/scattering target, and a reverse channel to the receive sensor. Both the forward and reverse channels will be jointly parameterized by a parametric model with parameter θ.

The target will be considered point-like in nature. For the ith transmit sensor and jth receive sensor the scattering function will be denoted as $a_{j,i}$. The received signal at the jth receive sensor due to the ith transmit waveform can be espressed as

$$r_{j,i}(t) = s_i(t, \theta, j) a_{j,i} + n_j(t) \tag{63}$$

The term $n_j(t)$ is an additive noise process independent of the target scattering function $a_{j,i}$. The term $s_i(t, \theta, j)$ represents the ith transmitted signal modified according to the parametric channel model with parameter θ for the jth receive sensor. Because ther are N_T transmit signals, the received signal at the jh receive sensor is the linear combination of all such signals as in Eq. (4). Also the received signal will most likely be sampled, so we can use matrix vector notation to represent the received signal as an $N \times 1$ vector [5]

$$\mathbf{r}_j = \mathbf{S}(\theta, j)\mathbf{a}_j + \mathbf{n}_j \tag{64}$$

where $\mathbf{S}(\theta, j) = [s_1(\theta, j), s_2(\theta, j), \ldots, s_{N_T}(\theta, j)]$. There are a total of N_R receive sensors, the data from each can be composed into a single vector size $NN_R \times 1$

$$\mathbf{S}(\theta)\mathbf{a} + \mathbf{n} \tag{65}$$

The desription of probability density function (pdf) for the data has been provided in [5]. A common pdf used in radar signal processing is the complex Gaussian distribution. The received data vecor has the distribution

$$\mathbf{r} \sim \mathcal{CN}(0, \sigma_s^2 \mathbf{S}(\theta)\mathbf{\Sigma}\mathbf{S}^H(\theta) + \sigma_n^2 \mathbf{I}_{NN_R}) \tag{66}$$

where $\sigma_s^2 \mathbf{\Sigma}$ is the covariance matrix of the target response vector and $\sigma_n^2 \mathbf{I}_{NN_R}$ denotes the covariance matrix of the noise vector.

D. MIMO parametric channel model

we introduce a parametric model that describes how the transmit signals appear at the receive sensors. The parts of this model include signal transmission, signal propagation, signal reflection, and signal reception.

1) Transmit signal model: MIMO radars consist of coherent networks of transmit and receive sensors. Both transmit and receive sensors spatial location will have Cartesian coordinates. These coordinates are refrenced to a predefined origin shared by both the sensors and the target. All sensors operate at the same carrier frequency f_c that is refrenced to the same phase angle on transmit and receive for all sensors. It is assumed that the signal bandwidth satisfies the condition $B/2 < f_c$. Under this assumption, the common complex envelope notation can be applied to the form of the transmitted signal

$$g_i(t) = 2\text{Re}\{s_i(t)e^{j2\pi f_c t}\} \tag{67}$$

where $s_i(t)$ is the complex envelope of the ith waveform.

2) Channel and Target models: For each transmit/receive sensor pair there exists a forward transmit channel to the target and a reverse receive channel from the target. These channels are modeled as lossless time delay and phase shift channels. All other losses will be assumed to be due to the target reflection process. The target model applied here is point-like target moving with constant velocity. Point targets will be described by a parameter vector θ consisting of a position vecor component $\mathbf{p}$ and velocity vector component $\mathbf{v}$. Depending on the array configuration, each target could be identified by up to six unambiguous parametrs, three for position and three for velocity. In general, if the signal $h(t)$ is transmitted from sensor i, reflects off a target, and is received by sensor j, the response can be described by

$$\beta(\theta, i, j)h(t - \delta(t, \theta, \mathbf{x}_{i,T}, \mathbf{x}_{j,R})) \tag{68}$$

From [5], we have

$$\delta(t, \theta, \mathbf{x}_{i,T}, \mathbf{x}_{j,R}) \approx \tau_{i,j}(\mathbf{p}) - \frac{f(\theta)}{f_c}(t - \tau_{i,j}(\mathbf{p})) \tag{69}$$

The term $\tau_{i,j}(\mathbf{p})$ is simply the two-way time delay due to a target located at $\mathbf{p}$ with transmit and receive sensors at $\mathbf{x}_{i,T}$ and $\mathbf{x}_{j,R}$

$$\tau_{i,j}(\mathbf{p}) = \tau_i(\mathbf{p}) + \tau_j(\mathbf{p}) = \frac{\|\mathbf{p} - \mathbf{x}_{i,T}\|}{c} + \frac{\|\mathbf{p} - \mathbf{x}_{j,R}\|}{c} \tag{70}$$

The term $f(\theta)$ is the frequency shift caused by a target moving with velocity vector $\mathbf{v}$ and position $\mathbf{p}$. $f(\theta)$ is defined as

$$f_{i,j}(\theta) = \frac{1}{\lambda}\frac{d}{dt}R_T + \frac{1}{\lambda}\frac{d}{dt}R_R = f_i(\theta) + f_j(\theta) \tag{71}$$

Also the stretch factor is defined as

$$\gamma_{i,j}(\theta) = 1 + \frac{f_{i,j}(\theta)}{f_c} \tag{72}$$

3) Received signal parametric model: Using the parametric model described above, the received signal at the jth receive sensor after demodulation to baseband is

$$r_j(t,\theta) = \sum_{i=1}^{N_T} a_{j,i}\sqrt{\gamma_{i,j}(\theta)}s_i(\gamma_{i,j}(\theta)(t - \tau_{i,j}(\mathbf{p})))e^{-j2\pi\tau_{i,j}(\mathbf{p})(f_c+f_{i,j}(\theta))}e^{j2\pi f_{i,j}(\theta)t} + n_j(t) \tag{73}$$

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