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A probabilistic tool for monitoring the US-Canadian dollar exchange rate

A derivative based approach with the Heston model

David W. Maybury
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Scientific Report
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Abstract

Using the Heston model stochastic volatility model with publicly available US-Canadian dollar exchange rate (USDCAD) option price data, I provide a client service tool that elicits the implied USDCAD risk neutral density function. The program displays this information through user specified dates for extracting the probability density functions, and through a heat map over the time horizon of the input data. Senior decision makers can use the information provided by this tool to monitor exchange rate risk over the fiscal year.

Significance for defence and security

In this paper I provide a client service tool that monitors the US-Canadian dollar exchange rate (USDCAD). I extract the USDCAD risk neutral probability measure using the Heston stochastic volatility model in conjunction with public data from the Canadian Derivatives Exchange. The program displays exchange rate risk information across the expiry dates of derivative contracts on the exchange, and in particular the model:

- extracts and interpolates the USDCAD risk neutral probability density function from derivative prices; and
- displays the USDCAD probability density as a heat map over the expiry interval of the derivative contract data.

Résumé

En me servant du modèle à volatilité stochastique de Heston et des données accessibles au public sur le prix de l'option du taux de change entre les devises américaine et canadienne (USD-CAD), je fournis un outil de service à la clientèle qui permet de dériver la fonction implicite de densité neutre au risque USD CAD. Le programme affiche ces renseignements grâce aux dates précisées par l'utilisateur pour extraire les fonctions de densité de probabilité et grâce à un diagramme d'exposition au risque dans l'horizon temporel des données d'entrée. Les principaux décideurs peuvent utiliser les renseignements fournis par cet outil pour suivre le risque du cours de change durant l'année financière.

Importance pour la défense et la sécurité

Dans ce document, je présente un outil de service à la clientèle qui permet d'assurer le suivi du taux de change entre le dollar américain et le dollar canadien (USD-CAD). J'extrais la mesure de probabilité neutre à l'égard du risque des USD-CAD à l'aide du modèle à volatilité stochastique de Heston avec les données publiques provenant de la Bourse canadienne de produits dérivés. Le programme affiche les renseignements sur le risque du cours de change selon l'échéance des contrats dérivés en bourse, plus particulièrement le modèle :

- permet d'extraire et d'interpoler la fonction de densité de probabilité neutre à l'égard du risque des USD-CAD à partir du cours des produits dérivés ;
- affiche la densité de probabilité des USD-CAD sous forme de diagramme d'exposition au risque durant la période de validité des données sur les contrats dérivés.

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1 Introduction

As exemplified by the Arctic/Offshore Patrol Ship, the Next Generation Fighter, and the Single Class Surface Combatant programs, the Department of National Defence (DND) has embarked on an ambitious capital renewal path for the Canadian Armed Forces (CAF). In this new procurement environment, DND's project risk management expertise will play a central role for final delivery success. Senior decision makers need a detailed understanding of project financial risks. Given the amount of foreign military equipment expected within each capital project—especially from the US—DND requires a monitoring process for foreign exchange risk. In this paper, we provide a tool for monitoring USDCAD on horizons of less than one year.

Canada's monetary and fiscal authorities do not target a Canadian dollar exchange rate with respect to foreign currencies. Instead, Canada relies on market forces to set the value of the Canadian dollar. The floating exchange rate regime employed by Canada allows for automatic price adjustments which reflect Canada's economic fundamentals, fiscal and monetary policy, and trade balances with the rest of the world. While a market determined exchange rate provides an economic benefit to Canada, DND projects must cope with uncertainty in foreign acquisition costs.

Over the last six years, the Centre for Operational Research and Analysis (CORA) has advanced DND's understanding of foreign exchange transaction risk in operational planning and military procurement. In 2007, CORA provided a Value-at-Risk (VaR) tool for foreign exchange transactions with both the National Procurement and Capital accounts [1]. Following this work, CORA extended [2] the VaR models to allow for simultaneous multiple currency exposures within projects. In 2011, and in 2013, CORA provided complete counterfactual derivative based hedging examples [3], [4] for the Assistant Deputy Minister of Materiel (ADM(Mat))'s US dollar obligations over both short and long horizons (November 2009–July 2010, and FY 2009–2013). As a proof of concept, CORA created a high level prototype software tool [5] in 2013, which implemented a set of financial models to extract the risk neutral probability distribution of the USDCAD from the Canadian Derivatives Exchange. In this paper, I follow the analysis found in [5] by providing a much more detailed client service tool for decision makers. I extract the USDCAD risk neutral probability measure using the Heston stochastic volatility model [6], replicating a function within the Bloomberg terminal.

1.1 Scope and proposals

CORA's client, Directorate Costing Services (DCostS), requires a USDCAD exchange rate monitoring tool. I provide a client service solution, based on the Heston model [6] and data from the Canadian Derivatives Exchange, which:

- extracts and interpolates the USDCAD risk neutral probability density function from derivative prices; and
- displays the USDCAD risk as heat map over the expiry interval of the derivative contract data.

2 The model

In this section, I outline the theory behind derivative pricing along with the Heston model option pricing solution. Readers who are not interested in the theory can skip this section.

Consider a set of $N+1$ assets $\{S_t^0, S_t^1, \ldots S_t^N\}$, in which S_t^0 represents the bank account numeraire,

$$dS_t^0 = rS_t^0\,dt, \tag{1}$$

with the discount factor, $D(0,t) \equiv 1/S_t^0$.

Suppose that asset prices are positive semimartingale processes adapted to a probability space $(\Omega, \mathscr{F}, \mathbb{P})$, with a right continuous filtration $\mathbb{F} = \{\mathscr{F}_t : 0 \leq t \leq T\}$ in which T represents the time horizon. A trading strategy consists of a vector ϕ of predictable, bounded processes, $\{\phi_t^n : 0 \leq t \leq T\}$, with a value process given by,

$$V_t(\phi) = \sum_{n=0}^{N} \phi_t^n S_t^n, \tag{2}$$

and with an associated gains process,

$$G_t(\phi) = \sum_{n=0}^{N} \int_0^t \phi_t^n\, dS_t^n. \tag{3}$$

We say that a trading strategy is self-financing if $V(\phi) \geq 0$ and

$$V_t(\phi) = V_0(\phi) + G_t(\phi). \tag{4}$$

An arbitrage opportunity exists if $V_0(\phi) = 0$, with $\mathbb{P}(V_T \geq 0) = 1$, i.e., it is possible to set up a portfolio with zero initial cost and to proceed with a trading strategy that is guaranteed not to lose money, yet has a positive probability of earning money. If an equivalent probability measure $\mathbb{Q}$ exists ($\mathbb{Q}(A) = 0$ iff $\mathbb{P}(A) = 0$), with the Radon-Nikodym derivative $d\mathbb{Q}/d\mathbb{P}$ square integrable with respect to $\mathbb{P}$, such that $D(0,\cdot)S$ is an $(\mathbb{F}, \mathbb{Q})$ martingale, then the market model will not permit arbitrage. For details further see [7], [8], and [9].

A contingent claim is a square integrable random variable on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$. Roughly, a contingent claim is an insurance contract. We say that a contingent claim, C, is attainable if there exists a self-financing trading strategy ϕ with $V_T(\phi) = C$ and time t value function $\pi_t = V_t(\phi)$. An attainable contingent claim implies that its trading strategy replicates, or hedges, the pay-off in every possible state of the world. If an equivalent martingale measure, $\mathbb{Q}$, exists then, for an attainable contingent claim C, the associated price π_t is given by,

$$\pi_t = V_t(\phi) = \mathbb{E}^{\mathbb{Q}}[D(t,T)C|\mathscr{F}_t]. \tag{5}$$

In eq.(5) we see that the value function of the trading strategy that replicates the pay-off, C, has the form of an expectation under an equivalent martingale measure, yielding the no arbitrage price. Any price different from π_t would allow a trader to take a starting position in the trading strategy ϕ without cost, thereby permitting the possibility of riskless gains.

A financial market is complete iff every contingent claim is attainable, that is, every contingent claim can be replicated or hedged. In such markets, the equivalent martingale measure $\mathbb{Q}$ is unique. Markets in which more than one martingale measure exists, while arbitrage-free, are not complete, implying that not all contingent claims can be hedged.

To see the implication of these ideas, consider the Black-Scholes-Merton (BSM) market model for an asset $S(t)$,

$$dS(t) = \alpha S(t)\,dt + \sigma S(t)\,dW(t), \tag{6}$$

in which $dW(t)$ denotes the differential element for Brownian motion (see [10], [11] for details on stochastic calculus, and [12] for applications in a financial setting), α is the asset specific growth rate, and σ represents the asset's volatility. All parameters of the model are constants, and we assume a continuous compounding interest rate, r.

Consider the contingent claim $c(S,T) = (S_T - K)^+$, a contract called a European call option, (the put option has the form $c(S,T) = (K - S_T)^+$) which pays the positive difference between the terminal asset price and some set price K (called the strike price). We wish to attain this contingent claim with a self-financing trading strategy and we have two assets from which to construct a strategy—a bank account, which earns riskless interest, and the asset itself $(N+1=2)$. To begin, we split a portfolio, $X(t)$, at time t, between the asset and the bank account. At time t, let $\Delta(t)$ denote the number of shares of the asset. After a small time increment, the portfolio, $X(t)$, becomes,

$$\begin{aligned} dX(t) &= \Delta(t)\,dS(t) + r(X(t) - \Delta(t)S(t))\,dt \\ &= rX(t)dt + \Delta(t)(\alpha - r)S(t)\,dt + \Delta(t)\sigma S(t)\,dW(t), \end{aligned} \tag{7}$$

in which we applied eq.(6). The differential of the discounted asset price, $f(S,t) = e^{-rt}S(t)$ reads,

$$\begin{aligned} d(e^{-rt}S(t)) &= df(S,t) \\ &= \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial S}dS + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}dS(t)dS(t) \\ &= (\alpha - r)e^{-rt}S(t)\,dt + \sigma e^{-rt}S(t)\,dW(t), \end{aligned} \tag{8}$$

leading to the relationship,

$$d(e^{-rt}X(t)) = \Delta(t)d(e^{-rt}S(t)). \tag{9}$$

As expected, we see that the discounted asset grows at the rate $(\alpha - r)$, and the changes in the discounted portfolio come only from changes in the discounted stock price. Similarly, we see that the call option at $t < T$ obeys the relationship,

$$\begin{aligned} dc(S(t),t) &= \frac{\partial c}{\partial t}dt + \frac{\partial c}{\partial S}dS + \frac{1}{2}\frac{\partial^2 c}{\partial S^2}(dS)^2 \\ &= \left[\frac{\partial c}{\partial t} + \alpha S(t)\frac{\partial c}{\partial S} + \frac{1}{2}\sigma^2 S^2(t)\frac{\partial^2 c}{\partial S^2}\right]dt \\ &\quad + \sigma S(t)\frac{\partial c}{\partial S}dW(t), \end{aligned} \tag{10}$$

yielding the differential of the contigent claim price,

$$\begin{aligned} d(e^{-rt}c(S(t),t)) &= e^{-rt}\left[-rc + \frac{\partial c}{\partial t} + \alpha S(t)\frac{\partial c}{\partial S} + \frac{1}{2}\sigma^2 S^2(t)\frac{\partial^2 c}{\partial S^2}\right]dt \\ &\quad + e^{-rt}\sigma S(t)\frac{\partial c}{\partial S}dW(t). \end{aligned} \tag{11}$$

An attainable contingent claim requires the matching condition,

$$d(e^{-rt}X(t)) = d(e^{-rt}c(S(t),t)) \text{ for all } t \in [0,T) \tag{12}$$

with $X(0) = c(S(0),0)$, representing the initial investment needed to start the self-financing strategy. Equating the two differentials, we find

$$\begin{aligned} &\Delta(t)(\alpha - r)S(t)\,dt + \Delta(t)\sigma S(t)\,dW(t) \\ &= \left[-rc + \frac{\partial c}{\partial t} + \alpha S(t)\frac{\partial c}{\partial S} + \frac{1}{2}\sigma^2 S^2(t)\frac{\partial^2 c}{\partial S^2}\right]dt \\ &\quad + \sigma S(t)\frac{\partial c}{\partial S}dW(t). \end{aligned} \tag{13}$$

The $dW(t)$ and dt terms respectively yield,

$$\Delta(t) = \frac{\partial c}{\partial S} \quad \text{for all } t \in [0,T), \tag{14}$$

and

$$rc = \frac{\partial c}{\partial t} + rS(t)\frac{\partial c}{\partial S} + \frac{1}{2}\sigma^2 S^2(t)\frac{\partial^2 c}{\partial S^2} \quad \text{for all } t \in [0,T). \tag{15}$$

With terminal condition $c(S,T) = (S-K)^+$, eq.(15), called the Black-Scholes-Merton equation, gives the time evolution of the contingent claim while eq.(14) provides the no arbitrage hedging rule, which makes the claim attainable. With these rules, we have a replicating portfolio and hence a hedge to the short position. Notice that eq.(15) does not contain the asset growth parameter, α, only the riskless interest rate, r. The replicating portfolio (trading strategy) does not depend on subjective opinions regarding the prospects of the asset. In light of eq.(5), the contingent claim price has an expression under an equivalent martingale measure, the risk neutral measure ($\mathbb{Q}$), which ensures the absence of arbitrage. Thus, the solution of eq.(15) has the form,

$$c(S(t),t) = \mathbb{E}^{\mathbb{Q}}[e^{-r(T-t)}(S-K)^{+}|\mathscr{F}_t]. \tag{16}$$

The connection between stochastic calculus and partial differential equations rests on the Feynman-Kac Lemma—a remarkable result used extensively in derivative pricing problems.

Independent of the market model, call and put options have important market implications. The equivalent martingale measure expectation, eq.(16), has the form

$$c(K,T) = e^{-r(T-t)} \int_0^\infty (y-K)^{+} f(y,T;S_t)\, dy, \tag{17}$$

in which $f(y,T;S_t)$ denotes the probability density of the risk neutral measure conditioned on the information at time t. Differentiating eq.(17) with respect to the strike price, K, yields the distribution function relationship,

$$\int_0^x f(y,T;S_t) dy = F(x) = 1 + e^{r(T-t)} \frac{\partial c(x,T)}{\partial x}. \tag{18}$$

For put options we have the analogous result,

$$p(K,T) = e^{-r_d(T-t)} \int_0^\infty (K-y)^{+} f(y,T;S_t)\, dy, \tag{19}$$

yielding,

$$\int_0^x f(y,T;S_t) dy = F(x) = e^{r(T-t)} \frac{\partial p(x,T)}{\partial x}. \tag{20}$$

We see that using market prices for calls and puts allows one, in principle, to construct the risk neutral probability distribution. In this paper, I report the risk neutral density function of the USDCAD by calibrating the Heston model to observed derivative market prices quoted on the Canadian Derivatives Exchange.

2.1 The Heston model

The Black-Scholes-Merton market model does not capture the details of real foreign exchange derivative markets. On a technical note, the result should not surprise us—attainable claims are perfectly redundant to their underlying replicating trading strategies. Redundant assets have no reason to trade as separate securities; thus the existence of a foreign exchange derivative market tells us that real markets are more complicated than the BSM or any fully hedgeable model[1]. Furthermore, eq.(6) implies a lognormal distribution for asset

[1] In reality, traders use the BSM model in reverse by quoting implied volatility, σ. The resulting volatility smile and volatility surface contains information that allows traders to construct sophisticated approximate hedging strategies and provide security price interpolation. For details in a foreign exchange context see [13].

prices, which the empirical data rejects. The Heston model [6] attempts to capture realistic features of foreign exchange derivative prices by promoting the volatility parameter of the BSM model to a stochastic process, namely

$$dS(t) = \mu S(t)\,dt + \sqrt{\nu} S(t)\,dW_1 \tag{21}$$

$$d\nu = \kappa(\theta - \nu)dt + \sigma\sqrt{\nu} dW_2 \tag{22}$$

$$dW_1\,dW_2 = \rho\,dt \tag{23}$$

in which ρ represents the correlation between the two driving Brownian motions and κ, θ, ν, and σ are model parameters. The parameter μ leaves the solution under the risk neutral measure. As before, we approach the problem by setting up a replication trading strategy. Proceeding similarly to the BSM model, the replicating portfolio, for some general terminal pay-off $g(T,S) = U(T,\nu,S)$, takes the form,

$$dX = \Delta\,dS + \Gamma\,dZ + r_d(X - \Gamma Z)\,dt - (r_d - r_f)\Delta\,dt \tag{24}$$

in which S is the underlying asset (in this case, the foreign currency), Z is some other derivative or asset, and r_d and r_f are the domestic and foreign interest rates respectively. The variables Δ and Γ represent the positions in the underlying assets S, and Z. The replicating portfolio requires a second asset to hedge the uncertainty in the Brownian motion driving the volatility process. Attainability requires that we have as many assets as we have sources of uncertainty. Specializing the contingent claim $U(T,\nu,S)$ to the call option, we can write,

$$\begin{aligned} c(t,K) &= e^{-r_d\tau}\mathbb{E}^{\mathbb{Q}}[(S_T - K)^+|\mathscr{F}_t] \\ &= e^{-r_d\tau}\mathbb{E}^{\mathbb{Q}}[S_T\mathbb{I}_{S_T>K}|\mathscr{F}_t] - e^{-r_d\tau}\mathbb{E}^{\mathbb{Q}}[\mathbb{I}_{S_T>K}|\mathscr{F}_t], \end{aligned} \tag{25}$$

in which $\tau = T - t$ denotes the time left to maturity, and $\mathbb{I}_{(\cdot)}$ represents the indicator function. The matching portfolio method results in the solution,

$$c(t,K) = e^x e^{-r_f\tau} P_1 - e^{-r_d\tau} K P_2 \tag{26}$$

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathscr{R}\left(\frac{e^{i\phi\ln(K)} f_j(\phi;\nu,x)}{i\phi}\right) d\phi, \tag{27}$$

in which $x = \ln(S_t)$ and

$$f_j(\phi;x,\nu) = \exp\left[C_j(\tau,\phi) + D_j(\tau,\phi)\nu + i\phi x\right], \tag{28}$$

with

$$D_j = \frac{b_j - \rho\sigma i\phi + d_j}{\sigma^2}\left(\frac{1 - e^{d_j\tau}}{1 - g_j e^{d_j\tau}}\right) \tag{29}$$

$$C_j = (r_d - r_f)i\phi\tau + \frac{a}{\sigma^2}\left[(b_j - \rho\sigma i\phi + d_j)\tau - 2\ln\left(\frac{1 - g_j e^{d_j\tau}}{1 - g_j}\right)\right], \tag{30}$$

with

$$
\begin{aligned}
b_1 &= \kappa - \sigma\rho && (31)\\
b_2 &= \kappa && (32)\\
d_j &= \sqrt{(\rho\sigma i\phi - b_j)^2 - \sigma^2(2u_j i\phi - \phi^2)} && (33)\\
g_j &= \frac{b_j - \rho\sigma i\phi + d_j}{b_j - \rho\sigma i\phi - d_j}. && (34)
\end{aligned}
$$

A similar result holds for the put option; for solution details, see [6], and [14]. The Heston model is not complete—we do not have the necessary asset, Z, to hedge the uncertainty arising from the stochastic volatility process, leaving us with an undetermined market price of volatility. Calls and puts have a special symmetry which allows us to eliminate the market price of volatility; see [14] for technical details. Furthermore, we impose the restriction,

$$2\kappa\theta \geq \sigma^2, \tag{35}$$

to ensure positivity of the volatility process [15].

3 Data example: March 20, 2015

The Canadian Derivatives Exchange quotes the USDCAD call and put contracts under the ticker symbol USX. All USX contracts are European style with an expiry cycle which has, at a minimum, the nearest three expiries (the third Friday of the month) plus the next two expiries in the designated quarterly cycle: March, June, September, December [16]. Furthermore, trading ceases at 12:00 p.m. on the third Friday of the contract month, provided it is a business day. If it is not a business day, trading will cease at 12:00 p.m. on the first preceding business day [16]. The Canadian Derivatives Exchange lists all relevant contract data on their website [16] which I scrape and parse using a purpose built script. I also scrape US London Interbank Offer Rate (LIBOR) data from [17], which I convert to continuous time from US LIBOR's day count convention. Since the discontinuation of Canadian LIBOR [18], I interpolate between the Canadian overnight lending rate and the two year Canadian government bond rate as a proxy for the domestic interest rate.

To perform the fit to data, I acquire vectors of market call and put prices from USX, eliminating stale or illiquid quotes by rejecting contracts with bid/ask spreads larger than 20% of the midpoint price. Using the price vectors, $(c_1, c_2, \ldots, c_n)$, $(p_1, p_2, \ldots, p_m)$, I minimize, in a least square sense, the difference between the Heston determined prices, c_H, p_H, and the empirical midpoint price vectors with respect to the model parameters, $\Omega = \{\kappa, \theta, \rho, \sigma, \nu\}$, for each contract month expiry, T_M:

$$\min_{\Omega_M} \sum_i^n \omega_i^M \left(c_H(K_i, T_M) - c(K_i, T_M)\right)^2 + \sum_i^m \xi_i^M \left(p_H(K_i, T_M) - p(K_i, T_M)\right)^2. \tag{36}$$

In eq.(36), ω_i^M, and ξ_i^M denote weights given by the inverse square of the bid-ask spread of each contract, thereby adjusting the fit quality to price uncertainty. The optimization problem is not convex; to solve eq.(36), I use Nelder-Mead optimization in conjunction with simulated annealing. Calibration to market data yields the risk neutral probability function at each expiry month.

As a demonstration of the model, I use public market data [16] at noon on March 20 2015, which contains contracts for the expiry months April, May, June, September, and December. In Figures 1–2 we see the Heston model fits to the April contract price data; the high fidelity fits result from a simulated annealing cooling schedule that takes approximately 1000 seconds to complete on a desktop computer. Notice the trade-off fit between the calls and puts with the December contracts in Figures 3–4. This result arises from the limited data and the wide bid/ask spreads at the latest date in the option price data set. From the model parameters, we can extract the implied risk neutral density function. Figures 5 and 6 display the results for the two end months in the contract data (April and December) and we clearly see the leptokurtic and skewed nature of the implied distributions. For completeness, I include the 1-standard deviation interval along with the expectation (the forward price); Table 1 summarizes the results. By interpolating the density functions across time[2], I develop a heat map, shown in Figure 8, of the USDCAD over the range of contract expiry dates, giving a high level overview of the risk of large price movements. In addition to the heat map, the interpolation procedure allows the user to display the probability density function for any intermediate date, see Figure 7.

Decision makers can run the model each day and observe changes in the risk neutral probability density. This information would help project managers understand the risks to their contingency as payments come due over the course of the fiscal year.

Table 1: March 20, 2015: Summary of the central tendency of the USDCAD risk neutral probability function derived from the Heston model fits to exchange quoted option price data at the expiry months.

	Measures of central tendency	
Expiry date	Forward Price	1-Standard Deviation
April 17 2015	125.85	[121.88–129.82] (74%)
May 15 2015	125.91	[120.66–131.15] (73%)
June 19 2015	125.98	[119.25–132.71] (75%)
September 18 2015	126.14	[116.78–135.51] (75%)
December 18 2015	126.27	[115.29–137.26] (73%)

[2]The technical set-up proceeds as follows: I construct cubic spline interpolants of the cumulative distribution function (CDF) at each expiry month. The CDF computation comes from the Heston Greeks (Delta and Dual-Delta). I interpolate the CDFs from each expiry across time using linear interpolation. The resulting CDF has a cubic spline representation which results in the density function on spline differentiation. This procedure ensures that probabilities sum to unity and that expectations result in the forward rate.

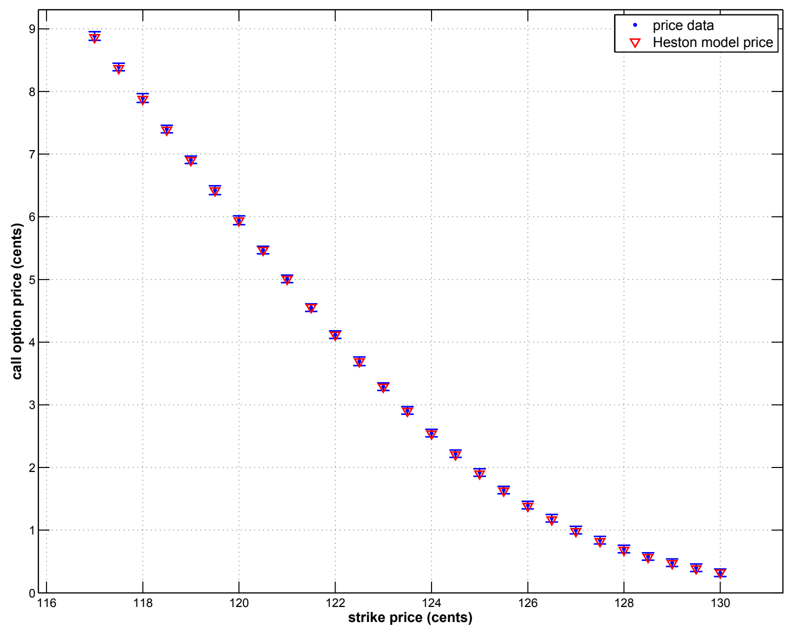

Figure 1: Call option price data with Heston model fit: April 2015 contracts. Bid/ask spreads indicated by the error bar.

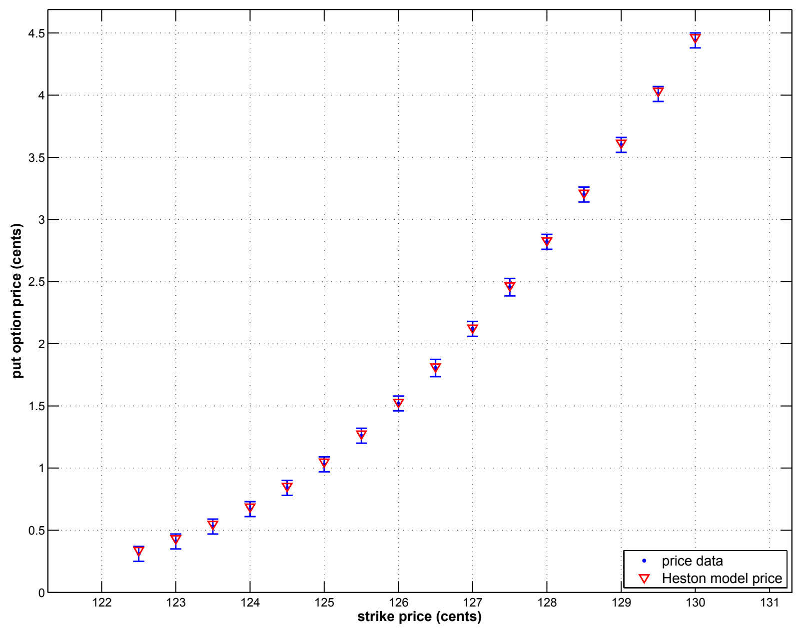

Figure 2: Put option price data with Heston model fit: April 2015 contracts. Bid/ask spreads indicated by the error bar.

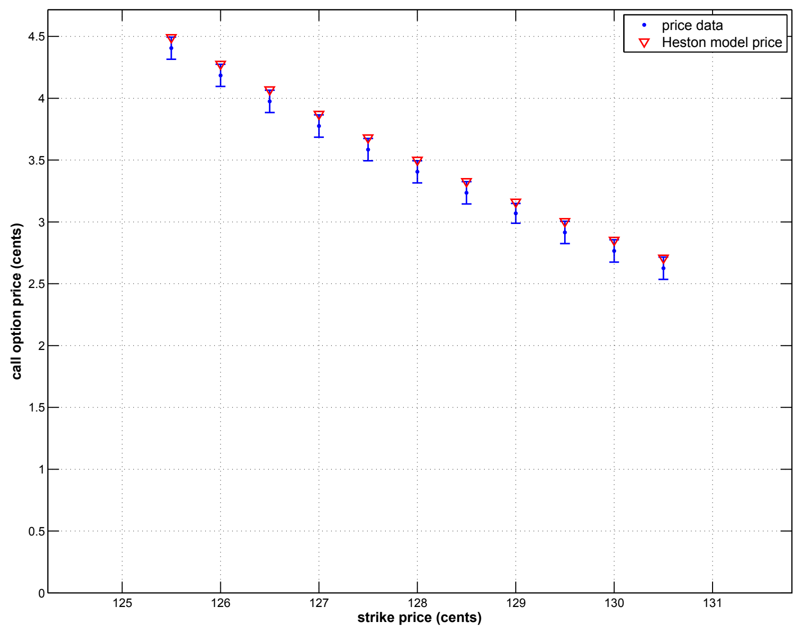

Figure 3: Call option price data with Heston model fit: December 2015 contracts. Bid/ask spreads indicated by the error bar.

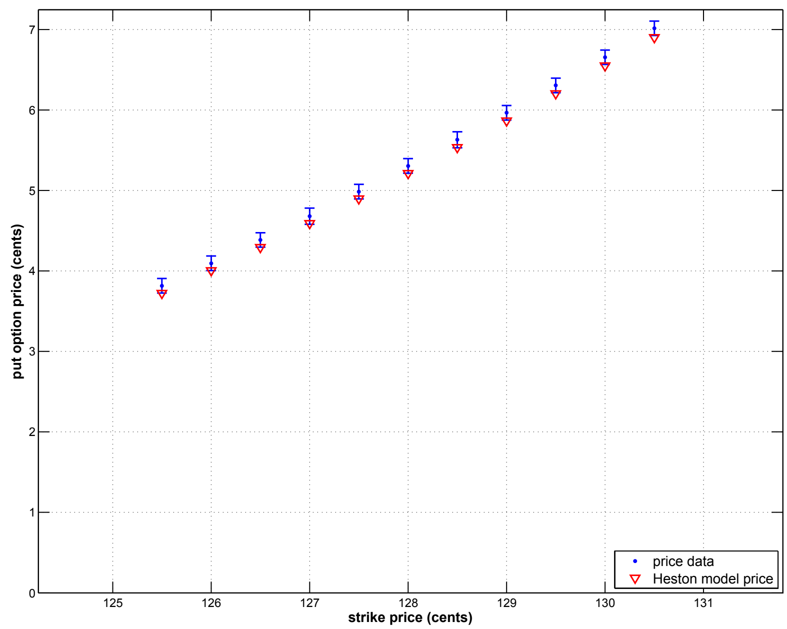

Figure 4: Put option price data with Heston model fit: December 2015 contracts. Bid/ask spreads indicated by the error bar.

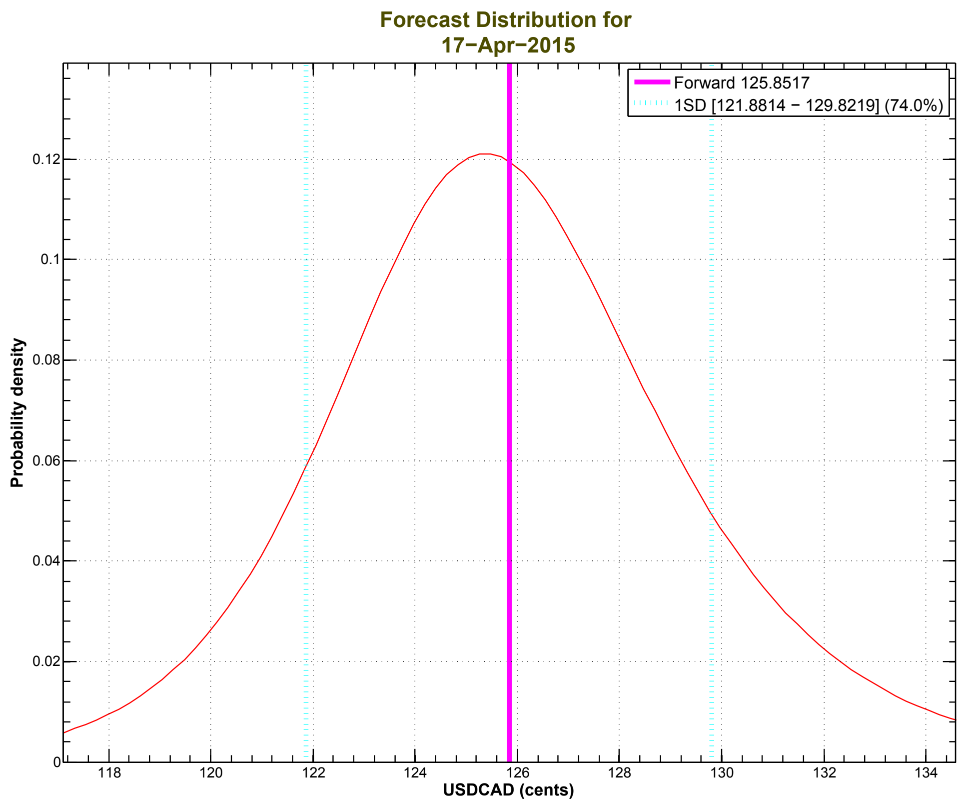

Figure 5: The Heston model implied risk neutral probability distribution for April 17 2015 with the 1-standard deviation level indicated, as of March 20 2015.

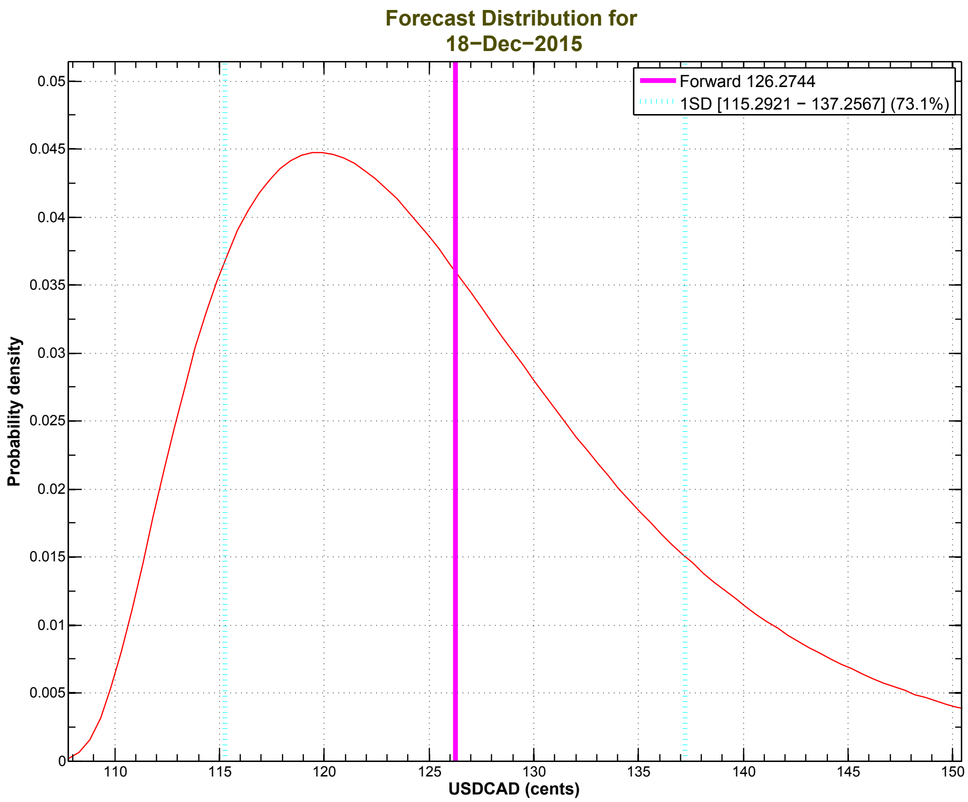

Figure 6: The Heston model implied risk neutral probability distribution for December 18 2015 with the 1-standard deviation level indicated, as of March 20 2015.

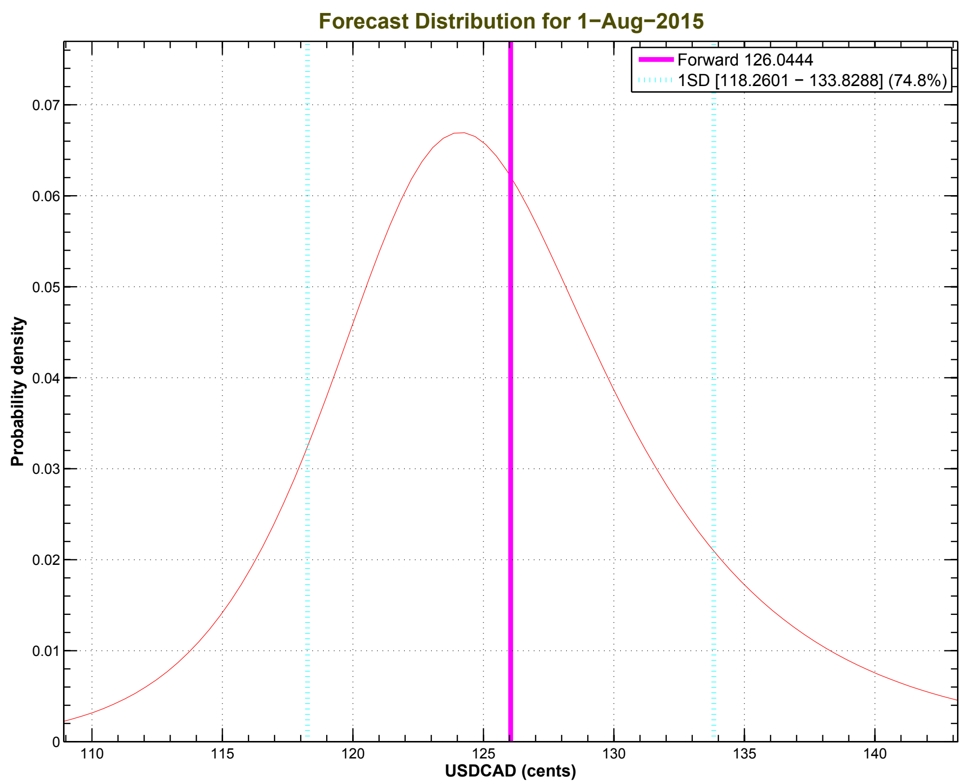

Figure 7: The interpolated Heston model implied risk neutral probability distribution for August 1 2015 as of March 20 2015.

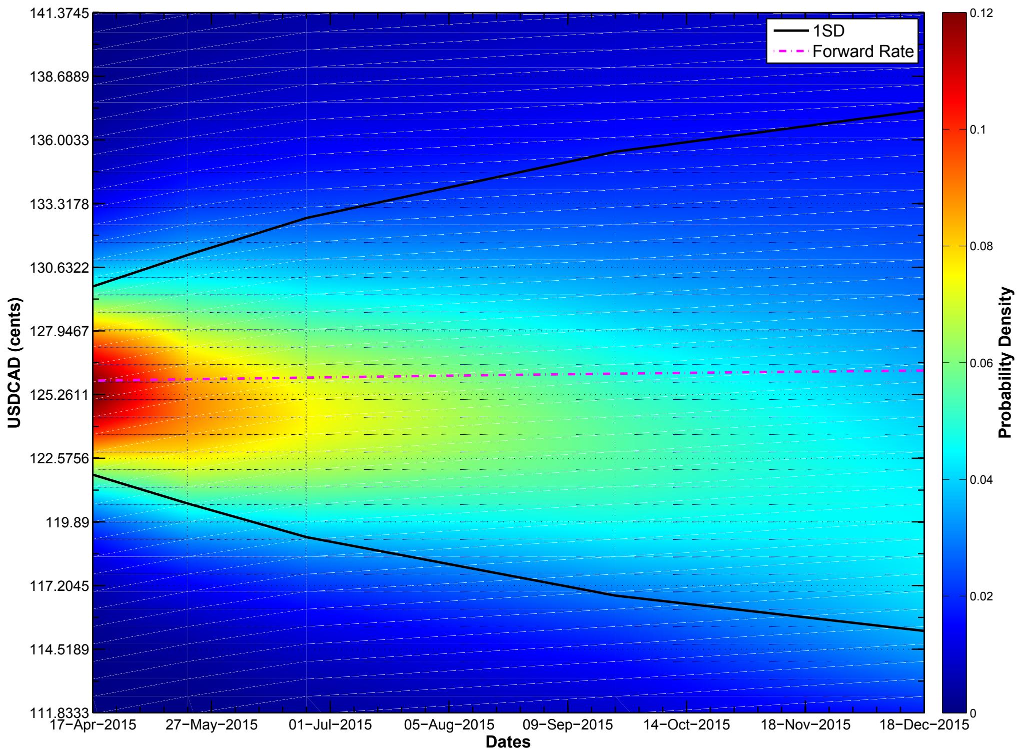

Figure 8: The Heston model implied risk neutral probability distribution interpolated across the expiry months as of March 20 2015. The 1-standard deviation line and the forward rate are indicated.

4 Discussion

Using the Heston model and USDCAD option price data from the Canadian Derivatives Exchange, I have developed a program that elicits the USDCAD risk neutral density function over the time interval contained in the data. The program displays this information through date specified probability density functions and through a heat map.

Currently, DCostS monitors foreign exchange risk by aggregating consensus industry forecasts with its internal historical data analysis. Each quarter, DCostS provides an update to their foreign exchange risk forecasts. I suggest a complementary if not alternative approach. Foreign exchange markets exhibit high efficiency, implying that no one has the ability to forecast exchange rates with any degree of accuracy or reliability [19], [20]. In [21], the authors make the salient point,

"Overall, the conclusion emerges that, although the theory of exchange rate determination has produced a number of plausible models, empirical work on exchange rates still has not produced models that are sufficiently statistically satisfactory to be considered reliable and robust... In particular, although empirical exchange rate models occasionally generate apparently satisfactory explanatory power in-sample, they generally fail badly in out-of-sample forecasting tests in the sense that they fail to outperform a random walk."

Using industry forecasts places DCostS and DND in the awkward position of accepting speculative market predictions generated largely for sales volume and trading purposes. The research field has arrived at the consensus that the explanatory power of expert forecasts is essentially zero. From a more practicable position, we must ask: "If industry leaders can forecast exchange rates, why would they tell us instead of trading on the information themselves?" In place of forecasts and speculation, we can use calibrated arbitrage-free market models to elicit the risk neutral probability density. The advantage of this strategy rests in its reliance on the widest aggregation possible—the whole market, under a risk neutral evaluation.

As a risk monitoring device, our clients can request the model output from CORA's Defence Economics Team to use in conjunction with with their foreign exchange reports for decision makers. CORA's contribution allows decision maker to now examine how the market implied risk neutral density changes over time. If, over the course of several weeks, decision makers observe lifting tails in the USDCAD risk neutral distribution, they will have the confidence of the entire market that risk has increased. Not only does this tool provide real time results, but it replicates the function within the Bloomberg terminal of most interest to DND decision makers during a trial use period [3].

As DND enters a renewal phase, foreign exchange risk will present project managers with budgetary challenges. This tool helps project staff understand foreign exchange developments in real time, providing insight into the risk of cost escalation.

5 Abbreviations and acronyms

ADM(Mat) Assistant Deputy Minister of Materiel

CAF Canadian Armed Forces

CORA Centre for Operational Research and Analysis

DCostS Directorate Costing Services

DND Department of National Defence

DRDC Defence Research and Development Canada

LIBOR London Interbank Offer Rate

USDCAD US-Canadian dollar exchange rate

VaR Value-at-Risk

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Using the Heston model stochastic volatility model with publicly available US-Canadian dollar exchange rate (USDCAD) option price data, I provide a client service tool that elicits the implied USDCAD risk neutral density function. The program displays this information through user specified dates for extracting the probability density functions, and through a heat map over the time horizon of the input data. Senior decision makers can use the information provided by this tool to monitor exchange rate risk over the fiscal year.

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Financial derivatives; Financial engineering; Foreign currency risk