GPS-based Attitude Determination Using RLS and LAMBDA Methods

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Abstract— **This work presents a procedure for getting 3D attitude from GPS raw measurements. The method is based on differential GPS and uses four receivers; although it can be easily generalized. The disposition of the receivers is fixed in the body frame, as attached to a rigid platform that moves freely in the local navigation frame. Baseline estimation is performed through a Recursive Least Squares (RLS) algorithm using code and carrier phase measurements. This work is an extension of [1] to multiple roving receivers, and where the LAMBDA method has been used for integer ambiguity resolution. The dispositions of receivers are known a priori, and this information used to constrain the solution at each epoch. Finally, 3D attitude is determined from the estimated baselines using the SVD method. The performance of the proposed method is evaluated through simulation. The results show significant enhancement with respect to the original method as well as with comparison with previous work in literature.**

Keywords- GNSS, GPS, Attitude determination, Ambiguity resolution, LAMBDA algorithm

I. INTRODUCTION

Precise 3D attitude determination of a platform (such as a vehicle, an UAV, etc.) is needed for countless navigation applications, either space, aerial, marine or terrestrial, and for both civil and military purposes [2-4]. Moreover, the integration of attitude determination systems in compact and lightweight devices, such as smartphones, tablets, or wearable devices has extended the fields of application to new areas that is limited by the accuracy requirements only.

Traditionally, the problem of accurate attitude estimation has been solved using a combination of inertial sensors and magnetometer [5]. However, these systems suffer from either lack of long term accuracy (medium grade systems) or high cost and complexity of use.

Multi-antenna GNSS systems, which integrate three or more GNSS antennas into a single receiver, has recently received great attention for accurate attitude determination [3],[6]. This kind of systems use GNSS signals to determine the attitude of a moving object in addition to providing position, velocity and time with high long-term accuracy. Besides that, GNSS-based systems do not need costly calibration procedures, and even current low-cost receivers attain 1 second TTFF (Time-To-First-Fix). However the price of these multi-antenna receivers is still high compared to single-antenna GNSS receivers. In this work, we propose using several low-cost single-antenna GNSS receivers for attitude determination. The purpose is to keep all the benefits of multi-antenna GNSS receivers while reducing the cost of the system.

GNSS-based attitude determination methods have been divided into two main categories in the literature: direct methods and methods based on baseline estimation. In the first category, attitude parameters are directly estimated from measurements. For example, [7] uses an Extended Kalman Filter (EKF) to estimate the attitude quaternion matrix. In this method, the Kalman state vector includes the attitude quaternion. However, this method requires an accurate initial guess for the EKF to converge. This effect is called the linearization limitation [7]. Another example of direct method can be found in [8]. This work applies a batch algorithm that simultaneously solves attitude quaternion and carrier phase ambiguity.

In the second and biggest category, attitude is estimated in two steps: 1) estimating baseline vectors in local frame from GNSS measurements; and 2) estimating attitude from baseline vectors. Since baseline vectors are known in the body frame, the second step leads to Wahba's problem [9], whose solution yields the minimum-error rotation matrix from local to body frame. The methods proposed in [10-15] are examples of this category.

This paper proposes an efficient method for baseline estimation. This method is based on least squares estimation, which is a common method for kinematic position estimation. Typically, this means a differential positioning with a stationary receiver whose position is known and a roving receiver with an unknown position [10]. Another common approach is to use Kalman filter, but it requires prior dynamics information.

This work is an extension of [1], but herein, we not only use four roving receivers, but also we use this method for a very short baseline configuration, a square platform (1 m^2) Besides we fix the ambiguity in carrier phase measurements as an integer number and we take advantage from knowledge

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about platform geometry in order to restrain the solution and increase accuracy and convergence speed.

This paper is organized as follows: First, in Section 2 we present the global structure of the proposed navigation system. Then, we explain briefly the mathematical procedure of the recursive least squares method [1]. After that, we describe the algorithm used to solve the carrier phase ambiguity, i.e., Least squares AMBiguity Decorrelation Adjustment (LAMBDA) method [16]. The section also shows restrictions application to the solution which are the baseline lengths. Moreover, a robust attitude determination algorithm is presented. Section 3 presents and discusses some simulation results to show the performance of the proposed algorithm. We also compare our method with several methods in literature. Section 4 summarizes some relevant conclusions from this study and suggests interesting future work.

II. Structure of the Proposed Navigation System

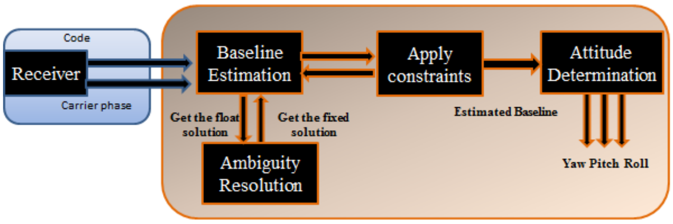

Figure 1. Structure of the proposed navigation system

The overall idea of the proposed navigation system is depicted in Fig. 1. The system is composed of 4 distinguishable steps:

1) Baseline estimation: Estimates baseline vectors in navigation frame based on code and phase measurements.

2) Ambiguity resolution: Fixes float ambiguity solutions to integer numbers, as a part of the baseline estimation process.

3) Apply constraints to the baseline lengths: This method fixes the norms of baseline vectors while estimating them.

4) Attitude determination. Computes attitude from estimated baselines.

The following sections describe in detail the mathematical procedure and function of each step.

A. Baseline estimation

Baseline estimation procedure exploits differential GPS carrier phase measurement technique. This technique has some advantages over more conventionally used pseudorange measurement technique, such as lower noise and multipath error. However, the increased accuracy is attained at the cost of a higher complexity, and only if the integer ambiguity is accurately estimated.

The baseline estimation method is mainly based on [1]. The goal is to find baseline vectors in navigation frame. The equations relating baseline vectors and measurements are, at the *k*-th epoch:

$$y_k^{\phi} = E_k x_k + a + e\beta_k^{\phi} + v_k^{\phi} \tag{1}$$

$$y_k^{\rho} = E_k x_k + a + e\beta_k^{\rho} + v_k^{\rho} \tag{2}$$

where:

y_k^{ϕ} is the vector of single difference carrier phase measurements from satellite one to m,

$$y_k^{\phi} \equiv \begin{bmatrix} \phi_k^1 & \cdots & \phi_k^m \end{bmatrix}^T \tag{3}$$

y_k^{ρ} is the vector of single difference code measurements from satellite one to m,

$$y_k^{\rho} \equiv \begin{bmatrix} \rho_k^1 & \cdots & \rho_k^m \end{bmatrix}^T \tag{4}$$

x_k is the baseline vector,

a is the vector of single difference ambiguities, N^i,

$$a \equiv \begin{bmatrix} N^1 & \cdots & N^m \end{bmatrix}^T \tag{5}$$

e is the all-ones vector,

β_k^{ϕ} is the error vector for phase measurements (single difference receiver clock error, receiver hardware delay, and the initial phase of the generated receiver carrier signal at the initial time)

β_k^{ρ} is the error vector for code measurements (single difference receiver clock error, and receiver hardware delay),

v_k^{ϕ} is the carrier phase measurement noise including multipath error, with $v_k^{\phi} \sim N\left(0, \sigma_{\phi}^2 I_m\right)$,

v_k^{ρ} is the code measurement noise including multipath error, with $v_k^{\rho} \sim N\left(0, \sigma_{\rho}^2 I_m\right)$,

E_k is a matrix defined as:

$$E_k \equiv \lambda^{-1} \begin{bmatrix} \left(\omega_k^1 e_k^1\right)^T \\ \vdots \\ \left(\omega_k^m e_k^m\right)^T \end{bmatrix} \tag{6}$$

where:

λ is the wavelength,

e_k^i is the unit vector pointing from the midpoint of the baseline toward the satellite m (i varies from 1 to m),

ω_k^i is a vector defined as:

$$\omega_k^i \equiv \frac{\left\|2h_r^i - x_k\right\|}{\left\|h_r^i\right\| + \left\|h_r^i - x_k\right\|} \tag{7}$$

and

h_r^i is the vector from receiver r to satellite i.

In order to eliminate β_k^ϕ from the carrier phase measurement equation (1), the following Householder transformation, P, has been used:

$$P \equiv I - \frac{2uu^T}{u^T u} \tag{8}$$

also,

$$Pe = \sqrt{m} e_1 \tag{9}$$

and

$$u \equiv e_1 - \frac{1}{\sqrt{m}} e \tag{10}$$

where e_1 is $\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}^T$, and we partition P as:

$$P \equiv \begin{bmatrix} P^T \\ \bar{P} \end{bmatrix} \tag{11}$$

Then we multiply (1) by the P matrix, we only take the lower part of the multiplied equation so:

$$\bar{P} y_k^\phi = \bar{P} E_k x_k + \bar{P} a + \bar{P} v_k^\phi \tag{12}$$

In order to achieve double differences without losing the integer nature of single difference ambiguity vector a, we define a matrix F as:

$$F \equiv I_{m-1} - \frac{ee^T}{m - \sqrt{m}} \tag{13}$$

So by definition of F we have:

$$\bar{P} a = Fz \tag{14}$$

By replacing (14) into (12), it can be written as:

$$\bar{P} y_k^\phi = \bar{P} E_k x_k + Fz + \bar{P} v_k^\phi \tag{15}$$

Similarly, we can eliminate β_k^ρ from the code measurement equation (2), and get:

$$\bar{P} y_k^\rho = \bar{P} E_k x_k + \bar{P} v_k^\rho \tag{16}$$

In order to have the same covariance matrix for both $\bar{P} y_k^\phi$ and $\bar{P} y_k^\rho$, (16) is multiplied by $\sigma \equiv \sigma_\phi / \sigma_\rho$, yielding:

$$\begin{bmatrix} \bar{P} y_k^\phi \\ \sigma \bar{P} y_k^\rho \end{bmatrix} = \begin{bmatrix} \bar{P} E_k \\ \sigma \bar{P} E_k \end{bmatrix} x_k + \begin{bmatrix} F \\ 0 \end{bmatrix} z + \begin{bmatrix} \bar{P} v_k^\phi \\ \sigma \bar{P} v_k^\rho \end{bmatrix} \tag{17}$$

From this equation we estimate z, the float solution used by LAMBDA method in order to fix the ambiguity.

B. Ambiguity resolution-LAMBDA method

One of the most powerful methods for fixing the ambiguity vector as an integer is the LAMBDA method [17]. This method is well-known as an efficient method with success rate maximization, and differs from FASF, FARA, and AFM in that it performs search space transformation [18]. Each baseline can be written as a linearized model:

$$Y = Aa + Bb + e \tag{18}$$

where Y is the difference between measurement and estimate of GPS carrier phase double difference, a is the double difference ambiguity vector, b is the estimated baseline vector, e is the noise vector, and A and B are the design matrices [19].

LAMDBA method first computes least squares estimates of vector a using sample estimate of its covariance matrix, while constraining its integer nature, i.e.:

$$\min \left\|\hat{a} - a\right\|^2_{Q_{\hat{a}}^{-1}} \quad a \in Z^n \tag{19}$$

Due to the cross-correlation of the ambiguities in the original search space, the search space is extremely elongated and it takes a long time to accurately determine the ambiguity [20]. LAMBDA overcomes this problem by performing a Z -transformation in order to decorrelate the cross-correlation between ambiguities while preserving the integer nature of the problem. In other words, LAMBDA converts the elongated space to a round (spherical) one. Then, the search space is aligned to the grid axes, and simply estimated by rounding to the nearest integer [12].

Here is a summary of the LAMBDA method implementation steps:

1) Center the ambiguities to the center of the search space by an integer shift. This is a necessary step before

decorrelation. This shift is saved in order to add it back at the very last step to the estimation;

2) Appling Z-transformation;

3) Inspecting the search space to find a suitable initial size of the search space, which depends on the number of candidates;

4) Solving the integer minimization problem by a discrete search over the ellipsoidal region, that is, the best integer vector nearest to the float solution;

5) Inverting the Z transformation;

6) Shifting back the ambiguities using the vector saved in step 1.

C. Apply constraints to the baseline lengths

After fixing the integer ambiguity through LAMBDA method, we obtain the constrained least squares estimate solution from (17), which satisfies the restriction on the baseline length:

$$x_s^2 + y_s^2 + z_s^2 = \left(baseline \quad length\right)^2 \qquad (20)$$

D. Attitude determination

Wahba's problem is a minimization problem to find a rotation matrix A, which minimizes the following weighted least squares cost function:

$$L\left(A\right) \equiv \frac{1}{2}\sum_i a_i \left|b_i - Ar_i\right|^2 \qquad (21)$$

where b_i are baseline vectors in the body frame, r_i are baseline vectors in the navigation frame, and a_i are non-negative weights. In our case, we use inverse squared baseline lengths as weights. There are many methods to solve the Wahba's problem, such as Singular Value Decomposition (SVD) [21], QUaternion ESTimator (QUEST) [22], EStimator of the Optimal Quaternion (ESOQ) [23], ESOQ2 [24], and Fast Optimal Attitude Matrix (FOAM) [25]. The most robust estimator for minimizing Wahba's cost function is the SVD method [12]. QUEST, ESOQ and ESOQ2 are faster methods, but computational cost is only an issue when we deal with numerous baselines. Since we deal with a reduced number of baselines, we choose the SVD method. In this way we introduce matrix B:

$$B \equiv \sum_i a_i b_i r_i^T \qquad (22)$$

which has singular value decomposition as:

$$B = U\Sigma V^T = Udiag\left[\Sigma_{11} \quad \Sigma_{22} \quad \Sigma_{33}\right]V^T \qquad (23)$$

Then, the optimal rotation matrix is [12]:

$$R = Udiag\left[1 \quad 1 \quad \left(\det U\right)\left(\det V\right)\right]V^T \qquad (24)$$

which applies the yaw, pitch and roll angles as:

$$\psi = \text{Arctan}(R\ (2,1)/R\ (1,1)) \qquad (25)$$

$$\theta = \text{Arctan}(-R(3,1))\Big/\sqrt{R(3,2)^2 + R(3,3)^2} \qquad (26)$$

$$\phi = \text{Arctan}(R\ (3,2)/R\ (3,3)) \qquad (27)$$

These three Euler angles yaw, pitch and roll define the attitude.

III. SIMULATION RESULTS

In order to demonstrate the performance enhancement with respect to the original method [1] we use a set of data which simulates a moving platform where the four receivers are located with synchronized with a common clock. One receiver is located at the origin (master receiver) and the three baselines in body frame, which is aligned to navigation frame at t=0, are x1 = (1,0,0), x2 = (1,1,0) and x3 = (0,1,0). This coplanar configuration is just an example of application and not a requirement of our algorithm, which can be applied to any antenna configuration. The platform moves counter-clockwise by 90° about y axis from horizontal position to vertical position. During the first 100 epochs the platform is not moving; then, it starts to rotate at 1/3° per epoch. Data rate is 1 Hz, so one epoch equals 1 second. Measurement noise has been simulated with a standard deviation of 1m for the code measurements and 10^{-2} m for the carrier phase measurement. Standard deviation phase to code measurements ratio is 100 which is a typical value for low-cost GNSS receivers.

Fig. 2 presents the convergence speed of our baseline estimation method for the three baselines. Here we assumed that 9 satellites are visible during the whole data record so that there is no rising or setting of satellites. To fix ambiguities, we set to 9 the number of epochs required in order to estimate covariance matrixes in LAMBDA method. That is why our method requires 9 epochs to converge, as can be seen in Fig. 2.

Fig. 3 represents the magnitude of the baseline estimation error for the rest of epochs not shown in Fig. 2 (from epoch 12 to epoch 250). The average errors are 0.0067, 0.0055, 0.0076(m) and standard deviations are 0.0330, 0.0249 and 0.0343 (m) for baseline one to three respectively. This result has an average error of 0.0067, 0.0039, and 0.0076 (m) relative to the baseline length, which directly affects the attitude accuracy.

In Fig. 4 we can see the convergence time for our 3D attitude determination algorithm. As expected, the convergence time matches the one attained when computing the baseline vectors, as no additional delay is introduced by our algorithm. Fig. 5 represents the Euler angles error for the rest of epochs. In this case, the biases are -0.0629°, 0.0892°, 0.0016° and the standard deviations are 1.0729°, 1.4314° and 0.5119° for roll, pitch and yaw respectively.

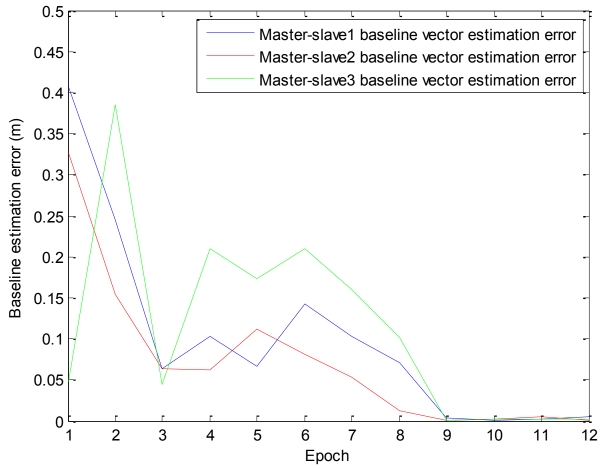

Figure 2. Baseline estimation convergence time

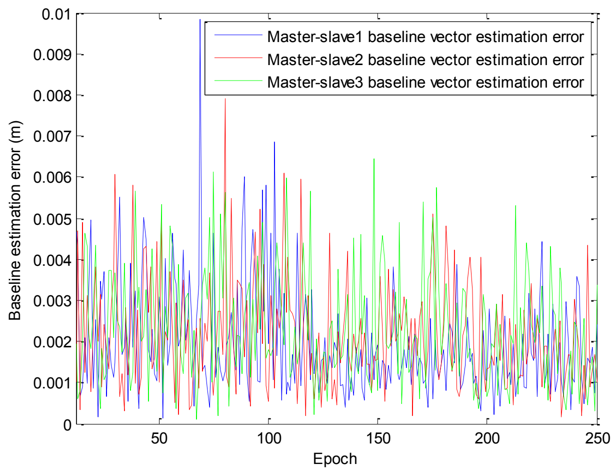

Figure 3. Baseline estimation error

Next, we compare the proposed method to the original one [1]. Fig. 6 and Fig. 7 represent the convergence time and estimation error of both methods when estimating first baseline. Results clearly show a significant performance improvement in convergence time as well as in baseline estimation error. For the sake of completion, Fig. 8 and Fig. 9 make the comparison of convergence time and estimation error, for the pitch angle, between our proposed method and the original one. As expected, major enhancement in Euler angles estimation are achieved as well.

In order to show the performance of our proposed method, we made a comparison between previous works and ours in terms of cost and accuracy. Fig. 10 shows a comparison between the estimated prices for the entire designed attitude determination packages with respect to their Root Mean Square Error (RMSE) per baseline length. Here the yaw angle or heading has been chosen for comparison because it is the most common angle to take into consideration for different application of attitude determination systems.

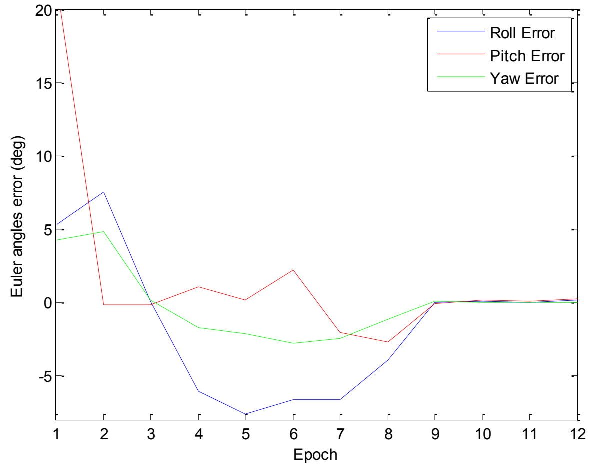

Figure 4. Euler angles estimation convergence time

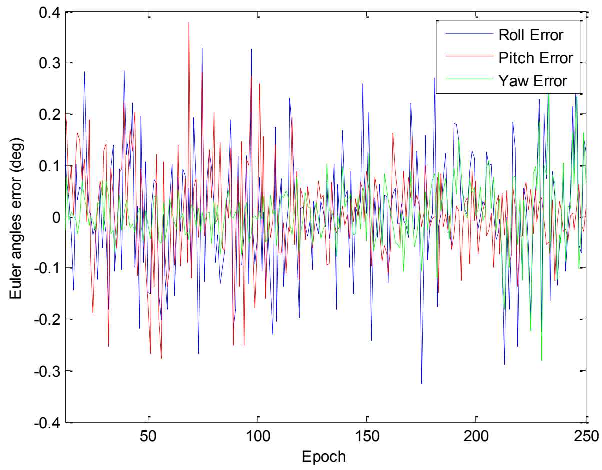

Figure 5. Euler angles estimation error

This figure also shows significant improvement, based on a trade-off between price and accuracy. Euler angles's RMSE of our proposed method after convergence are 0.0621°, 0.0964° and 0.1202° for yaw, pitch and roll respectively. This estimated price is included all GNSS receivers, antennas and all aided navigation sensors which have been used in each paper.

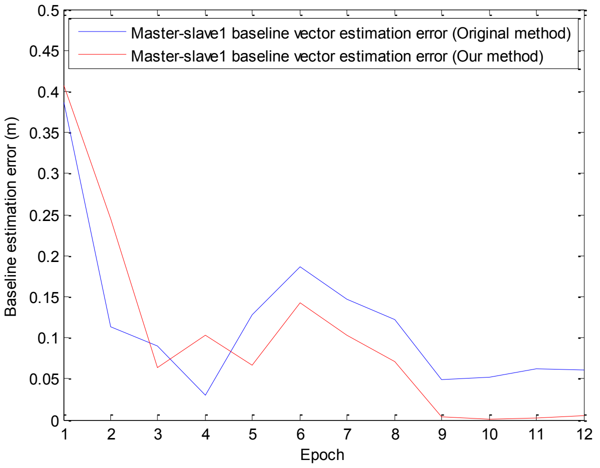

Figure 6. Baseline estimation convergence time comparison

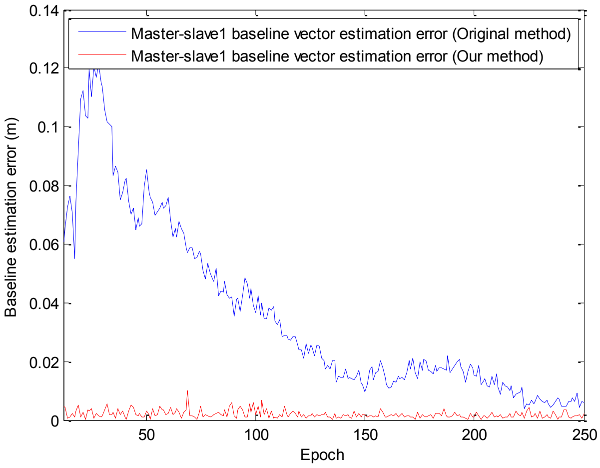

Figure 7. Baseline estimation error comparison

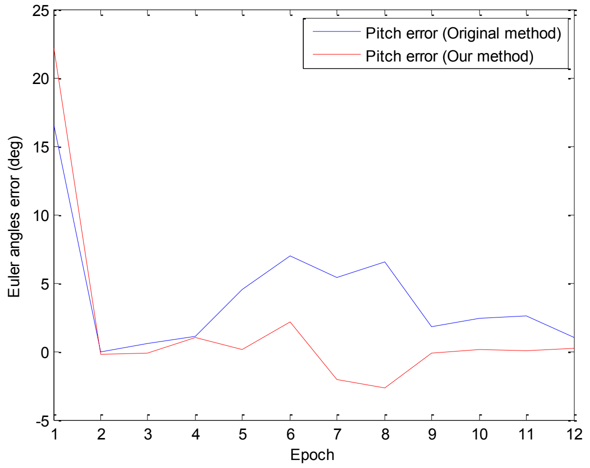

Figure 8. Euler angle estimation convergence time comparison

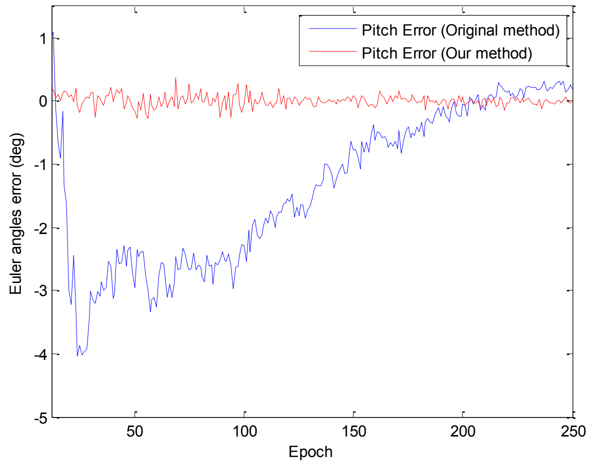

Figure 9. Euler angle estimation error comparison

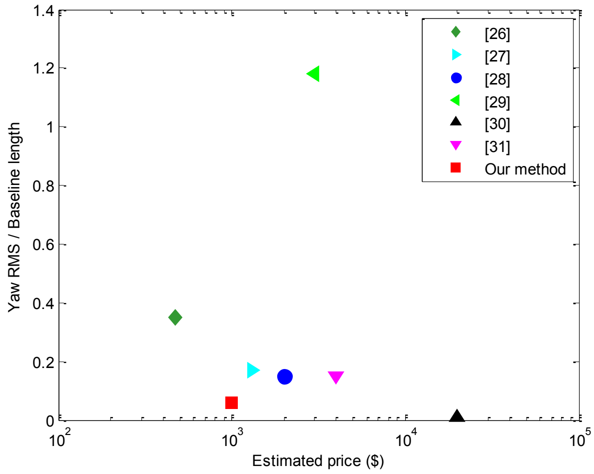

Figure 10. Accuracy vs. price trade-off in comparison with previous works [26-31]

IV. CONCLUSION

In this work we applied a recursive least squares method for baseline estimation using both code and carrier phase measurements to an attitude determination problem. The resulting algorithm benefits from all advantages of that method, such as numerical reliability and computational and storage efficiency.

The proposed algorithm employs LAMBDA method to fix ambiguities as integer vectors. Afterwards, baselines are obtained as solutions of least squares estimation problems, on which constraints to the estimated baseline length have been applied. As a result, baseline estimation and attitude accuracy are significantly improved with respect to the original method. The proposed method requires 9 epochs to converge due to the LAMBDA method. One of the interesting future works is applying this method to real data.

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