The Multi-Depot Split-Delivery Vehicle Routing Problem: Model and Solution Algorithm

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Abstract

Logistics and supply-chain management may generate notable operational cost savings with increased reliance on shared serving of customer demands by multiple agents. However, traditional logistics planning exhibits an intrinsic limitation in modeling and implementing shared commodity delivery from multiple depots using multiple agents. In this paper, we investigate a centralized model and a heuristic algorithm for solving the multi-depot logistics delivery problem including depot selection and shared commodity delivery. The contribution of the paper is threefold. First, we elaborate a new integer linear programming (ILP) model, namely: Multi-Depot Split-Delivery Vehicle Routing Problem (MDSDVRP) which allows establishing depot locations and routes for serving customer demands within the same objective function. Second, we illustrate a fast heuristic algorithm leveraging knowledge gathering in order to find near-optimal solutions. Finally, we provide performance results of the proposed approach by analyzing known problem instances from different VRP problem classes. The experimental results show that the proposed algorithm exhibits very good performance when solving small and medium size problem instances and reasonable performance for larger instances.

Keywords: Multi-Depot Vehicle Routing Problem, Integer Linear Programming, Heuristic Algorithm, Supply Chain Management, Operations Research, etc.

1. Introduction

The technological transformations that are taking place over the last few decades have brought about new challenges to the conventional supply chain operations in organizations ranging from private enterprises to governmental institutions. The changing global economy and agile infrastructure have placed high demand for systematic and automated planning of large-scale commodity delivery operations. In this respect, academic and industrial research and development efforts are being pursued for logistics operational plan generation. In this article, we intend to explore a subset of such large-scale planning requirements and analyze how multiple distribution centers (depots) and vehicle routing paths can be derived together, if possible, in a centralized planning environment to deliver commodities within predefined constraints and limited vehicle capacity. The specific focus of this study is the multi-depot split-delivery and location routing problem. The proposed technique can efficiently compute near-optimal solutions for problem instances where the combined cost (distribution center establishment and vehicle routing) needs to be minimized.

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1.1. Motivation & Background

Business organizations address their logistics pursuits at three different levels. These levels are often referred as: Strategic, Operational and Tactical [1]. Strategically, decision makers locate depots for serving the customers. Various network partitioning and resource allocation algorithms are applied on external inputs to choose depot locations in the vicinity of the customers. Once the depots are chosen, operational decision makers solve the underlying routing problem as per the requirements. The problem types at this level are often characterized as: traveling salesman problem (TSP), vehicle routing problem (VRP), traveling repairman problem (TRP), etc. Analytical, heuristic and meta-heuristic algorithms are used to solve the routing problems where the typical objective is to minimize routing cost. The results of these planning processes are a set of vehicle routes. Finally, the tactical officers proceed to execute the routing tasks in compliance with previously taken decisions. The need for large-scale quick logistic delivery planning is vital for situations like humanitarian aid distribution, disaster relief, rescue operations and national crises. However, such a multi-level decision making exhibits notable gaps to find optimal partitioning and routing in the transportation network for commodity delivery [2]. Furthermore, traditional logistics planning and its subsequent execution phase(s) heavily depend on human expertise in decision making that exhibit intrinsic limitations in handling large and complex operations. In this respect, an efficient and sufficiently automated mechanism for sharing responsibilities in commodity delivery may offer better situational response.

A relevant situation can be mentioned from the experience of the well-known Haiti disaster in the aftermath of the January 2010 earthquake [3]. In this crisis situation, the response operations started the delivery of essential commodities to more than 300,000 injured and 1.5 million homeless people [3]. Several organizations, such as: International Rescue Committee (IRC), Management Sciences for Health (MSH), International Federation of Red Cross (IFRC), etc., teamed up to deliver drugs and supplies to out-of-stock clinics and health facilities from multiple operational Emergency Response Units[1]. It was well documented that the overwhelming emergency requirements caused delay in shelter preparation [3]. Renowned news channels also reported on the mismanagement in cooperation for shared delivery arrangement among participating organizations[2]. The logistics delivery planning is also an important research problem for the supply chain management. In the commercial sectors, trade related surface transportation has been constantly increasing in North America. Between 2009 and 2010, the total value-added of the Transportation and Warehousing sector has been growing approximately by 4.3% [4]. In Canada, the Gross Domestic Product (GDP) in the Transportation and Warehousing sector has increased from $50.2 billion in 2001 to $58.4 billion in 2010 [4]. The United States Department of Transportation has issued a notable report stating that the surface transportation trade between North American Free Trade Agreement (NAFTA) partners has been increased by 11.5% in January 2012 compared to January 2011 at $75.5 billion [5]. Alongside, a Gartner report reveals that the market for intelligent transportation planning software holds the key to the success of the multi-organizational response. The report also indicates a 20.6% increase of worldwide Transport Management System software revenue from 2007 ($538 million) to 2008 ($648 million) and a growth in the field through 2012 (up to $963 million) [6].

1.2. Problem Elaboration

Multi-Depot Split-Delivery Vehicle Routing Problem (MDSDVRP) handles commodity delivery to customers (demand points) that are represented as nodes in a complete graph named as transport network. Given a set of nodes (V) and a set of edges (E), where E is a relation in ($V \times V$), a transport network is a complete graph $\mathrm{G} = (V, E)$. Each edge of the graph provides the traversal cost (c_{ij}) between the corresponding two nodes i and j. Usually, a transport network is composed of different node types: Customers (N) and Depots (D). While customer nodes are characterized with deterministic demand (integer) for commodity (δ_i), depot nodes (having no demand) alternatively host vehicles ($k = 1, 2, \ldots K$) to supply customers. In case of predefined depots and customers, a solution for an MDSDVRP instance gives the routes for each vehicle that minimizes the overall routing cost to serve all customer demands. In our proposed formulation,

[1] MSH and IRC to Partner in Haiti; link: `http://www.msh.org/news-bureau/msh-and-irc-to-partner-in-haiti.cfm`
[2] BBC, What is delaying Haiti's aid?; link: `http://news.bbc.co.uk/2/hi/americas/8472670.stm`

we further consider that if the depots are not predefined, the solution to MDSDVRP will determine the optimal location(s) of the depot(s) within the set of customer nodes. In this case, assuming that the newly found depot(s) will serve their own need(s), the goal of problem is then to minimize the combined depot establishment and routing cost.

In actuality, vehicle routing can be seen as a core problem in supply chain/logistics planning, with conceptual, empirical and behavioral aspects. A holistic view of the supply chain process offers an overarching perspective spanning over various facets such as facility location, vehicle routing and environmental impact. In this respect, focusing on a single aspect, for example minimizing the routing cost without considering facility locations may result in higher warehousing cost and larger externalities such as: pollution, congestion, etc. In the usual setup, the problem of multi-depot split delivery vehicle routing is considered with the common assumptions of Split-Delivery VRP (SDVRP) and Multi-Depot VRP (MDVRP) under which we essentially consider a vehicle routing problem involving commodity delivery as an abstract conceptual optimization problem [7] with few empirical details. The participating entities are depots (as starting/ending points for vehicles), customers (with deterministic demand) and vehicles (with predefined and available capacity). Typical abstractions are observed in terms of unlimited route length (not considering required stop-overs for rest, etc.) as well as deterministic infrastructure analysis (fixed traversal cost across transport network nodes, etc.). Another prevalent abstraction is to consider the problem of facility location separate as specifically employed by cluster first-route second approaches [2]. However, in this work, we emphasize the importance of considering together the problems of location allocation and vehicle routing.

The optimization goal of VRP is the overall cost minimization based on the cost assigned on each edge of the transportation network. In the literature the deterministic capacitated-VRP (CVRP) is a well-studied NP-hard combinatorial optimization problem having several variants and extensions [8]. In fact, the CVRP is composed of two problems: Bin-Packing and Routing. The Bin Packing Problem (BPP) addresses an optimal allocation of commodity to vehicles having deterministic capacity. The routing problem deals with the most efficient routing possible using the loaded vehicles. We may note that in shared commodity delivery settings (which represent practical aspects at the requirements level), it is possible to determine the feasibility of a problem instance by requiring the total vehicle capacity to be greater or equal to the total demand. In other words, MDSDVRP will always yield a solution if the total available capacity is equal or more than the total demand. In this respect, MDSDVRP is less restrictive than some of the other VRP variants for which there may be no feasible solution (e.g some customers having demands larger than the capacity of a single vehicle). However, MDSDVRP still belongs to the NP-hard class of problems [9, 10] and is therefore intractable when approached with an exact algorithm. It is worthy to mention that it has a notable larger solution space since splitting the delivery among different vehicles is subject to combinatorial explosion. Consequently, we detail an effective heuristic technique that yields good near-optimal solutions.

1.3. Objectives

In the scope of this article, we aim at investigating an advanced decision support platform to address a combined problem of depot assignment and logistics delivery planning. Currently available transport management systems exhibit notable gaps in optimal partitioning of transport network for shared delivery of logistics/commodities [6]. In order to bridge the gap, we introduce a linear model of the combined problem and propose a generic solution search technique for multi-depot vehicle routing problems that may employ shared delivery of commodities, if needed. The solutions to this problem is expected to efficiently use a number of vehicles of predefined capacity to serve geographically distant customers of known demands. The objectives of this paper can be summarized as follows:

- Elaborate an ILP model to find locations for the depots and the vehicle routes of commodity delivery.
- Propose an efficient fast-convergent heuristic-based mechanism to solve the model near-optimally.
- Generate solution benchmarks for known problem instances and compare with existing results.
- Analyze the performance and provide other notable insights of the proposed solution.

The core contribution of this paper includes the elaboration of an integer linear programming (ILP) optimization model for multi-depot, multi-vehicle per depot vehicle routing with split delivery. A notable contribution relates to the flexibility of the proposed model. This allows to customize it via small modifications (according to the need) in order to address specific problems of the VRP family that are within the scope of the proposed model. These include MDVRP (no split delivery), SDVRP (only one depot), CVRP (no split delivery and only one depot), etc. Moreover, the concept of location routing allows to consider both location allocation and vehicle routing as part of the same objective function. In this context, it is also possible to customize the depot establishment cost values such as to predefine depots at specific locations. With respect to the related heuristics algorithm, it allows to generate vehicle routes with near-optimal cost while serving the customers by multiple vehicles belonging to the same or different depots.

1.4. Article Organization

The remainder of the paper is organized as follows. Section 2 offers an overview of the related work on various types of vehicle routing problems. Section 3 elaborates the proposed model for MDSDVRP and our solution generation approach. It describes a generic heuristic based searching mechanism designed to solve vehicle routing problem instances, MDSDVRP instances in particular. Along with the algorithm, we also discuss two improvement techniques over the initially derived solutions. Section 4 presents a relevant case study problem illustrating CVRP, MDVRP and MDSDVRP in order to demonstrate solution generation using the proposed approach. In Section 5, we provide the results and compare them to existing benchmark values. We further conduct an analysis of the results in Section 6 to determine appropriate ranges for the parameter values used in the solution approach. Finally, we summarize our findings in Section 7 by highlighting the benefits and the limitations of the proposed procedure and conclude with future work.

2. Related Work

The transportation management and logistics delivery problems are known for their practical relevance and high computational complexity. They have been extensively studied across the scientific community all around the world for more than half a century. Many of these problems commonly exhibit NP-hard complexity and are often modeled from centralized perspective. In the literature, there are different research articles discussing multi-stage approaches to solve logistics delivery planning. Numerous research initiatives propose multi-stage approaches [1, 8] that require network partitioning and choosing the facility locations at the first stage. Then, in the latter stage, they consider various VRP variants to deliver commodities.

The transport network partitioning problem belongs to the more general graph partitioning problem whose objective is to partition a graph into approximately equal parts with the least number of interconnections. The graph partitioning problem is known to be NP-complete [11]. Thus, there is no general tractable procedure that would allow to efficiently perform optimal graph partitioning for large problem instances. Nonetheless, specific approaches allow for graph partitioning in geographical information systems, telecommunication networks, clustering, image processing and many other areas [12, 13, 14], including operations research. Jarrah and Bard published a heuristic approach [15] for graph partitioning using contiguous geographic clustering for pickup and delivery VRP via network route segmentation. Many algorithms addressing network partitioning exist in the literature, such as: K-Means clustering [16], DB-Scan algorithm, shortest path algorithm [17] etc. However, the underlying limitations of the "cluster-first, route-second" approaches stems from the fact that the best depot locations obtained by partitioning at the strategic level may not always optimize the cost at the operational level since the depot locations are generally chosen without considering the potential routing cost. Salhi and Rand [2] show that the best solution after facility location stage does not necessarily lead to the lowest cost solution after the routing stage.

VRP aims at commodity delivery to a set of customers by a set of vehicles, from one or many depots over a transport network characterized by a full mesh graph. Early on, Dantzig and Ramser [18] formally introduced the vehicle routing problem in their pioneering work on truck dispatching. VRP entails combinatorial optimization to reach optimal routing cost solution. In 2002, Toth and Vigo elaborated an extensive classification of VRP family [8]. Subsequently, Golden *et al.* documented the more recent advancements of the last decade in [19]. In this article, we focus on capacitated-VRP (CVRP). In its original form, CVRP includes the bin packing problem, which need to be solved along with the routing in order to optimally use the available capacity of the vehicles and optimally serve the customers. CVRP is usually expressed as a linear optimization problem with several constraints represented through linear equations and inequalities. Three commonly used CVRP models include the Vehicle Flow Model, Commodity Flow Model and Set Partitioning Model [8]. SDVRP is related to CVRP as it aims at minimizing the total traveling cost for commodity delivery but it allows to serve individual customer demands by more than one vehicle. SDVRP instances observe relaxed bin-packing constraints and a feasible solution always exists if the overall customer demand is less or equal to the overall capacity available. The concept of split-delivery was first introduced by Dror and Trudeau [20] and later further elaborated by Archetti and Speranza [21]. A survey on the progress in SDVRP has been recently published by Archetti *et al.* [22] where the benefits of shared delivery are illustrated on various problem instances. However, an important limitation of SDVRP lies in the availability of a single depot. In this article, we take the problem of SDVRP into our MDSDVRP model, which is addressing in addition the depot location allocation problem while allowing vehicles from the same or different depots to participate in shared commodity delivery. In the proposed MDSDVRP model, we employ split-delivery vehicle routing and also address facility location by choosing depots from a subset of customer nodes (based on their corresponding depot establishment cost). The model also allows the use of pre-established depots by setting the corresponding depot establishing cost to zero. Toward this end, we incorporate idea from the Location Routing Problem (LRP) [1] which combines location allocation and vehicle routing. This involves determining depots and routes for a fixed number of vehicles to serve customers. Although LRP definition is elaborated, it suffers from traditional key limitation that the set of candidate depots is generally pre-established. In our previous work, we presented the concept of MDSDVRP by extending the aforementioned ideas [23].

Recently, Gulczynski et al. [10] investigated a version of MDSDVRP by extending SDVRP. Their work relates to our problem to some extent. The authors provide a mixed integer programming optimization model that is essentially addressing route cost minimization by applying split delivery among vehicles from the same or different depots. However, the employed approach is multi-stage as it considers first the assignment of customers to depots using a distance based approximation, solving then the split-delivery VRP for each depot. Thereafter, further improvements are pursued by creating inter depot routes. The authors combine a mixed integer programming with a variable length record-to-record travel algorithm for which experimental results shows the cost reduction from splitting the deliveries among vehicles from different depots. However, the work has a number of limitations as follows. First, it employs an apriori, rule-based allocation of customers to depots by favoring the assignment to the nearest depots. In case of insufficient vehicle capacity, this will lead to a higher cost. Such concerns were addressed in [24] where a distant depot is required to serve a customer that is closest to another depot in order to achieve overall cost reduction. Second, the proposed solving technique leverages the Clarke and Wright (CW) saving mechanism that is limited in its applicability to the situations where the triangle inequality is satisfied [25]. Furthermore, the model considers only predefined depot locations on the transport network.

VRPs represent a well studied class of NP-hard problems [8]. Traditional (analytical) solving techniques such as branch and bound [19], are intractable for medium and large scale problems. Therefore, solution generation often involves various supervised solution searching procedures such as: heuristic and meta-heuristic algorithms which are commonly used to find near-optimal solutions for NP hard problems [26]. Concerning customer allocation, Chan and Kumar [27] developed ant-colony based meta-heuristic optimization for managing customer demands whereas Zhou *et al.* [28] used a genetic algorithm based approach for customers allocation to their distribution center. Heuristics and mathematical programming for cargo loading are studied in [26] and [29]. In the case of SDVRP, Archetti *et al.* showed that the SDVRP can be solved in polynomial time if and only if common vehicle capacity (C) is 2 [22] whereas the problem becomes NP-hard for $C > 2$. Dror *et al.* described a local search algorithm based on specific SDVRP properties [30]. Archetti *et al.* obtained improved results using Tabu search [21]. Chen *et al.* proposed a hybrid algorithm, additionally using the standard Clarke and Wright saving algorithm in order to solve SDVRP.

In the context of logistics planning for natural disasters, Ozdamar *et al.* presented an approach of planning in emergency situation [31]. Yi and Kumar also used an ant colony based approach for optimizing disaster relief operations [32]. In the case of MDSDVRP, we opted for near-optimal solution generation using heuristic procedures. Heuristic methodologies usually involve a directed search procedure based on knowledge gathering. Meta-heuristics, generally involves comparing and iteratively enhancing candidate solutions with respect to a defined quality function measure. In our research context, generational heuristic algorithms followed by solution refinements are more suitable especially to address adaptive route planning. A preliminary version of our proposed heuristic algorithm has been introduced in [33].

3. Proposed Approach

In the following, we present an overview of the approach to solve the aforementioned research problem. Under a set of assumptions, we propose an ILP model for the multi-depot vehicle routing problem allowing joint serving of customer demands using vehicles from multiple depots. The model allows to identify the problem requirements in terms of variables and parameters. However, MDSDVRP belongs to the problem class NP-hard [10]. Therefore, no scalable exact solution algorithm exists to efficiently find the optimal solution. Consequently, we investigate a heuristic algorithm that can efficiently explore a large portion of the solution space in order to find a good near-optimal solution. The proposed search procedure is guided by a learning mechanism that allows to steer the search toward the most probable area of the solution space where near optimal solutions are likely to be found. We also employ a stochastic technique to prevent the premature convergence of the algorithm to a local optimal solution.

3.1. Assumptions

Classical VRP generally refers to the capacitated vehicle routing problem (CVRP) where each location (customer) has a finite demand (integer value) for the same type of commodity. A maximum number of vehicles having finite capacity (integer value) can start from and return to a single depot, with no restriction on the route length. In addition, each location is served by only one vehicle and the sum of the demands served by a vehicle does not exceed the vehicle capacity. It is also customarily assumed that the vehicle fleet is homogeneous, that is all vehicles have the same capacity. Moreover, MDVRP changes the assumption of a single depot and considers the availability of more than one depot, each of which can serve any of the customers. In this setup it is also customarily assumed that the vehicle fleet is homogeneous across depots (the same maximum number of homogeneous vehicles). Capacitated VRP with split delivery (SDVRP) shares most assumptions of CVRP except that each customer can be served by one or more vehicles that jointly satisfy the total customer demand. This allows to address problems where the individual capacity of the vehicles can be less than some (or all) of the customer demands. MDSDVRP combines the assumptions of MDVRP with those of SDVRP such that each customer can be served by one or more vehicles, each of which can belong to the same or different depots. In addition to the aforementioned constraints, we intend to address the situation where there is are no pre-established depot(s) in the problem. The depots locations are then chosen from the set of customer locations. Consequently, the demands at the node locations chosen as depot locations are considered to be served by the respective depot itself. In order to solve the problem, the heuristic procedure may be exercised by one or more decision maker(s). The corresponding MDSDVRP solution algorithm assumes that all input information is exact. We assume that each decision maker has the knowledge of all available vehicles in every depot along with their capacities. S/he also knows the cost of routing across every edge of the transport network. In case of more than one decision maker, we consider the existence of a centralized result sharing platform where multiple decision makers can share information while searching a partially different solution space.

3.2. MDSDVRP Modeling

In a complete directed graph G = (V, E) of a transport network, let c_{ij} be an input cost matrix derived from a composed cost function (depending on various parameters) for all node pairs. We extend this transport network graph G to $G' = (V', E')$ wherein an arbitrary node 0 is added in the transport graph such that $V' = V \cup \{0\}$. The concept was earlier introduced by Yu *et al.* [34]. The intent of including node 0 as a virtual node is to carefully capture the facility (depot) location subproblem as a part of the routing

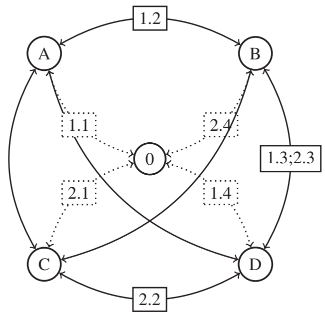


Figure 1: An example transport network of customers and depots

model. The inclusion of additional edges between 0 and existing customer nodes is associated to two new pair of cost values (c_{i0} and c_{0j}). The establishment cost for node j is represented as (EC_j) where $EC_j = c_{0j}$. Consequently, we write the cost function c'_{ij} as follows:

$$c'_{ij} = \begin{cases} c_{ij} & \text{if } \{i, j\} \subset V \\ EC_j & \text{if } i = 0 \textit{ and } j \in V \\ c_{i0} & \text{if } j = 0 \textit{ and } i \in V \end{cases}$$

Figure 1 shows a sample graph in the proposed setting with customers (B and D), depots (A and C) and two vehicles. Actual delivery routes can be assumed as: AB→BD→DA and CD→DB→BC. The changes to the original transport network is considered with addition of a virtual node 0 and the inclusion of dotted paths. In this case, a solution from the MDSDVRP model would result in the paths: 0A→AB→BD→D0 and 0C→CD→DB→B0. The proposed model evaluates a cost function based on its original first set of routes rather than the later derived set of routes. Actually, we incorporate additional routes 0A and 0C to reflect EC_A and EC_C in the cost function and replace the cost of paths D0 and B0 with DA and BC respectively. Toward this consideration, we associate an additional set of boolean variables w_i on top of the usual SDVRP formulation [9] to determine the depot locations.

In formulation of the problem, we consider three given input parameters. First, the establishment cost for creating depot on node i is termed as EC_i. Second, the demand level at each node i is: d_i for all $i \in V$. Finally, the let us consider that the maximum vehicles available is K and each of them has capacity C_k. The aims of the modeling can be summarized as follows:

- Determining an optimal number of depots and their locations, in absence of predefined depots;
- Computing the optimal cost of the overall customer serving;
- Evaluating optimal number of tasked vehicles;
- Elaborating routes (potentially shared) for each tasked vehicle for overall optimal delivery cost.

The decision variables are as follows:

- $x_{ijk} \in \{0, 1\}$ are boolean variables to determine routes (1 if the edge (i, j) is taken by vehicle k).
- $y_{ik} \in N$ is an integer amount of resource deposited at node i by vehicle k.
- $w_i \in \{0, 1\}$ is 1 if node i is a depot.

We present the ILP model formulation as follows:

$$min \sum_{i \in V} EC_i w_i + \sum_{i \in V} \sum_{j \in V} c_{ij} \sum_{k \in K} x_{ijk}, \qquad \forall i \neq j \tag{1}$$

Subject to:
Flow conservation:

$$\sum_{i\in V'}\sum_{k\in K} x_{ijk} \geq 1, \quad \forall j \in V', i \neq j \tag{2}$$

$$\sum_{j\in V}\sum_{k\in K} x_{0jk} \leq |K| \tag{3}$$

$$x_{0ik} = x_{i0k} \ \forall i \in V \ and \ k \in K \tag{4}$$

$$\sum_{i\in V'} x_{ipk} = \sum_{j\in V'} x_{pjk} \ \forall p \in V' and \ k \in K, \ i, j \neq p \tag{5}$$

Sub-tour elimination:

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} - \sum_{j\in S} x_{0jk} \leq |S| - 1, \quad S \subseteq V, \ |S| \geq 2, k \in K \ and \ i \neq j \tag{6}$$

Capacity restriction:

$$\sum_{i\in V} y_{ik} \leq C_k, \quad \forall k \in K \tag{7}$$

$$\sum_{k\in K} y_{ik} = d_i(1 - w_i), \quad \forall i \in V \tag{8}$$

$$y_{ik} \leq d_i \sum_{j\in V'} x_{ijk}, \quad \forall i \in V \ and \ k \in K \tag{9}$$

Depot assignment:

$$\sum_{k\in K} x_{0ik} \geq w_i, \quad \forall i \in V \tag{10}$$

$$x_{0ik} \leq w_i, \quad \forall i \in V, k \in K \tag{11}$$

Variables:

$$x_{ijk} \in \{0, 1\}; \ where \ i, j \in V', i \neq j, k \in K \tag{12}$$

$$w_i \in \{0, 1\}; \ where \ i \in V \tag{13}$$

$$y_{ik} \geq 0; \ where \ i \in V, i \neq j, k \in K \tag{14}$$

The objective function (1) minimizes total depot establishment and routing costs. With respect to the constraints, the equations (2) and (5) impose that each customer is visited by at least one vehicle. The equations (3) and (4) set the limit of maximum vehicles that can be used in solution and make sure that all vehicles in operation finally return back to node 0. Equation (6) is a modified version of generalized sub-tour elimination constraint from [35] in order to accommodate that all routes start and finishes at node 0 and passes through a determined depot before reaching node 0. The equations (7),(8) and (9) impose that serving a customer on a route takes place if and only if the route is selected and the total on-route serving does not exceed vehicle capacity, while ensuring that the total demands of each customer are met. Finally, the equations (10) and (11) assure that a vehicle, if serving at least a customer, must start and finish through a determined depot location. These ILP constraints further satisfy the following compound relations:

- If x_{0jk} is 1 then $\sum_{i\in v} x_{ijk} = 1$ and $x_{j0k} = 1$; i.e. a route for vehicle k will start and end with through a proposed depot j.
- $w_i = 1$ if $x_{oik} = 1$, i.e. a node i is a depot if and only if it is directly connected to node 0 in the solution.
- $\sum_{k\in K} y_{ik} = 0$ if $w_i = 1$, i.e. the demand of a prospective depot is 0 during computation of the routes.

The proposed model is flexible and extensible, allowing to capture richer real-world problems.

- MDSDVRP can be tailored to select pre-established depots by suitably choosing low establishment cost for some favored nodes and high establishment cost for others (*see* Table 4).
- With many vehicles and one depot in configuration, this model may express an SDVRP.
- One can provide product or service through the same model. For simplicity, we assume that service delivery (e.g. surveillance) resembles product delivery but the vehicle capacity (C_k) is not reduced after visiting the demand nodes. However it must meet the service requirements (y_{ik}). Thus, we change Equation (7) as follows:

$$y_{ik} \leq C_k, \quad \forall k \in K \text{ and } \forall i \in V \tag{15}$$

3.3. Solving MDSDVRP

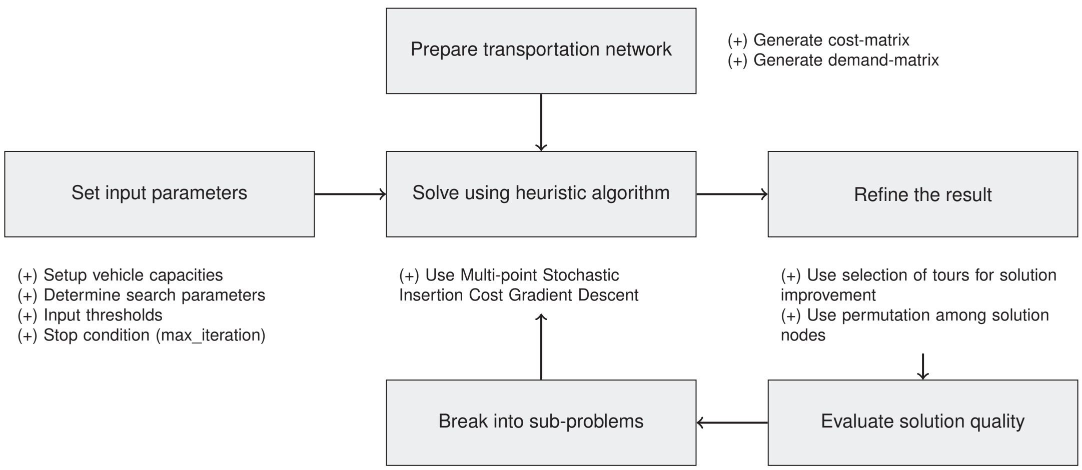


Figure 2: An overview of solution generation technique

MDSDVRP can be solved either analytically or using heuristics and meta-heuristics techniques. Analytically, MDSDVRP requires solving a set of linear equations as created in the model. The procedure is practical as long as the problem is smaller. Usually, ILP models are represented using a suitable language used to describe a set of linear equations in a readable manner by both human and machine. It is also necessary to use analytical techniques like Branch & Bound, Branch & Cut to tighten the initial linear programming relaxations. We initially selected AMPL (*A Mathematical Programming Language*) to represent the problem. Then, we chose GNU Linear Programming Kit (GLPK) as a freely available solver module for AMPL based ILP formulation. GLPK uses the revised simplex method, the primal-dual interior point method for non-integer problems and the branch-and-bound algorithm along with Gomory's mixed integer cuts for (mixed) integer problems. We may additionally employ MIR cut [36], Cover cut and Clique cut [37], which are helpful when solving ILP models. However, the complexity of MDSDVRP increases exponentially with problem size. Therefore, we investigate solution finding mechanism through generative heuristics. This essentially involves the exploration of candidate solutions which are "grown" from dynamically generated solution fragments ranked on their cost. The process involves a guided search whereby the potentially good (cost effective) "fragments" are marked beneficial for subsequent exploration and retained in the data structures. The costlier fragments are continuously discarded. In this way, the grown solutions are also cost effective since only the cost effective fragments have been retained during the search.

The solution generation technique requires a preparation procedure which analyzes the transport network graph and the customer demands at the nodes in order to establish an ordered traversal map (sorted based on cost) and respectively a demand map for all customer demands. Moreover, different solution search input parameters are also required to be set before the algorithm run. After a careful analysis on various heuristic algorithms, we arrived at a modified multi-point stochastic insertion cost gradient descent algorithm to address solution search from multiple depots. The search allows the insertion of customer nodes in the explored set of route fragments, subsequently boosting the more cost effective set of routes iteratively.

Figure 2 presents the solution generation approach which assures that a ready solution is always available after the first pass. The algorithm is also expected to help in cooperative solving of compound routing problems by a team of potentially remotely located agents. In this setting, during the search process, progressively better upper bounds found by different agents can be exchanged for improved convergence. The heuristic solution can be further improved using meta-heuristic like techniques such as permutation of adjacent nodes in routes, etc. Also, the approach allows the use of a divide and conquer policy in order to handle large scale problems whereby sub-problems involving a subset of the routes will be subjected to the same algorithm with the potential to yield better overall results. In the following, we discuss the details of the multi-point stochastic insertion cost gradient descent algorithm.

3.4. Algorithm Design

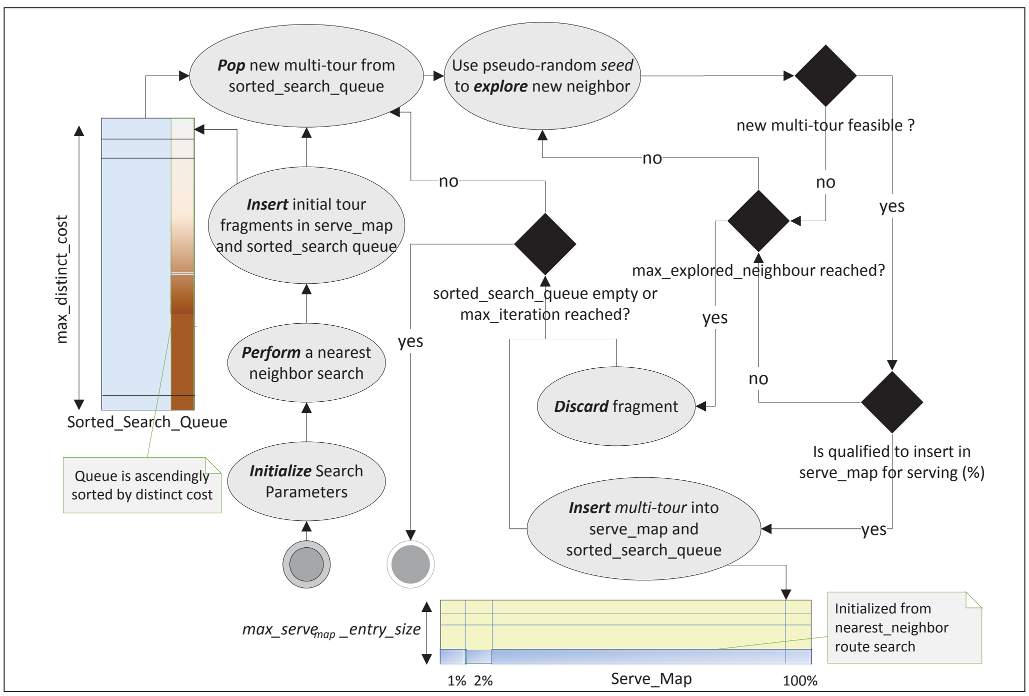


Figure 3: Heuristic procedure of route generation

Heuristics is often employed to obtain near-optimal solution where exact algorithms and equation solving are expensive in terms of memory and time allocated to the computation. The stochastic multi-point insertion cost gradient descent is one such search technique. It uses a seed based pseudo-random number generator to steer the solution search while allowing to reproduce (for a given problem instance) the same solution by using the same seed to solve the problem. Figure 3 presents the components of the high-level procedure. It starts by determining a nearest neighbor-based solution (which is computationally inexpensive to obtain), denoted by (S_{nn}) and representing an initial upper bound reference. Then, the main search starts with a base fragment (multi-tour consisting of initial vehicle locations in their depot(s)) inserted in *sorted_search_queue*. The latter keeps the fragments inserted into it in an ascending cost oder while the fragments with the same cost are arranged in descending order of the amount of their total demand served.

Algorithm 1 : MDSDVRP Heuristics

```
Input: max_iteration,S_nn,max_distinct_cost(mdc), max_explored_neighbor(maxnbr),
        max_serve_map_entry_size(msset), init_fragment, seed, usesplit
Global Knowledge: transport_network_graph(G), demand_map(dmap)
Output: S*
Initialize: S* ← S_nn; sorted_search_queue(s_que) ← ∅; serve_map ← {}; D* ← GetAllDemand(dmap);
Insert(init_fragment, s_que);
while max_iteration ≥ 0 and s_que is not empty do
  pop MultiTour s from top of s_que;
  if s contains more than one tour then
    Use Shuffle(seed) to randomize their order;
  end if
  for selectedTour in s do
    Find next customer: nextDst ← GetNextCustomer(G,LastInsertedElementOf(selectedTour));
    maxNN ← maxnbr;
    while maxNN > 0 and CountDistinctCostEntries(s_que) ≤ mdc do
      Find demand to be served: nextServeNeed ← GetDemandOf(nextDst, dmap);
      if nextServeNeed > 0 then
        if usesplit or nextServeNeed ≤ GetRemainingCapacity(selectedTour) then
          InsertInTour(nextDst, selectedTour) ;
        end if
        if CostOf(s) > CostOf(S*) then
          continue;
        end if
        if GetServeAmt(s) = D* or GetRemainingCapacity(s) = 0 then
          S* ← s;
          Remove each multi-tour(s′) fragments from s_que where CostOf(s′) > CostOf(s);
        end if
        if SizeOf(GetEntry(GetServeAmt(s),serve_map)) < msset or
        CostOf(s) ≤ GetMaxValueIn(GetEntry(GetServeAmt((s),serve_map)) then
          Insert(CostOf(s), GetEntry(GetServeAmt(s), serve_map));
          Insert(s, s_que);
        end if
        if SizeOf(serve_set(GetServeAmt(s)))> msset then
          RemoveMaxValueIn(GetEntry(GetServeAmt(s),serve_map));
        end if
      end if
      maxNN ← maxNN - 1;
    end while
  end for
  max_iteration ← max_iteration - 1;
end while
return S*;
```

At each iteration, the topmost fragment in the *sorted_search_queue* is selected by popping it out and exploring it in order to insert a neighbor node not yet served or partially served into one of its considered tours. The neighbor is identified among the unserved demand nodes by exploring them progressively up to a bound of *maxnbr* in an ascending order of traversal cost from the last inserted element in the considered tour. An neighbor that is explored can be inserted in the tour as per the vehicle's ability to serve the node (enough remaining capacity when split delivery is not used or non-empty capacity otherwise). After inserting the neighbor in the selected fragment, the latter is updated with a corresponding increased cost of serving and increased amount of serving. The updated fragment is then qualified for storing in the *sorted_search_queue* by examining if its cost of serving fits within the bounds of the corresponding $serve_{map}$ entry. The $serve_{map}$ data structure is essentially employed to build up and represent the knowledge related to the specific topology and serving availability characterizing the problem instance being solved. The knowledge gathered is represented by a set of adjustable cost bounds corresponding to particular percentages of total demand serving as discovered during fragment generation. This knowledge is used to continuously guide the search procedure by qualifying or disqualifying potential fragments while they are being explored. Thus, the $serve_{map}$ keeps entries related to the cost of serving at each related serving percentage (granularity dependent). Each entry holds a set of different cost values (for the same serving percentage) with a maximum cardinality of $max\_serve_{map}\_entry\_size$. A qualified fragment will update the corresponding $serve_{map}$ entry.

During solution search, the $serve_{map}$ entries are populated by progressively smaller cost bounds in ascending order of cost. When the maximum entry size is reached, the highest value is removed from the entry set updating the knowledge related to serving the corresponding percentage of total demand. This in turn places tighter selection pressure on subsequently explored fragments with the same serving amount. The fragments placed in the *sorted_search_queue* are stored until the *max_distinct_cost* bound is reached. Subsequently, a fragment is discarded if its updated cost is higher than the maximum cost value of the stored fragments. When a fragment is updated such that is forms a complete solution, any member of *sorted_search_queue* that has a higher cost can be removed since a complete solution with lower cost has been found. The procedure continues as long as the *sorted_search_queue* is not empty and alongside progressively lower cost complete solutions can be identified, with the one having the lowest cost remaining as the final result of solution search. The effectiveness of this solution generation procedure stems from the following. First, the heuristic employs an evolving selection pressure to identify better quality fragments leveraging knowledge gathering based on the corresponding cost values stored in the $serve_{map}$. The fragments selected in this manner are potentially more able to eventually develop good near-optimal solutions. Second, the procedure exhibits a thorough local search characteristic since all the fragments in *sorted_search_queue* are explored and updated according to their cost. Finally, the gradual solution generation trajectory leverages bounded local neighbor exploration which allows for faster convergence.

Algorithm 1 elaborates in pseudo-code the aforementioned concept by extending our previous work [33]. We describe next the notation used for the input parameters and the output. An upper bound solution denoted by S_{nn} provides an initial reference that can be quickly determined using the nearest neighbor. A demand map (*dmap*) holds the demands of each node. The *sorted_search_queue* is an ascendantly sorted queue of solution fragments based on the cost. The $serve_{map}$ represents an associative array used for knowledge gathering where each entry contains an ordered set (of parameterized maximum cardinality - $max\_serve_{map}\_entry\_size$) containing fragment serving cost values corresponding to a related percentage of total serving. Seed (*seed*) represents a unique number used to generate repeatable (for the same seed value) pseudo-random choices. The maximum number of neighbors to be considered in fragment exploration is represented by *maxnbr*. The *usesplit* is a binary input that selects whether the heuristic algorithm considers split-delivery. The algorithm is presented at a high level of abstraction with self explanatory names for the called procedures which follow the convention of having the first letter capitalized.

3.5. Property Analysis

Heuristic algorithms provide practical means to approximately solve optimization problems in short time and bounded memory with a trade-off in solution quality [38]. Moreover, specific challenges are faced during an extensive assessment of the properties characterizing heuristic algorithms. In this respect, our technique has a similar profile. Thus, in the scope of this paper, we provide three important insights with respect to the termination, convergence and solution quality.

- *Termination: Every execution of MDSDVRP heuristic will eventually stop.*
- *Convergence: Any execution of MDSDVRP heuristic for a feasible problem will converge toward a competitive solution if the search is not stopped by the maximum iteration count.*

- *Solution quality: The solution found by executing the MDSDVRP heuristic represents the lowest local optimal within the scope of the solution search space delineated by the underlying search parameters.*

With respect to the first property, every selected multi-tour fragment is restricted to explore only within a set of customer nodes that are among the closest unserved (or partially served) *maxnbr* neighbors. Therefore, for a feasible problem, the search procedure only evaluates and stores distinct fragments that can grow at most to full solutions (all customers fully served). In this respect, the fragment exploration procedure either reduces (eventually down to 0) the remaining demand unserved or discards the disqualified fragments. Since the solution space of MDSDVRP is finite, albeit potentially very large, at the extreme (for a sufficiently large values of the search parameters), the algorithm will stop after an exhaustive evaluation of all competitive solutions within the search space. However, with reasonable parameter values, the heuristic will only search a subset of the solution space, bounded in memory and time.

Concerning the second property, given the dynamics of the search technique, the potentially promising multi-tours will evolve similarly, (with respect to their granularity based serving percentage) before growing to a full solution. This growth characteristic is stemming from the fact that the *serve_map* restricts the storage of multi-tour fragments over a cost bounded percentage of serving and the *sorted_search_queue* stores the multi-tours in an ascending order of serving cost. Therefore, for each subsequent solution found, the probability of finding a better solution within a fixed delineated search space decreases successively and the solution improvement margin follows a natural logarithmic path. Figure 9 depicts the convergence characteristic (*see* Section 6). Our analysis on various problem instances reveals empirically that the solution cost (y) convergence curve over time (t) can be approximated as: $y = -C_1 \times ln(t) + C_2$, ($C_1$, C_2 are positive constants) with Pearson Coefficient of Determination (R^2) value of a few percentage points under unity.

Finally, the algorithm handles premature convergence by competitively ranking different potentially promising multi-tours based on their cost. The corresponding fragments are qualified by the bounds maintained in *serve_map* according to the percentage of serving. In essence, *serve_map* supports a guided learning over the heuristic procedure in order to promote the growth of potentially good multi-tour fragments from diverse exploration points within evolving tightness bounds. This guidance benefits the multi-point gradient descent such that each of the growing multi-tours leads toward a local optimal solution bounded by the search constraints. Therefore, the final solution emerges as the lowest one among all the local optimal solutions, generated from the diversely explored multi-tours.

3.6. Refinement Technique

The initial heuristic technique we introduced in [33] included solution refinement techniques for improving the routing cost, including localized node permutation and a Density Based Clustering tour refinement. The latter was used to dynamically generate traversal cost (distance equivalent) clusters over vehicle tour nodes. This was aimed at inter-dependent route identification using incremental clustering distances over related complete solution tour pairs until all nodes of a tour belong to the same cluster. Then, if any node (except for directly density connected ones) binds two otherwise separate clusters, then the two tours are likely to allow for solution improvement by solving the corresponding sub-problem. Thus, better (lower cost) routing is likely to be identified if available. In this work, we retain the localized node permutation refinement and introduce an alternate (more scalable) non-deterministic tour delineated sub-problem refinement. Both of these refinements are detailed next. We employ the following schemes in order to locally improve the heuristic solution as follows:

- *Selective Localized Permutation Refinement*: We generate node permutations (up to a predefined threshold) around adjacent tour nodes trying to obtain a lower routing cost in the scope of a given vehicle tour. The permutation procedure is continued successively around adjacent neighbors until no further gain can be achieved.

- *Non-deterministic tour delineated sub-problem refinement*: we proceed to non-deterministically delineate tour pairs for iterative improvement by selecting for successive iterations only those that lead to cost saving along with all the other pairs that share a member with one of the tours improved in the current iteration. This way, better solution can be progressively identified while reducing the number of tour pairs selected for further improvement over multiple iterations until no further cost savings can be obtained.

4. Case Study

In this section, we apply the proposed algorithms on a running example of a transport network in various experimental setups. The selected problem is modified from the original CVRP problem instance: (E016-03m) as published by Golden *et al.* [39]. The configuration of the transport network and customer demand of this new problem are presented below.

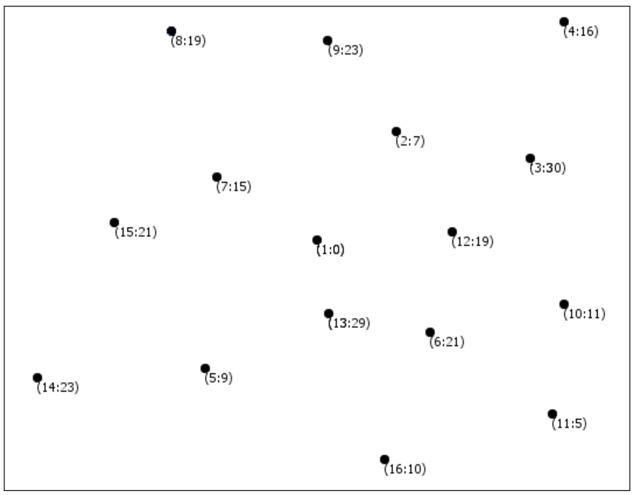


Figure 4: Transport network and customer demands

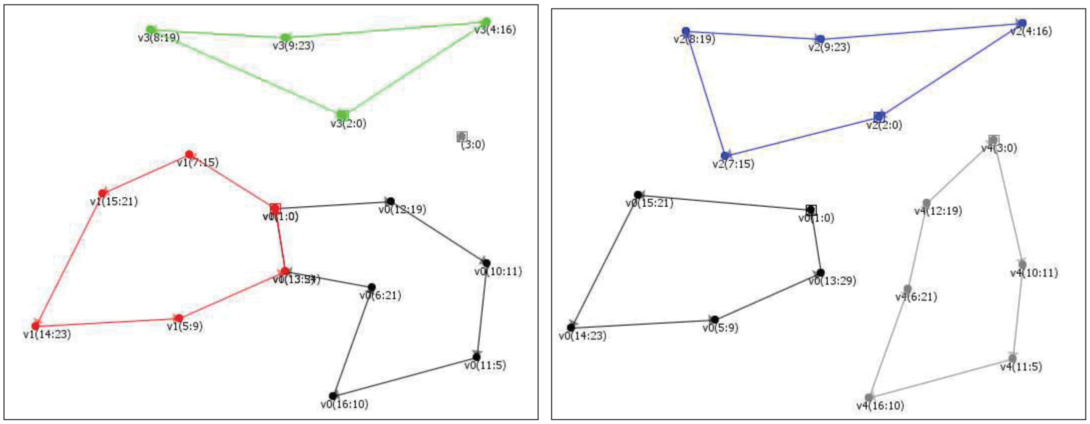
Node	X	Y	Demand	EC
1	300	400	0	0
2	370	520	7	0
3	490	490	30	0
4	520	640	16	1000
5	200	260	9	1000
6	400	300	21	1000
7	210	470	15	1000
8	170	630	19	1000
9	310	620	23	1000
10	520	330	11	1000
11	510	210	5	1000
12	420	410	19	1000
13	310	320	29	1000
14	50	250	23	1000
15	120	420	21	1000
16	360	160	10	1000

Table 1: Problem Instance Data

Figure 4 presents the example problem in a 2-Dimensional Euclidean graph. The customer nodes and their demands are presented in the format of ([node no.]:[serving]). We formed the problem such that the depots may use at most two vehicles. All of them have capacity of delivering 90 units of commodity.

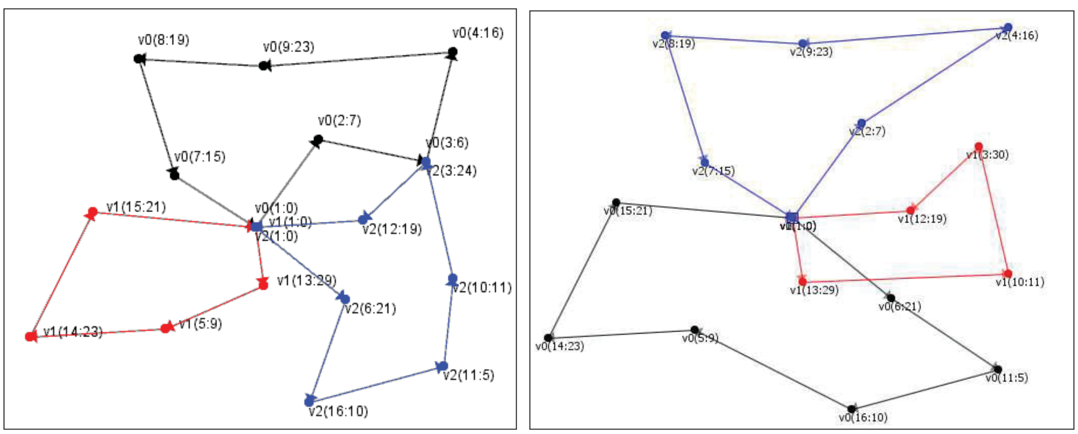
With no restriction on the number of depots, the proposed heuristic mechanism considers nodes 1, 2 and 3 as depots. Therefore, the heuristic algorithm starts finding routes after removing the demand from these nodes after considering that they are self-served. The cost of the near-optimal solutions found with and without using split-delivery are 2373 and 2402 respectively. Figure 5(a) depicts the solution computed with split delivery. For this solution, it should be noted that a split delivery is formed at customer node 13 and that depot 3 was not used in serving any customers. Figure 5(b) represents the solution found without split-delivery which uses all depots. Afterward, we test the same example with the restriction of single depot. The setup allows to verify the performance of the proposed procedure on SDVRP and CVRP problem instances.

With a restriction allowing one depot, the heuristic algorithm may perform on the case study problem instance similar to SDVRP. In such situation, we may additionally opt out for split delivery and use the same heuristic algorithm to solve problem instance as CVRP. We compare such solutions as presented in Figure 6. In this example, we setup a Split-Delivery VRP with one depot where depot establishment cost is 0 for node 1. The solutions found using heuristic algorithm are 2721 (*see* Figure 6(a)) and 2786 (*see* Figure 6(b)) using and without using split delivery respectively. The split delivery is slightly beneficial here as it can create better solution (with lower cost) than the optimal value achieved using CVRP [40]. The lower cost is achieved due to splitting the delivery in node 3 where vehicles v0 and v2 deliver 6 and 24 respectively to meet the demand.



(a) Solution using Split-Delivery in heuristics (Cost: 2373) (b) Solution using No-Split Delivery in heuristics (Cost: 2402)

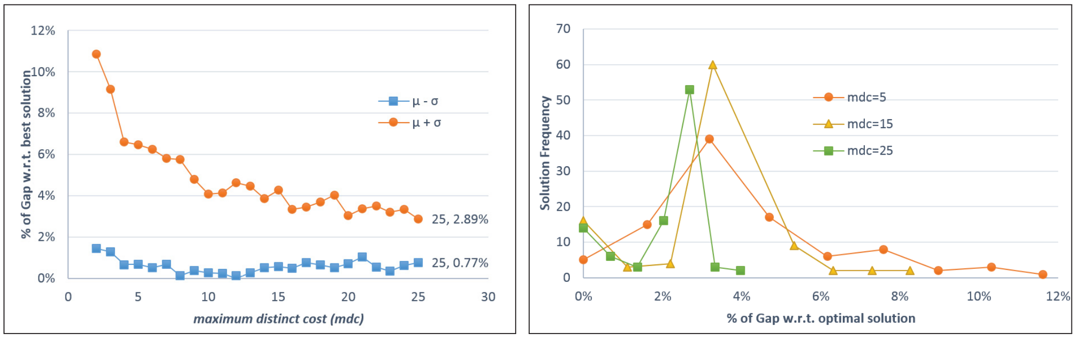
Figure 5: 3-depot heuristic solution on modified-*E016-03m* problem.



(a) Solution using Split-Delivery Heuristics (Cost: 2721) (b) Solution using Heuristics (Cost: 2786)

Figure 6: 1-depot 3-vehicle solution of modified-*E016-03m* using MDSDVRP.

Figure 7 depicts the results obtained after performing convergence analysis on the proposed heuristic approach for the MDVRP setup of the case study problem (*see* Figure 5(b)). In this setting, we explore an increasingly larger search scope of the solution space by incrementing the value of the maximum distinct cost (*mdc*) parameter. The latter represents the dominant factor in delimiting the scope of the solution search. The maximum explored neighbors (*maxnbr*) parameter is set to 3 since this value was found to perform well in benchmarks. The maximum serve map entry size (*msset*) is set to 50 accordingly. In Figure 7(a), we can see the solution generation evolution profile, in terms of standard deviation (σ) excursions from the mean (μ), corresponding to successive solution population batches. Each batch consists of 100 individual solutions obtained by applying the heuristic procedure repeatedly for the same *mdc* value but with different randomly selected seeds. We can initially note large ($\mu - \sigma$) and ($\mu + \sigma$) excursions that progressively narrow and finally flatten for the larger values of the *mdc*. In addition, it is worthy to emphasize that early on, the ($\mu - \sigma$) excursions indicate that competitive solution are also being found albeit dispersed in population



(a) Convergence Analysis of Case Study

(b) Detailed Convergence Frequency Distribution

Figure 7: 3-depot 1-vehicle per depot convergence study of modified-*E016-03m* heuristics.

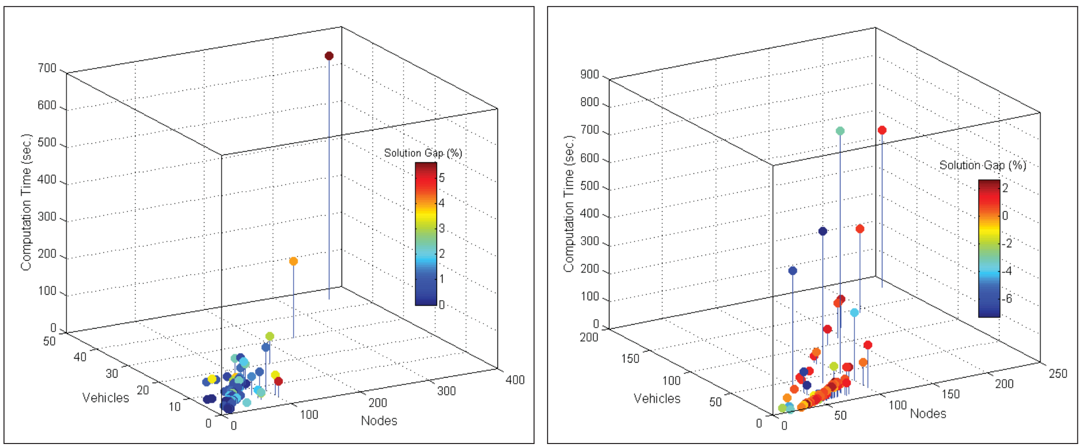
Table 2: Benchmark on known MDSDVRP problem instances [41]

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (bestHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
SQ1 (32)	2400 (2)	12 (100)	1.0 (yes)	1058 (1048)	1072 (1056.38)	-0.10 (1.00)
SQ2 (48)	3600 (3)	12 (100)	1.0 (yes)	1589 (1588)	1607 (1596.25)	0.51 (1.13)
SQ3 (64)	4800 (4)	12 (100)	1.0 (yes)	2131 (2116)	2182 (2152.25)	1.01 (2.63)
SQ4 (80)	6000 (5)	12 (100)	1.0 (yes)	2662 (2665)	2706 (2692.13)	1.16 (5.63)
SQ5 (64)	4800 (2)	25 (100)	0.96 (yes)	3422 (3446)	3481 (3461.50)	1.16 (8.63)
SQ6 (96)	7200 (3)	25 (100)	0.96 (yes)	5135 (5153)	5235 (5197.75)	1.27 (24.38)
SQ7 (128)	9600 (4)	25 (100)	0.96 (yes)	6860 (6929)	7028 (6970.25)	1.61 (57.88)
SQ8 (160)	12000 (5)	25 (100)	0.96 (yes)	8573 (8638)	8787 (8729.25)	1.84 (101.63)
SQ9 (96)	7200 (2)	36 (100)	1.0 (yes)	7051 (7047)	7074 (7062.75)	0.21 (41.13)
SQ10 (144)	10800 (3)	36 (100)	1.0 (yes)	10578 (10587)	10668 (10638.25)	0.63 (127.75)
SQ11 (192)	14400 (4)	36 (100)	1.0 (yes)	14117 (14152)	14296 (14234.13)	0.89 (302.75)
SQ12 (240)	18000 (5)	36 (100)	1.0 (yes)	17645 (17780)	17886 (17829.25)	1.07 (566.75)

with less competitive mean value. Moreover, for the larger *mdc* values, the $(\mu - \sigma)$ and $(\mu + \sigma)$ excursions are distinctly narrow and positioned around competitive mean solution values. Figure 7(b) provide further insight with respect to the convergence of the procedure to near optimal solution. It depicts the solution frequency histogram for small (5), medium (15) and large (25) *mdc* values. We can see that for $mdc = 5$, the solution frequency distribution contains a wide spectrum spanning over many less competitive solution with few hits on the best solution and many hits on poor solutions. For $mdc = 15$, we note that the solution frequency distribution spectrum is less wide, having more hits on the best solution albeit it still includes less competitive solutions. Finally, for $mdc = 25$, we observe an even narrower spectrum exhibiting the most competitive solution frequency distribution along with notable hits on the best solution.

5. Experimental Results

We present our result in Table 2 by applying the proposed algorithm on known MDSDVRP instances published previously by Gulczynski *et al.* [10, 41]. The first, second and third column define the problem instance. The `totalDem` parameter represents the combined demands of all customers while `vehCnt` and `vehCap` provide the maximum number of vehicles and related capacities. In the fourth column, the tightness of an instance represents a ratio between total customer demands and total capacity available [44] while `(split)` conveys whether the heuristic solution employs shared delivery. The fifth and sixth columns offer results from our proposed approach and compare with currently best-known values. In every run, the search is invoked eight times in parallel with different seed values in eight cores of an *Intel core i7* machine. The `bestHeurVal`, `maxHeurVal` and `avgHeurVal` denotes the best, worst and average routing cost for a problem instance. The `avgGap[%]` in last column defines the percentage of the average gap of our solution with respect to the best known value. `avgTime[sec]` is the average time taken to solve the problem instance. The underlined values in column five and seven indicate finding of better result and average than previously known solutions of the corresponding problem instances. The heuristic solutions are refined by performing localized permutation on up to 4 adjacent nodes in a route. The routing details for the underlined results are presented in the appendix. To solve the SQ problem series, the proposed algorithm uses $mdc = 5$, $msset = 100$ and $maxnbr = 1$.



(a) Time (sec.) taken to solve instances without split delivery

(b) Time (sec.) taken to solve instances with split delivery

Figure 8: Comparative Study of solution quality and time.

Similarly we solve known MDVRP instances [45] produced by Cordeau *et al.* [42]. We run the proposed heuristic algorithm by setting (*usesplit*) input parameter false in the heuristic procedure. For these problem instances, the algorithm uses $mdc = 25$, $msset = 100$ and $maxnbr = 3$. During the solution enhancement, we perform localized permutation up to 4 adjacent nodes in a route. Table 3 shows the results. In certain cases, we find similar or better results than the best known solutions published in literature [45]. With the same input parameters as used for solving the aforementioned MDVRP instances, Table 4 elaborates the result of applying heuristics over SDVRP instances introduced by Dror *et al.* [46]. These problem instances are carefully designed such that capacitated vehicle routes require sharing of commodity delivery in order to reach optimal routing. However, all the problem setups consist of one depot. In order to solve SDVRP instances, we place a restriction over depot deployment cost and start heuristic search directly from a known

Table 3: Benchmark on known MDVRP problem instances [42, 43]

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
p01 (50)	777 (4)	4 (80)	0.607 (no)	577 (577)	588 (583.38)	1.13 (2.38)
p02 (50)	777 (4)	2 (160)	0.607 (no)	474 (472)	484 (477.88)	0.86 (3.75)
p03 (75)	1364 (5)	3 (140)	0.649 (no)	641 (638)	648 (643.13)	0.38 (12.75)
p04 (100)	1458 (2)	8 (100)	0.911 (no)	1002 (997)	1014 (1007.13)	0.52 (52.13)
p05 (100)	1458 (2)	5 (200)	0.729 (no)	750 (749)	774 (758.63)	1.17 (41.00)
p06 (100)	1458 (3)	6 (100)	0.81 (no)	877 (890)	906 (897.75)	2.36 (40.63)
p07 (100)	1458 (4)	4 (100)	0.911 (no)	886 (883)	909 (897.00)	1.25 (25.13)
p12 (80)	432 (2)	5 (60)	0.72 (no)	1319 (1314)	1331 (1319.88)	0.11 (13.75)
p15 (160)	864 (4)	5 (60)	0.72 (no)	2505 (2539)	2614 (2583.25)	3.08 (74.13)
p18 (240)	1296 (6)	5 (60)	0.72 (no)	3702 (3835)	3872 (3855.75)	4.03 (206.25)
p21 (360)	1944 (9)	5 (60)	0.72 (no)	5475 (5737)	5862 (5799.25)	5.64 (657.13)

Table 4: Benchmark on known SDVRP problem instances [30]

Problem (nodes)	totalDem	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
eil22 (21)	22500	4(6000)	0.937 (no)	375 (375)	379 (378.00)	0.82 (1.00)
eil23 (22)	10189	3(4500)	0.754 (no)	569 (570)	570 (570.00)	0.20 (1.00)
eil30 (29)	12750	3(4500)	0.944 (yes)	510 (510)	511 (510.50)	0.10 (1.25)
eil33 (32)	29370	4(8000)	0.917 (no)	835 (841)	843 (842.33)	0.93 (3.00)
eil51 (50)	777	5 (160)	0.971 (no)	521 (521)	533 (525.67)	0.90 (13.67)
eilA76 (75)	1364	10 (140)	0.974 (yes)	832 (831)	841 (836.25)	0.55 (57.63)
eilA101 (100)	1458	8 (200)	0.911 (no)	817 (822)	831 (827.25)	1.29 (115.75)
eilB76 (75)	1364	14 (100)	0.974 (yes)	1023 (1010)	1032 (1024.63)	0.17 (34.38)
eilB101 (100)	1458	14 (112)	0.929 (yes)	1077 (1088)	1095 (1090.60)	1.28 (155.40)
eilC76 (75)	1364	8 (180)	0.947 (yes)	735 (741)	747 (745.00)	1.40 (47.00)
eilD76 (75)	1364	7 (220)	0.885 (no)	683 (691)	695 (692.63)	1.46 (49.38)
S51D1 (50)	402	3 (160)	0.837 (no)	458 (464)	481 (467.75)	2.13 (4.75)
S51D2 (50)	1415	9 (160)	0.982 (yes)	726 (707)	715 (711.00)	-2.06 (5.00)
S51D3 (50)	2275	15 (160)	0.947 (yes)	972 (953)	970 (959.75)	-1.22 (8.00)
S51D4 (50)	4317	27 (160)	0.999 (yes)	1677 (1561)	1581 (1569.75)	-6.79 (75.00)
S51D5 (50)	3645	23 (160)	0.99 (yes)	1440 (1337)	1351 (1344.25)	-7.09 (31.88)
S51D6 (50)	6459	41 (160)	0.984 (yes)	2327 (2182)	2196 (2187.25)	-6.35 (418.63)
S76D1 (75)	614	4 (160)	0.959 (no)	594 (601)	628 (612.38)	3.04 (17.63)
S76D2 (75)	2383	15 (160)	0.992 (yes)	1147 (1091)	1108 (1099.25)	-4.29 (36.37)
S76D3 (75)	3542	23 (160)	0.962 (yes)	1474 (1440)	1456 (1448.25)	-1.74 (82.00)
S76D4 (75)	5765	37 (160)	0.973 (yes)	2257 (2096)	2115 (2102.25)	-7.31 (547.25)
S101D1 (100)	788	5 (160)	0.985 (no)	716 (733)	748 (740.80)	3.40 (53.80)
S101D2 (100)	3064	20 (160)	0.957 (yes)	1393 (1383)	1403 (1395.00)	0.20 (82.63)
S101D3 (100)	4841	31 (160)	0.976 (yes)	1975 (1889)	1904 (1897.38)	-4.05 (244.63)
S101D5 (100)	7679	48 (160)	0.999 (yes)	2915 (2814)	2866 (2828.63)	-3.00 (874.63)

depot. With the presented input parameters, we achieve better results for many of these instances. Finally, we also solve CVRP Augerat *et. al* [44] A, B and P problem set by restricting search from a given depot and without using *split*. Table 5, Table 7 and Table 8 elaborate the results presented in the Appendix.

6. Results & Analysis

Figure 8 depicts an overall estimate of the time taken in solving all the problem instances considered in Section 5. In both sub-figures, the solution time has been calculated for all the solved problem instances with respect to number of customer nodes and vehicles. We depict the results by category based on the use of split-delivery in solution. Figure 8 shows that the proposed technique is successful in solving CVRP, SDVRP, MDVRP and MDSDVRP instances reasonably fast for small and medium scale problems. The solution generation is faster especially in the cases where split-delivery is not used (see Figure 8(a)). However, split-delivery (see Figure 8(b)) allows to generate good quality solutions which are some times better than the best known values for these instances. After a careful analysis of the results, it becomes apparent that the solving time increases notably with respect to customer nodes. On the other side, the increase in vehicles also adversely affects the solution time.

In analyzing the proposed procedure, we tested its performance using 18 different parameter combinations for *mdc*, *maxnbr* and *msset* as follows: *mdc*: {5,15,25}; *maxnbr*: {1,3,5}; *msset*: {50,100}. We selected a representative SDVRP instance (S76D2) [46] consisting of 76 nodes and 15 vehicles. Figure 9 illustrates our findings for the best solution values obtained from 8 execution runs for each parameter combination. The results are represented in two separate graphs corresponding to *msset* 50 and 100. We can notice that the algorithm converges reasonably fast during the solution search and improvement.

To further analyze the convergence characteristic, we evaluate the trend-lines for the parameter combinations $(25 \times 3 \times 50)$ and $(25 \times 3 \times 100)$. Both trend-lines represent logarithmic curves: $y = -4.213ln(t) + 7.5327$ and $y = -3.468ln(t) + 6.0566$ with R^2 value 0.9609 and 0.9867 respectively. The findings indicate that (i) the general nature of convergence curve is approximately logarithmic and (ii) the coefficients (corresponding to the search parameters) determines the approximate speed of convergence and quality of the final solutions.

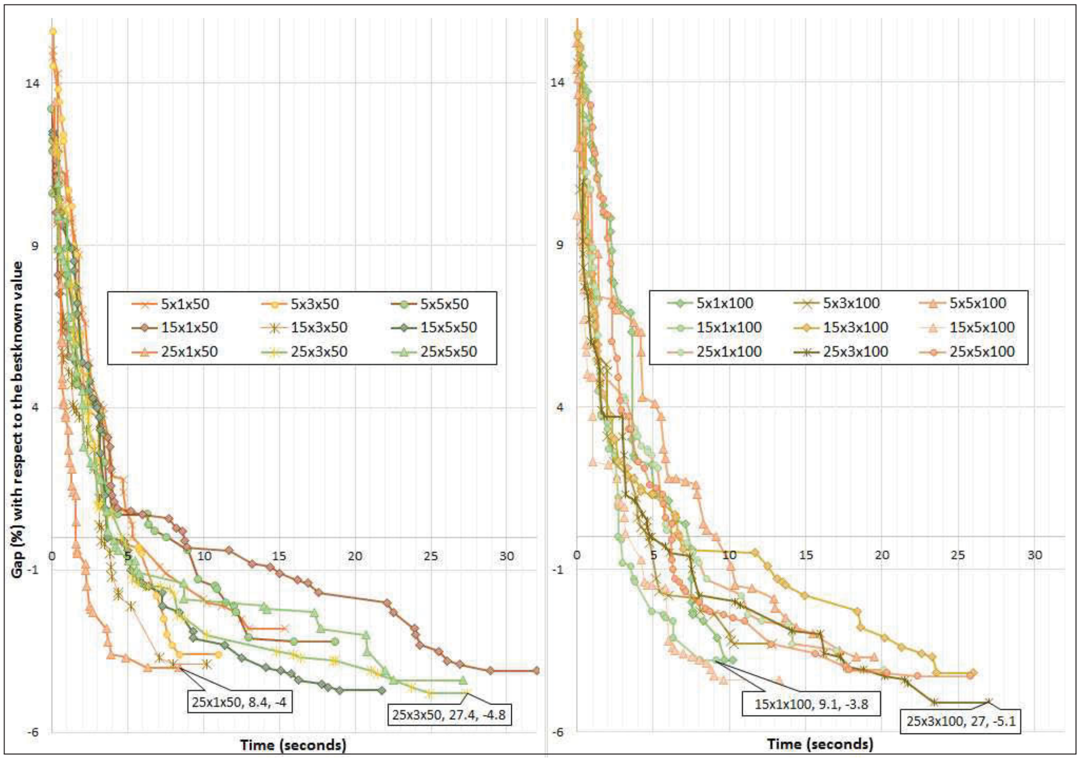


Figure 9: Convergence Study on SDVRP instance S76D2 [46] for multiple parameter values

In fact, faster convergence corresponds to diminished solution quality. Conversely, longer search time leads to better solutions for appropriate parameter combinations. The lowest computation time is obtained with parameter combinations $(25 \times 1 \times 50)$ and $(15 \times 1 \times 100)$ in the left and the right sub-figures respectively. Likewise, the best solutions are obtained with parameter combinations $(25 \times 3 \times 50)$ and $(25 \times 3 \times 100)$ in the left and the right sub-figures respectively. We may notice the level of dissimilarity with respect to the solution finding trajectory when comparing the left (less similar) and right (more similar) sides of the figure. Thus, we emphasize the selection of parameter combinations depending on the need in terms of time and quality. We favored the combination $(25 \times 3 \times 100)$ for conducting the bulk of our benchmark experiments.

Figure 10 shows a performance evaluation with respect to average gap values on 18 parameter combinations over a set of 3 CVRP series (A, B and P-series [44]) consisting of known problem instances for which optimal solutions are available in the literature. We aimed at finding appropriate parameter combinations that may lead the solution generating procedure closer to optimality for a large number of problem instances. In the upper half of Figure 10, we can see that the larger values for the 3 parameters used for solution generation help in bringing average gap close to 1% for each of the 3 series. However, we can notice a gradual increase in the average time for larger values of the parameters as depicted in Figure 10-lower half. With respect to the latter, the y-axis represents the average computation time ratio normalized by the maximum average computation time which was obtained for the larger values of the parameters. Since we observe a plateau of the average gap values in the neighborhood of 1% while reaching a *mdc* of 25 and *maxnbr* of 3, we favor the combinations for which the average computation time ratio is lower.

Thus, from the experiments conducted, a defined range of parameter values can be seen to correspond to finding good near-optimal solutions. For the *mdc*, we note a snap region for values over 15 which gradually reaches a plateau around a value of 25. With respect to *maxnbr*, a range from 3 to 5 appears to

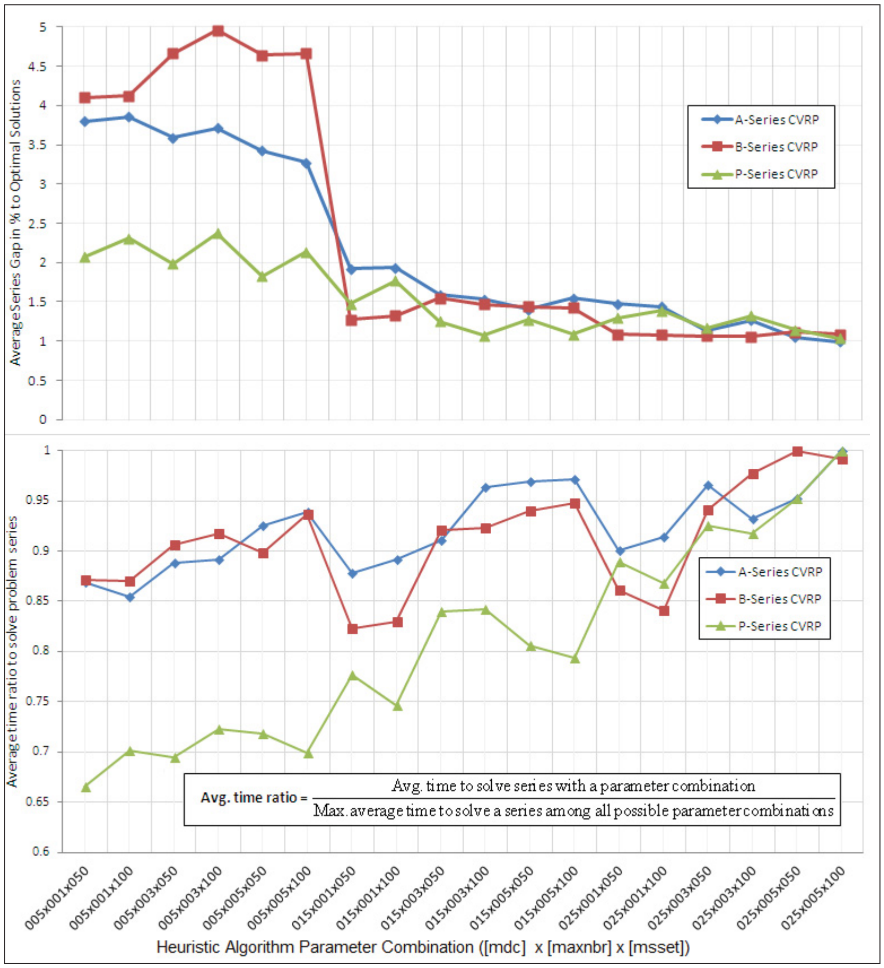


Figure 10: Performance comparisons of input parameters on CVRP instances

be most beneficial. In this context, we can estimate that values larger than 5 would lead to a certain amount of fragments grown from more distant neighbors, many of which will not eventually lead to competitive solutions. Concerning the *msset*, we can note that in some cases the lower value of 50 corresponds to better results while in other cases, the value of 100 is better suited. This indicates that a more strict (smaller *msset*) guided search may be more appropriate than a less strict (higher *msset*) for some problems and vice versa.

7. Conclusion and Future Work

In this article, we presented a generalized VRP model (MDSDVRP) suitable for multi-depot, multi-vehicle and split delivery along with a heuristic solution generation approach with efficiency refinements. The proposed approach can provide competitive solutions for diverse instances of the VRP family both in terms of cost as well as computation time. In this respect, we provided extensive benchmark results for known problem instances belonging to different VRP variants, including CVRP, MDVRP, SDVRP and MDSDVRP. Location routing represents another important feature allowing to optimize depot location and vehicle routing in a single objective function. The integer linear programming model is general and can be customized to accommodate any of the aforementioned VRP family members. The accompanying solution generation approach combines a generational stochastic cost insertion gradient descent technique with iterative solution improvement in order to produce competitive solutions. In addition, the heuristic solution finding technique exhibits good scalability and performs well over various benchmark problem sets yielding good near-optimal solutions. The use of split-delivery may help in better serving from a practical perspective and our results are especially notable in this context when compared to previous best known values. The presented technique can be suitable for various transport management systems to quickly plan cost effective vehicle routes for product delivery. The proposed heuristic approach is flexible and allows for further adaptation to accommodate other VRP settings.

The benchmark results show clear benefits in deriving routes using the proposed MDSDVRP solving approach. In this respect, the discussed heuristics yields near optimal or even optimal values for many instances. However, a trade-off exists between faster convergence and improved solution quality as determined by different values of the search parameters. It is important to emphasize that the node coordinates are not needed in the search process and only the cost matrix is used. Consequently, the procedure is not adversely affected by geometric considerations. The proposed technique is applicable in both Euclidean non-Euclidean settings. This is an important aspect since in [25], it is shown that the solution approaches based on the assumption of triangle inequality (e.g. Clarke-Wright Savings Algorithm) can be adversely affected in terms of solution quality in case where this assumption does not hold. More specifically, the solution quality increasingly degrades with the number of triangle inequality violations and in practice these situations commonly arise in many real world vehicle routing scenarios. Our solution generation approach is assessed up to hundreds of nodes and tens of vehicles. Also, it allows for parallelization (using different randomizing seeds) and solution regeneration / re-tracing (when using the same randomizing seed). However, the proposed approach has some limitations in terms of single commodity delivery and the absence of time-windows, which are the subject of future work. Other future work directions include extending the technique to handle maximum vehicle tour cost and stochastic customer demands in centralized and distributed setting.

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Appendix

Table 5: Benchmark on known CVRP problem instances: A-Set from Augerat *et al.*[44]- Part 1

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
A-n32-k5 (31)	410 (1)	5 (100)	0.82 (no)	784 (784)	791 (785.31)	0.17 (2.31)
A-n33-k5 (32)	446 (1)	5 (100)	0.892 (no)	661 (661)	669 (663.25)	0.34 (1.50)
A-n33-k6 (32)	541 (1)	6 (100)	0.901 (no)	742 (742)	745 (742.31)	0.06 (1.38)
A-n34-k5 (33)	460 (1)	5 (100)	0.92 (no)	778 (778)	791 (784.92)	0.92 (1.42)
A-n34-k5 (33)	460 (1)	5 (100)	0.92 (yes)	778 (780)	783 (782.25)	0.60 (2.25)
A-n36-k5 (35)	442 (1)	5 (100)	0.884 (no)	799 (799)	829 (818.53)	2.41 (4.13)
A-n36-k5 (35)	442 (1)	5 (100)	0.884 (yes)	799 (820)	820 (820.00)	2.60 (3.00)
A-n37-k5 (36)	407 (1)	5 (100)	0.814 (no)	669 (670)	691 (680.19)	1.68 (5.19)
A-n37-k6 (36)	570 (1)	6 (100)	0.95 (no)	949 (955)	972 (965.00)	1.70 (2.75)
A-n37-k6 (36)	570 (1)	6 (100)	0.95 (yes)	949 (948)	968 (958.00)	0.99 (5.00)
A-n38-k5 (37)	481 (1)	5 (100)	0.962 (no)	730 (730)	739 (731.80)	0.28 (3.40)
A-n38-k5 (37)	481 (1)	5 (100)	0.962 (yes)	730 (724)	745 (730.17)	0.08 (4.50)
A-n39-k5 (38)	475 (1)	5 (100)	0.95 (no)	822 (822)	830 (826.56)	0.58 (3.89)
A-n39-k5 (38)	475 (1)	5 (100)	0.95 (yes)	822 (825)	840 (829.71)	0.99 (4.86)
A-n39-k6 (38)	526 (1)	6 (100)	0.876 (no)	831 (833)	841 (834.79)	0.50 (5.50)
A-n39-k6 (38)	526 (1)	6 (100)	0.876 (yes)	831 (834)	834 (834.00)	0.40 (4.00)
A-n44-k6 (43)	570 (1)	6 (100)	0.95 (no)	937 (937)	955 (943.69)	0.73 (7.08)
A-n44-k6 (43)	570 (1)	6 (100)	0.95 (yes)	937 (937)	938 (937.33)	0.07 (4.33)
A-n45-k6 (44)	593 (1)	6 (100)	0.988 (no)	944 (948)	966 (952.88)	0.99 (6.63)
A-n45-k6 (44)	593 (1)	6 (100)	0.988 (yes)	944 (932)	943 (938.63)	-0.54 (9.13)
A-n45-k7 (44)	634 (1)	7 (100)	0.905 (no)	1146 (1151)	1164 (1157.91)	1.07 (6.82)
A-n45-k7 (44)	634 (1)	7 (100)	0.905 (yes)	1146 (1154)	1171 (1159.80)	1.24 (11.40)
A-n46-k7 (45)	603 (1)	7 (100)	0.861 (no)	914 (915)	948 (920.36)	0.75 (9.00)
A-n46-k7 (45)	603 (1)	7 (100)	0.861 (yes)	914 (926)	935 (929.00)	1.66 (10.20)
A-n48-k7 (47)	626 (1)	7 (100)	0.894 (no)	1073 (1073)	1112 (1101.62)	2.64 (13.08)
A-n48-k7 (47)	626 (1)	7 (100)	0.894 (yes)	1073 (1078)	1085 (1082.67)	0.97 (12.00)
A-n53-k7 (52)	664 (1)	7 (100)	0.948 (no)	1010 (1014)	1036 (1025.25)	1.52 (13.88)
A-n53-k7 (52)	664 (1)	7 (100)	0.948 (yes)	1010 (1008)	1023 (1015.75)	0.60 (16.50)
A-n54-k7 (53)	669 (1)	7 (100)	0.955 (no)	1167 (1173)	1190 (1180.13)	1.19 (16.88)
A-n54-k7 (53)	669 (1)	7 (100)	0.955 (yes)	1167 (1171)	1179 (1174.50)	0.69 (29.38)
A-n55-k9 (54)	839 (1)	9 (100)	0.932 (no)	1073 (1074)	1103 (1082.78)	0.93 (10.00)
A-n55-k9 (54)	839 (1)	9 (100)	0.932 (yes)	1073 (1074)	1093 (1082.00)	0.86 (16.71)
A-n60-k9 (59)	829 (1)	9 (100)	0.921 (no)	1354 (1357)	1377 (1364.45)	0.83 (27.18)
A-n60-k9 (59)	829 (1)	9 (100)	0.921 (yes)	1354 (1357)	1375 (1363.20)	0.72 (38.00)
A-n61-k9 (60)	885 (1)	9 (100)	0.983 (no)	1035 (1038)	1052 (1043.38)	0.84 (19.13)
A-n61-k9 (60)	885 (1)	9 (100)	0.983 (yes)	1034 (1022)	1028 (1025.63)	-0.74 (36.63)
A-n62-k8 (61)	733 (1)	8 (100)	0.916 (no)	1290 (1310)	1325 (1319.46)	2.28 (37.69)
A-n62-k8 (61)	733 (1)	8 (100)	0.916 (yes)	1290 (1314)	1321 (1316.67)	2.10 (42.33)
A-n63-k9 (62)	873 (1)	9 (100)	0.97 (no)	1616 (1630)	1648 (1633.13)	1.10 (26.13)
A-n63-k9 (62)	873 (1)	9 (100)	0.97 (yes)	1616 (1625)	1633 (1627.50)	0.76 (36.13)
A-n63-k10 (62)	932 (1)	10 (100)	0.932 (no)	1315 (1321)	1330 (1325.50)	0.86 (23.63)
A-n63-k10 (62)	932 (1)	10 (100)	0.932 (yes)	1315 (1312)	1329 (1321.50)	0.54 (38.25)
A-n64-k9 (63)	848 (1)	9 (100)	0.942 (no)	1402 (1427)	1450 (1437.13)	2.51 (29.63)
A-n64-k9 (63)	848 (1)	9 (100)	0.942 (yes)	1402 (1410)	1443 (1429.00)	1.95 (42.25)

Table 6: Benchmark on known CVRP problem instances: A-Set from Augerat *et al.*[44]- Part 2

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
A-n69-k9 (68)	845 (1)	9 (100)	0.938 (no)	1159 (1171)	1181 (1174.89)	1.39 (35.00)
A-n69-k9 (68)	845 (1)	9 (100)	0.938 (yes)	1159 (1168)	1179 (1174.00)	1.30 (38.57)
A-n80-k10 (79)	942 (1)	10 (100)	0.942 (no)	1764 (1799)	1823 (1806.33)	2.40 (67.78)
A-n80-k10 (79)	942 (1)	10 (100)	0.942 (yes)	1764 (1785)	1816 (1799.86)	2.03 (97.00)

Table 7: Benchmark on known CVRP problem instances: B-Set from Augerat *et al.*[44]

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
B-n31-k5 (30)	412 (1)	5 (100)	0.824 (no)	672 (672)	675 (672.44)	0.08 (1.38)
B-n34-k5 (33)	457 (1)	5 (100)	0.914 (no)	788 (789)	789 (789.00)	0.20 (1.88)
B-n34-k5 (33)	457 (1)	5 (100)	0.914 (yes)	788 (782)	783 (782.50)	-0.65 (3.25)
B-n35-k5 (34)	437 (1)	5 (100)	0.874 (no)	955 (956)	979 (962.60)	0.86 (3.47)
B-n35-k5 (34)	437 (1)	5 (100)	0.874 (yes)	955 (976)	976 (976.00)	2.20 (4.00)
B-n38-k6 (37)	512 (1)	6 (100)	0.853 (no)	805 (805)	809 (806.80)	0.27 (4.73)
B-n38-k6 (37)	512 (1)	6 (100)	0.853 (yes)	805 (807)	807 (807.00)	0.30 (7.00)
B-n39-k5 (38)	440 (1)	5 (100)	0.88 (no)	549 (549)	571 (560.67)	2.11 (4.11)
B-n39-k5 (38)	440 (1)	5 (100)	0.88 (yes)	549 (550)	555 (552.57)	0.70 (4.29)
B-n41-k6 (40)	567 (1)	6 (100)	0.945 (no)	829 (834)	844 (838.20)	1.12 (4.50)
B-n41-k6 (40)	567 (1)	6 (100)	0.945 (yes)	829 (827)	839 (831.67)	0.37 (8.50)
B-n43-k6 (42)	521 (1)	6 (100)	0.868 (no)	742 (742)	749 (744.69)	0.42 (9.77)
B-n43-k6 (42)	521 (1)	6 (100)	0.868 (yes)	742 (741)	746 (743.33)	0.23 (11.67)
B-n44-k7 (43)	641 (1)	7 (100)	0.915 (no)	909 (909)	932 (925.79)	1.86 (8.71)
B-n44-k7 (43)	641 (1)	7 (100)	0.915 (yes)	909 (927)	933 (930.00)	2.30 (10.00)
B-n45-k5 (44)	486 (1)	5 (100)	0.972 (no)	751 (760)	772 (765.11)	1.90 (8.89)
B-n45-k5 (44)	486 (1)	5 (100)	0.972 (yes)	751 (758)	768 (763.29)	1.67 (11.57)
B-n45-k6 (44)	592 (1)	6 (100)	0.986 (no)	678 (678)	691 (682.50)	0.68 (8.88)
B-n45-k6 (44)	592 (1)	6 (100)	0.986 (yes)	678 (674)	677 (675.38)	-0.32 (10.13)
B-n50-k7 (49)	609 (1)	7 (100)	0.87 (no)	741 (741)	744 (741.71)	0.13 (15.14)
B-n50-k7 (49)	609 (1)	7 (100)	0.87 (yes)	741 (743)	744 (743.50)	0.40 (19.50)
B-n50-k8 (49)	735 (1)	8 (100)	0.918 (no)	1312 (1319)	1332 (1327.75)	1.26 (14.88)
B-n50-k8 (49)	735 (1)	8 (100)	0.918 (yes)	1312 (1293)	1330 (1314.38)	0.22 (23.88)
B-n51-k7 (50)	684 (1)	7 (100)	0.977 (no)	1032 (1032)	1047 (1036.75)	0.47 (11.00)
B-n51-k7 (50)	684 (1)	7 (100)	0.977 (yes)	1032 (1026)	1042 (1034.75)	0.31 (19.38)
B-n52-k7 (51)	606 (1)	7 (100)	0.865 (no)	747 (748)	753 (751.54)	0.62 (15.69)
B-n52-k7 (51)	606 (1)	7 (100)	0.865 (yes)	747 (751)	753 (752.00)	0.70 (16.67)
B-n56-k7 (55)	616 (1)	7 (100)	0.88 (no)	707 (709)	716 (712.93)	0.87 (25.87)
B-n56-k7 (55)	616 (1)	7 (100)	0.88 (yes)	707 (717)	717 (717.00)	1.40 (20.00)
B-n57-k7 (56)	697 (1)	7 (100)	0.995 (no)	1153 (1158)	1192 (1173.13)	1.75 (22.00)
B-n57-k7 (56)	697 (1)	7 (100)	0.995 (yes)	1153 (1147)	1159 (1153.00)	0.05 (32.38)
B-n57-k9 (56)	803 (1)	9 (100)	0.892 (no)	1598 (1601)	1628 (1612.25)	0.93 (15.25)
B-n57-k9 (56)	803 (1)	9 (100)	0.892 (yes)	1598 (1594)	1613 (1601.75)	0.29 (28.25)
B-n63-k10 (62)	922 (1)	10 (100)	0.922 (no)	1496 (1537)	1548 (1542.38)	3.05 (29.75)
B-n63-k10 (62)	922 (1)	10 (100)	0.922 (yes)	1496 (1484)	1547 (1515.25)	1.29 (38.13)
B-n64-k9 (63)	878 (1)	9 (100)	0.975 (no)	861 (867)	881 (875.75)	1.71 (30.50)
B-n64-k9 (63)	878 (1)	9 (100)	0.975 (yes)	861 (861)	869 (865.13)	0.54 (49.00)
B-n66-k9 (65)	861 (1)	9 (100)	0.956 (no)	1316 (1318)	1332 (1323.00)	0.60 (36.63)
B-n66-k9 (65)	861 (1)	9 (100)	0.956 (yes)	1316 (1315)	1322 (1318.38)	0.24 (46.88)
B-n67-k10 (66)	907 (1)	10 (100)	0.907 (no)	1032 (1065)	1078 (1072.70)	3.83 (36.90)
B-n67-k10 (66)	907 (1)	10 (100)	0.907 (yes)	1032 (1040)	1075 (1059.33)	2.58 (53.17)
B-n68-k9 (67)	837 (1)	9 (100)	0.93 (no)	1272 (1287)	1294 (1289.88)	1.44 (40.75)
B-n68-k9 (67)	837 (1)	9 (100)	0.93 (yes)	1272 (1270)	1290 (1281.00)	0.74 (52.50)
B-n78-k10 (77)	937 (1)	10 (100)	0.937 (no)	1221 (1237)	1254 (1244.13)	1.91 (76.00)
B-n78-k10 (77)	937 (1)	10 (100)	0.937 (yes)	1221 (1222)	1247 (1232.50)	0.97 (97.38)

Table 8: Benchmark on known CVRP problem instances: P-Set from Augerat *et al.*[44]

Problem (nodes)	totalDem (depots)	vehCnt (vehCap)	tightness (split)	bestKnown (minHeurVal)	maxHeurVal (avgHeurVal)	avgGap[%] (avgTime[sec])
P-n16-k8 (15)	246 (1)	8 (35)	1.13 (no)	450 (450)	450 (450.00)	0.00 (1.00)
P-n16-k8 (15)	246 (1)	8 (35)	1.13 (yes)	450 (440)	440 (440.00)	-2.20 (1.00)
P-n19-k2 (18)	310 (1)	2 (160)	1.03 (no)	212 (212)	212 (212.00)	0.00 (1.00)
P-n19-k2 (18)	310 (1)	2 (160)	1.03 (yes)	212 (205)	205 (205.00)	-3.40 (1.00)
P-n20-k2 (19)	310 (1)	2 (160)	1.03 (no)	216 (217)	217 (217.00)	0.50 (1.00)
P-n21-k2 (20)	298 (1)	2 (160)	1.07 (no)	211 (211)	211 (211.00)	0.00 (1.00)
P-n22-k2 (21)	308 (1)	2 (160)	1.03 (no)	216 (216)	216 (216.00)	0.00 (1.00)
P-n22-k8 (21)	22500 (1)	8 (3000)	1.06 (no)	603 (603)	603 (603.00)	0.00 (1.00)
P-n22-k8 (21)	22500 (1)	8 (3000)	1.06 (yes)	603 (575)	586 (577.38)	-4.38 (1.00)
P-n23-k8 (22)	313 (1)	8 (40)	1.02 (no)	529 (529)	533 (529.50)	0.10 (1.00)
P-n23-k8 (22)	313 (1)	8 (40)	1.02 (yes)	529 (511)	519 (512.75)	-3.15 (1.00)
P-n40-k5 (39)	618 (1)	5 (140)	1.13 (no)	458 (458)	464 (459.27)	0.29 (4.87)
P-n45-k5 (44)	692 (1)	5 (150)	1.08 (no)	510 (510)	520 (516.08)	1.22 (5.92)
P-n50-k10 (49)	951 (1)	10 (100)	1.05 (no)	696 (697)	707 (702.25)	0.92 (2.63)
P-n50-k10 (49)	951 (1)	10 (100)	1.05 (yes)	696 (692)	699 (696.25)	0.08 (4.50)
P-n50-k7 (49)	951 (1)	7 (150)	1.1 (no)	554 (556)	565 (560.07)	1.12 (8.47)
P-n50-k8 (49)	951 (1)	8 (120)	1.0 (no)	631 (638)	645 (641.38)	1.65 (4.50)
P-n50-k8 (49)	951 (1)	8 (120)	1.0 (yes)	631 (618)	622 (619.50)	-1.82 (10.38)
P-n51-k10 (50)	777 (1)	10 (80)	1.02 (no)	741 (741)	756 (747.25)	0.86 (3.50)
P-n51-k10 (50)	777 (1)	10 (80)	1.02 (yes)	741 (730)	739 (733.88)	-0.94 (4.50)
P-n55-k10 (54)	1042 (1)	10 (115)	1.1 (no)	694 (696)	702 (700.00)	0.89 (5.36)
P-n55-k15 (54)	1042 (1)	15 (70)	1.0 (no)	989 (996)	1067 (1024.75)	3.49 (14.00)
P-n55-k15 (54)	1042 (1)	15 (70)	1.0 (yes)	989 (922)	937 (928.00)	-6.52 (11.38)
P-n55-k7 (54)	1042 (1)	7 (170)	1.14 (no)	568 (575)	579 (576.20)	1.48 (16.07)
P-n60-k10 (59)	1134 (1)	10 (120)	1.05 (no)	744 (750)	756 (752.50)	1.16 (8.50)
P-n60-k10 (59)	1134 (1)	10 (120)	1.05 (yes)	744 (742)	755 (749.25)	0.74 (17.50)
P-n60-k15 (59)	1134 (1)	15 (80)	1.05 (no)	968 (975)	980 (976.25)	0.93 (9.50)
P-n60-k15 (59)	1134 (1)	15 (80)	1.05 (yes)	968 (965)	971 (968.25)	0.09 (10.38)
P-n65-k10 (64)	1219 (1)	10 (130)	1.06 (no)	792 (800)	806 (802.33)	1.32 (19.08)
P-n70-k10 (69)	1313 (1)	10 (135)	1.02 (no)	827 (835)	845 (837.25)	1.26 (28.13)
P-n70-k10 (69)	1313 (1)	10 (135)	1.02 (yes)	827 (825)	837 (830.25)	0.41 (43.25)
P-n76-k4 (75)	1364 (1)	4 (350)	1.02 (no)	593 (598)	614 (606.31)	2.24 (19.23)
P-n76-k5 (75)	1364 (1)	5 (280)	1.02 (no)	627 (630)	648 (638.43)	1.83 (22.07)
P-n101-k4 (100)	1458 (1)	4 (400)	1.09 (no)	681 (696)	735 (718.31)	5.23 (40.75)

Table 9: Route details for selected solutions from SQ-Series with better than the best known cost.

Problem[sol]	Solution Details (segments): tour serve / tour cost								
SQ1 (split) [1048]	Route	0, 1, 2,0	(3)	Route	0,11, 3,0	(3)	Route	33,31,23,33	(3)
	Serve	0,80,20,0	100	Serve	0,90,10,0	100	Serve	0,60,40, 0	100
	Cost	14,10,10	34	Cost	28,14,14	56	Cost	20,10,10	40
	Route	0, 8, 7,0	(3)	Route	0, 9, 1,0	(3)	Route	33,28,20,33	(3)
	Serve	0,85,15,0	100	Serve	0,90,10,0	100	Serve	0,60,40, 0	100
	Cost	14,10,10	34	Cost	28,14,14	56	Cost	20,10,10	40
	Route	0, 3, 5,0	(3)	Route	0, 8,16,27,0	(4)	Route	33,18,26,33	(3)
	Serve	0,80,20,0	100	Serve	0, 5,85,10,0	100	Serve	0,35,65, 0	100
	Cost	14,10,10	34	Cost	14,14, 2,30	60	Cost	10,10,20	40
	Route	0, 4, 6,0	(3)	Route	0, 6,14,25,0	(4)	Route	33,29,21,33	(3)
	Serve	0,20,80,0	100	Serve	0,10,85, 5,0	100	Serve	0,60,40, 0	100
	Cost	10,10,14	34	Cost	14,14, 2,30	60	Cost	20,10,10	40
	Route	0, 2,10,0	(3)	Route	33,19,18,33	(3)	Route	33,25,17,33	(3)
	Serve	0,40,60,0	100	Serve	0,75,25, 0	100	Serve	0,90,10, 0	100
	Cost	10,10,20	40	Cost	14,10,10	34	Cost	28,14,14	56
	Route	0, 4,12,0	(3)	Route	33,17,20,33	(3)	Route	33,32,24,33	(3)
	Serve	0,40,60,0	100	Serve	0,80,20, 0	100	Serve	0,90,10, 0	100
	Cost	10,10,20	40	Cost	14,10,10	34	Cost	28,14,14	56
	Route	0, 5,13,0	(3)	Route	33,21,24,33	(3)	Route	33,19,27,33	(3)
	Serve	0,40,60,0	100	Serve	0,20,80, 0	100	Serve	0,15,85, 0	100
	Cost	10,10,20	40	Cost	10,10,14	34	Cost	14,14,28	56
	Route	0,15, 7,0	(3)	Route	33,23,22,33	(3)	Route	33,22,30,33	(3)
	Serve	0,55,45,0	100	Serve	0,20,80, 0	100	Serve	0,10,90, 0	100
	Cost	20,10,10	40	Cost	10,10,14	34	Cost	14,14,28	56

Pb.[sol]	Solution Details (segments): tour serve / tour cost								
SQ2 (split) [1588]	Route	0, 8, 7,0	(3)	Route	49,20,17,49	(3)	Route	50,38,36,50	(3)
	Serve	0,90,10,0	100	Serve	0,20,80, 0	100	Serve	0,90,10, 0	100
	Cost	14,10,10	34	Cost	10,10,14	34	Cost	14,10,10	34
	Route	0, 2, 1,0	(3)	Route	49,18,19,49	(3)	Route	50,33,34,50	(3)
	Serve	0,20,80,0	100	Serve	0,60,40, 0	100	Serve	0,80,20, 0	100
	Cost	10,10,14	34	Cost	10,10,14	34	Cost	14,10,10	34
	Route	0, 4, 6,0	(3)	Route	49,21,24,49	(3)	Route	50,37,35,50	(3)
	Serve	0,20,80,0	100	Serve	0,30,70, 0	100	Serve	0,20,80, 0	100
	Cost	10,10,14	34	Cost	10,10,14	34	Cost	10,10,14	34
	Route	0, 3, 5,0	(3)	Route	49,23,22,49	(3)	Route	50,40,39,50	(3)
	Serve	0,80,20,0	100	Serve	0,20,80, 0	100	Serve	0,80,20, 0	100
	Cost	14,10,10	34	Cost	10,10,14	34	Cost	14,10,10	34
	Route	0,10, 2,0	(3)	Route	49,28,20,49	(3)	Route	50,44,36,50	(3)
	Serve	0,60,40,0	100	Serve	0,60,40, 0	100	Serve	0,50,50, 0	100
	Cost	20,10,10	40	Cost	20,10,10	40	Cost	20,10,10	40
	Route	0, 4,12,0	(3)	Route	49,31,23,49	(3)	Route	50,34,42,50	(3)
	Serve	0,40,60,0	100	Serve	0,60,40, 0	100	Serve	0,40,60, 0	100
	Cost	10,10,20	40	Cost	20,10,10	40	Cost	10,10,20	40
	Route	0,13, 5,0	(3)	Route	49,29,44,21,49	(4)	Route	50,39,47,50	(3)
	Serve	0,60,40,0	100	Serve	0,65, 5,30, 0	100	Serve	0,40,60, 0	100
	Cost	20,10,10	40	Cost	20, 2,12,10	44	Cost	10,10,20	40
	Route	0,15, 7,0	(3)	Route	49,19,26,15,49	(4)	Route	50,37,45,50	(3)
	Serve	0,50,50,0	100	Serve	0,30,65, 5, 0	100	Serve	0,40,60, 0	100
	Cost	20,10,10	40	Cost	14,14, 2,22	52	Cost	10,10,20	40
	Route	0,11, 3,0	(3)	Route	49,17,25,49	(3)	Route	50,35,43,50	(3)
	Serve	0,90,10,0	100	Serve	0,10,90, 0	100	Serve	0,10,90, 0	100
	Cost	28,14,14	56	Cost	14,14,28	56	Cost	14,14,28	56
	Route	0, 1, 9,0	(3)	Route	49,24,32,49	(3)	Route	50,33,41,50	(3)
	Serve	0,10,90,0	100	Serve	0,20,80, 0	100	Serve	0,10,90, 0	100
	Cost	14,14,28	56	Cost	14,14,28	56	Cost	14,14,28	56
	Route	0,14,25, 6,0	(4)	Route	49,22,30,49	(3)	Route	50,40,48,50	(3)
	Serve	0,85, 5,10,0	100	Serve	0,10,90, 0	100	Serve	0,10,90, 0	100
	Cost	28, 2,16,14	60	Cost	14,14,28	56	Cost	14,14,28	56
	Route	0,16,27,0	(3)	Route	49,19,27,49	(3)	Route	50,46,32,50	(3)
	Serve	0,85,15,0	100	Serve	0,20,80, 0	100	Serve	0,85,15, 0	100
	Cost	28, 2,30	60	Cost	14,14,28	56	Cost	28, 2,30	60
SQ3 (split) [2116]	Route	0, 5, 3,0	(3)	Route	65,18,26,65	(3)	Route	66,48,39,66	(3)
	Serve	0,15,85,0	100	Serve	0,45,55, 0	100	Serve	0,90,10, 0	100
	Cost	10,10,14	34	Cost	10,10,20	40	Cost	28,22,10	60
	Route	0, 7, 8,0	(3)	Route	65,31,23,65	(3)	Route	66,43,64,66	(3)
	Serve	0,25,75,0	100	Serve	0,60,40, 0	100	Serve	0,85,15, 0	100
	Cost	10,10,14	34	Cost	20,10,10	40	Cost	28, 2,30	60
	Route	0, 2, 1,0	(3)	Route	65,28,20,65	(3)	Route	66,33,41,36,66	(4)
	Serve	0,20,80,0	100	Serve	0,60,40, 0	100	Serve	0, 5,85,10, 0	100
	Cost	10,10,14	34	Cost	20,10,10	40	Cost	14,14,22,10	60
	Route	0, 4, 6,0	(3)	Route	65,21,29,44,65	(4)	Route	66,46,32,66	(3)
	Serve	0,20,80,0	100	Serve	0,30,65, 5, 0	100	Serve	0,85,15, 0	100
	Cost	10,10,14	34	Cost	10,10, 2,22	44	Cost	28, 2,30	60
	Route	0,13, 5,0	(3)	Route	65,19,27,65	(3)	Route	67,55,54,67	(3)
	Serve	0,55,45,0	100	Serve	0, 5,95, 0	100	Serve	0,15,85, 0	100
	Cost	20,10,10	40	Cost	14,14,28	56	Cost	10,10,14	34
	Route	0, 2,10,0	(3)	Route	65,17,25,65	(3)	Route	67,56,55,67	(3)
	Serve	0,40,60,0	100	Serve	0,10,90, 0	100	Serve	0,90,10, 0	100
	Cost	10,10,20	40	Cost	14,14,28	56	Cost	14,10,10	34
	Route	0, 4,12,0	(3)	Route	65,24,32,65	(3)	Route	67,50,51,67	(3)
	Serve	0,40,60,0	100	Serve	0,20,80, 0	100	Serve	0,20,80, 0	100
	Cost	10,10,20	40	Cost	14,14,28	56	Cost	10,10,14	34
	Route	0,26,15, 7,0	(4)	Route	65,22,30,65	(3)	Route	67,52,49,67	(3)
	Serve	0,10,55,35,0	100	Serve	0,10,90, 0	100	Serve	0,25,75, 0	100
	Cost	22, 2,10,10	44	Cost	14,14,28	56	Cost	10,10,14	34
	Route	0, 9, 1,0	(3)	Route	66,37,40,66	(3)	Route	67,52,60,67	(3)
	Serve	0,90,10,0	100	Serve	0,10,90, 0	100	Serve	0,35,65, 0	100
	Cost	28,14,14	56	Cost	10,10,14	34	Cost	10,10,20	40
	Route	0, 8,16,0	(3)	Route	66,33,34,66	(3)	Route	67,61,53,67	(3)
	Serve	0,15,85,0	100	Serve	0,85,15, 0	100	Serve	0,60,40, 0	100
	Cost	14,14,28	56	Cost	14,10,10	34	Cost	20,10,10	40
	Route	0, 3,11,57,0	(4)	Route	66,37,35,66	(3)	Route	67,55,63,67	(3)
	Serve	0, 5,85,10,0	100	Serve	0,10,90, 0	100	Serve	0,35,65, 0	100
	Cost	14,14, 2,30	60	Cost	10,10,14	34	Cost	10,10,20	40
	Route	0,14,25, 6,0	(4)	Route	66,38,39,66	(3)	Route	67,58,50,67	(3)
	Serve	0,85, 5,10,0	100	Serve	0,90,10, 0	100	Serve	0,60,40, 0	100
	Cost	28, 2,16,14	60	Cost	14,10,10	34	Cost	20,10,10	40
	Route	65,24,21,65	(3)	Route	66,37,45,66	(3)	Route	67,54,62,67	(3)
	Serve	0,70,30, 0	100	Serve	0,40,60, 0	100	Serve	0, 5,95, 0	100
	Cost	14,10,10	34	Cost	10,10,20	40	Cost	14,14,28	56
	Route	65,20,17,65	(3)	Route	66,36,44,66	(3)	Route	67,49,57,67	(3)
	Serve	0,20,80, 0	100	Serve	0,50,50, 0	100	Serve	0,15,85, 0	100
	Cost	10,10,14	34	Cost	10,10,20	40	Cost	14,14,28	56
	Route	65,18,19,65	(3)	Route	66,34,42,66	(3)	Route	67,59,51,67	(3)
	Serve	0,15,85, 0	100	Serve	0,45,55, 0	100	Serve	0,90,10, 0	100
	Cost	10,10,14	34	Cost	10,10,20	40	Cost	28,14,14	56
	Route	65,23,22,65	(3)	Route	66,47,39,66	(3)	Route	67,53,64,67	(3)
	Serve	0,20,80, 0	100	Serve	0,60,40, 0	100	Serve	0,20,80, 0	100
	Cost	10,10,14	34	Cost	20,10,10	40	Cost	10,22,28	60

Pb.[sol]	Solution Details (segments):	tour serve / tour cost							
	Route	0,28,36,0	(3)	Route	0, 4, 1,0	(3)	Route	97,76,84,92,97	(4)
	Serve	0,60,40,0	100	Serve	0,60,40,0	100	Serve	0,30,10,60, 0	100
	Cost	40,10,50	100	Cost	10,10,14	34	Cost	40,10,10,60	120
	Route	0,39,31,0	(3)	Route	0, 6,15,0	(3)	Route	97,82,90,47,97	(4)
	Serve	0,60,40,0	100	Serve	0,50,50,0	100	Serve	0,30,65, 5, 0	100
	Cost	50,10,40	100	Cost	14,14,20	48	Cost	50,10, 2,62	124
	Route	0,21,29,37,0	(4)	Route	0, 9, 1,0	(3)	Route	97,72,88,97	(3)
	Serve	0,20,60,20,0	100	Serve	0,50,50,0	100	Serve	0,10,90, 0	100
	Cost	30,10,10,50	100	Cost	28,14,14	56	Cost	42,28,71	141
	Route	0,26,34,18,0	(4)	Route	0, 8,16,0	(3)	Route	97,70,86,97	(3)
	Serve	0,40,40,20,0	100	Serve	0,10,90,0	100	Serve	0,20,80, 0	100
	Cost	40,10,20,30	100	Cost	14,14,28	56	Cost	42,28,71	141
	Route	0,15,23,24,0	(4)	Route	0, 3,11,0	(3)	Route	97,65,81,97	(3)
	Serve	0,10,30,60,0	100	Serve	0,50,50,0	100	Serve	0,10,90, 0	100
	Cost	20,10,30,42	102	Cost	14,14,28	56	Cost	42,28,71	141
	Route	0,22,30,0	(3)	Route	0, 6,14,0	(3)	Route	97,83,67,97	(3)
	Serve	0,10,90,0	100	Serve	0,40,60,0	100	Serve	0,90,10, 0	100
	Cost	42,14,57	113	Cost	14,14,28	56	Cost	71,28,42	141
	Route	0,32,24,0	(3)	Route	0,12,20,0	(3)	Route	97,91,67,97	(3)
	Serve	0,90,10,0	100	Serve	0,60,40,0	100	Serve	0,90,10, 0	100
	Cost	57,14,42	113	Cost	20,10,30	60	Cost	85,42,42	169
	Route	0,25,17,0	(3)	Route	0,13,21,0	(3)	Route	97,65,89,97	(3)
	Serve	0,90,10,0	100	Serve	0,60,40,0	100	Serve	0,20,80, 0	100
	Cost	57,14,42	113	Cost	20,10,30	60	Cost	42,42,85	169
	Route	0,19,27,0	(3)	Route	0,10,18,0	(3)	Route	97,96,72,97	(3)
	Serve	0,10,90,0	100	Serve	0,60,40,0	100	Serve	0,90,10, 0	100
	Cost	42,14,57	113	Cost	20,10,30	60	Cost	85,42,42	169
	Route	0,26,34,42,0	(4)	Route	0,14,22,0	(3)	Route	97,86,94,97	(3)
	Serve	0,20,20,60,0	100	Serve	0,30,70,0	100	Serve	0,10,90, 0	100
	Cost	40,10,10,60	120	Cost	28,14,42	84	Cost	71,14,85	170
	Route	0,23,31,47,0	(4)	Route	0, 9,17,0	(3)	Route	97,52,55,97	(3)
	Serve	0,30,20,50,0	100	Serve	0,40,60,0	100	Serve	0,60,40, 0	100
	Cost	30,10,20,60	120	Cost	28,14,42	84	Cost	10,14,10	34
	Route	0,37,45,0	(3)	Route	0,11,19,0	(3)	Route	97,53,56,97	(3)
	Serve	0,40,60,0	100	Serve	0,40,60,0	100	Serve	0,10,90, 0	100
SQ9 (split) [7047]	Cost	50,10,60	120	Cost	28,14,42	84	Cost	10,10,14	34
	Route	0,20,36,44,0	(4)	Route	97,66,74,82,97	(4)	Route	97,53,51,97	(3)
	Serve	0,20,20,60,0	100	Serve	0,10,60,30, 0	100	Serve	0,10,90, 0	100
	Cost	30,20,10,60	120	Cost	30,10,10,50	100	Cost	10,10,14	34
	Route	0,17,33,0	(3)	Route	97,68,76,84,97	(4)	Route	97,50,49,97	(3)
	Serve	0,20,80,0	100	Serve	0,20,30,50, 0	100	Serve	0,20,80, 0	100
	Cost	42,28,71	141	Cost	30,10,10,50	100	Cost	10,10,14	34
	Route	0,24,40,0	(3)	Route	97,69,77,85,97	(4)	Route	97,54,55,97	(3)
	Serve	0,20,80,0	100	Serve	0,20,40,40, 0	100	Serve	0,90,10, 0	100
	Cost	42,28,71	141	Cost	30,10,10,50	100	Cost	14,10,10	34
	Route	0,19,35,0	(3)	Route	97,79,87,97	(3)	Route	97,50,58,97	(3)
	Serve	0,20,80,0	100	Serve	0,50,50, 0	100	Serve	0,40,60, 0	100
	Cost	42,28,71	141	Cost	40,10,50	100	Cost	10,10,20	40
	Route	0,22,38,0	(3)	Route	97,69,72,97	(3)	Route	97,53,61,97	(3)
	Serve	0,10,90,0	100	Serve	0,40,60, 0	100	Serve	0,40,60, 0	100
	Cost	42,28,71	141	Cost	30,30,42	102	Cost	10,10,20	40
	Route	0,33,41,0	(3)	Route	97,60,68,65,97	(4)	Route	97,49,57,97	(3)
	Serve	0,10,90,0	100	Serve	0,10,40,50, 0	100	Serve	0,10,90, 0	100
	Cost	71,14,85	170	Cost	20,10,30,42	102	Cost	14,14,28	56
	Route	0,35,43,0	(3)	Route	97,78,70,97	(3)	Route	97,63,71,97	(3)
	Serve	0,10,90,0	100	Serve	0,90,10, 0	100	Serve	0,50,50, 0	100
	Cost	71,14,85	170	Cost	57,14,42	113	Cost	20,10,30	60
	Route	0,40,48,91,0	(4)	Route	97,72,80,97	(3)	Route	97,55,64,97	(3)
	Serve	0,10,85, 5,0	100	Serve	0,10,90, 0	100	Serve	0,10,90, 0	100
	Cost	71,14, 2,86	173	Cost	42,14,57	113	Cost	10,22,28	60
	Route	0,46,89,0	(3)	Route	97,75,67,97	(3)	Route	97,60,62,97	(3)
	Serve	0,85,15,0	100	Serve	0,90,10, 0	100	Serve	0,50,50, 0	100
	Cost	85, 2,86	173	Cost	57,14,42	113	Cost	20,20,28	68
	Route	0, 7, 8,0	(3)	Route	97,65,73,97	(3)	Route	97,59,66,97	(3)
	Serve	0,60,40,0	100	Serve	0,10,90, 0	100	Serve	0,50,50, 0	100
	Cost	10,10,14	34	Cost	42,14,57	113	Cost	28,22,30	80
	Route	0, 5, 8,0	(3)	Route	97,63,71,79,87,95,97	(6)	Route	97,67,59,97	(3)
	Serve	0,60,40,0	100	Serve	0,10,10,10,10,60, 0	100	Serve	0,60,40, 0	100
	Cost	10,10,14	34	Cost	20,10,10,10,10,60	120	Cost	42,14,28	84
	Route	0, 2, 3,0	(3)	Route	97,93,85,77,97	(4)	Route	97,62,70,97	(3)
	Serve	0,60,40,0	100	Serve	0,60,20,20, 0	100	Serve	0,40,60, 0	100
	Cost	10,10,14	34	Cost	60,10,10,40	120	Cost	28,14,42	84

Table 10: Route details for notable solutions from SDVRP instances with better than the best known cost.

Problem[sol]	Solution Details (segments):			tour serve / tour cost		
eilB76 (split) [1010]	Route	1,59,11,32,56,26,1	(6)	Route	1,46,30,16,58,55,14,28,1	(8)
	Serve	0,21,26,25, 7,14,0	93	Serve	0,21,12, 8,14,16,12,17,0	100
	Cost	20, 6,13,22, 9,33	103	Cost	14, 4,10, 4,14, 8, 7,16	77
	Route	1,76, 5,53,35,1	(5)	Route	1, 7,34, 2,57,24,64,1	(7)
	Serve	0,20,30,19,19,0	88	Serve	0,19,27,11,26, 6,11,0	100
	Cost	3, 5, 9, 4,10	31	Cost	9, 9, 8,16, 6, 9,22	79
	Route	1,27,13,41,18,1	(5)	Route	1,52,17,50,25,19,51,33,1	(8)
	Serve	0,18,16,33,20,0	87	Serve	0,12,19, 5,27,13,22, 2,0	100
	Cost	6, 8, 5, 7, 8	34	Cost	11, 9, 9, 7,13, 6, 8,22	85
	Route	1,68,47, 9,36, 8,1	(6)	Route	1,54,12,60,15,20,1	(6)
	Serve	0,30,27,16,10,15,0	98	Serve	0,22, 8,24,31,15,0	100
	Cost	5, 6, 5, 5, 5,14	40	Cost	23, 8,15,11, 9,23	89
	Route	1,69, 3,63,29,75,31,1	(7)	Route	1,22,62,70,48,49,31,1	(7)
	Serve	0,10,26,15,29,10,10,0	100	Serve	0,28,15, 8,19,20,10,0	100
	Cost	7, 7, 8, 6, 6, 7,14	55	Cost	27,11,14,11, 6, 7,14	90
	Route	1,73,40,10,33,45, 4,1	(7)	Route	1,30, 6,38,21,71,61,72,37,31,1	(10)
	Serve	0, 1,16,29,26,17,11,0	100	Serve	0, 1,21,14,22,11,13, 3,12, 2,0	99
	Cost	21, 5, 4, 7, 5, 3,20	65	Cost	18, 7, 7, 6, 6, 4, 5, 7,19,14	93
	Route	1,39,66,67,12,1	(5)	Route	1,74,2,44,42,43,65,23,63,1	(9)
	Serve	0,24, 9,37,29,0	99	Serve	0, 6,7,18,15,11,28,12, 3,0	100
	Cost	27, 5, 7, 7,29	75	Cost	21, 5,7, 4, 4, 9,14, 8,22	94
S51D2 (split) [707]	Route	1,46,34,40,31,11, 6,1	(7)	Route	1,17,51,35,22,30, 3,1	(7)
	Serve	0,43,19,47,18,20,12,0	159	Serve	0,18,20,18,47,33,23,0	159
	Cost	31, 7,14,12, 9,14,14	101	Cost	22, 6, 6, 9, 7, 9,21	80
	Route	1,13,38,16,45,18,48,1	(7)	Route	1,49,24, 8,44,25,1	(6)
	Serve	0,17,25,41,17,45,15,0	160	Serve	0,19,20,47,43,23,0	152
	Cost	8,10, 7, 6, 9, 9, 9	58	Cost	16, 9, 6,12,12,25	80
	Route	1,33,12,39,10,50, 6,1	(7)	Route	1,23, 4,37,36,21,33,1	(7)
	Serve	0,15,46,18,24,47, 9,0	159	Serve	0,18,46,22,45,21, 8,0	160
	Cost	10, 6, 7, 7, 6, 8,14	58	Cost	21,12,12, 6, 7,22,10	90
	Route	1, 7,15,26,14,19,47,1	(7)	Route	1,48, 5,42,20,41,43,1	(7)
	Serve	0,21,19,43,37,18,19,0	157	Serve	0,18,28,36,32,20,18,0	152
	Cost	11,10, 6,13,14,16, 2	72	Cost	9, 8,13, 5,11,16,31	93
	Route	1,28, 9,27,32,29, 2,1	(7)			
	Serve	0,24,22,18,37,23,33,0	157			
	Cost	8,14, 7,10, 6,16,14	75			
S51D3 (split) [953]	Route	1,13,48,19,1	(4)	Route	1, 2,23,21, 3,1	(5)
	Serve	0,25,76,59,0	160	Serve	0,18,72,43,27,0	160
	Cost	8, 6, 8,15	37	Cost	14, 7,15,12,21	69
	Route	1,33,12,39,1	(4)	Route	1, 6,50,31,35,10,39,1	(7)
	Serve	0,79,31,50,0	160	Serve	0,20,51,26,33,20, 7,0	157
	Cost	10, 6, 7,16	39	Cost	14, 8,10, 7, 9, 7,16	71
	Route	1,47,1	(2)	Route	1,24,44,25,1	(4)
	Serve	0,79,0	79	Serve	0,18,68,70,0	156
	Cost	2, 2	4	Cost	22,13,12,25	72
	Route	1, 7,15,26,19,1	(5)	Route	1,17,51,22,30,1	(5)
	Serve	0,31,74,23,20,0	148	Serve	0,32,78,21,25,0	156
	Cost	11,10, 6,11,15	53	Cost	22, 6, 8, 7,29	72
	Route	1,18,43, 5,1	(4)	Route	1,38,34,40,11,1	(5)
	Serve	0,20,72,68,0	160	Serve	0, 5,28,56,56,0	145
	Cost	17,14,16,17	64	Cost	18,18,14,10,28	88
	Route	1,49, 8,27, 9,1	(5)	Route	1, 4,37,36,21,1	(5)
	Serve	0,53,31,52,24,0	160	Serve	0,30,76,43,11,0	160
	Cost	16,11,11, 7,22	67	Cost	33,12, 6, 7,32	90
	Route	1,16,46,45,38,1	(5)	Route	1, 5,20,41,42,14,1	(6)
	Serve	0,29,43,61,27,0	160	Serve	0, 3,25,69,37,20,0	154
	Cost	25, 7,10, 7,18	67	Cost	17,15,11,12, 9,29	93
	Route	1,28, 9,32,29, 2,1	(6)			
	Serve	0,21,27,32,78, 2,0	160			
	Cost	8,14, 9, 6,16,14	67			

Problem[sol]	Solution Details (segments):			tour serve / tour cost		
S51D4 (split) [1561]	Route	1,47, 33,1	(3)	Route	1,19,42,1	(3)
	Serve	0,36,124,0	160	Serve	0,89,71,0	160
	Cost	2, 9, 10	21	Cost	15,17,30	62
	Route	1,13, 48,1	(3)	Route	1,16, 46,13,1	(4)
	Serve	0,46,114,0	160	Serve	0,34,112,14,0	160
	Cost	8, 6, 9	23	Cost	25, 7, 23, 8	63
	Route	1,48, 5,1	(3)	Route	1, 9, 29, 2,1	(4)
	Serve	0,17,143,0	160	Serve	0, 5,127,28,0	160
	Cost	9, 8, 17	34	Cost	22,13, 16,14	65
	Route	1, 38,13,1	(3)	Route	1,51,22,17,12,1	(5)
	Serve	0,134,26,0	160	Serve	0,30,69,42,19,0	160
	Cost	18, 10, 8	36	Cost	26, 8,10,10,12	66
	Route	1,28,49, 7,1	(4)	Route	1,39,10,31,11,1	(5)
	Serve	0,58,81,21,0	160	Serve	0, 5,23,63,69,0	160
	Cost	8, 9, 9,11	37	Cost	16, 7, 8, 9,28	68
	Route	1, 15, 7,1	(3)	Route	1,47, 34,13,1	(4)
	Serve	0,127,33,0	160	Serve	0,16,136, 8,0	160
	Cost	18, 10,11	39	Cost	2,32, 27, 8	69
	Route	1, 2, 23,33,1	(4)	Route	1,39,31,35,51,1	(5)
	Serve	0,13,137,10,0	160	Serve	0,10,45,92,13,0	160
	Cost	14, 7, 12,10	43	Cost	16,15, 7, 6,26	70
	Route	1,39,50, 6,1	(4)	Route	1, 8, 44, 7,1	(4)
	Serve	0,62,70,28,0	160	Serve	0,27,117,16,0	160
	Cost	16, 8, 8,14	46	Cost	26,12, 23,11	72
	Route	1,19,26,15,1	(4)	Route	1,43,20,42,1	(4)
	Serve	0,54,94,12,0	160	Serve	0,63,40,57,0	160
	Cost	15,11, 6,18	50	Cost	31, 9, 5,30	75
	Route	1,38, 45,18,1	(4)	Route	1, 6,11, 40,1	(4)
	Serve	0, 9,106,45,0	160	Serve	0, 7,53,100,0	160
	Cost	18, 7, 9,17	51	Cost	14,14,10, 38	76
	Route	1, 7,25,24,1	(4)	Route	1,33, 3,21,36, 4,1	(6)
	Serve	0,19,93,48,0	160	Serve	0, 8,12,45,84,11,0	160
	Cost	11,14, 9,22	56	Cost	10,11,12, 7,10,33	83
	Route	1, 9, 27,1	(3)	Route	1, 2, 4, 37,1	(4)
	Serve	0,36,124,0	160	Serve	0, 2,14,141,0	157
	Cost	22, 7, 28	57	Cost	14,19,12, 44	89
	Route	1, 3, 30,12,1	(4)	Route	1,14, 41,42,1	(4)
	Serve	0,31,118,11,0	160	Serve	0,25,127, 8,0	160
	Cost	21, 9, 17,12	59	Cost	29,19, 12,30	90
	Route	1,32, 9,1	(3)			
	Serve	0,70,90,0	160			
	Cost	30, 9,22	61			
S51D5 (split) [1337]	Route	1,13,47,1	(3)	Route	1,18, 43,48,1	(4)
	Serve	0,70,90,0	160	Serve	0,40,111, 9,0	160
	Cost	8, 7, 2	17	Cost	17,14, 22, 9	62
	Route	1,28, 33,1	(3)	Route	1, 29,32,1	(3)
	Serve	0,52,108,0	160	Serve	0,108,52,0	160
	Cost	8, 8, 10	26	Cost	30, 6,30	66
	Route	1,48, 5,1	(3)	Route	1,50,35,10,1	(4)
	Serve	0,52,108,0	160	Serve	0,54,91,15,0	160
	Cost	9, 8, 17	34	Cost	22,14, 9,23	68
	Route	1,12, 39,1	(3)	Route	1,19,14,42,1	(4)
	Serve	0,53,104,0	157	Serve	0,47,59,54,0	160
	Cost	12, 7, 16	35	Cost	15,14, 9,30	68
	Route	1, 7,15,1	(3)	Route	1, 3,30,22,17,1	(5)
	Serve	0,94,66,0	160	Serve	0,33,60,64, 3,0	160
	Cost	11,10,18	39	Cost	21, 9, 7,10,22	69
	Route	1, 2,23,33,47,1	(5)	Route	1,24,44,25,1	(4)
	Serve	0,59,78, 3,20,0	160	Serve	0, 6,58,93,0	157
	Cost	14, 7,12, 9, 2	44	Cost	22,13,12,25	72
	Route	1,15,26,19,1	(4)	Route	1, 6,34,46,1	(4)
	Serve	0,45,63,52,0	160	Serve	0,23,56,66,0	145
	Cost	18, 6,11,15	50	Cost	14,21, 7,31	73
	Route	1,49, 8,24,1	(4)	Route	1, 3,21, 4,1	(4)
	Serve	0,60,54,46,0	160	Serve	0,36,68,56,0	160
	Cost	16,11, 6,22	55	Cost	21,12, 8,33	74
	Route	1,13,38,16,45,1	(5)	Route	1,10,31,40,1	(4)
	Serve	0, 8,49,52,51,0	160	Serve	0,34,70,55,0	159
	Cost	8,10, 7, 6,25	56	Cost	23, 8,12,38	81
	Route	1,17, 51,10,1	(4)	Route	1,18,20,41,42,1	(5)
	Serve	0,50,106, 4,0	160	Serve	0,22,52,79, 1,0	154
	Cost	22, 6, 6,23	57	Cost	17,17,11,12,30	87
	Route	1,47,50, 11, 6,1	(5)	Route	1,21,36, 37,1	(4)
	Serve	0, 1,24,109,26,0	160	Serve	0, 1,54,100,0	155
	Cost	2,19, 8, 14,14	57	Cost	32, 7, 6, 44	89
	Route	1,49,27, 9,1	(4)			
	Serve	0,11,51,96,0	158			
	Cost	16,13, 7,22	58			

Problem[sol]	Solution Details (segments):		tour serve / tour cost			
S51D6 (split) [2182]	Route	1, 13,1	(2)	Route	1,17, 51,10,1	(4)
	Serve	0,131,0	131	Serve	0,34,118, 8,0	160
	Cost	8, 8	16	Cost	22, 6, 6,23	57
	Route	1, 48,1	(2)	Route	1,50, 11,1	(3)
	Serve	0,140,0	140	Serve	0,49,111,0	160
	Cost	9, 9	18	Cost	22, 8, 28	58
	Route	1,28, 7,1	(3)	Route	1, 14,19,1	(3)
	Serve	0,39,121,0	160	Serve	0,114,46,0	160
	Cost	8, 9, 11	28	Cost	29, 14,15	58
	Route	1,12,33,1	(3)	Route	1,12, 30,17,1	(4)
	Serve	0,77,83,0	160	Serve	0, 5,106,49,0	160
	Cost	12, 6,10	28	Cost	12,17, 9,22	60
	Route	1,28, 2,1	(3)	Route	1, 42, 5,1	(3)
	Serve	0,42,118,0	160	Serve	0,129,31,0	160
	Cost	8, 8, 14	30	Cost	30, 13,17	60
	Route	1,12, 39,1	(3)	Route	1,10, 31,1	(3)
	Serve	0,33,127,0	160	Serve	0,25,135,0	160
	Cost	12, 7, 16	35	Cost	23, 8, 31	62
	Route	1,19, 5,1	(3)	Route	1,18, 43,1	(3)
	Serve	0,50,110,0	160	Serve	0,35,123,0	158
	Cost	15, 8, 17	40	Cost	17,14, 31	62
	Route	1,18, 38,1	(3)	Route	1,16, 46,1	(3)
	Serve	0,36,117,0	153	Serve	0,21,136,0	157
	Cost	17, 5, 18	40	Cost	25, 7, 31	63
	Route	1, 3,33,1	(3)	Route	1, 22,17,1	(3)
	Serve	0,118,42,0	160	Serve	0,125,35,0	160
	Cost	21, 11,10	42	Cost	32, 10,22	64
	Route	1,19, 15,1	(3)	Route	1,10, 35,1	(3)
	Serve	0,47,113,0	160	Serve	0,29,131,0	160
	Cost	15,10, 18	43	Cost	23, 9, 32	64
	Route	1,33, 23,1	(3)	Route	1,18, 20, 5,1	(4)
	Serve	0,18,142,0	160	Serve	0,48,110, 2,0	160
	Cost	10,12, 21	43	Cost	17,17, 15,17	66
	Route	1,28, 9,1	(3)	Route	1,29, 32,1	(3)
	Serve	0,20,140,0	160	Serve	0,35,125,0	160
	Cost	8,14, 22	44	Cost	30, 6, 30	66
	Route	1,24,49,1	(3)	Route	1, 34, 6,1	(3)
	Serve	0,78,82,0	160	Serve	0,142,18,0	160
	Cost	22, 9,16	47	Cost	34, 21,14	69
	Route	1,15, 26,1	(3)	Route	1, 7, 44,24,1	(4)
	Serve	0,29,131,0	160	Serve	0, 9,137,13,0	159
	Cost	18, 6, 23	47	Cost	11,23, 13,22	69
	Route	1, 47,1	(2)	Route	1, 29, 4,1	(3)
	Serve	0,121,0	121	Serve	0,104,56,0	160
	Cost	2, 2	4	Cost	30, 9,33	72
	Route	1,38, 45,1	(3)	Route	1, 21, 4,1	(3)
	Serve	0,26,134,0	160	Serve	0,119,41,0	160
	Cost	18, 7, 25	50	Cost	32, 8,33	73
	Route	1, 6,50,10,1	(4)	Route	1,11, 40,1	(3)
	Serve	0,33,76,51,0	160	Serve	0,27,133,0	160
	Cost	14, 8, 6,23	51	Cost	28,10, 38	76
	Route	1,49, 8,1	(3)	Route	1,30, 36,1	(3)
	Serve	0,46,114,0	160	Serve	0,31,129,0	160
	Cost	16,11, 26	53	Cost	29,16, 39	84
	Route	1, 6,16,1	(3)	Route	1,42, 41,20,1	(4)
	Serve	0,65,95,0	160	Serve	0,13,139, 8,0	160
	Cost	14,15,25	54	Cost	30,12, 11,32	85
	Route	1, 25,24,1	(3)	Route	1, 4, 37,1	(3)
	Serve	0,131,29,0	160	Serve	0,17,143,0	160
	Cost	25, 9,22	56	Cost	33,12, 44	89
	Route	1, 27,28,1	(3)			
	Serve	0,139,21,0	160			
	Cost	28, 20, 8	56			

Problem[sol]	Solution Details (segments):		tour serve / tour cost			
S76D2 (split) [1091]	Route	1,68,35,47,53, 5,1	(6)	Route	1,64,24,57, 2,7,1	(6)
	Serve	0,18,46,40,46,10,0	160	Serve	0,19,46,47,40,8,0	160
	Cost	5, 5, 2, 5, 9, 7	33	Cost	22, 9, 6,16,16,9	78
	Route	1,76,31,49,30,46,5,1	(7)	Route	1,75,29,23,62,22,31,1	(7)
	Serve	0,18,10,44,43,37,8,0	160	Serve	0,40,26,29,16,23,26,0	160
	Cost	3,11, 7, 6, 4, 7,7	45	Cost	20, 6, 9,12,11,13,14	85
	Route	1,69, 3,63,74,34,1	(6)	Route	1,27,59,11,32,66,39,1	(7)
	Serve	0,28,22,40,23,47,0	160	Serve	0, 2,29,17,39,19,47,0	153
	Cost	7, 7, 8, 5, 5,18	50	Cost	6,14, 6,13,20, 5,27	91
	Route	1,27, 8,36,20, 9,1	(6)	Route	1,2,44,42,43,65,1	(6)
	Serve	0,33,27,35,47,18,0	160	Serve	0,4,27,43,46,38,0	158
	Cost	6,10, 5, 7, 8,16	52	Cost	25,7, 4, 4, 9,43	92
	Route	1,18,41,10,40,73,13,1	(7)	Route	1,8,54,15,60,67,12,1	(7)
	Serve	0,13,37,44,31,16,19,0	160	Serve	0,7,26,18,20,47,42,0	160
	Cost	8, 7,10, 4, 5, 9,12	55	Cost	14,9, 7,11,15, 7,29	92
	Route	1,46, 6,38,37,48,1	(6)	Route	1,18,33,26,56,19,51,45,1	(8)
	Serve	0, 1,47,47,33,25,0	153	Serve	0, 9,29,35,21,23,29,14,0	160
	Cost	14,11, 7, 8, 6,27	73	Cost	8,14,12, 9,14, 6,10,21	94
	Route	1,28,16,58,14,55,53,1	(7)	Route	1,16,21,71,61,72,70,31,1	(8)
	Serve	0,43,44,20,35,16, 1,0	159	Serve	0, 2,22,46,45,18,22, 5,0	160
	Cost	16,12, 4, 9, 8,13,14	76	Cost	27,11, 6, 4, 5, 9,23,14	99
	Route	1, 7,17,50,25,45, 4,52,1	(8)			
	Serve	0,12,20,32,24,27,22,23,0	160			
	Cost	9,12, 9, 7,15, 3,10,11	76			
S76D3 (split) [1440]	Route	1,13,40,32,56,26,10,1	(7)	Route	1,46,30, 6,37,48,49,1	(7)
	Serve	0,19,23,20,38,31,21,0	152	Serve	0, 9,25,17,68,29, 7,0	155
	Cost	12,10,16,22, 9,10,24	103	Cost	14, 4, 7,10, 6, 6,21	68
	Route	1,76, 5,1	(3)	Route	1,17,50,25, 4,1	(5)
	Serve	0,75,76,0	151	Serve	0,21,46,61,24,0	152
	Cost	3, 5, 7	15	Cost	19, 9, 7,14,20	69
	Route	1,69, 7,1	(3)	Route	1,45,19,51,1	(4)
	Serve	0,68,62,0	130	Serve	0,19,67,74,0	160
	Cost	7, 5, 9	21	Cost	21,14, 6,30	71
	Route	1,68,35,53,1	(4)	Route	1,64,24,57,1	(4)
	Serve	0,44,59,55,0	158	Serve	0,52,20,75,0	147
	Cost	5, 5, 4,14	28	Cost	22, 9, 6,37	74
	Route	1,68, 9,47,1	(4)	Route	1,12,67,66,1	(4)
	Serve	0,17,64,79,0	160	Serve	0,21,44,79,0	144
	Cost	5,10, 5,11	31	Cost	29, 7, 7,32	75
	Route	1,18,33,41,1	(4)	Route	1,49,22,62,1	(4)
	Serve	0,30,79,47,0	156	Serve	0,32,32,78,0	142
	Cost	8,14, 9,14	45	Cost	21, 9,11,34	75
	Route	1,27,59,73,13,1	(5)	Route	1,44,42,43, 2,1	(5)
	Serve	0,19,57,57,27,0	160	Serve	0,26,23,79,27,0	155
	Cost	6,14, 5, 9,12	46	Cost	32, 4, 4,11,25	76
	Route	1,31,75,29, 3,1	(5)	Route	1,70,72,38,1	(4)
	Serve	0,38,32,76,14,0	160	Serve	0,73,69,17,0	159
	Cost	14, 7, 6,10,15	52	Cost	37, 9,10,32	88
	Route	1,52,34,74,63,1	(5)	Route	1, 3,63,23,65,1	(5)
	Serve	0,33,24,58,45,0	160	Serve	0,23, 6,46,77,0	152
	Cost	11,10, 5, 5,22	53	Cost	15, 8, 8,14,43	88
	Route	1,54,15,36,1	(4)	Route	1,46,30,21,71,61,1	(6)
	Serve	0,59,79,22,0	160	Serve	0, 2, 3,37,37,79,0	158
	Cost	23, 7,10,18	58	Cost	14, 4,18, 6, 4,43	89
	Route	1,59,11,39,1	(4)	Route	1, 8,60,20,55,53,1	(6)
	Serve	0, 2,77,72,0	151	Serve	0,22,45,20,67, 6,0	160
	Cost	20, 6, 7,27	60	Cost	14,24,18, 9,13,14	92
	Route	1,28,14,58,16,46,1	(6)			
	Serve	0,20,27,70,20,23,0	160			
	Cost	16, 7, 9, 4,13,14	63			

Problem[sol]	Solution Details (segments):			tour serve / tour cost		
	Route	1,51,56,19,25,1	(5)			
	Serve	0,35,55,26,44,0	160	Route	1,28, 58,1	(3)
	Cost	30,15,14,13,33	105	Serve	0,25,135,0	160
	Route	1, 68,1	(2)	Cost	16,12, 28	56
	Serve	0,143,0	143	Route	1,27, 12,1	(3)
	Cost	5, 5	10	Serve	0,24,133,0	157
	Route	1, 69,1	(2)	Cost	6,24, 29	59
	Serve	0,138,0	138	Route	1,53,55,20,1	(4)
	Cost	7, 7	14	Serve	0,32,41,82,0	155
	Route	1, 18,1	(2)	Cost	14,13, 9,23	59
	Serve	0,135,0	135	Route	1,13,40,11,59,1	(5)
	Cost	8, 8	16	Serve	0, 1,77,33,49,0	160
	Route	1, 8,1	(2)	Cost	12,10,12, 6,20	60
	Serve	0,143,0	143	Route	1, 2, 44,34,1	(4)
	Cost	14, 14	28	Serve	0,24,126,10,0	160
	Route	1, 46, 5,1	(3)	Cost	25, 7, 14,18	64
	Serve	0,126,34,0	160	Route	1,41, 26,33,1	(4)
	Cost	14, 7, 7	28	Serve	0,32,124, 4,0	160
	Route	1,47, 9,36,1	(4)	Cost	14,19, 12,22	67
	Serve	0,31,41,88,0	160	Route	1,75, 62,1	(3)
	Cost	11, 5, 5,18	39	Serve	0,38,121,0	159
	Route	1,13, 73,1	(3)	Cost	20,15, 34	69
	Serve	0,21,139,0	160	Route	1, 76,1	(2)
	Cost	12, 9, 21	42	Serve	0,124,0	124
	Route	1, 7,74, 3,1	(4)	Cost	3, 3	6
	Serve	0,39,74,46,0	159	Route	1,39,66,67,12,1	(5)
	Cost	9,12, 9,15	45	Serve	0,25,49,72, 5,0	151
S76D4	Route	1,35,53, 14,28,1	(5)	Cost	27, 5, 7, 7,29	75
(split)	Serve	0,24,17,102,17,0	160	Route	1,42,43, 2,1	(4)
[2096]	Cost	10, 4, 9, 7,16	46	Serve	0,96,27,37,0	160
	Route	1, 64,34,1	(3)	Cost	36, 4,11,25	76
	Serve	0,131,21,0	152	Route	1,36,15, 60,1	(4)
	Cost	22, 6,18	46	Serve	0,30,29,101,0	160
	Route	1,41,45, 4,1	(4)	Cost	18,10,11, 38	77
	Serve	0, 1,20,139,0	160	Route	1,48,37,70,31,1	(5)
	Cost	14, 9, 3, 20	46	Serve	0,13,37,89,19,0	158
	Route	1,36, 54,1	(3)	Cost	27, 6, 7,23,14	77
	Serve	0,15,143,0	158	Route	1, 5, 72,37,1	(4)
	Cost	18, 7, 23	48	Serve	0,14,139, 5,0	158
	Route	1,63,29,75,1	(4)	Cost	7,33, 7,33	80
	Serve	0,60,47,53,0	160	Route	1,52,17,24,57, 2,1	(6)
	Cost	22, 6, 6,20	54	Serve	0,13,25,35,73,14,0	160
	Route	1,75, 22,31,1	(4)	Cost	11, 9,13, 6,16,25	80
	Serve	0,12,143, 5,0	160	Route	1,30,21,71,38,1	(5)
	Cost	20, 8, 13,14	55	Serve	0, 8,37,87,28,0	160
	Route	1,40,10,33,1	(4)	Cost	18,18, 6, 9,32	83
	Serve	0,66,51,43,0	160	Route	1, 2, 65,23,1	(4)
	Cost	22, 4, 7,22	55	Serve	0,11,121,25,0	157
	Route	1, 50,52,1	(3)	Cost	25,18, 14,30	87
	Serve	0,141,19,0	160	Route	1, 6,61,38,1	(4)
	Cost	28, 17,11	56	Serve	0,36,98,24,0	158
	Route	1, 5,30,48,49,1	(5)	Cost	25,18,12,32	87
	Serve	0,17,42,73,28,0	160	Route	1,73, 32,66,1	(4)
	Cost	7,11,11, 6,21	56	Serve	0, 4,100,56,0	160
	Route	1,30, 16,28,1	(4)	Cost	21,16, 20,32	89
	Serve	0,34,118, 8,0	160			
	Cost	18,10, 12,16	56			

Problem[sol]	Solution Details (segments):			tour serve / tour cost		
S101D3 (split) [1889]	Route	1,11,64,65,50,1	(5)	Route	1, 7,94,86,92,62, 6,1	(7)
	Serve	0,44,20,69,24,0	157	Serve	0,15, 3,40,35,31,36,0	160
	Cost	25, 9,14,13,44	105	Cost	11, 9, 3, 3, 6, 7,21	60
	Route	1, 2,21,67,72,66,36,1	(7)	Route	1,42,23,75,1	(4)
	Serve	0, 4,52,24,21,30,29,0	160	Serve	0,22,79,59,0	160
	Cost	15,16, 9, 9,10,12,41	112	Cost	29, 4, 3,25	61
	Route	1,54,27,29,1	(4)	Route	1,61,6,85,18,1	(5)
	Serve	0,29,74,57,0	160	Serve	0,48,1,24,79,0	152
	Cost	4, 8, 8, 6	26	Cost	18, 4,4, 6,30	62
	Route	1,14,95,1	(3)	Route	1,34,82,10,52,1	(5)
	Serve	0,74,64,0	138	Serve	0,45,28,60,20,0	153
	Cost	11, 4,12	27	Cost	25, 3, 6, 6,27	67
	Route	1,90, 7,97,1	(4)	Route	1,30,25,55,1	(4)
	Serve	0,52,30,78,0	160	Serve	0,58,60,39,0	157
	Cost	9, 5, 4,15	33	Cost	30, 7,10,23	70
	Route	1,59, 3,41,1	(4)	Route	1,33,91,11,1	(4)
	Serve	0,59,33,68,0	160	Serve	0,75,71,14,0	160
	Cost	9, 9, 9,11	38	Cost	34, 4, 7,25	70
	Route	1,28,70, 2,51,1	(5)	Route	1,88,43,44,16,58,1	(6)
	Serve	0,38,26,17,79,0	160	Serve	0,31,15,23,60,31,0	160
	Cost	5, 7, 4, 6,17	39	Cost	18, 7, 9, 7, 7,23	71
	Route	1,96,98,93,1	(4)	Route	1,29,80,35,79,1	(5)
	Serve	0,41,74,45,0	160	Serve	0, 8,24,36,76,0	144
	Cost	15, 3, 3,18	39	Cost	6,19,11, 5,31	72
	Route	1,41,22,73,1	(4)	Route	1,63,12,20,1	(4)
	Serve	0, 7,77,76,0	160	Serve	0,34,68,52,0	154
	Cost	11, 7, 4,22	44	Cost	25, 8, 7,32	72
	Route	1, 4,78,77,29,1	(5)	Route	1,43,15,45,1	(4)
	Serve	0,79,18,60, 3,0	160	Serve	0,34,69,57,0	160
	Cost	22, 3, 4, 9, 6	44	Cost	25, 9, 6,32	72
	Route	1,13,81,69,1	(4)	Route	1, 9,47,46,84,1	(5)
	Serve	0,38,35,79,0	152	Serve	0,33,70,17,27,0	147
	Cost	15, 6, 2,21	44	Cost	26, 9,11, 8,21	75
	Route	1,100,94,99,1	(4)	Route	1,55,56,26,40, 5,1	(6)
	Serve	0, 48,60,52,0	160	Serve	0,36,20,69, 4,29,0	158
	Cost	17, 3, 3,21	44	Cost	23, 8, 4, 9, 9,25	78
	Route	1,53, 8,89,1	(4)	Route	1,49,48,37,1	(4)
	Serve	0,35,75,50,0	160	Serve	0,57,21,79,0	157
	Cost	11,10, 6,19	46	Cost	28, 6, 7,41	82
	Route	1,19,83,1	(3)	Route	1,94,17,87,39,45,1	(6)
	Serve	0,77,79,0	156	Serve	0, 1,44,77,22,14,0	158
	Cost	16, 9,23	48	Cost	20, 9, 6,13,11,32	91
	Route	1,60,38,101,99,93,1	(6)	Route	1,41,74,75,76,24,68,40,57,1	(9)
	Serve	0,20,33, 79, 3,23,0	158	Serve	0, 3,21, 7,22,44,20,21,22,0	160
	Cost	18, 4, 3, 3, 3,18	49	Cost	11, 9, 4, 4, 8,12,10, 7,29	94
	Route	1,32,71,31,1	(4)			
	Serve	0,33,48,59,0	140			
	Cost	17, 7, 5,25	54			

Problem[sol]	Solution Details (segments):			tour serve / tour cost		
S101D5 (split) [2814]	Route	1,52,72, 66,21,1	(5)	Route	1,73,75,23,1	(4)
	Serve	0,16,27,111, 6,0	160	Serve	0,25,48,87,0	160
	Cost	27,13,10, 21,32	103	Cost	22, 3, 3,27	55
	Route	1,12,65,50,1	(4)	Route	1,34, 80,1	(3)
	Serve	0,16,61,83,0	160	Serve	0,56,104,0	160
	Cost	34,13,13,44	104	Cost	25, 6, 26	57
	Route	1,54,29,1	(3)	Route	1,84,46, 9,1	(4)
	Serve	0,76,84,0	160	Serve	0, 8,75,77,0	160
	Cost	4, 7, 6	17	Cost	21, 8, 6,26	61
	Route	1,28, 70,1	(3)	Route	1, 55,56,1	(3)
	Serve	0,53,107,0	160	Serve	0,102,58,0	160
	Cost	5, 7, 12	24	Cost	23, 8,30	61
	Route	1,59,41,1	(3)	Route	1,51,34,82,52,1	(5)
	Serve	0,91,69,0	160	Serve	0, 2, 4,71,83,0	160
	Cost	9, 4,11	24	Cost	17, 8, 3, 7,27	62
	Route	1,90, 7,95,1	(4)	Route	1,61, 6,85,18,1	(5)
	Serve	0,77,31,52,0	160	Serve	0,34,21,52,52,0	159
	Cost	9, 5, 3,12	29	Cost	18, 4, 4, 6,30	62
	Route	1,27,13,29,1	(4)	Route	1,57, 5,1	(3)
	Serve	0,84,57,19,0	160	Serve	0,71,89,0	160
	Cost	11, 7, 9, 6	33	Cost	29, 8,25	62
	Route	1,14, 98,1	(3)	Route	1,69,30, 25,1	(4)
	Serve	0,52,108,0	160	Serve	0, 1,57,102,0	160
	Cost	11, 6, 17	34	Cost	21, 9, 7, 30	67
	Route	1,53,19,1	(3)	Route	1,100,17,45,38,1	(5)
	Serve	0,77,83,0	160	Serve	0, 31,17,93,19,0	160
	Cost	11, 8,16	35	Cost	17, 12, 6,11,21	67
	Route	1,97,60,96,1	(4)	Route	1,83,49,48,1	(4)
	Serve	0,46,93,21,0	160	Serve	0,17,53,90,0	160
	Cost	15, 3, 4,15	37	Cost	23, 5, 6,34	68
	Route	1, 51, 2,1	(3)	Route	1,70,31, 33,1	(4)
	Serve	0,108,52,0	160	Serve	0, 4,45,111,0	160
	Cost	17, 6,15	38	Cost	12,13,10, 34	69
	Route	1,97,100,94, 7,1	(5)	Route	1,11,64,91,1	(4)
	Serve	0, 6, 46,98,10,0	160	Serve	0,24,72,64,0	160
	Cost	15, 2, 3, 9,11	40	Cost	25, 9, 4,32	70
	Route	1,41,22, 74,1	(4)	Route	1,83, 47, 9,1	(4)
	Serve	0,32,23,105,0	160	Serve	0,43,111, 6,0	160
	Cost	11, 7, 3, 20	41	Cost	23,13, 9,26	71
	Route	1,95,96,38,99,93,1	(6)	Route	1,62,17,87, 6,1	(5)
	Serve	0,10,31,31,38,50,0	160	Serve	0,19,47,63,31,0	160
	Cost	12, 3, 6, 1, 3,18	43	Cost	25, 4, 6,16,21	72
	Route	1, 4,78,77,1	(4)	Route	1,32,12,20, 8,1	(5)
	Serve	0,62,91, 7,0	160	Serve	0,24,47,57,32,0	160
	Cost	22, 3, 4,16	45	Cost	17,17, 7,11,21	73
	Route	1,19, 84,61,1	(4)	Route	1,58,16,42,23,1	(5)
	Serve	0,28,101,31,0	160	Serve	0,13,61,69,17,0	160
	Cost	16, 7, 4,18	45	Cost	23, 7,12, 4,27	73
	Route	1,81,69,77,29,1	(5)	Route	1,40,26,56,1	(4)
	Serve	0,53,57,47, 3,0	160	Serve	0,56,55,49,0	160
	Cost	21, 2, 8, 9, 6	46	Cost	34, 9, 4,30	77
	Route	1, 3,58,88,1	(4)	Route	1,88,43, 44,15,1	(5)
	Serve	0,59,86,15,0	160	Serve	0,10,13,100,37,0	160
	Cost	18, 6, 7,18	49	Cost	18, 7, 9, 11,32	77
	Route	1,88,43,1	(3)	Route	1,29,80,79,35,10,1	(6)
	Serve	0,69,91,0	160	Serve	0, 4, 7,55,65,29,0	160
	Cost	18, 7,25	50	Cost	6,19, 6, 5,11,32	79
	Route	1,93,99,92,101,38,1	(6)	Route	1,31,21,67,1	(4)
	Serve	0, 2,21,70, 53,14,0	160	Serve	0,13,80,67,0	160
	Cost	18, 3, 4, 3, 3,21	52	Cost	25, 7, 9,40	81
	Route	1, 7,62,86,60,1	(5)	Route	1,8,49,48, 37,1	(5)
	Serve	0,22,87,50, 1,0	160	Serve	0,2,32,18,108,0	160
	Cost	11,14, 4, 5,18	52	Cost	21,7, 6, 7, 41	82
	Route	1,71,11,32,1	(4)	Route	1,98,15,39,1	(4)
	Serve	0,56,64,40,0	160	Serve	0, 2,69,89,0	160
	Cost	21, 8, 8,17	54	Cost	17,15,11,42	85
	Route	1,54,76,73,22,1	(5)	Route	1,52,72,36,10,1	(5)
	Serve	0,15,92,23,30,0	160	Serve	0,12,50,75,23,0	160
	Cost	4,23, 5, 4,18	54	Cost	27,13, 7, 9,32	88
	Route	1,89,63, 8,1	(4)	Route	1,73,76,24,68,40, 5,1	(7)
	Serve	0,52,79,29,0	160	Serve	0, 7, 7,50,50,32,14,0	160
	Cost	19, 6, 9,21	55	Cost	22, 5, 8,12,10, 9,25	91