OPT-*i*
An International Conference on
Engineering and Applied Sciences Optimization
M. Papadrakakis, M.G. Karlaftis, N.D. Lagaros (eds.)
Kos Island, Greece, 4-6, June 2014

# ON THE PRIORITIZATION OF THE REVIEW OF CONTINGENCY PLANS FOR THE CANADIAN ARMED FORCES

**Raman Pall and Jean-Denis Caron**

Defence R&D Canada – Centre for Operational Research and Analysis
National Defence Headquarters
Ottawa, ON K1A OK2, Canada

e-mails: {Raman.Pall, Jean-Denis.Caron}@drdc-rddc.gc.ca

**Keywords:** Multi-Criteria Decision Analysis, MCDA, Contingency Plans, Knapsack Problem, Rankings, Decision Support

**Abstract.** *The Canadian Armed Forces (CAF) have contingency plans (CONPLANs) in place to deal with a variety of scenarios, ranging from disaster relief and humanitarian aid to international counter-terrorism. These CONPLANs are periodically reviewed and updated to ensure they remain relevant to ever-changing operating environments. The establishment of a CONPLAN review schedule is a challenging process for the CAF given the status and importance of each CONPLAN and the limited capacity of the staff available to conduct the reviews. This paper addresses the joint problem of prioritization of the CONPLAN reviews and calculation of the optimal set of the plans to review within a given time period. The CONPLANs were evaluated against a set of criteria by military subject matter experts, with weights of the criteria provided on an ordinal scale. The volume of the weight-space associated to each of the possible rankings of the CONPLANs was computed through Monte Carlo simulation, and the expected rank of each CONPLAN was determined by viewing the results of this multi-criteria decision analysis in terms of the probability associated to each ranking. The optimal set of CONPLANs to review within a given time period (with an associated resource capacity) was calculated by formulating the problem as an instance of the knapsack problem. A recent application of this methodology is provided in this paper.*

DRDC-RDDC-2014-P119

# 1 INTRODUCTION

## 1.1 Background

The Canadian Armed Forces (CAF) must be capable of responding to a broad range of potential scenarios; ranging from disaster relief and humanitarian aid to international counter-terrorism. To that end, the CAF are required to be proactive and have contingency plans (CONPLANs) in place to deal with such scenarios. The CAF maintain dozens of CONPLANs that must be periodically reviewed and updated to ensure they remain relevant to ever-changing operating environments and to address lessons identified during recent operations and exercises. The establishment of a CONPLAN review schedule is a challenging process for the CAF given the status and importance of each CONPLAN and the limited capacity of the staff available to conduct the reviews.

## 1.2 Overview

In this paper, we present a three-step approach developed to address the joint problem of prioritization of the CONPLAN reviews and calculation of the optimal set of the plans to review within a given time period. First, the CONPLANs are evaluated against a set of criteria and an associated framework by military Subject Matter Experts (SMEs), with weights of the criteria provided on an ordinal scale. Second, the volume of the weight-space associated to each of the possible rankings of the CONPLANs is computed through Monte Carlo simulation, and the expected rank of each CONPLAN is determined by viewing the results of this multi-criteria decision analysis (MCDA) in terms of the probability associated to each ranking. Finally, the optimal set of CONPLANs to review within a given time period is calculated by formulating the problem as an instance of the knapsack problem. A notional example is provided to illustrate the results of a recent application of this approach for the CAF.

# 2 SELECTION CRITERIA AND ASSESSMENT

## 2.1 List of Criteria

Six criteria were identified to prioritize the reviews of the CONPLANs. This list of criteria, detailed in Table 1, was developed by the authors in concert with CAF military SMEs.

## 2.2 Assessment Scales

The SMEs evaluated the CONPLANs by selecting the most appropriate values amongst a set of specified assessment levels on ordinal scales for each of the criteria. These sets of levels are presented in Figure 1, with each level specified as a dot along each scale. The levels on the left hand side of the scales are the most desirable, in that they show the lowest levels of concern necessitating a CONPLAN review; whereas the levels on the right hand side of the scales are the least desirable, associated with CONPLANs most in need of a review.

The assessed levels of criteria 1, 3, 4, 5 and 6 were selected from amongst a set of specified options. For instance, for criterion 1 (the time elapsed since the CONPLAN was last issued and signed), the user was to choose a value amongst the following: “Less than 1 year”, “Between 1 and 2 years”, “Between 2 and 3 years”, “Between 3 and 4 years”, “Between 4 and 5 years” and “More than 5 years”. Conversely, criterion 2 was evaluated by having the SMEs specify whether or not there have been changes in up to three specified areas which could affect the plan: identification of lessons learned, changes in the CAF’s organizational structure, and changes in the operating environment or threat; the number of which determined the criterion’s assessed

| | Criterion | Description |
|---|---|---|
| 1 | Time since last review | The period of time elapsed since the plan was last issued and signed. |
| 2 | Changes in situational understanding | The number of changes in the situational understanding relating to the plan due to lessons learned from previous operations; modifications in the organizational structure (and associated relevant Command and Control (C2) elements); or changes in the current operating environment and threat impacting the plan. |
| 3 | Frequency | The frequency at which the plan is expected to be implemented. |
| 4 | Importance | The negative consequences of the event covered by the plan. |
| 5 | Completeness | The quality and the completeness of the plan considering the workability and the ease of implementability of the plan in its current (pre-reviewed) state. |
| 6 | Strategic direction and guidance | The highest level of strategic direction and guidance received regarding this plan. |

Table 1: The list of criteria used to evaluate the need for reviews of the contingency plans.

level.

### 2.3 Raw Assessments Provided by the SMEs

The CONPLANs are assessed by the military SMEs most familiar with the plans. To illustrate the approach described in this paper, we use a list of 16 generic CONPLANs, denoted as CONPLANs “A” through “P”, having notional raw scores as shown in Table 2. Also included in the table is the number of person-months (PMs) required to review each CONPLAN (in last column); these values will be used later in Section 4.

The scores in the table are displayed using a “red-yellow-green” ramp to give a simple visual indication on the strength of requirement for review for each of the CONPLAN. On this scale, “red” represents the highest strength of requirement for review and “green” represents the converse. For example, for criterion 3 (the frequency at which the CONPLAN is expected to be implemented), CONPLANs assessed as occurring on an annual basis (*i.e.*, “Every Year”, such as CONPLAN C) are coloured in red, whereas CONPLAN assessed as occurring very infrequently (*i.e.*, “Less than once every 10 years”, such as CONPLAN O) are coloured in green.

Intuitively, the number of cells coloured red in the rows of Table 2 should be indicative of being the CONPLANs with the highest requirements for a review. However, the relative importance of the criteria can also affect the prioritization of a CONPLAN’s review. Hence, during the assessment process, the SMEs also provided an ordered ranking of the criteria by specifying the weights of the criteria on the following ordinal scale:

$$\text{High} > \text{Medium} > \text{Low}.$$

In this paper, we use the notional relative weights presented in Table 3. “Frequency” and “Strategic direction and guidance” are identified as the most important criteria; while “Time since last review” and “Completeness” are deemed as the least important amongst the six criteria.

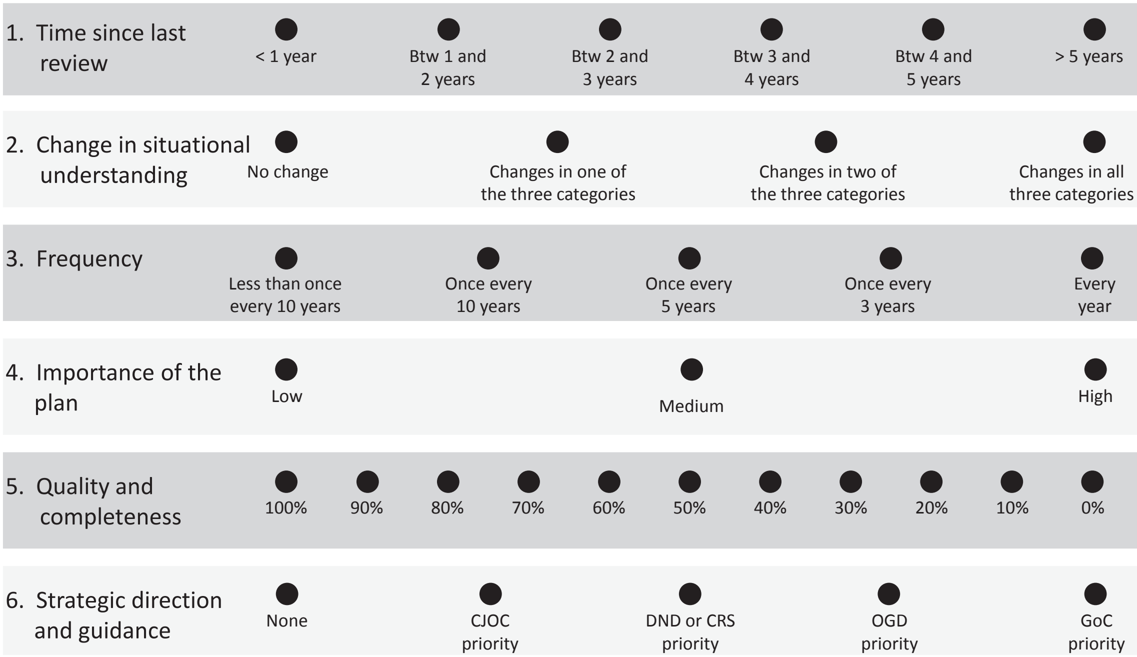


Figure 1: The ordinal assessment scales for the criteria.

| **CONPLAN** | **Criteria Assessment** | | | | | | **PMs Required** |
|---|---|---|---|---|---|---|---|
| | Time Since Last Review | Changes in Situational Understanding* | Frequency | Importance of the Plan | Quality and Completeness | Strategic Direction and Guidance | |
| **A** | Btw 2 and 4 yrs | OS; OE/T | Every 5 yrs | High | 70% | High | 20 |
| **B** | Btw 1 and 2 yrs | OS; OE/T | Every 10 yrs | High | 80% | High | 15 |
| **C** | Btw 1 and 2 yrs | OS | Annually | High | 90% | High | 4 |
| **D** | Btw 1 and 2 yrs | LL; OS | Annually | Medium | 90% | Medium-High | 8 |
| **E** | More than 5 yrs | LL; OS | Every 5 yrs | Low-Medium | 70% | Medium-High | 10 |
| **F** | Btw 1 and 2 yrs | OS | Every 3 yrs | Medium | 90% | High | 4 |
| **G** | Btw 1 and 2 yrs | OS | Every 3 yrs | High | 90% | High | 12 |
| **H** | Btw 1 and 2 yrs | OS | Every 3 yrs | Medium | 90% | High | 4 |
| **I** | Btw 1 and 2 yrs | OS | Every 5 yrs | Medium | 90% | High | 4 |
| **J** | Btw 3 and 4 yrs | LL; OS | Every 3 yrs | Medium | 80% | High | 25 |
| **K** | Btw 3 and 4 yrs | OS | Every 5 yrs | High | 90% | High | 20 |
| **L** | More than 5 yrs | OS | Every 5 yrs | Medium | 60% | Medium | 15 |
| **M** | Less than 1 yr | LL; OS; OE/T | Every 3 yrs | Medium | 100% | Low | 8 |
| **N** | Btw 2 and 3 yrs | None | Every 10 yrs | Low-Medium | 20% | Medium-High | 8 |
| **O** | Btw 3 and 4 yrs | OE/T | < Every 10 yrs | Low | 0% | Low | 12 |
| **P** | Less than 1 yr | LL; OS; OE/T | Annually | Medium-High | 80% | Low-Medium | 6 |
| | | | | | | *Total*: | 175 |

* LL: changes in lessons learned – OS: changes in organizational structure – OE/T: changes in the operating environment or threat.

Table 2: The notional assessments of the CONPLANs.

| | Criterion | Relative Weight |
|---|---|---|
| 1 | Time since last review | Low |
| 2 | Change in situational understanding | Medium |
| 3 | Frequency | High |
| 4 | Importance | Medium |
| 5 | Completeness | Low |
| 6 | Strategic direction and guidance | High |

Table 3: The relative weights (notional) of the criteria on an ordinal scale (High, Medium, Low).

### 2.4 Conversion of the Assessed Ordinal Levels to Numerical Values

Given that the CONPLANs are assessed against criteria on ordinal scales, the first step in determining their overall assessments is to assign numerical values to the assessed ordinal levels. Such an assignment function must be based on the position of the assessed level in the scale. Moreover, it need not be linear – all that is known with certainty is that the numerical values associated to the criteria should be related to their assessments on the ordinal scales through a monotonically increasing function.

In particular, the relationship in the assessment scheme may be *convex* (meaning that a change in the underlying criterion value from a middling to a high level results in an increase in CONPLAN review priority that is greater than a change in value from a low to a middling level); or the opposite may be true, and the assessment scheme would be termed *concave*. Example conversion functions that would have properties include a logarithmic function for the convex case, and an exponential function for the concave case. In the language of Keeney and Raiffa [1], one would say the criteria are assessed by a *risk-averse* decision-maker in the first case, and a *risk-prone* decision-maker in the second case. Maybury and Van Bavel [2] argue that in defence contexts decision-makers are often risk-averse. However, in the absence of any information on the risk profiles of the decision-makers conducting the assessments of the criteria, the authors used a naïve approach of assigning a numerical value to a CONPLAN's assessed criteria levels: that of using linear functions. Mathematically, the functions used to determine the numerical values of the criteria are of the following form:

$$\varphi(x\,;\,n) = \frac{x-1}{n-1}, \tag{1}$$

where $x$ is the position of the assessed level in the scale, and $n$ is the number of levels in the scale. Hence all criteria are given values ranging on a common scale of 0 to 1, where 1 represents the highest requirement for a review of the CONPLAN.

The numerical values associated with the notional assessments provided in Table 2 are given in Table 4. As an example, CONPLAN A was assessed as occurring once every 5 years. As the criterion in question (criterion 3: Frequency) has five levels, this CONPLAN was assessed numerically as $\varphi(3\,;\,5) = \frac{3-1}{5-1} = 0.5$ on criterion 3.

## 3 PRIORITIZATION

### 3.1 Problem Formulation

The CONPLAN prioritization problem described in this paper is a typical MCDA problem, as the problem amounts to ranking the alternatives (in this case, the 16 CONPLANs) using the

| | Criterion | | | | | |
|---|---|---|---|---|---|---|
| **CONPLAN** | **1** | **2** | **3** | **4** | **5** | **6** |
| A | 0.5 | 0.67 | 0.5 | 1 | 0.3 | 1 |
| B | 0.2 | 0.67 | 0.25 | 1 | 0.2 | 1 |
| C | 0.2 | 0.33 | 1 | 1 | 0.1 | 1 |
| D | 0.2 | 0.67 | 1 | 0.5 | 0.1 | 0.75 |
| E | 1 | 0.67 | 0.5 | 0.25 | 0.3 | 0.75 |
| F | 0.2 | 0.33 | 0.75 | 0.5 | 0.1 | 1 |
| G | 0.2 | 0.33 | 0.75 | 1 | 0.1 | 1 |
| H | 0.2 | 0.33 | 0.75 | 0.5 | 0.1 | 1 |
| I | 0.2 | 0.33 | 0.5 | 0.5 | 0.1 | 1 |
| J | 0.6 | 0.67 | 0.75 | 0.5 | 0.2 | 1 |
| K | 0.6 | 0.33 | 0.5 | 1 | 0.1 | 1 |
| L | 1 | 0.33 | 0.5 | 0.5 | 0.4 | 0.5 |
| M | 0 | 1 | 0.75 | 0.5 | 0 | 0 |
| N | 0.4 | 0 | 0.25 | 0.25 | 0.8 | 0.75 |
| O | 0.6 | 0.33 | 0 | 0 | 1 | 0 |
| P | 0 | 1 | 1 | 0.75 | 0.2 | 0.25 |

* For display purposes, values are rounded to the nearest 0.01.

Table 4: The numerical values associated to the criteria.

six decision criteria in the Section 2.1, which may or may not be of equal importance.

In order to simplify the subsequent sections, we present some mathematical notation here: we will let $p$ represent the index of an individual CONPLAN ($p = 1, \ldots, 16$), $c$ be the index of an individual criterion ($c = 1, \ldots, 6$), and $s_{p,c}$ be the assessment given by the SMEs to CONPLAN $p$ with respect to criterion $c$ (as per Table 4). More succinctly, we can represent the set of assessments for any given CONPLAN $p$ by a vector

$$\mathbf{s_p} = \left(\varphi_1(s_{p,1}),\, \varphi_2(s_{p,2}),\, \varphi_3(s_{p,3}),\, \varphi_4(s_{p,4}),\, \varphi_5(s_{p,5}),\, \varphi_6(s_{p,6})\right),$$

where the six $\varphi_c$ functions are calculated using equation 1 that converts the assessed ordinal values of the criteria to numerical values. For example, we see in Table 4 that CONPLAN A has numerical assessments of 0.50, 0.67, 0.50, 1.00, 0.30, and 1.00 along the six criteria. Hence we write $\mathbf{s_1} = (0.50,\, 0.67,\, 0.50,\, 1.00,\, 0.30,\, 1.00)$.

### 3.2 Possible Approaches

The traditional approach to MCDA problems is to use an additive weighted scoring rule, where weights are assigned to the criteria, and the alternatives are rated on each criterion. However, it can be difficult in practice to objectively quantify the importance of criteria by specifying their respective weights: Arbel and Vargas [4] and Borcherding *et al.* [5] showed that weight values that are determined by subjective judgment suffer from internal consistency and validity problems. Hence, one can unintentionally promote a portion of the alternatives by inadvertently overemphasizing given criteria.

There are a number of other, more sophisticated, methods commonly used to solve MCDA problems [6]. These include multi-objective mathematical programming techniques such as the one described by Evans and Steuer [7]; multi-attribute utility theory (MAUT) methods such as Keeney and Raiffa's trade-off method and pricing-out method [1]; Saaty's analytical hierarchy process (AHP) [8]; outranking methods [9] and the Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) method developed by Brans *et al.* [10]; and a consensus method for determining the ranking of the alternatives without specifying the relative strengths of the preferences of the decision-makers, developed by Emond *et al.* [11, 12, 13].

### 3.3 Determining of the Weights

Many methods seek to improve on the simplest of the methods outlined above, the additive weighted scoring rule, by employing an array of sophisticated techniques for determining the required weights using the rankings of the criteria. One such approach, described by Stillwell *et al.* [14], proposes using the rank reciprocal, rank linear, and rank exponent functions to convert ordinal rankings to cardinal weights.

Conversely, Barron [15] proposed using the Fundamental Weight Simplex (FWS) – a projection of the $n$-dimensional polytope connecting the points $(1, 0, \ldots, 0)$, $(0, 1, \ldots, 0)$, and $(0, 0, \ldots, 1)$ to determine the weights. Ranking the criteria yields a set of constraints on the weights (e.g., $w_1 \leq w_2 \leq \ldots \leq w_n$), restricting the feasible region to a particular subset of the FWS determined by the constraints on the weights. Barron argued that the centroid of this feasible region – the average of the vertices of the feasible region – is the most meaningful weighting that should be considered.

Furthermore, Alfares and Duffuaa have given empirical evidence for the use of linear weights when an ordinal ranking is given on the criteria, based on several survey-based experiments [17]; whereas Solymosi and Dombi argued for a centroid weight method [18]; Lootsma and Bots proposed the use of geometric-based weights [19]; and Hunter and Emond presented a case for using rank-based linear and power functions to determine the weights [16].

More recently, novel approaches have been employed which do not require the explicit determination of the weights on the criteria. Butler *et al.* presented an approach for using random weights generated on the FWS, and employing a Monte Carlo simulation approach to determine the distribution of the ranks for each option [20].

In a similar vein to Butler's work, Kaluzny detailed a method to determine a winner amongst the alternatives when provided with an ordered ranking of the criteria by analytically determining the proportion of the space occupied by each ranking of the objects using a computational geometry approach [21]. Kaluzny's method, dubbed QuBE, does not require a pre-selection of criteria weights, and can consider a set of constraints on the criteria – giving higher-ranked criteria more weight than lower-ranked criteria, and thus constraining the problem to one region of the FWS [22].

Building on Kaluzny's work, Pall extended QuBE by viewing the volume of the weight-space associated to each of the possible rankings of the options as a probability measure in the mathematical sense, where the volumes represent the chance of obtaining a particular ranking given randomly chosen (feasible) weights [23]. Analogues to various probabilistic concepts can then be defined and extended to the results obtained using QuBE; in particular, the expected ranks of the options could be considered.

### 3.4 Methodology

The approach we propose to determine the ranking on the CONPLAN reviews is a modification of the approaches described above due to Butler *et al.*, Kaluzny, and Pall [20, 21, 23].

A Monte Carlo simulation approach is used to generate a set of weight schemes for the criteria under the additional condition that the relative importance of the criteria was preserved. As such, only the feasible region of the FWS are considered. This approach has the added advantage over the computationally-intensive approach of QuBE in that it does not require the exact computation of the volumes occupied by each possible ranking on the CONPLANs – potentially quite a cumbersome feat. For the example in this paper, 16 CONPLANs to be ranked amounts to a total of $16!$ ($\approx 2.1 \times 10^{13}$) different possible rankings[1].

The ranks of the contingency plans (CONPLANs) under each of these weighting systems is then determined, which allowed for the calculation of the expected rank of each CONPLAN, as well as their associated 95% Confidence Intervals (CIs).

#### 3.4.1 Generating the Weight Schemes

Recall from Table 3 that there are six criteria; two specified as having "High" importance, two as having "Medium" importance, and two with "Low" importance. Let $w_c$ represent the weight of criterion $c$ ($c = 1, \ldots, 6$). The constraints specified on the weights, that criteria with high importance are given more weight than those with low importance, can be written mathematically as follows:

$$\left\{ \begin{array}{c} w_3 \\ w_6 \end{array} \right\} > \left\{ \begin{array}{c} w_2 \\ w_4 \end{array} \right\} > \left\{ \begin{array}{c} w_1 \\ w_5 \end{array} \right\} \geq 0. \tag{2}$$

Furthermore, we assume that criteria of equal importance have equal weight, and so we can write the weights of the criteria more succinctly as follows:

$$w_H := w_3 = w_6; \quad w_M := w_2 = w_4; \quad w_L := w_1 = w_5 \tag{3}$$

Finally, the sum of the weights must equal one, *i.e.*,

$$\sum_{c=1}^{6} w_c = 1. \tag{4}$$

From equations 3 and 4, we have the following simple relation on the weights:

$$w_H + w_M + w_L = 0.5. \tag{5}$$

A total of 100,000 different weight systems are generated uniformly distributed over the feasible region of the FWS. The specific random weight generation technique used is due to Wang and Zoints [24], and is built on work by Butler *et al.* [20] and Rubenstein [25]. This technique samples weights from the constrained simplex efficiently without generating infeasible weight schemes for the criteria; using the property expressed in equation 5, and the fact that there are an equal number of criteria at each level of importance[2].

Note that the weights span a triangle in the weight-space; the vertices of the triangle are the most extreme weight schemes satisfying constraints 2 and 5. They are given by $(w_H, w_M, w_L) = (0.5, 0, 0)$; $(0.25, 0.25, 0)$; and $(0.167, 0.167, 0.167)$. Moreover, the mean weights on the simplex (*i.e.*, its centroid) are as follows: $w_H = {}^{11}\!/_{36} = 0.305$, $w_M = {}^{5}\!/_{36} = 0.135$, and $w_L = {}^{1}\!/_{18} = 0.055$.

[1] Ties in the rankings are not explicitly considered here. If ties between the CONPLANs were considered, the number of rankings would increase.

[2] More sophisticated techniques exist for random weight generation in cases where there are an unequal number of criteria at each level of importance, and are presented in [24] for the interested reader.

### 3.4.2 Determining the Ranks of the CONPLANs

Denoting the weight of criterion $c$ in weight scheme $i$ by $w_c^i$, we can denote the individual weight schemes by

$$\begin{aligned} \mathbf{w^i} &= (w_1^i, w_2^i, w_3^i, w_4^i, w_5^i, w_6^i) \\ &= (w_H^i, w_M^i, w_L^i, w_M^i, w_H^i, w_L^i) \end{aligned} \tag{6}$$

for $i = 1, \ldots, 100,000$. The weighted sums for CONPLAN $p$ under weighting scheme $i$ can be determined as the inner product of the weight scheme and its assessment vector $\mathbf{s_p}$ (defined previously in Section 3.1):

$$\begin{aligned} x_p^i &= \mathbf{s_p} \cdot \mathbf{w^i} \\ &= \sum_{c=1}^{6} \varphi_c(s_{p,c})\, w_c^i \end{aligned} \tag{7}$$

We can define the rank $r_p^i$ of CONPLAN $p$ under weight scheme $i$ as follows:

$$r_p^i = \text{rank}\left(x_p^i\, ;\, \{x_1^i, x_2^i, \ldots, x_{16}^i, \}\right)$$

where the $\text{rank}$ function assigns a value of 1 to the largest value in the list, and a value on 16 to the smallest value in the list. The mean ranks of the CONPLANs can then be used to specify the ranking imparted on the CONPLANs.

## 3.5 CONPLAN Prioritization Results

All results presented in this paper are based on computations performed in the R programming language and software environment for statistical computing and graphics.

There were 168 different rankings found to have non-zero volume among the 100,000 weight systems considered. A subset of the unique rankings, presented in decreasing order of volume occupied in the simplex, can be found in Table 5. The table should be read as follows: the first (non-header) row represents the ranking with CONPLAN C in first place, G in second place, J in third place, and so on. From the table, we see that this ranking occupies approximately 7% of the feasible region of the weight-space.

The mean ranks of the CONPLANs were calculated, as were their associated 95% CIs. These results are presented in Figure 2, in decreasing order of the expected ranks of the CONPLANs; and are also included in the last row of Table 5. Note that CONPLAN C has 89% probability of ranking first; the only other CONPLAN that has a possibility of ranking first is CONPLAN A (with a probability of 11%). Furthermore, the majority of the time (72% probability) the top four ranking CONPLANs are C, J, G, and A.

## 3.6 Convergence of the Volumes Occupied by the Rankings

Given the stochastic nature of the procedure used to determine the volumes occupied by the various rankings on the CONPLANs, it is necessary to ensure that a sufficient number of weight systems are used in the computations for stability in the results. The number of unique rankings found after using a given number of weight systems is presented in Figure 3. Note that this number quickly converges to its final quantity (168); indeed, no new unique rankings are found once $\approx 50,000$ (of the total of 100,000) weight systems are considered. This result imparts credence to the idea that a sufficient number of weight systems were considered to ensure convergence of the ranking procedure, and hence stability in the results.

| | Rank of CONPLAN | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Ranking** | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | **Volume** |
| 1 | 7 | 11 | 1 | 4 | 12 | 5.5 | 2 | 5.5 | 9 | 3 | 8 | 13 | 15 | 14 | 16 | 10 | 0.07 |
| 2 | 5 | 10 | 1 | 4 | 12 | 6.5 | 2 | 6.5 | 11 | 3 | 8 | 13 | 14 | 15 | 16 | 9 | 0.06 |
| 3 | 7 | 10 | 1 | 4 | 12 | 5.5 | 3 | 5.5 | 9 | 2 | 8 | 13 | 15 | 14 | 16 | 11 | 0.05 |
| 4 | 2 | 7 | 1 | 6 | 11 | 9.5 | 4 | 9.5 | 12 | 3 | 5 | 13 | 14 | 15 | 16 | 8 | 0.03 |
| 5 | 2 | 8 | 1 | 5 | 12 | 9.5 | 3 | 9.5 | 11 | 4 | 7 | 14 | 13 | 15 | 16 | 6 | 0.03 |
| 6 | 4 | 10 | 1 | 5 | 12 | 7.5 | 2 | 7.5 | 11 | 3 | 6 | 13 | 14 | 15 | 16 | 9 | 0.02 |
| 7 | 3 | 9 | 1 | 5 | 11 | 7.5 | 4 | 7.5 | 12 | 2 | 6 | 13 | 15 | 14 | 16 | 10 | 0.02 |
| 8 | 5 | 10 | 1 | 4 | 12 | 6.5 | 3 | 6.5 | 9 | 2 | 8 | 13 | 15 | 14 | 16 | 11 | 0.02 |
| 9 | 3 | 10 | 1 | 5 | 9 | 7.5 | 4 | 7.5 | 12 | 2 | 6 | 13 | 15 | 14 | 16 | 11 | 0.02 |
| 10 | 2 | 8 | 1 | 5 | 12 | 9.5 | 3 | 9.5 | 11 | 4 | 6 | 14 | 13 | 15 | 16 | 7 | 0.02 |
| 11 | 2 | 8 | 1 | 5 | 11 | 9.5 | 4 | 9.5 | 12 | 3 | 6 | 13 | 14 | 15 | 16 | 7 | 0.02 |
| 12 | 1 | 7 | 2 | 6 | 9 | 10.5 | 4 | 10.5 | 13 | 3 | 5 | 12 | 14 | 15 | 16 | 8 | 0.02 |
| 13 | 2 | 7 | 1 | 5 | 11 | 9.5 | 4 | 9.5 | 12 | 3 | 6 | 13 | 14 | 15 | 16 | 8 | 0.02 |
| 14 | 7 | 11 | 1 | 4 | 10 | 5.5 | 3 | 5.5 | 9 | 2 | 8 | 13 | 15 | 14 | 16 | 12 | 0.02 |
| 15 | 2 | 8 | 1 | 5 | 11 | 9.5 | 3 | 9.5 | 12 | 4 | 6 | 13 | 14 | 15 | 16 | 7 | 0.02 |
| 16 | 2 | 8 | 1 | 5 | 12 | 9.5 | 3 | 9.5 | 11 | 4 | 6 | 13 | 14 | 15 | 16 | 7 | 0.02 |
| 17 | 7 | 11 | 1 | 4 | 12 | 5.5 | 3 | 5.5 | 9 | 2 | 8 | 13 | 15 | 14 | 16 | 10 | 0.02 |
| 18 | 2 | 9 | 1 | 5 | 11 | 7.5 | 4 | 7.5 | 12 | 3 | 6 | 13 | 14 | 15 | 16 | 10 | 0.02 |
| 19 | 2 | 7 | 1 | 6 | 12 | 9.5 | 3 | 9.5 | 11 | 4 | 8 | 14 | 13 | 15 | 16 | 5 | 0.02 |
| 20 | 7 | 11 | 1 | 4 | 12 | 5.5 | 2 | 5.5 | 9 | 3 | 8 | 13 | 14 | 15 | 16 | 10 | 0.02 |
| *148 other rankings, occupying a total volume of 0.46* | | | | | | | | | | | | | | | | | |
| *Expected* | *3.6* | *8.9* | *1.1* | *5.0* | *11.0* | *8.0* | *3.1* | *8.0* | *11.1* | *2.9* | *6.6* | *13.0* | *14.2* | *14.6* | *16.0* | *8.9* | – |

Table 5: The unique rankings, in decreasing order of volume occupied in the weight-space.

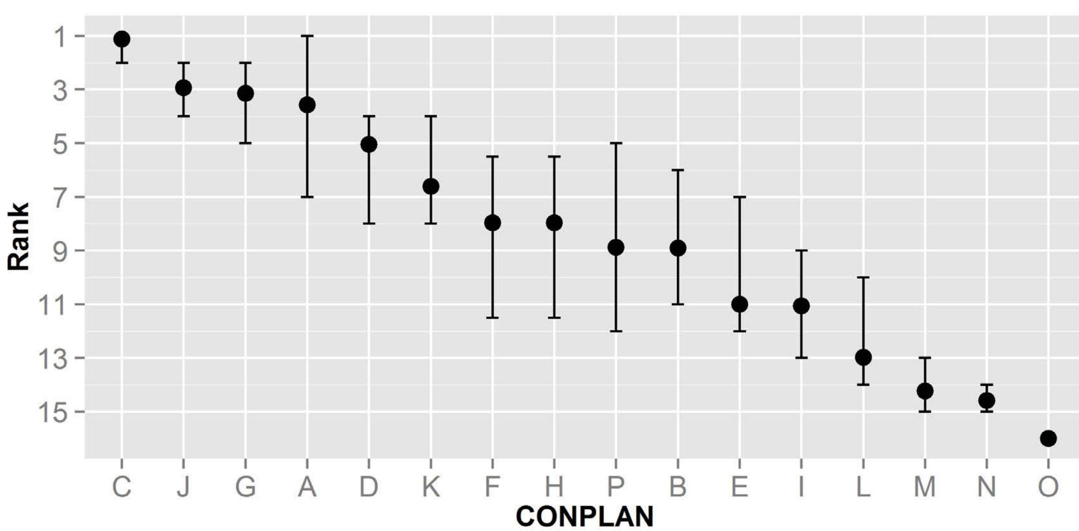


Figure 2: The expected ranks of the CONPLANs, along with their associated 95% confidence intervals.

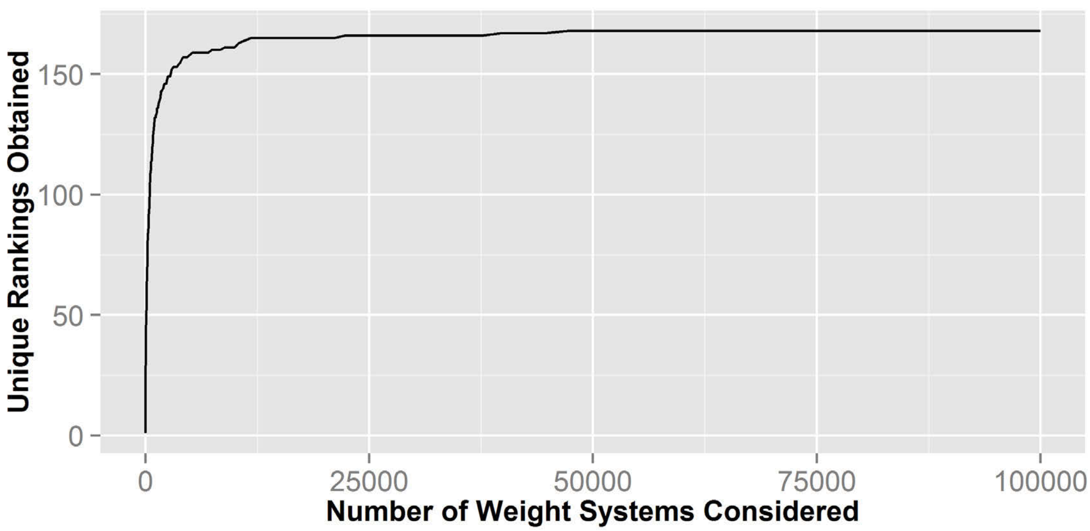


Figure 3: The number of unique rankings found as a function of the number of weight systems considered.

## 4 OPTIMIZED SCHEDULING OF THE STAFF EFFORT

In the previous section, we provided a method to rank the CONPLANs based on the importance of their reviews. The main challenge that the CAF face is that it has a limited capacity of the staff available to conduct the reviews. Here, we describe a method for calculating an optimized CONPLAN review schedule considering the ranked list and the limited staff within a given time period. The problem is formulated as an instance of the knapsack problem.

### 4.1 The *Knapsack Problem*

The knapsack problem is a common problem in the field of combinatorial optimization that can be defined as follows: given a set of items, each with a weight and a value, determine the number of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible [26, 27]. In our particular case, the problem being solved is an instance of the *binary*, or *0-1 knapsack problem*, which restricts the number of copies of each kind of item to zero or one.

Let there be $n$ items, $z_1$ to $z_n$ where $z_i$ has a value $u_i$ and weight $k_i$. Let us denote the weight constraint, *i.e.*, the maximum weight allowable, by $K$. Mathematically, we can formulate the 0-1-knapsack problem as:

$$\text{Maximize} \sum_{i=1}^{n} u_i a_i \qquad \text{such that} \qquad \sum_{i=1}^{n} k_i a_i \leq K, \qquad a_i \in \{0,1\}, \tag{8}$$

where the $a_i$ are binary decision variables indicating whether or not item $z_i$ is placed in the knapsack or not. In other words, we wish to maximize the sum of the values of the items in the knapsack under the constraint that the sum of the weights must be less than the knapsack's capacity.

In our instance, the knapsack represents the set of CONPLANs to review in the calendar year; the items are the CONPLAN reviews themselves; their "weights " are the time required to complete their reviews; and their "values" are functions of their importance (which, intuitively, should be related to their expected ranks).

### 4.2 Time Requirements for the Reviews

The expected time to complete each individual CONPLAN review as well as the total manpower available to perform the reviews (within a time period of interest) was provided by military SMEs. The total resources required to review each individual CONPLAN are shown in the last column of Table 2, where the values are expressed in terms of PMs. For example, 20 PMs are required to review CONPLAN A.

We also assume that the total resources available within the CAF to conduct all the reviews is 70 PMs, which is less than the total staff effort required to review all CONPLANs (175 PM, as per Table 2); justifying the need for an approach to optimize the CONPLAN review schedule.

### 4.3 Utility of the CONPLAN Reviews

The "importance" of the reviews are inversely associated with the ranks of the reviews. For example, CONPLAN J, of high importance, has an expected rank of 2.9. As the knapsack problem requires high values to be associated with objects that are desirable in the knapsack, a function is required to translate the expected ranks to their values, deemed here as their *utility*. We define the utility of the CONPLANs as a linear function of their expected ranks as follows:

$$\begin{aligned} \text{Utility}(\text{CONPLAN } p) &= \frac{(16+1) - \text{Expected Rank}(\text{CONPLAN } p)}{\sum_{p=1}^{16} p} \\ &= \frac{17 - \text{mean}_i(r_p^i)}{136} \end{aligned} \tag{9}$$

This definition ensures that if all reviews were completed the total utility achieved would equal $(1 + 2 + \ldots + 16)/136 = 100\%$. Hence, the utility of each CONPLAN represents its fraction of the total utility achievable given no resource constraint. The utility of the reviews of each of the CONPLANs is provided in Table 6.

| CONPLAN | Utility | CONPLAN | Utility | CONPLAN | Utility | CONPLAN | Utility |
|---|---|---|---|---|---|---|---|
| A | 9.9% | E | 4.4% | I | 4.3% | M | 2.1% |
| B | 6.0% | F | 6.6% | J | 10.4% | N | 1.8% |
| C | 11.7% | G | 10.2% | K | 7.6% | O | 0.7% |
| D | 8.8% | H | 6.6% | L | 2.9% | P | 6.0% |

Table 6: The utility of the reviews of the CONPLANs, obtained using equation 9.

### 4.4 Solutions Obtained Through Optimization

The optimal solution considering a total of 70 PMs available for review was found to include nine CONPLANs. In decreasing order of expected rank, they are CONPLANs: C, G, A, D, F, H, P, I and M. These CONPLANs have a total utility of 66.2%, and require all 70 PMs available. Of interest is the fact that some highly-ranked CONPLANs are not included in the optimal solution due to their high resource requirement for review (*e.g.*, CONPLAN J, with an expected rank of 2.9, and an resource requirement of 25 PMs).

A list of near-optimal solutions is presented in Table 7. Each row in the table is one possible set of reviews, shown with associated PMs required and utility achieved. The optimal solution is listed in the first row, and all subsequent rows are shown in decreasing order of utility achieved.

| | CONPLAN Review Included in Solution | | | | | | | | | | | | | | | | PMs | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Solution** | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | **Required** | **Utility** |
| 1 | X | | X | X | | X | X | X | X | | | | X | | | X | 70 | 66.2% |
| 2 | X | | X | X | | X | X | X | X | | | | | X | | X | 70 | 65.9% |
| 3 | | | X | X | | X | X | X | X | X | | | | | | X | 67 | 64.6% |
| 4 | | X | X | X | X | X | X | X | X | | | | | | | X | 67 | 64.6% |
| 5 | X | | X | X | X | X | X | X | | | | | | | | X | 68 | 64.2% |
| 6 | | | X | X | | X | X | X | X | | X | | X | | | X | 70 | 64.0% |
| 7 | | | X | X | | X | X | X | X | | X | | | X | | X | 70 | 63.7% |
| 8 | X | | X | X | X | X | X | X | X | | | | | | | | 66 | 62.6% |
| 9 | | | X | X | X | X | X | X | X | | | | X | X | | X | 68 | 62.5% |
| 10 | | X | X | X | | X | X | X | X | | | | X | | | X | 65 | 62.3% |
| 11 | | | X | X | X | X | X | X | | | X | | | | | X | 68 | 62.0% |
| 12 | | X | X | X | | X | X | X | X | | | | | X | | X | 65 | 62.0% |
| 13 | X | | X | X | X | X | X | | X | | | | | | | X | 68 | 61.9% |
| 14 | X | | X | X | X | | X | X | X | | | | | | | X | 68 | 61.9% |
| 15 | | | X | X | X | X | X | X | X | | | X | | | | X | 67 | 61.6% |
| 16 | X | | X | X | | X | | X | X | | X | | | | | X | 70 | 61.5% |
| 17 | X | X | X | | | X | X | X | X | | | | | | | X | 69 | 61.3% |
| 18 | | X | X | X | | X | X | X | X | | | | | | X | X | 69 | 61.0% |
| 19 | | | X | X | | X | X | X | X | X | | | X | | | | 69 | 60.7% |
| 20 | | X | X | X | X | X | X | X | X | | | | X | | | | 69 | 60.7% |
| *3177 other sub-optimal solutions* | | | | | | | | | | | | | | | | | | |

Table 7: The optimal and near-optimal solutions for the CONPLAN reviews. CONPLANs listed with an "X" indicate that their reviews are included in the particular solution.

## 5 CONCLUSIONS

The methodology described and illustrated in this paper is robust, in that it implicitly considers a multitude of weighting systems on the criteria; and general, in that it can be applied to a variety of problems in which objects are to be selected based on both a set of criteria as well as an independent resource constraint. However, a number of modifications can be made to extend the methodology further. In particular, the optimization problem could then be formulated as an instance of the *multiple* knapsack problem such that the objects being ranked can be associated with additional resource constraints. Such constraints could pertain to the particular types of expertise required for the review – perhaps logistical, strategic, or political. In addtion, a more refined scheduling of the objects contained in the optimal solution could be incorporated into the final results. While requiring information on the schedules of the variety of individuals required for the CONPLAN reviews, this type of result could provide the decision-maker with a schedule of exactly what individuals are to complete which reviews at specific points of the year.

## REFERENCES


[1] R.L. Keeney, H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Trade-Offs*. Cambridge University Press, 1993.

[2] D.W. Maybury, G.H. van Bavel, *Marine Builder's Risk Insurance for the Joint Support Ship Contract: A Utility Theory Approach to Risk Analysis*, DRDC – Centre for Operational and Analysis, Technical Memorandum, DRDC CORA TM 2009-41, 2009.

[3] B. Roy, *Méthodologie Multicritère d'Aide à la Décision*. Economica, Paris, 1985.

[4] A. Arbel, L.G. Vargas, *Preference Simulation and Preference Programming: Robustness Issues in Priority Derivation*, European Journal of Operational Research, **69**, 200–209, 1993.

[5] K. Borcherding, T. Eppel, D. Von Winterfeldt, *Comparison of weighting judgments in multiattribute utility measurement*, Management Science, **37(12)**, 1603–1619, 1991.

[6] C. Zopounidisn M. Doumpos, *Multicriteria classification and sorting methods: A literature review*, European Journal of Operational Research, **138(2)**, 229–246, 2002.

[7] J. Evans, R. Steuer, *A Revised Simplex Method for Linear Multiple Objective Programs*, Mathematical Programming, **5**, 54–72, 1973.

[8] T.L. Saaty, *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw-Hill, New York, 1980.

[9] P. Vincke, *Multi-criteria Decision-Aid*. John Wiley, Chichester, 1992.

[10] J.P. Brans, P.H. Vincke, B. Mareschal, *How to select and how to rank projects: The PROMETHEE method*, European Journal of Operational Research, **24**, 228–238, 1986.

[11] E.J. Emond, D.W. Mason, *A New Rank Correlation Coefficient with Application to the Consensus Ranking Problem*, Journal of Multi-Criteria Decision Analysis, **11**, 17–28, 2002.

[12] E.J. Emond, *Developments in the Analysis of Rankings in Operational Research*, DRDC – Centre for Operational and Analysis, Technical Report, DRDC CORA TR 2006-37, 2006.

[13] T. Yazbeck, E.J. Emond, *Multi-Criteria Analysis and Ranking Consensus Unified Solution (MARCUS) – User Guide*, Operational Research Division, Research Note, ORD RN 2004/13, 2004.

[14] W.G. Stillwell, D.A. Seaver, W. Edwards, *A comparison of weight approximation techniques in multiattribute utility decision making*, Organizational Behavior and Human Performance, **28(1)**, 62–77, 1981.

[15] F.H. Barron, Acta Psychologica. *Selecting a best multiattribute alternative with partial information about attribute weights*, **80(1-3)**, 91–103, 1992.

[16] D.G. Hunter, E.J. Emond, *Analytical Support to PMO JSS – Assigning Weights to a Hierarchy Using Ordinal Rankings*, DRDC – Centre for Operational and Analysis Technical Memorandum,DRDC CORA TM 2005-32, 2005.

[17] H. Alfares, S. Duffuaa, *Assigning cardinal weights in multi-criteria decision making based on ordinal ranking*, Journal of Multi-Criteria Decision Analysis, **15(5)**, 125–133, 2008.

[18] T. Solymosi, J. Dombi, *A method for determining the weights of criteria: The centralized weights*, European Journal of Operational Research, **26(1)**, 35–41, 1986.

[19] F.A. Lootsma, P.W.G. Bots, *The assignment of scores for output-based research funding*, Journal of Multi-Criteria Decision Analysis, **8(1)**, 44–50, 1999.

[20] J.C. Butler, J. Jia, J.S. Dyer, *Simulation Techniques for the Sensitivity Analysis of Multi-criteria Decision Models*, European Journal of Operational Research, **103(3)**, 531–545, 1997.

[21] B.L. Kaluzny, R.H.A. David Shaw, *Sensitivity analysis of additive weighted scoring methods: How to fool your friends (again)*, DRDC – Centre for Operational and Analysis Technical Report, DRDC CORA TR 2009-002, 2009.

[22] B.L. Kaluzny, *A qualitative bidder evaluation method: A new scoring method void of cardinal weight assignment*, DRDC – Centre for Operational and Analysis, Technical Memorandum, DRDC CORA TM 2010-007, 2010.

[23] R. Pall, *On the Selection of a Location in Europe for an Operational Support Hub – A Multi-Criteria Decision Analysis*, DRDC – Centre for Operational and Analysis, Technical Memorandum, DRDC CORA TM 2012-175, 2012.

[24] J. Wang, S. Zionts, *Random-Weight Generation in Multiple Criteria Decision Models*, MCDM 2006, Chania, Greece, June 19-23, 2006.

[25] R. Rubinstein, *Generating Random Vectors Uniformly Distributed Inside and On the Surface Of Different Regions*, European Journal of Operational Research, **10(2)**, 205–209, 1982.

[26] H. Kellerer. U. Pferschy, D. Pisinger, *Knapsack Problems*. Springer-Verlag, Heidelberg, 2004.

[27] B. Karte, J. Vygen, *Combinatorial Optimization: Theory and Algorithms*. 3rd Edition, Springer-Verlag, Heidelberg, 2006.