Detailed maintenance planning for military systems with random lead times and cannibalization

Rui Zhang, Ahmed Ghanmi

Center for Operational Research and Analysis
Defence Research and Development Canada
Ottawa Ontario, Canada, K1A 0K2

Rui.Zhang@forces.gc.ca
Ahmed.Ghanmi@forces.gc.ca

Abstract: Detailed maintenance planning under uncertainty is one of the most important topics in military research and practice. As one of the fastest ways to recover failed weapon systems, cannibalization operations are commonly applied by maintenance personnel. Due to the additional complexities introduced by these operations, detailed maintenance and repair decision making with cannibalization was rarely studied in the literature. This paper proposed an analytical model for studying the maintenance planning problem of military systems with random lead times and cannibalization. The objective of the problem is to maximize fleet reliabilities under operating costs constraints. A complementary problem that minimizes total operating costs under fleet reliabilities constraints was also constructed. A polynomial algorithm was proposed to solve the minimization problem and determine optimal decision strategies. This algorithm was also used as a subroutine in a binary-search algorithm to solve the maximization problem.

Keywords: Reliability, Maintenance, Optimization, Cannibalization, Multi-stage, Multi-item

1. Introduction

1.1 Background

Manufactured products can fail due to different processes such as corrosion, wear and tear, as well as fatigue. Products such as domestic electronics and appliances are generally discarded and replaced upon failure because they are inexpensive. However, capital goods such as defence weapon systems are repaired because of their high replacement cost. Repair/maintenance decisions often involve removal and replacement of failed parts. Research on maintenance decision making problems have spawned several noteworthy papers in the open literature over the last five decades, starting with the textbook of Barlow and Proschan (1965) and the paper of McCall (1965). More recent research studies on a variety of models and solution methodologies for determining and/or comparing the best corrective, preventive and opportunistic maintenance policies could be found in the review papers (Dekker 1996), (Wang 2002) and (Nicolai and Dekker 2008), respectively.

In the military context, the research of maintenance systems focused on the level of repair analysis (LORA) problems (Basten et al. 2009), the spare parts stocking (SPS) problems (Tang and Liu 2011), and the combined LORA-SPS problems (Basten et al 2012). In these problems, repair decisions were studied at the strategic level using aggregated approaches. However, operating managers are under increased pressures to improve fleet reliabilities through their detailed (daily) repair decisions. This paper examined some of the repair decision making problems at the operational level and developed an optimization model to determine detailed maintenance planning strategies for military systems with random lead times and cannibalization.

1.2 Cannibalization

In this study, a support network of one operating base and one repair depot was considered and a finite number of repair decision periods were used. In the network, when a defence weapon system or prime equipment (PE) fails, one and only one responsible part or line replaceable unit (LRU) is identified. The failed LRU will be separated from the PE and sent to the depot for repair. The left LRU hole will be filled up with a functioning LRU. If a spare LRU is not available, cannibalizing a functioning LRU from other failed PEs is also acceptable in practice. As indicated by the General

Accounting Office of the United States (GAO U.S. 2001), the Air Force and the Navy reported 376,000 and 468,000 cannibalizations between 1996 and 2000, respectively. In the Canadian Armed Forces (CAF), the cannibalization of the Eryx missile system is being considered as the weapon will be retired in 2016.

Figures 1 and 2 describe an example of cannibalization, where the fleet includes three PEs, i.e., PE-1, PE-2, and PE-3, and each PE includes three LRUs, i.e., L-1, L-2 and L-3. In the figures, a functioning LRU is denoted by a solid-double-line rectangle; otherwise a dashed-double-line rectangle is used. Figure 1 has PE-1, PE-2 and PE-3 waiting for L-3, L-2 and L-1 replacement, respectively. The two arrows indicate that PE-1 and PE-3 can cannibalize L-3 and L-1 from PE-2, respectively. With these cannibalizations, PE-1 and PE-3 get back to work immediately, while PE-2 is in an even worse situation with three LRU “holes”. Figure 2 shows the resulting status of the fleet. It is clearly observed that the number of functioning PEs increases from zero (Figure 1) to two (Figure 2).

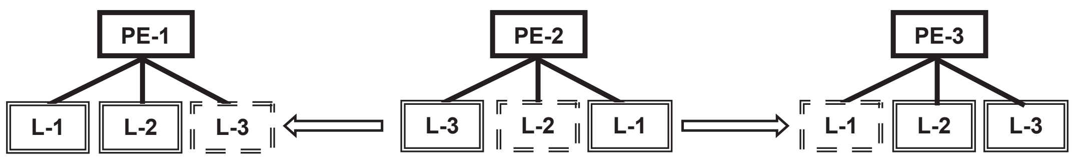

Figure 1: PE-1 and PE-3 can cannibalize L-3 and L-1 from PE-2, respectively.

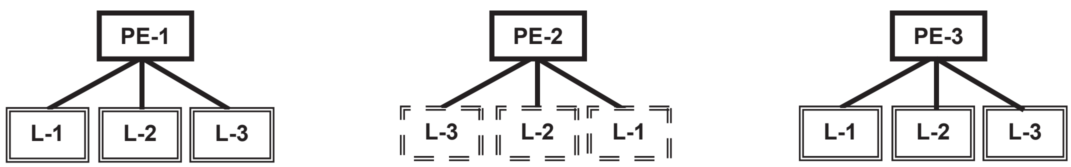

Figure 2: PE-1 and PE-3 are recovered after cannibalization operations.

This example demonstrates that cannibalization is a useful operation when there is no sufficient spare LRU; this example also explains why cannibalization operations are favoured by military maintenance personnel in practice. However, due to the additional complexities introduced by cannibalization, detailed maintenance/repair decision making with cannibalization was rarely studied in the literature.

1.3 Structure

This paper is organized as follows. Section 2 describes the maintenance planning problem with random lead times and cannibalization. Section 3 presents a mathematical formulation of the maintenance planning problem and Section 4 proposes a binary-search algorithm to determine approximate solutions to the problem. Concluding remarks and future work are provided in Section 5.

2. Preliminaries

2.1 Maintenance system

A pictorial representation of the maintenance support network is shown in Figure 3. Initially, there are N ($0 < N < \infty$) failed PEs ($E_1, E_2, \ldots, E_N$) installed in the operating base; each PE is made up of M ($0 < M < \infty$) distinct LRUs ($L_1, L_2, \ldots, L_M$). Among the LRU positions, there is at least one malfunctioning LRU or LRU hole on each PE.

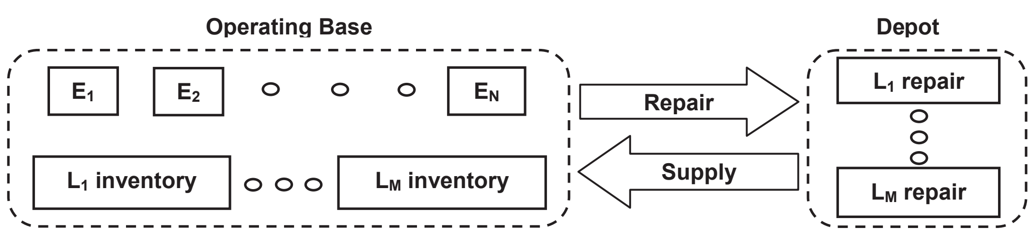

Figure 3: Maintenance support network with one operating base and one depot.

In the network, all failed PEs and spare LRUs are installed in the operating base and all repair operations are conducted in the depot (assuming that the warehouse space in the base and the repair capacity in the depot are unlimited).

Let f_m and g_m be the number of functioning and malfunctioning L_m in the base, respectively. Let q_m be the number of such due-in L_m, which are either under repair in the depot or in transshipment between the base and depot. Thus, the total number of spare (or *individually existed*) L_m can be calculated as:

$$s_m = f_m + g_m + q_m - N.$$

The use of the term "individually existed" is due to the observation that by default a PE should include a PE frame, on which LRUs are installed. When the individually existed LRUs are first introduced in the system, they are not attached to any PE frame. Assuming that there are no external supplies or demands for any individual L_m, s_m remains constant during the whole decision horizon.

A system is called *effective* if all failed PEs, whose L_m demands can be satisfied using the stocked L_m or the functioning L_m on other failed PEs (via cannibalization), are recovered. Given that all in-base operations are generally controlled by operating managers, the PE-recovery operations can be treated as zero-time and zero-cost operations. Assuming that the system is initially effective, $N > 0$ implies that there is at least one L_m such that $f_m = 0$.

Let $[1, T + 1)$, where $0 < T < \infty$, be the decision horizon including T mutually disjoint time/decision periods. Figure 4 depicts an example of time period t ($1 \le t \le T$), where repair decisions are made at the beginning of the period (i.e., just after time t), and PE failures occur at the end of the period (i.e., just before time t + 1).

Figure 4: Decisions are made at the beginning of period t and PE failures occur at the end of period t.

For a given decision period, let x_m be the decision variable denoting the number of L_m sent to the depot for repair and let k_m be the number of malfunctioning L_m introduced into the system by the PE failures, where $x_m \ge 0$ and $k_m \ge 0$ for all m = 1, 2, …, M. Based on the assumption that there is exactly one responsible LRU for each PE failure, the number of newly-failed PEs (or PE frames) is given by:

$$k = k_1 + k_2 + \ldots + k_M.$$

The number of functioning L_m along with the newly-failed PEs is $K_m = k - k_m$. Therefore, the number of recovered PEs (n) can be determined as follows (assuming zero spare L_m and zero repaired L_m returning/arriving during the period for all m = 1, 2, …, M):

$$n = \min\{f_1 + K_1, f_2 + K_2, \ldots, f_M + K_M\}.$$

Assuming that the system is effective, the recovered PEs will leave the system immediately. Thus, by the end of the period, only the number of unrecovered PEs is recorded. This paper assumes independent and identically distributed (IID) PE failures for all time periods.

2.2 Model definition

Let c^t be the total operating cost of period t, which is the sum of transportation (fixed and period-dependent) and repair (variable and LRU-dependent) costs. Let h^t be the transportation cost of period t if there is some malfunctioning LRUs sent to the depot for repair and let r_m be the per-unit repair cost for the made L_m-repair decisions. Thus, $c^t = 0$ (if $x_m = 0$ for all m) and $c^t = h^t + r_1x_1 + r_2x_2 + \ldots + r_Mx_M$ (if there is at least one $x_m > 0$). Let γ be the per-period interests gained on per unused fund. The present value of c^t at time is: $pv^t = c^t/(1+ \gamma)^{t-1}$, and the present value of the total operating cost over T periods is:

$$pv = pv^1 + pv^2 + \ldots + pv^T.$$

On the other hand, let rc^t be the recovering ratio of the first t periods, which can be used to evaluate the fleet reliability of the first t periods (t = 1, 2, …, T). Let k^t and n^t be the number of newly-failed and recovered PEs of period t, respectively. Then, the value of rc^t can be easily calculated as:

$$rc^t = \frac{\sum_{s=1}^{t} n^s}{N + \sum_{s=1}^{t} k^s}$$

Let B ($0 < B < \infty$) be the total available operating budget at time one. The problem that maximizes fleet reliabilities with constraints on total operating costs (denoted by MaxRatio) can be formulated as:

Max: rc
S.T.: $pv \le B$
$rc^t \ge rc$, for all t = 1, 2,…, T.

In order to solve MaxRatio, a complementary problem is constructed to minimize total operating costs with constraints on fleet reliabilities. Similarly, this problem (denoted by MinCost) can be written as:

Min: pv
S.T.: $rc^t \ge A$, for all t = 1, 2,…, T.

Note that A ($0 < A \le 1$) is the designated level for fleet reliabilities.

3. Optimal solutions to MinCost

3.1 Scenario tree formulation

The T-stage/period MinCost can be modelled on a (T+1)-level scenario tree, which is branched by PE failures. Let PE failures with small chances be zero-probability events. By assuming homogeneous Poison processes (HPPs) for PE failures during each period, the maximum number of failed PEs during each period, K ($0 < K < \infty$), can be easily determined for any given small chance. Therefore, the number of different PE failures during each period and the number of branches of the tree can also be determined. Note that in military maintenance problems, it is commonly adopted that IID-featured PE failures are described as HPPs.

Let W be the (T+1)-level scenario tree and V the set of nodes on W. Let l(j) be the level of node j, i.e., $1 \le l(j) \le T+1$ for all j in V. In particular, j is called a *leaf* node if l(j) = T+1 and a *root* node if l(j) = 1. The index 1 is reserved for the root node; l(1) = 1. Let b(j) be the *direct* ancestor node of j and let B(j) = {k|b(k) = j, for all k in V} be the set of *direct* descendant nodes of j. It is obvious that b(1) = ∅ and B(j) = ∅ for all leaf node j. For a non-leaf node j, let W(j) be the sub-tree rooted on j and let V(j) be the set of nodes on W(j). In particular, W(1) = W and V(1) = V.

Let p_j be the probability of node j. Since node 1 represents the system at time one, where the status is well-defined, then $p_1 = 1$. Since all nodes in B(1) are branched from node 1, the sum of probabilities of nodes in B(1) is equal to one (the sum of probabilities of all leaf nodes is also equal to one).

For a node j, let P(j) be the path including all nodes from 1 to j. In particular, P(1) = {1}. If j is a leaf node, then P(j) is called a scenario. Note that a scenario is a series of events, and the probability of a scenario is represented by the probability of the leaf node, with which the scenario ends up. All reliability and cost calculations are based on either paths or scenarios.

In order to formulate MinCost on W, fleet reliability requirements are converted to LRU demands first. On W, let (k,j) be the edge connecting node k and j. Obviously, (k,j) denotes the PE failures occurring in period [l(k), l(j)). Let $D_{(j,m)}$ be the cumulative demands for L_m by node j. Let k_j be the number of failed PEs introduced by (k,j) and let $k_{(j,m)}$ be the number of malfunctioning L_m associated with (k,j). For any give node j > 1, $D_{(j,m)}$ can be easily determined as (assuming A = 1 and $s_m = 0$ for all m):

$$D_{(j,m)} = \max_{m=1,2,...,M}\{0, N + \sum_{i\in P(j)} k - f_{(1,m)} - \sum_{i\in P(j)} K_{(i,m)}\}.$$

Since PE failures always bring functioning LRUs and these LRUs can be used as spare LRUs, there might be some irregular cumulative demands, i.e., $D_{(j,m)} < D_{(b(j),m)}$ for some j and m. In this case, $D_{(j,m)}$ is re-assigned to $D_{(b(j),m)}$.

Let $x_{(j,m)}$ be the decision variable, which represents the number of L_m sending to the depot for repair on node j. Let y_j be the binary variable such that $y_j = 1$ if there is some m with $x_{(j,m)} > 1$ and $y_j = 0$ if $x_{(j,m)} = 0$ for all m. Let $d_{(j,m)} > 0$ be the lead time for the L_m-repair operations on node j. Due the fact that none out-base operations are controlled by operating managers, this paper assumes random $d_{(j,m)}$ variables. Note that $d_{(j,m)}$ are assumed to be positive integers. Let $F(j,m) = \{k|l(k) = l(j)+d_{(j,m)}$ and k in $V(j)\}$ be the set of nodes such that if a failed L_m is sent out for repair on node j, then the repaired L_m will be returned on all nodes in F(j,m). On the other hand, let $G(j,m) = \{k|l(j) = l(k)+d_{(k,m)}$ and k in $P(j)\}$ be the set of nodes such that if a failed L_m is sent out for repair on any node in G(j,m), then the repaired L_m will be returned on node j. Next, the scenario tree formulation of MinCost, denoted by MinCostST, is formulated as follows:

$$\text{Min:} \quad \sum_{j\in V:l(j)<T+1} p_j \frac{h_j y_j + \sum_{m=1}^{M} r_m x_{(j,m)}}{(1+\gamma)^{l(j)-1}} \quad (1)$$

$$\text{S.T.:} \quad D_{(j,m)} \le \sum_{i\in P(j):j\in V(h) \text{ where } h\in F(i,m)} x_{(i,m)} \text{ for all j,m} \quad (2)$$

$$x_{(j,m)} \le g_{(1,m)} + \sum_{i\in P(j)/\{1\}} (k_{(i,m)} - x_{(b(i),m)}) \text{ for all j,m} \quad (3)$$

$$x_{(j,m)} \le y_j Z \text{ for all j,m} \quad (4)$$

$$x_{(j,m)} \text{ are non-negative integers for all j,m} \quad (5)$$

$$y_j \text{ are binary variables for all j.} \quad (6)$$

Equation (1) is the objective to minimize the expected present value of the total costs. Constraints (2) and (3) represent the demand satisfactions and LRU-repair capacities, respectively. Constraint (4) uses the large Z value to model the relationship between $x_{(j,m)}$ and y_j. This makes MinCostST a non-trivial stochastic, production planning problem (PPP) with dynamically varied production capacities.

3.2 Polynomial algorithm

A deterministic PPP has all demands well-defined before time one. Using the Wagner-Whitin property (Wagner and Whitin 1958), if the problem has zero initial inventory, zero lead times and unlimited ordering capacities, then there exists an optimal plan such that the accumulated order quantity by period t is exactly the accumulated demand by some period s, where $0 < t < s < \infty$. However, with additional stochastic features, i.e., demands will be available only when times come, the Wagner-Whitin property is invalid (Ahmed 2003). In order to deal with this stochastic PPP, the Production-Path property was proved in (Guan and Miller 2008). In (Huang and Kucukyavuz 2008), the Production-Path property was modified to solve a more general stochastic PPP, where all lead times, production and inventory costs were random. Note that these PPPs consider only one type of product/item and the production/ordering capacities are unlimited. In MinCostST, however, each PE is made up of M LRUs and L_m-repair operations are upper-bounded by the number of malfunctioning L_m. The following proposition modifies the above properties to cooperate these stochastic features in MinCostST.

Proposition 1 For MinCostST, there exists an optimal solution $\Omega = \{x_{(j,m)}, y_j|$ for all j in V$\}$ such that if $x_{(j,m)} > 0$ for some j and m, then (I) $y_j = 1$; (II) $x_{(j,m)}$ satisfies the repair-capacity constraint (3); and (III) the demand-satisfaction constraint (2) is addressed by:

$$\sum_{k\in G(i,m)\cap P(j)} x_{(k,m)} = D_{(h,m)} - D_{(b(i),m)}, \text{ where } h \in V(i) \text{ and } i \in F(j,m).$$

Proof This proposition is proved by contradiction. An equivalent statement for this proposition is that for node j either there is no any LRU-repair operation or each LRU-repair operations are used to exactly satisfy the demands on some nodes at which the repaired LRUs will arrive later.

For L_m, let node j be the node with the smallest level such that a positive $x_{(j,m)}$ is not used to exactly satisfy the L_m demand on any node in V(k), where k could be any node in F(j,m). Note that V(k) is in V(j). Ω is optimal implies that all L_m demands on all nodes in V are satisfied by the current repair decisions in Ω. Reducing the value of $x_{(j,m)}$ to its minimum value such that any further reduction on $x_{(j,m)}$ would cause unsatisfied L_m demand on some node in V(k) for some k in F(j,m). In this case, there is at least one node in V(j), whose L_m demand is satisfied either by $x_{(j,m)}$ only or by a combination of $x_{(j,m)}$ and some later L_m-repair operations. In either case, contradiction is found as the conducted reduction on $x_{(j,m)}$ would not cause any increases on operating costs, but make the proposition satisfied.

Proposition 1 indicates that a dynamic programming algorithm, which evaluates possible LRU repair decisions on all non-leaf nodes in V, can be developed to search for optimal solutions to MinCostST. Let $v_j(\alpha)$, where α = (α(1), α(2), …, α(M)), be the optimal solution of the MinCostST problem defined on W(j), where $D_{(\alpha(m),m)}$ is satisfied by the L_m-repair decisions made on P(b(j)) for all nodes α(m) in α. Note that α might denote a single node (if α(1) = α(2) = … = α(M)) or at most M distinct nodes (if α(j) ≠ α(k) for any j and k with 1 ≤ j, k ≤ M and j ≠ k). Let $u_j(\alpha,\beta)$, where α = (α(1), α(2), …, α(M)) and β = (β(1), β(2), …, β(M)), be the objective value of the MinCostST problem defined on W(j), where $D_{(\alpha(m),m)}$ are satisfied by the L_m-repair decisions made on P(b(j)) for all α(m) in α and the LRU-repair decisions on node j would be either $x_{(j,m)} = D_{(\beta(m),m)} - D_{(\alpha(m),m)}$ or $x_{(j,m)} = 0$. Considering a positive $x_{(j,m)}$, the upper bound of $x_{(j,m)}$ can be calculated as:

$$g(j,m) = g_{(1,m)} + \sum_{i \in P(j)} \left(k_{(i,m)} - x_{(b(i),m)}\right).$$

Thus, the value of $u_j(\alpha,\beta)$ can be calculated as:

$$u_j(\alpha,\beta) = \begin{cases} p_j \dfrac{h_j + \sum_{m=1}^{M} x_{(j,m)} r_m}{(1+\gamma)^{l(j)-1}} + \sum_{k \in B(j)} v_k(\alpha), \text{ if for some m} \\ \quad \text{it has } 0 < D_{(\beta(m),m)} - D_{(\alpha(m),m)} \leq g_{(j,m)} \\ \\ \sum_{k \in B(j)} v_k(\alpha), \text{ if it has } D_{(\beta(m),m)} - D_{(\alpha(m),m)} = 0 \\ \quad \text{or } D_{(\beta(m),m)} - D_{(\alpha(m),m)} > g_{(j,m)} \text{ for all m} \end{cases} \qquad (7)$$

Note that $v_k(\alpha)$ is the optimal solution for the MinCostST problem defined on W(k), where $D_{(\alpha(m),m)}$ are satisfied by the L_m-repair decisions on P(b(k)) for all α(m) in α. It can be calculated as:

$$v_k(\alpha) = \min_{\beta(m) \in \{h | h \in V(k) \text{ and } l(h) \geq l(k) + d_{(k,m)}\}: D_{(\beta(m),m)} - D_{(\alpha(m),m)} \geq 0} \{u_k(\alpha,\beta)\}. \qquad (8)$$

The following backwards dynamic programming algorithm, denoted by Recursion, starts from $v_1(\bullet)$, where • is the dummy notation for the initial system status. Regarding boundary conditions, for node j in V, if $l(j) + d_{(j,m)} > T+1$, then the optimal L_m-repair decision is $x_{(j,m)} = 0$. Thus, if $l(\beta(m)) + d_{(\beta(m),m)} > T+1$ occurs for all β(m) in β, then $u_j(\alpha,\beta) = 0$ is determined for all feasible α.

<u>Recursion</u>

[Boundary Conditions]: For any pair of (α, β), if $l(\beta(m)) + d_{(\beta(m),m)} > T+1$ for m and all β(m) in β, then set $u_j(\alpha,\beta) = 0$; otherwise set the value of $u_j(\alpha,\beta)$ as undetermined.

[Recursion Procedure]: For each $u_j(\alpha,\beta)$, perform the calculations as described in Equation (7). For each k in B(j), perform the calculations as described in Equation (8).

[Optimal Solution]: Perform the calculations described in Equation (8), where $v_k(\alpha)$ and $u_k(\alpha,\beta)$ are replaced by $v_1(\bullet)$ and $u_1(\bullet,\beta)$, respectively. (Note that the assumption of all non-zero lead times gives $D_{(1,m)} = 0$ for all m.)

Theorem 1   Recursion can find an optimal solution to MinCostST in $O(|V|^{2M+2})$ time, where |V| denotes the total number of nodes in V.

Proof   The correctness of this theorem follows Proposition 1 and the discussions above. The total number of $u_j(\alpha,\beta)$ is upper-bounded by the production of the number of j, α and β, whose maximum values are |V|, $|V|^M$ and $|V|^M$, respectively. Since each $u_j(\alpha,\beta)$ value requires to evaluate exactly |B(j)| number of $v_k(\alpha)$, i.e., one evaluation for each node k in B(j), the total calculations require at most $O(|V|^{2M+2})$ such evaluations, where |B(j)| < |V| is the total number of nodes in B(j). Since computing $u_j(\alpha,\beta)$ dominates the run time of Recursion, the overall run time is $O(|V|^{2M+2})$.

4. Approximation solutions to MaxRatio

In MaxRatio, the goal is to determine a set of repair operations to maximize fleet reliabilities with constraints on operating costs. In military maintenance problems, however, if repair operations have direct/significant effects on fleet reliabilities, then there won't be any strict limit on operating costs. For example, in the CAF, *contingency funds* are always available for unexpected repair operations as long as there is a need. Thus, in the scenario tree formulation of MaxRatio (denoted by MaxRatioST), the *expected* value of the total T-period operating costs is constrained by the limited budget.

Let $D_{(j,m)}(r)$ denote the L_m demand by node j in V with respect to required fleet reliability level r, where m = 1, 2, …, M and 0 ≤ r ≤ 1. Note that the value of $D_{(j,m)}$ used in MinCostST is based on the fleet reliability level A, i.e., $D_{(j,m)} = D_{(j,m)}(A)$. As discussed in Section 3.1, for a given r, $D_{(j,m)}(r)$ can be easily determined. However, if $s_m > 0$ for some m or r < 1, then there would be no explicit formula to calculate $D_{(j,m)}(r)$. This makes MaxRatioST, which includes $D_{(j,m)}(r)$ as parts of demand satisfaction constraints, mathematically intractable. The MaxRatioST problem is formulated as follows:

$$\text{Max:}\quad r \tag{9}$$

$$\text{S.T.:}\quad D_{(j,m)}(r) \le \sum_{i\in P(a(j)):j\in V(k)\text{ where }k\in G(i,m)} x_{(i,m)} \text{ for all j,m} \tag{10}$$

$$\sum_{j\in V} p_j \frac{h_j y_j + \sum_{m=1}^{M} r_m x_{(j,m)}}{(1+\gamma)^{l(j)-1}} \le B \tag{11}$$

$$x_{(j,m)} \le g_{(1,m)} + \sum_{i\in P(j)} \left(k_{(i,m)} - x_{(b(i),m)}\right) \text{ for all j,m} \tag{12}$$

$$x_{(j,m)} \le y_j Z \text{ for all j,m} \tag{13}$$

$$0 \le r \le 1 \tag{14}$$

$$x_{(j,m)} \text{ are non-negative integers for all j,m} \tag{15}$$

$$y_j \text{ are binary variables for all j.} \tag{16}$$

Equation (9) indicates that the objective of the problem is to maximize the fleet reliability level and Equation (14) bounds r into a feasible range [0,1]. Constraints (10) and (11) describe the demand and the cost constraints, respectively. All other constraints (12), (13), (14) and (15) have the same meanings as in MinCostST.

Since MaxRhoSTM includes some implicit terms, it is unlikely that MaxRhoSTM can be directly solved using general optimization approaches, e.g., mathematical programming. In this case, combinatorial optimization approaches (e.g., dynamic programming) might be useful tools for optimal solutions. Generally, these approaches enumerate and compare all possible solutions to determine the optimal solution. This requires a finite number of value options for decision variables. In MaxRhoSTM, however, r could take any real value in [0,1] and the number of value options for r is infinity. It is impossible to develop an optimal algorithm for MaxRhoSTM using general combinatorial optimization approaches.

Next, a solution algorithm (BinRec) is developed to determine approximation solutions to MaxRatioST. In BinRec, a binary search is performed. In each iteration, an on-hand r value is first used to determine $D_{(j,m)}(r)$ and MinCostST(r), and then Recursion is called to solve MinCostST(r). If Recursion returns an *acceptable* result (i.e., a feasible solution is found to MinCostST(r), where the objective value is within the budget limit), then r is updated to a larger value for the next iteration. If Recursion returns no *acceptable* result (i.e., either none feasible solution is found to MinCostST(r) or the found feasible solution has the objective value over the budget limit), then r is updated to a smaller value for the next iteration.

Before presenting BinRec, let r* and r(ε) be the optimal and approximation solution values of MaxRatioST, respectively. It is obvious that r(ε) ≤ r*, e.g., r(ε) ≥ r* - ε for some ε > 0. In BinRec, let U and L be the upper and lower bounds of r*, respectively. Let e = U - L be the bound range. Assuming that there is at least one feasible solution to MaxRatioST, at least one failed PE can be recovered under the budget limit. Thus, the initial lower bound is L = 1/(TK+N). Note that K denotes the maximum for the number of failed PEs introduced in each period. It is also true that the initial upper bound is U = 1. Therefore, the initial range is determined as e = U - L = 1 - 1/(TK+N).

BinRec

[Preparation]: Set L = 1/(TK+N), U = 1, e = 1 - 1/(TK+N).
Set r(ε) = L, where ε is selected as 0 < ε < e.

[BinarySearch]: If e ≤ ε, then go to [FinalCheck]. Otherwise,

- Set r = (L + U)/2 and determine $D_{(j,m)}(r)$.
- Run Recursion to solve MinCostST(r).
    - If there are no acceptable solution, then set U = r.
    - If there is an acceptable solution, then set L = r and r(ε) = L.
- Calculate e = U - L and go to [BinarySearch].

[FinalCheck]: Determine $D_{(j,m)}(U)$, and run Recursion to solve MinCostST(U).

- If there is no acceptable result, then go to [solution].
- If there is an acceptable result, then set r(ε) = U.

[Solution]: Select r(ε) as the found fleet reliability level.
Trace back to obtain the corresponding repair decisions.

Theorem 2 For MaxRatioST, BinRec can find an approximation solution r(ε) such that r(ε) > r* - ε in $O([\ln(TK+N-1)/(\varepsilon TK+ \varepsilon N)]|V|^{2M+2})$ time.

Proof In [BinarySearch], the bounds are cut into half after each iteration by either increasing L or decreasing U. r(ε) is updated when a feasible solution is found. After [FinalCheck], it is determined that r* in [L,U) with L = r(ε). The stopping criteria (e ≤ ε) gives r* < U ≤ L + ε = r(ε) + ε, i.e., r(ε) > r* - ε. For complexity, in each iteration Recursion runs in $O(|V|^{2M+2})$ time to solve MinCostST(r). Since BinRec runs a binary search, Recursion is called O(ln[ln(TK+N-1)/(εTK+ εN)]) times. Thus, the overall run time is $O([\ln(TK+N-1)/(\varepsilon TK+ \varepsilon N)]|V|^{2M+2})$. Note that solution accuracy is traded-off with run time.

Note that BinRec has controllable run times and approximation ratios such that a pre-determined value of ε can be used to estimate the solution quality (i.e., r(ε) > r* - ε) and the run time (i.e., $O([\ln(TK+N-1)/(\varepsilon TK+ \varepsilon N)]|V|^{2M+2})$). This demonstrates that more accurate solutions require longer run times. As an extreme case, however, ε = 0 requires unlimited run time. This shows that BinRec cannot be used to search for optimal solutions to MaxRatioST.

5. Final remarks

This paper proposed an analytical model for multi-stage repair/maintenance decision making in a single-base, single-depot military support network, where cannibalization operations were allowed; failures were independent; repair lead times were random. The paper presented a solution algorithm for the problem to minimize total operating costs and an approximation algorithm for the problem to maximize fleet reliabilities. Future research could be to extend this study to more complex maintenance systems, where support networks are structured by several operating bases and/or repair depots. Another research direction could be to develop optimal algorithms for the fleet-reliability-maximization problem.

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