Time-Frequency Based Instantaneous Frequency Estimation of Sparse Signals from an Incomplete Set of Samples

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1

Abstract- **The estimation of time-varying instantaneous frequency for monocomponent signals with incomplete set of samples is considered. A suitable time-frequency distribution reduces the nonstationary signal into a local sinusoid over the lag variable prior to Fourier transform. Accordingly, the observed spectral content becomes sparse and suitable for compressive sensing reconstruction in the case of missing samples. Although the local bilinear or higher order autocorrelation functions will increase the number of missing samples, the analysis shows that accurate instantaneous frequency estimation can be achieved even if we deal with only few samples, as long as the auto-correlation function is properly chosen to coincide with signal's phase non-linearity. Additionally, by employing the sparse signal reconstruction algorithms, the ideal time-frequency representations are obtained. The presented theory is illustrated on several examples dealing with different auto-correlation functions and corresponding time-frequency distributions.**

Index Terms **– instantaneous frequency, time-frequency distributions, signal sparsity, autocorrelation function, reconstruction algorithms, orthogonal matching pursuit**

I. INTRODUCTION

Signals characterized by the instantaneous frequency (IF) laws arise in many sensing modalities and applications, including radar, biomedicine, ultrasound, and communications. The IF is typically estimated using quadratic or higher order time-frequency distributions (TFD) [1]-[4], which are obtained as the Fourier transforms of the second or higher order local autocorrelation functions (LAFs). In order to produce highly localized energy distributions, these functions must locally approximate a sinusoidal signal at each time sample. The sinusoidal frequencies over shifted data windows constitute the signal's overall IF characteristics. For chirp signals, in order to generate a sinusoid, it suffices to calculate the bilinear data products, as in the case of Wigner distribution (WD), whereas signals with higher order phase nonlinearities require the application of polynomial and other higher order distributions [2]-[11]. It is interesting to note that depending on the chosen distribution order, the concentration spread factor can be arbitrary decreased, yielding to high concentration even for signals whose IF varies very fast.

In this paper, the possibility of IF estimation is observed from the perspective of compressive sensing theory. The aim is to show that a reliable IF estimation can be obtained from a very small set of samples, if this available set is sufficient to provide successful CS reconstruction. The CS assumes that the signal is sparse, and thus we need to assure sparse highly concentrated spectral representation using proper LAF and TFD. As mentioned before, the choice of suitable LAF provides signal stationarization and sparse sinusoidal characteristics within the window, which is amenable to CS recovery algorithms [12]-[17]. Due to the coarsely under-sampled signal, the corresponding TFD will be seriously affected by the noise, with the amplitudes of components far below the exact values. Therefore, we apply the CS reconstruction to recover the missing LAF samples and to achieve an ideal time-frequency representation (TFR). In that sense, the orthogonal matching pursuit (OMP) algorithm [14] is applied to the bilinear or higher order LAFs (on a window by window basis), in lieu of the Fourier transform, to produce ideal TFR. Note that the OMP was successfully applied to another form of autocorrelation function in [18] where cyclo-stationarity is examined for bands detection in the radio spectrum.

The paper is organized as follows. A theoretical background about the IF estimation using time-frequency distributions is provided in Section II. The IF estimation based on a small set of samples has been considered in Section III, as well as the OMP based ideal time-frequency representation. The experimental results are presented and discussed in Section IV, while the concluding remarks are given in Section V.

II. Theoretical Background

Nonstationary signals have time-varying spectral contents. An important subclass of these signals consists of those which are uniquely characterized by their IF laws. Such signals arise in many applications, including radar, sonar, biomedicine and multimedia [19]-[22]. The signal's IF can assume linear, polynomial, or nonlinear behavior [1]-[6]. The IF has physical meaning only for monocomponent signals, while in the case of multicomponent signals, one can speak only about the IF of each component separately. For a signal $x(t)=Ae^{j\phi(t)}$, the instantaneous frequency is defined as:

$$\omega_i(t)=\frac{d\phi(t)}{dt}, \tag{1}$$

where A is a slow varying amplitude and $\phi(t)$ is a phase. Time-frequency distributions attempt to estimate the signal's IF by providing the joint-variable distribution:

$$TFD(t,\omega)=2\pi A^{2\alpha}\delta(\omega-\omega_i(t))*_\omega W(\omega)*_\omega FT\{e^{jQ(t,\tau)}\}, \tag{2}$$

where the distribution is centered around the IF. However, the level of concentration depends on spread factor $Q(t,\tau)$ and the Fourier transform of the employed window function $W(\omega)$. The factor $Q(t,\tau)$ is caused by higher order phase derivatives. The constant α is determined by the order of distribution, e.g., for quadratic distributions, we have $\alpha=1$. One of the most commonly used TFDs is the WD:

$$WD(t,\omega)=\int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau . \tag{3}$$

It is interesting that the WD can be defined using the STFT as follows:

$$WD(t,\omega)=\int_{-\infty}^{\infty} STFT(t,\omega+\theta)STFT^*(t,\omega-\theta)d\theta . \tag{4}$$

In this way, it is possible to avoid generating of the cross-terms which are the main WD drawback. The WD provide an ideal concentration for linear frequency modulated signals (chirps), while in the case of non-linear IF, higher order distributions should be used to reduce the spread factor $Q(t,\tau)$. An efficient higher order distribution is the L-Wigner distribution. It is derived from the standard WD by applying the frequency linearization around a considered time instant t:

$$LWD_L(t,\omega)=\int_{-\infty}^{\infty} w_L(\tau)x^L(t+\frac{\tau}{2L})x^{*L}(t-\frac{\tau}{2L})e^{-j\omega\tau}d\tau , \tag{5}$$

where w_L is a window function and L is an even integer greater than one. In practical applications, it is simpler to use the recursive formula given by:

$$LWD_L(t,\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} LWD_{L/2}(t,\omega+\theta)LWD^{*}_{L/2}(t,\omega-\theta)d\theta \, , \tag{6}$$

where $LWD_1=WD$.

In the case of signals with higher order polynomial phase function, the polynomial distributions (PD) can be used [5],[6]. A commonly used form of the PD is given by [5]:

$$PD(t,\omega) = \int_{-\infty}^{\infty} x^2(t+0.675\tau)x^{*2}(t-0.675\tau)x^{*}(t+0.85\tau)x(t-0.85\tau)e^{-j\omega\tau}d\tau \tag{7}$$

In the case of fast varying IF (e.g, sine of cosine modulated phase function), the complex-time distributions are usually considered [8]-[10]. As one of the most interesting and commonly used cases of complex-time distributions is defined as follows:

$$CTD_4(t,\omega) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{4})x^{-1}(t-\frac{\tau}{4})x^{-j}(t+j\frac{\tau}{4})x^{j}(t-j\frac{\tau}{4})e^{-j\omega\tau}d\tau. \tag{8}$$

The dominant term in the spread factor of CTD_4 is of the fifth order which assures an ideal concentration for signals with polynomial phase up to the fourth order. Therefore, the distribution (8) provides significant concentration improvement with respect to the quadratic distributions, but also improvements compared to the polynomial distribution (of the same order N=4).

Generally, all of the mentioned TFDs can be defined as the Fourier transform of the corresponding LAF:

$$TFR(t,\omega) = \int_{-\infty}^{\infty} \Re(t,\tau)e^{-j\omega\tau}d\tau \, , \tag{9}$$

where general form of the LAF is denoted as a function of t and τ, i.e, $\Re(t,\tau)$. Notations $TFR(t,\omega)$ and $\Re(t,\tau)$ will be used in the sequel for time-frequency representation and the LAF respectively, instead of each individual case considered in this Section.

III. TIME-FREQUENCY BASED IF ESTIMATION AND IDEAL TIME-FREQUENCY REPRESENTATION BASED ON A SMALL INCOMPLETE SET OF SAMPLES

A. IF estimation using a small incomplete set of signal samples

The local auto-correlation function that coincides with the signal's phase function allows us to linearize and sparsify frequency characteristics of signal within the window. Hence, the aim of the LAF is to shape local signal characteristics as close as possible to the sinusoidal characteristics, in order to be able to provide reliable representation using very limited set of samples. Now, assume that the LAF denoted as $\Re(t,\tau)$ has a sparse representation for a certain time instant t_j obtained using an $N\times N$ orthonormal basis matrix Ψ: $\Re=\Psi\theta$, such that only K out of N ($K<<N$) transform coefficients θ are nonzero [23], [24]. Furthermore, we consider that Ψ represents the inverse discrete Fourier transform matrix (IDFT). The idea of the proposed approach is to show that even if we do not have all N samples of $\Re(t,\tau)$, it is possible to achieve an efficient IF estimation using a small set of M samples in $\Re(t,\tau)$, where $M<<N$ holds. Hence, for an incomplete set of samples we can summarize two criteria which are required for the choice of distribution and the IF estimation:

1. The LAF $\Re$ should be directly related to the expected signal form, locally reducing the problem to the linear phase case. The auto-correlation function chosen in that way will provide a locally sparse signal with one frequency component, which needs just a couple of samples for its reconstruction [25]. In real applications, we usually can approximately assume the signal's IF law based on the nature of physical phenomena described by the signal. Therefore, we can say that a phase of certain signal is polynomial of order 3 or 4, or sine/cosine modulated in the case of some micro-Doppler radar signals, etc. Consequently, we can choose the LAF and corresponding TFD to eliminate the influence of higher order phase derivatives and to provide high concentration along the IF.
2. The LAF $\Re$ is defined in its generalized form as a product of P different signal terms x_i , i=1,…,P:

$$\Re(t,\tau)=x_1\cdot x_2\cdot\ldots\cdot x_P=\prod_{i=1}^{P}x_i=\prod_{i=1}^{P}x^{b_i}(t+a_i\tau), \tag{10}$$

where P is the order of the distribution, while the discrete signal terms are denoted by x_i. The samples in $\Re$ can be taken randomly from each signal term x_i. Namely, we consider the case of compressed sampled signal, meaning that x_i does not contain the entire set of samples, but just a small randomly positioned subset. The values at the positions of missing samples are set to zero. Here it is important to emphasize that the missing samples may appear as a consequence of random and compressive sensing/acquisition, or in some applications, due to the elimination of samples corrupted by noisy pulses.

If we would like to take independently randomized samples in each signal term, then the resulting $\Re$ should satisfy:

$$\left\|\Re(t,\tau)\right\|_{\ell_0} = \left\|\prod_{i=1}^{P} x_i(t)\right\|_{\ell_0} \geq M \ . \tag{11}$$

Recall that in the case of standard IF estimation, a monocomponent signal is assumed. Thus, if $\Re(t,\tau)$ provides an ideal representation, then K=1 holds, resulting in much relaxed conditions. In order to estimate the IF, we need just a few samples: $M = O(K\log(N/K))$ determined by K=1. The IF can be therefore estimated by:

$$IF(n) = \arg\max_{\omega}(TFR(t,\omega)) \ , \tag{12}$$

where $TFR(t,\omega)$ represents the corresponding time-frequency representation obtained as the Fourier transform of function $\Re(t,\tau)$ which has a small random set of non-zero values.

B. OMP based ideal time-frequency representation

The OMP algorithm has been known as the iterative signal reconstruction procedure. In each iteration, the OMP searches for the maximum correlation between the measurements and the transform matrix. Thus, through the iterations it selects a certain number of columns form the transform matrix, where this number is defined by the given number of iterations. The least square optimization is performed afterwards in the subspace spanned by all previously picked columns. In the case when we need the TFR, and not only the IF estimation, we may benefit from applying the OMP algorithm. Namely, by recovering the full auto-correlation function, we can produce an ideal TFR. It is important to emphasize that only the first iteration of OMP is used, which simplifies the calculation procedure.

If we observe the vector of the LAF samples for a single time instant t_j denoted as:

$$\Re(t_j,\tau)=\prod_{i=1}^{P} x^{b_i}(t_j+a_i\tau), \tag{13}$$

then the optimization problem can be defined as follows:

$$\min\left\|TFR(t_j,\omega)\right\|_{\ell_1} \quad s.t. \quad \Re(t_j,\tau)=\Theta\cdot TFR(t_j,\omega), \tag{14}$$

where $TFR(t_j,\omega)$ is a vector of ideal time-frequency representation at the time instant t_j, while Θ is the inverse Discrete Fourier Transform (DFT) based CS matrix. The OMP is a stepwise forward selection algorithm and can be easily implemented. Since we are looking for a single component at each considered instant t_j, the minimization procedure for obtaining the ideal TFR can be reduced to a simplified version of the OMP with a single iteration. The steps of the simplified OMP algorithm are given below:

Step 1: Set residual $r=\Re(t_j,\tau)$. Initial set of selected columns from Θ is: $c_0=\varnothing$, such that $\Theta(c_0)=\varnothing$.

Step 2: Find column u of Θ that solves the maximization problem: $\max_u\left|\Theta' r\right|$. Update the sets:

$$\Theta(c)=\Theta(c_0)\cup\Theta(u), \qquad c=c_0\cup u\ .$$

Step 3:

$$ITFR(t_j,\omega)=\left(\Theta'(c)\cdot\Theta(c)\right)^{-1}\cdot\Theta'(c)\cdot r\ .$$

Note that the spectral noise caused by missing samples does not influence the result of OMP algorithm as long as the signal component is dominant over noise components. This further means that we need to ensure a necessary and sufficient number of available samples, which is elaborated and verified in the experiments, using different scenarios, and different types of signals and distributions.

In order to summarize, the procedure for OMP based calculation of the ideal TFR, the flowchart is provided in Fig. 1. The variable P denotes the number of terms that will be multiplied within the LAF $\Re(t,\tau)$ (i.e., the order of the TFR), while a_i and b_i are parameters used for LAF calculation. The samples with non-zero values in $\Re(t,\tau)$ are available ones, and their positions are captured within vector ***pos***. DFT denotes the DFT transform matrix, while Θ is CS matrix obtained from DFT.

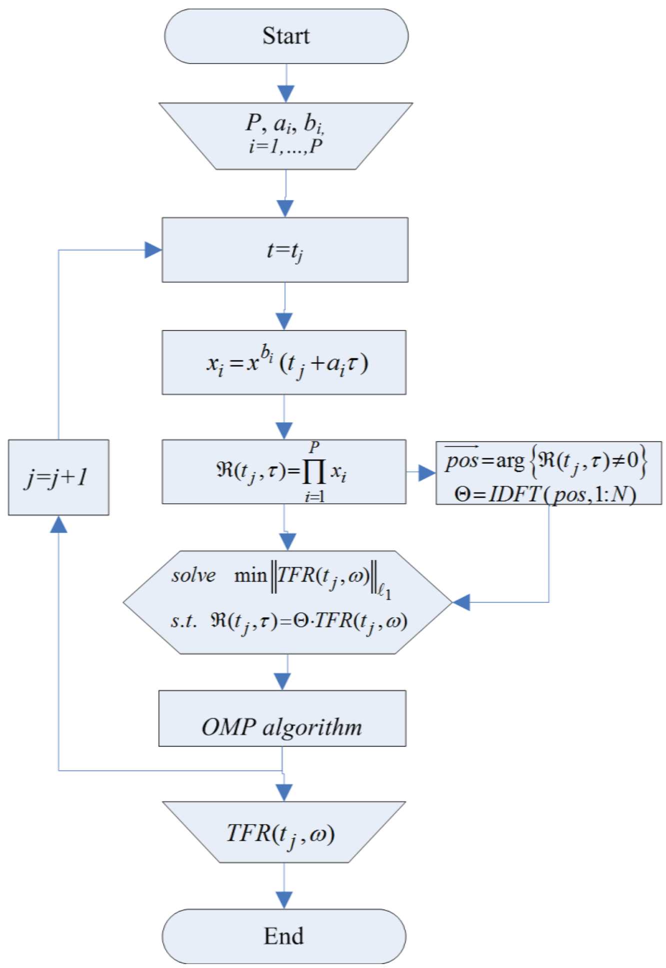

Fig. 1. Flowchart for the OMP based ideal time-frequency representation

IV. Simulation Results and Discussion

Example 1*:* In the following experiments, we will consider four different signals defined in the form:

$$\begin{aligned} &x_1(t) = e^{j64\pi t^2}, \; x_2(t) = e^{j16\pi t^3}, \\ &x_3(t) = e^{j96\pi t^3 - j48\pi t}, \; x_4(t) = e^{j10\sin(2\pi t) + j2\cos(\pi t)}. \end{aligned} \tag{15}$$

In order to estimate the IF, a suitable TFD should be chosen. Hence, the WD is applied to the signal $x_1(t)$, the L-WD and polynomial distribution (PD) are used for $x_2(t)$ and $x_3(t)$, respectively, while CTD is used for $x_4(t)$. Namely, as the phase nonlinearity increases from $x_1(t)$ to $x_4(t)$, we should increase the distribution order, as well. Snap shots for some of the distributions at a single time instant (that corresponds to single windowed signal segment) are illustrated in Fig. 2.

Let us firstly discuss the case when the randomly chosen samples have the same positions in different signal terms x_i. Only M=4 samples are used for all distributions to yield accurate IF estimation, as shown in Fig. 2. Note that 3 independent samples would be enough, because a sinusoidal function, can be completely specified by three non-zero samples taken at any 3 points in its period [25]. However, in practical applications, some additional samples are needed (e.g., 4 or 5 samples) in order to ensure that 3 of them would be independent within the period.

The detected positions of maximums depicted in Fig. 2 are 46, 108 and 51, respectively, being equal to the exact IF at the three considered time instants.

Furthermore, without loss of generality, we observe the case of the WD and $x_1(t)$, for different number of samples within the window: 12.5%, 25%, 20% and 25% (Fig. 2). We might observe how the peak detection becomes more reliable as the number of available samples increases. According to the previous analysis, if the number of non-zero samples in $\Re$ (defined by (11)) is at least M=4, the position of maximum in the transform domain will be exactly at the IF position. If the signal samples are independently and randomly selected in signal terms x_i, in order to be sure that we have $M \geq 4$ measurements in $\Re$ for all time instants, we need at least 25% of samples (note that in Fig. 3 we show that M=4 even for 12.5%, but this is just for particular time instant). Observe that, the peak component corresponding to the IF becomes more enhanced as the number of measurements increases (Fig. 3). For the signals $x_2(t)$, $x_3(t)$, and $x_4(t)$, we use higher order TFD to provide sparse representation. In the case of higher order distributions (PD and CTD) and independently random samples selection, we need 30-35% samples in each x_i to assure that the number of measurements in $\Re$ will be $M \geq 4$ for each time instant.

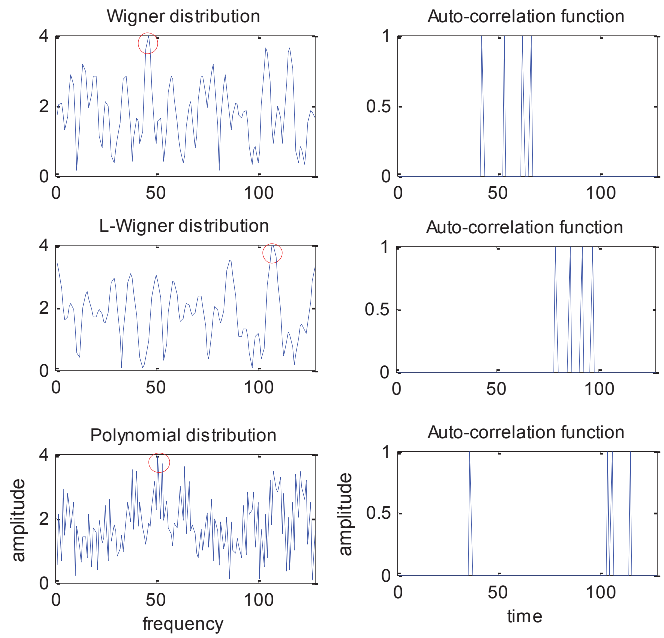

Fig. 2 TFRs for a single time instant (left column) obtained from M=4 samples of the corresponding auto-correlation functions (right column)

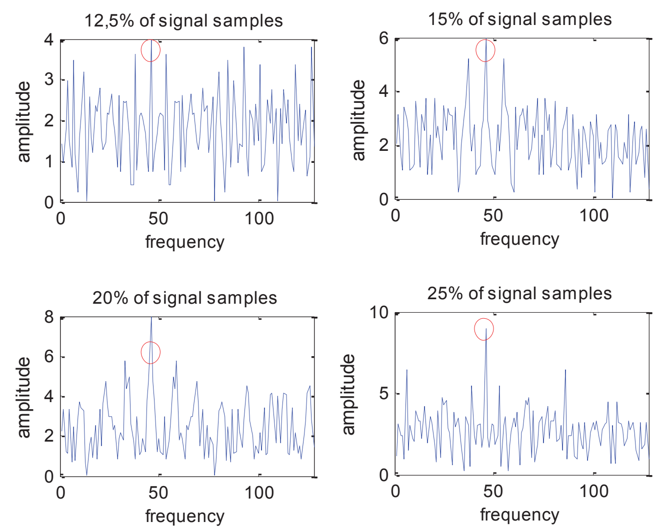

Fig. 3. WD for a single time instant calculated using: 12.5% of samples resulting in M=4 measurements in $\Re$, 15% samples resulting in M=5 measurements in $\Re$, 20% of samples (M=6), 25% of samples (M=8)

***Example 2*:** In the case when we need the exact amplitudes of the components, i.e., the ideal TFR, we might use the OMP signal reconstruction. The OMP is applied to each windowed signal part in order to recover sparse spectrum corresponding to each particular time instant. It means that in the case of monocomponent signal, a single frequency component is obtained for each time instant. All of them together will results in an ideal TFR.

The examples of the standard FT based distributions (with reduced set of samples) and the proposed OMP based ones are given in Fig. 4. The following cases are considered: the WD of the chirp signal $x_1(t)$ from *Example 1*, the PD applied to the cubic phase signal $x_3(t)$, and the CTD applied to the sine modulated signal x_4(t) having faster phase variations compared to other considered signals.

Example 3: In this example, we consider a scenario that might appear in real applications, when the whole signal could be compressive sampled at once. Let us assume that the vector of random positions q of length M defines the locations of available signal samples. The signal is given in the form: $x_1(t) = e^{j64\pi t^2}$, and consequently, we consider the case of the WD. In each time instant t_i the samples in $x(t-\tau/2)$ and $x(t+\tau/2)$ will be located at some random positions denoted by q_i^- and q_i^+, respectively. Now we should determine the minimal number of samples M such that the following condition is satisfied:

$$card\left(q_i^- \cap q_i^+\right) \geq 4,$$

for each time instant t_i. The above condition is required to allow an accurate IF estimation at t_i. Otherwise, the error occurs.

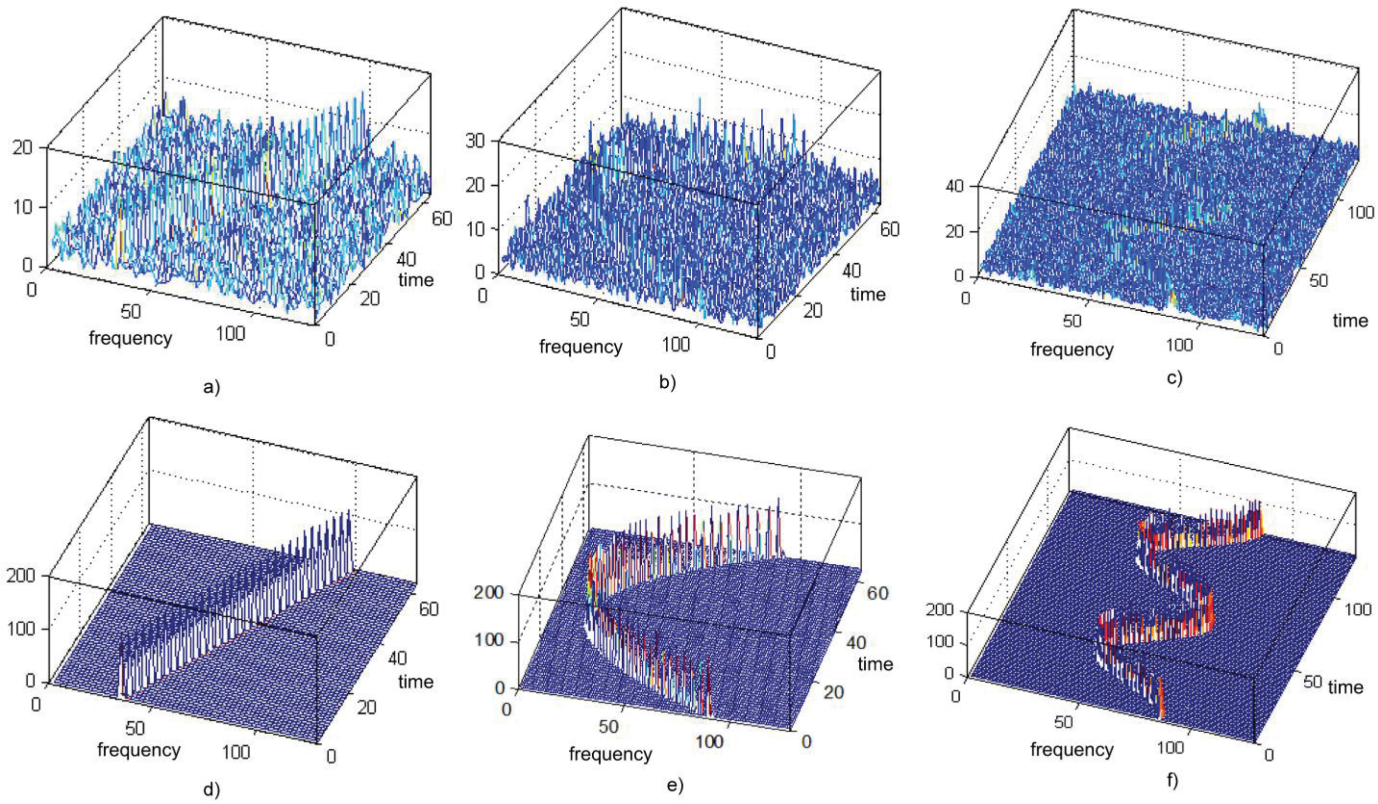

Fig. 4. a) Standard FT based WD for chirp signal $x_1(t)$, b) Standard FT based PD for signal $x_3(t)$, c) Standard FT based CTD for signal $x_4(t)$, d) OMP based reconstructed WD for $x_1(t)$, e) OMP based PD of $x_3(t)$, f) OMP based CTD of $x_4(t)$

TABLE 1. THE PROBABILITY OF ACCURATE IF ESTIMATION

	Number of faulty estimated IF instants	Trials
30% of samples used for IF estimation	0 errors	30%
	1 error	25%
	2 errors	22%
	3 errors	10%
35% of samples used for IF estimation	0 errors	74%
	1 error	16%
	2 errors	7%
	3 errors	3%
40% of samples used for IF estimation	0 errors	97%
	1 error	1%
	2 errors	1%
	3 errors	1%

We examined a number of random vectors q (trials in Table 1) defining the positions of available samples in x. For each trial we calculate the number of faulty estimated IF instants. In Table I, we present the most interesting cases obtained for M=0.3N, M=0.35N and M=0.4N.

We might conclude that, starting from M=0.4N (40% of samples), the estimated IF is accurate with high probability for any set of random samples defined by q. Further, even for M=0.3N we have 87% of trials with maximum 3 errors. Now, when observing the values of faulty estimated IF points in Fig. 5, we might say that small number of errors can be easily corrected. Namely, for a large number of signals observed in signal processing applications, the impulsive and transient changes are not common feature of the IF law, since the IF usually varies continuously with time. Consequently, the faulty IF points could be easily detected because they produce quite pronounced peaks, and could be interpolated using available neighboring samples.

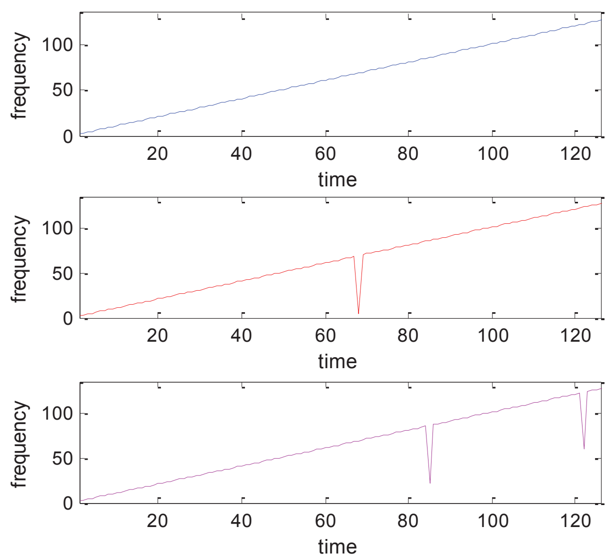

Fig. 5. IF estimation for signal $x_1(t)$: no errors (top), 1 error (middle), 2 errors (bottom)

Example 4: Let us consider the case when the IF estimation is done from the measurements corrupted by the Gaussian noise: $x_1(t)=e^{j64\pi t^2}+g(t)$ and $x_3(t)=e^{j96\pi t^3-j48\pi t}+g(t)$. The IF estimation based on the WD is considered for the samples of signal $x_1(t)$, while the PD is used for signal $x_3(t)$. The experiments have shown that the number of samples needed for an accurate IF estimation is M=10 (out of 128, which is less than 8% of samples). The SNR in the case of $x_1(t)$ and the WD can be as low as 4dB (Fig. 6), while in the case of $x_3(t)$ and the PD, the SNR should be higher (e.g., SNR=12 dB), since within the six-term LAF constituting the PD, the Gaussian input noise becomes highly impulsive.

The standard WD of noisy full data set $x_1(t)$ is given in Fig 7.a for a single time instant, while the WD calculated using reduced data set with M=10 is shown in Fig 7.b (note that the maximum corresponds to the exact IF). The OMP based ideal WD distribution for a single time instant is given in Fig 7.c. Similarly, the same analysis is performed for PD and shown in Figs d, e and f.

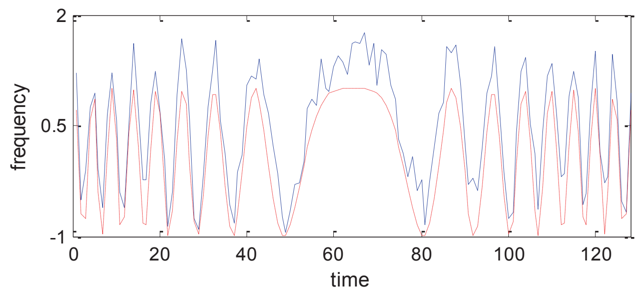

Fig. 6. Noisy signal (solid line) and non-noisy signal (dashed line) used for the calculation of the Wigner distribution

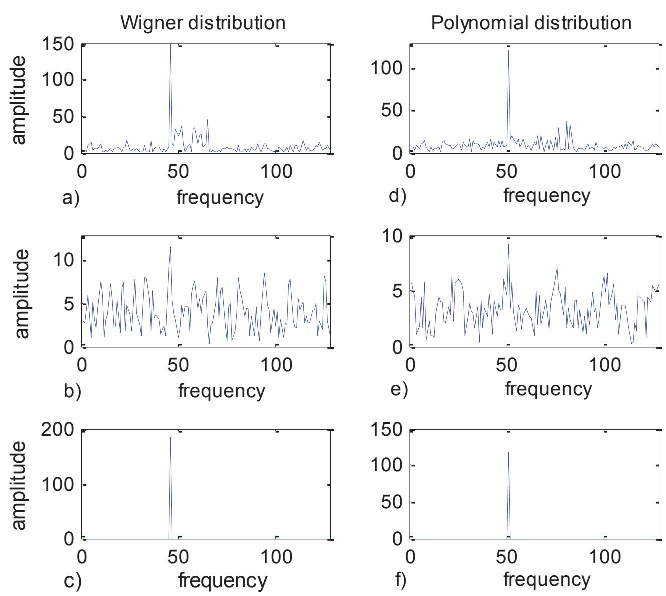

Fig. 7. Left column: Snap shots of WD calculated using: a) total set of 128 signal samples, b) M=10 randomly chosen signal samples, c) OMP and M=10 randomly chosen samples; Right column: Snap shots of PD calculated using: d) total set of 128 signal samples, e) 10 randomly chosen signal samples, f) OMP and 10 randomly chosen signal samples

V. Conclusion

The possibility of estimating signal's IF, in the case when we deal with an incomplete and random set of signal samples, is analyzed. The IF estimation based on the several time-frequency distributions is considered. It is shown that the IF can be accurately estimated by using only few samples, if the auto-correlation function, used to define certain time-frequency distribution, reduces frequency non-linearity to the sinusoids. Furthermore, it has been shown that applying the reconstruction algorithm to the auto-correlation function, which coincides with signal's phase nonlinearities, can lead to an ideal time-frequency representation.

References

[1] Boashash, B.: 'Estimating and Interpreting The Instantaneous Frequency of a Signal - Part 1: Fundamentals', Proceedings of the IEEE, 1992, 8, (4), pp. 520-538

[2] Boashash, B.: 'Time-Frequency Signal Analysis and Processing' (Elsevier, Amsterdam, 2003)

[3] Stanković, L., Daković, M., Thayaparan, T.: 'Time-frequency signal analysis with applications' (Artech House, Boston, 2013)

[4] Lerga, J., Sucic, V.: 'Nonlinear IF Estimation Based on the Pseudo WVD Adapted Using the Improved Sliding Pairwise ICI Rule' IEEE Signal Processing Letters, 2009, 16, (8), pp. 953-956.

[5] Boashash, B., O'Shea, P. J.: 'Polynomial Wigner-Ville distributions and their relationship to time-varying higher order spectra' IEEE Transactions on Signal Processing, 1994, 42, pp. 216-220.

[6] Ristic, B., Boashash, B.: 'Relationships between the Polynomial and Higher order Wigner-Ville Distribution', IEEE Signal Processing Letters, 1995, 2, (12), pp. 227-229

[7] Li, X., Bi, G., Stankovic, S., Zoubir, A. M.: 'Local polynomial Fourier transform: A review on recent developments and applications', Signal Processing, 2011, 91, (6), pp. 1370-1393

[8] Stankovic, S., Stankovic, L.: 'Introducing time-frequency distribution with a "complex-time" argument', Electronics Letters, 1996, 32, (14), pp.1265-1267

[9] Stankovic, S., Orovic, I., Ioana, C.: 'Effects of Cauchy Integral Formula Discretization on the Precision of IF Estimation: Unified Approach to Complex-lag Distribution and its L-Form', IEEE Signal Processing Letters, 2009, 16, (4), pp. 307-310.

[10] Salagean, M., Nafornita, I.: 'Time-Frequency Methods for Multicomponent Signals', International Symposium on Signals, Circuits and Systems, 2007, ISSCS 2007, pp.1-4.

[11] Omidvarnia, A., Azemi, G., O' Toole, J.M., Boashash, B.: 'Robust estimation of highly-varying nonlinear instantaneous frequency in monocomponent nonstationary signals using lower-order complex-time distribution', Signal Processing, 2013, 93, (11), pp. 3251-3260

[12] Baraniuk, R.: 'Compressive sensing', IEEE Signal Processing Magazine, 2007, 24, (4), pp. 118-121

[13] Stankovic, S., Orovic, I., Sejdic, E.: 'Multimedia Signals and Systems' (Springer 2012)

[14] Tropp, J. A., Gilbert, A. C.: 'Signal recovery from random measurements via Orthogonal Matching Pursuit', IEEE Transactions on Information Theory, 2007, 53, (12), pp. 4655-4666

[15] Stankovic, L., Stankovic, S., Orovic, I., Amin, M.: 'Robust Time-Frequency Analysis based on the L-estimation and Compressive Sensing', IEEE Signal Processing Letters, 2013, 20, (5), pp. 499-502

[16] Jokanovic, B., Amin, M., Stankovic, S.: 'Instantaneous frequency and time-frequency signature estimation using compressive sensing' SPIE Defense, Security and Sensing, Baltimore, Maryland, United States, 2013

[17] Zhang, Y. D., Amin, M. G., Himed, B.: 'Reduced interference time-frequency representations and sparse reconstruction of under sampled data', European Signal Processing Conference, Marrakech, Morocco, Sept. 2013

[18] Khalaf, Z., Nafkha, A. and Palicot, J.: 'Blind Spectrum Detector for Cognitive Radio Using Compressed Sensing and Symmetry Property of the Second Order Cyclic Autocorrelation' ICST Conf. on Cognitive Radio Oriented Wireless Networks and Communications, 2012, pp. 291-296

[19] Orovic, I., Stankovic, S., Thayaparan, T., Stankovic, L.: 'Multiwindow S-method for Instantaneous Frequency Estimation and its Application in Radar Signal Analysis', IET Signal Processing, 2010, 4, (4), pp: 363-370

[20] Zhang, J. J., Papandreou-Suppappola, A., Gottin, B., Ioana, C.: 'Time-Frequency Characterization and Receiver Waveform Design for Shallow Water Environments', IEEE Transactions on Signal Processing, 2009, 57, (8), pp.2973 – 2985

[21] Wang, R., Jiang, Y.: ‘ISAR Imaging of Ship Based on the Modified 4th Order Time-Frequency Distributions with Complex-lag Argument’, International Conference on Remote Sensing, Environment and Transportation Engineering, 2012, pp. 1-4

[22] Lee, L., Krishnan, S.: ‘Time-frequency signal synthesis and its application in multimedia watermark detection’, EURASIP Journal on Applied Signal Processing, 2006, Article ID 86712

[23] Candès, E., Romberg J., Tao, T.: ’Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information’ IEEE Transactions on Information Theory, 52, (2), 489-509

[24] Sejdic, E., Cam, A., Chaparro, L.F., Steele, C.M., Chau, T., ‘Compressive sampling of swallowing accelerometry signals using time-frequency dictionaries based on modulated discrete prolate spheroidal sequences’, EURASIP Journal on Advances in Signal Processing, 2012:101 doi:10.1186/1687-6180-2012-101.

[25] Das, S., Mohanty, N., Singh, A.: ‘The Sampling Theorem for Finite Duration Signals’, International Journal On Advances in Systems and Measurements, 2009, 2, (1), pp. 1-17