Simulation Results on Adaptive Transmit Waveforms (Final Report)

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Table of Contents

1. Introduction

This technical report, which presents the results obtained from the MATLAB simulations on adaptive radar transmit waveforms, serves as the final report to DRDC, Ottawa.

Using the new MATLAB program (described in Appendix 2), it is possible to design energy-constrained or amplitude-constrained waveforms so as to maximize the signal-to-interference ratio (SIR) for arbitrarily shaped clutter maps (subject to certain integrability constraints). The program provides a flexible mechanism for defining scenario parameters like clutter map shape, number of sub-pulses, amplitude constraints (if any), target SNR, target-to-clutter cross section ratio, initial waveform, etc. Clutter maps are assumed to be defined by a set of parallelograms (as required by DeLong et al. (1967, 1969)). In the program, the clutter maps are defined using a sequence of way-points. A sample scenario definition file, which corresponds to the experiments conducted in DeLong, et al. (1969), is given in Appendix 1.

The corresponding target clutter environment (plotted by the MATLAB program) is shown in Figure 1. Other clutter map shapes can be defined by changing the variable `clutter` above.

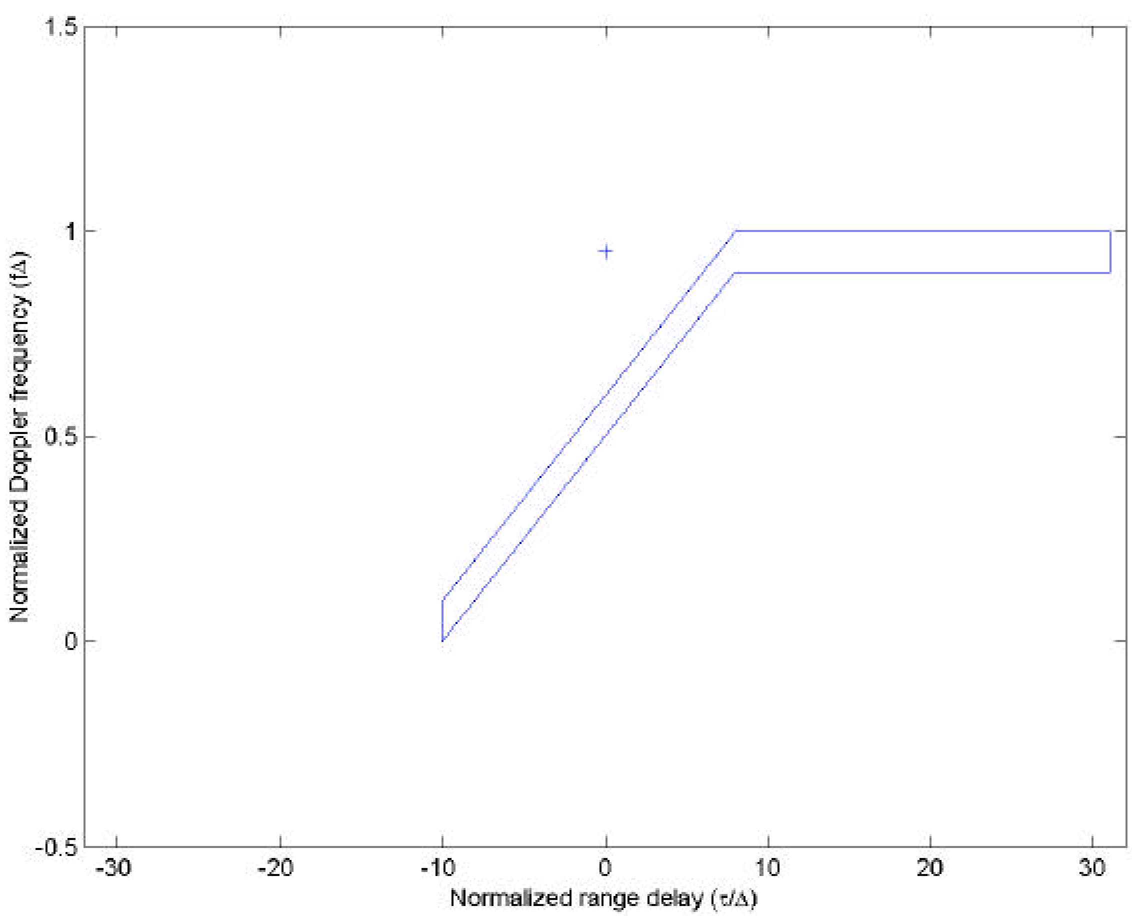

Figure 1 Target clutter environment for the first example from Delong et al. (1969)

2. Experiment 1: High SNR target with zero phase initial burst

As defined in the scenario file, the single-pulse SNR of the target is 5dB. The number of sub-pulses in the burst waveform of the radar is assumed to be 32. With 32 pulses it is possible to attain a maximum SNR of 20dB. The ratio of the clutter cross-section seen by the waveform to the target cross-section is 100. For the amplitude-constrained waveform, the minimum and maximum single pulse amplitudes are 0.1 and 1.0, respectively. Waveform adaptation is carried out for 30 iterations. For the energy-constrained waveform, the energy used by the last amplitude-constrained waveform is specified as the energy constraint.

In the first experiment, with amplitude-constrained waveform design, constant amplitude zero-phase burst was used as the initial waveform. For energy-constrained design, the last waveform from amplitude-constrained optimization is used as the initial waveform. Figure 2 shows the SIR values obtained with amplitude-constrained and energy-constrained waveforms. The former achieves a maximum SIR of 14.4dB after 30 iterations (compared with the maximum attainable SIR of 20dB), while the latter reaches up to 19.1dB. Energy-constrained optimum waveform design results in superior results and faster convergence to the optimal SIR as reported by DeLong et al. (1969). The SIR results with amplitude-constrained design are slightly better (compared with 11.6dB) than those reported by DeLong et al. (1969). The iterative search procedure has been modified slightly in our implementation to select variable step sizes thereby providing some improvement on the results reported by DeLong et al.

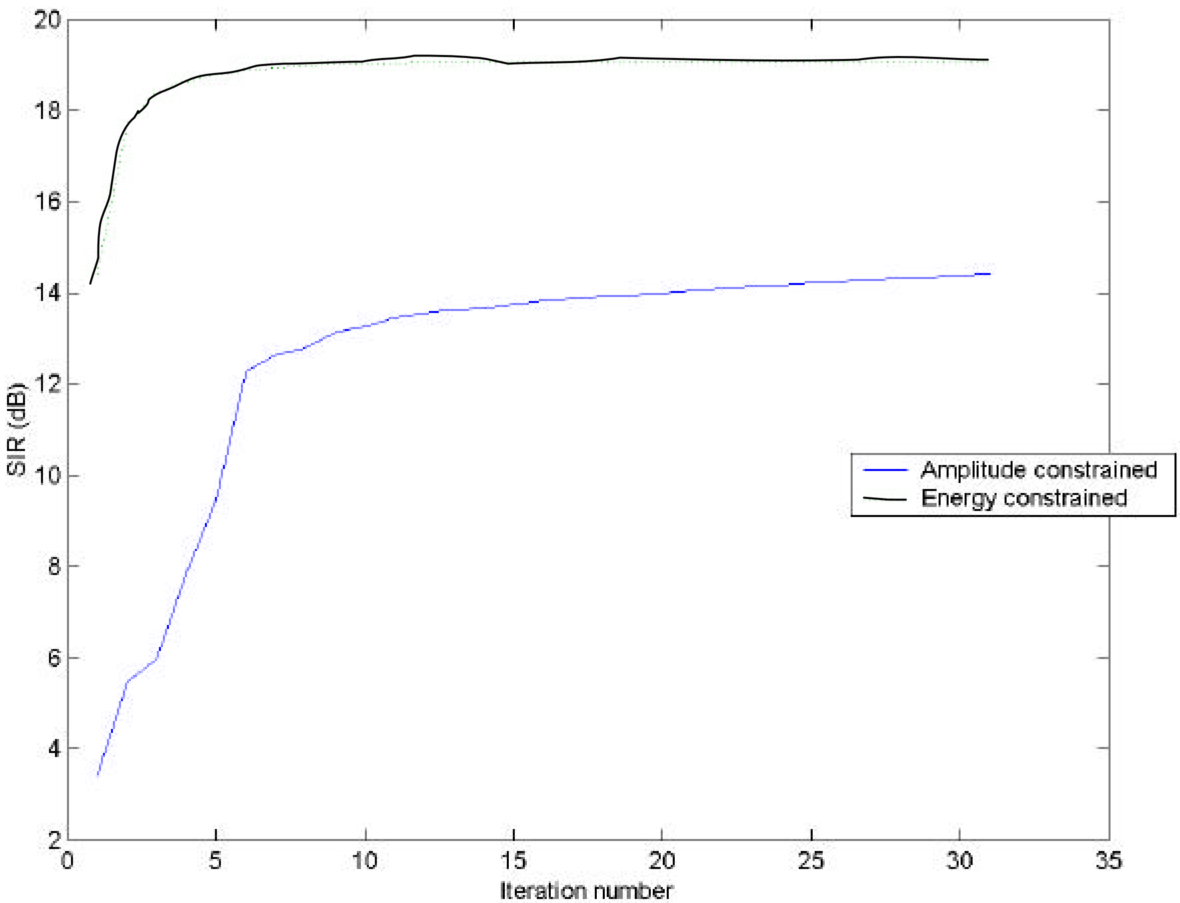

Figure 2 SIR values obtained by amplitude-constrained and energy-constrained waveform design with constant amplitude zero phase initial burst

The waveforms obtained at the end of 30 iterations from amplitude-constrained design are shown in Figure 3. These, together with the SIR values, compare very well with those obtained by DeLong et al. (1969). Note that the phase values can be unfolded to obtain a smooth progression. The corresponding optimal weighting vectors are shown in Figure 4.

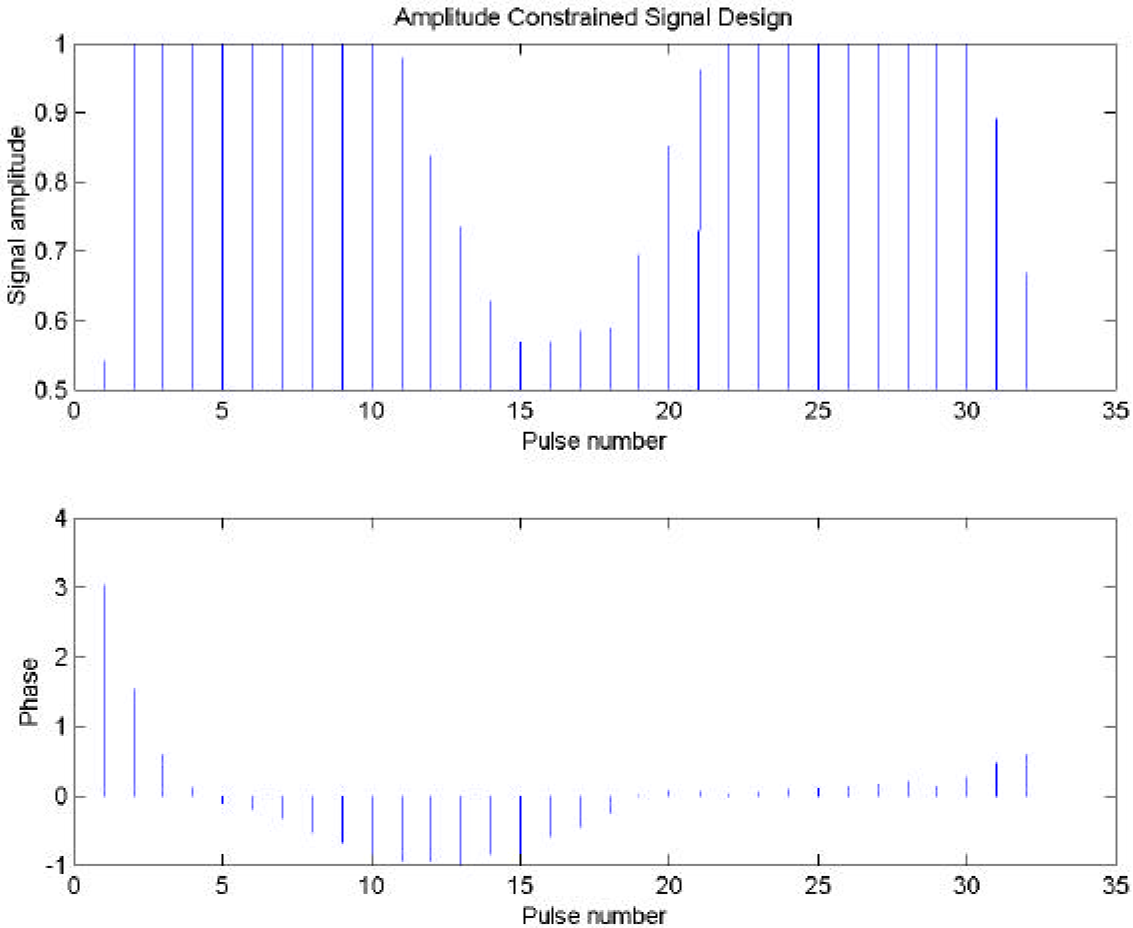

Figure 3 Pulse train at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude zero phase initial burst

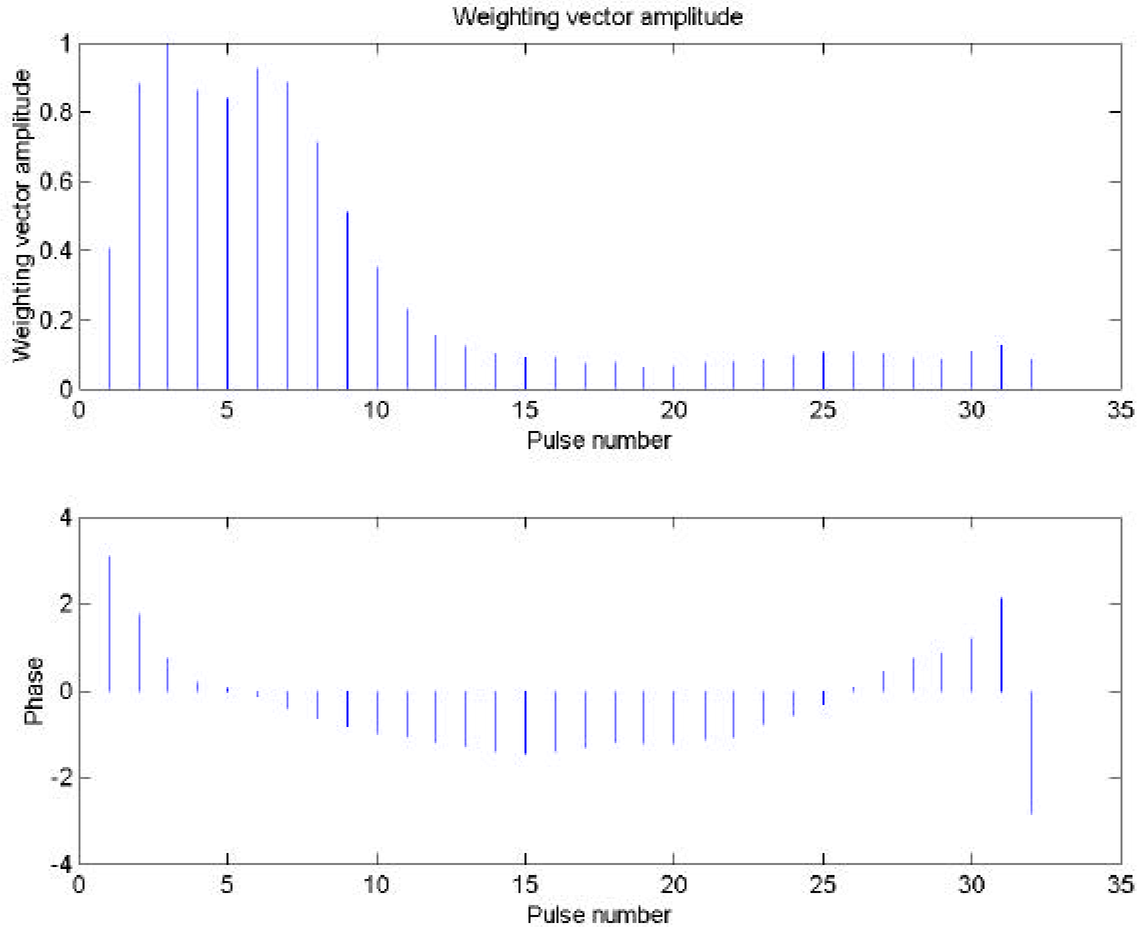

Figure 4 Optimal weighting vector at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude zero phase initial burst

The waveform obtained by energy-constrained optimization is shown in Figure 5. Note that, as expected, the amplitudes obtained by this method are higher than those obtained by amplitude-constrained optimization. Also note that that the optimal waveforms are obtained within almost 15 iterations.

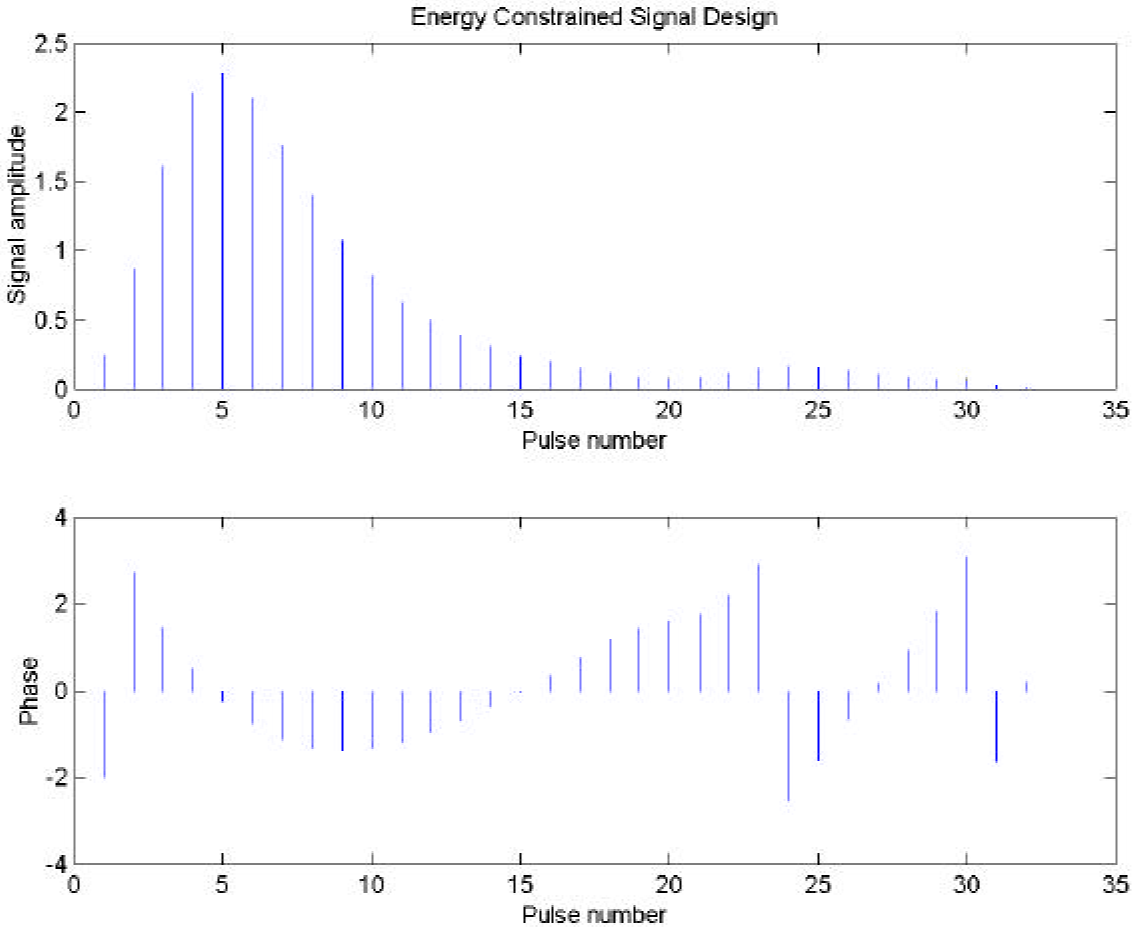

Figure 5 Pulse train at the end of 30 iterations of energy-constrained waveform design with constant amplitude zero phase initial burst

3. Experiment 2: High SNR target with quadratic phase initial burst

For experiment 2, the same scenario as above, with the exception of the initial signal, was used. The initial signal was constant amplitude quadratic phase burst, which provides a better ambiguity pattern than constant amplitude zero phase burst. This is apparent from Figure 6, which shows the SIR of amplitude-constrained and phase-constrained waveform design techniques with this initial waveform. Amplitude-constrained design reaches a maximum SIR of 16.9dB after 30 iterations, while energy-constrained design reaches 19.9dB compared with the 20dB, which is possible only with complete clutter suppression. With energy-constrained design, the maximum SIR is reached within about 5 iterations.

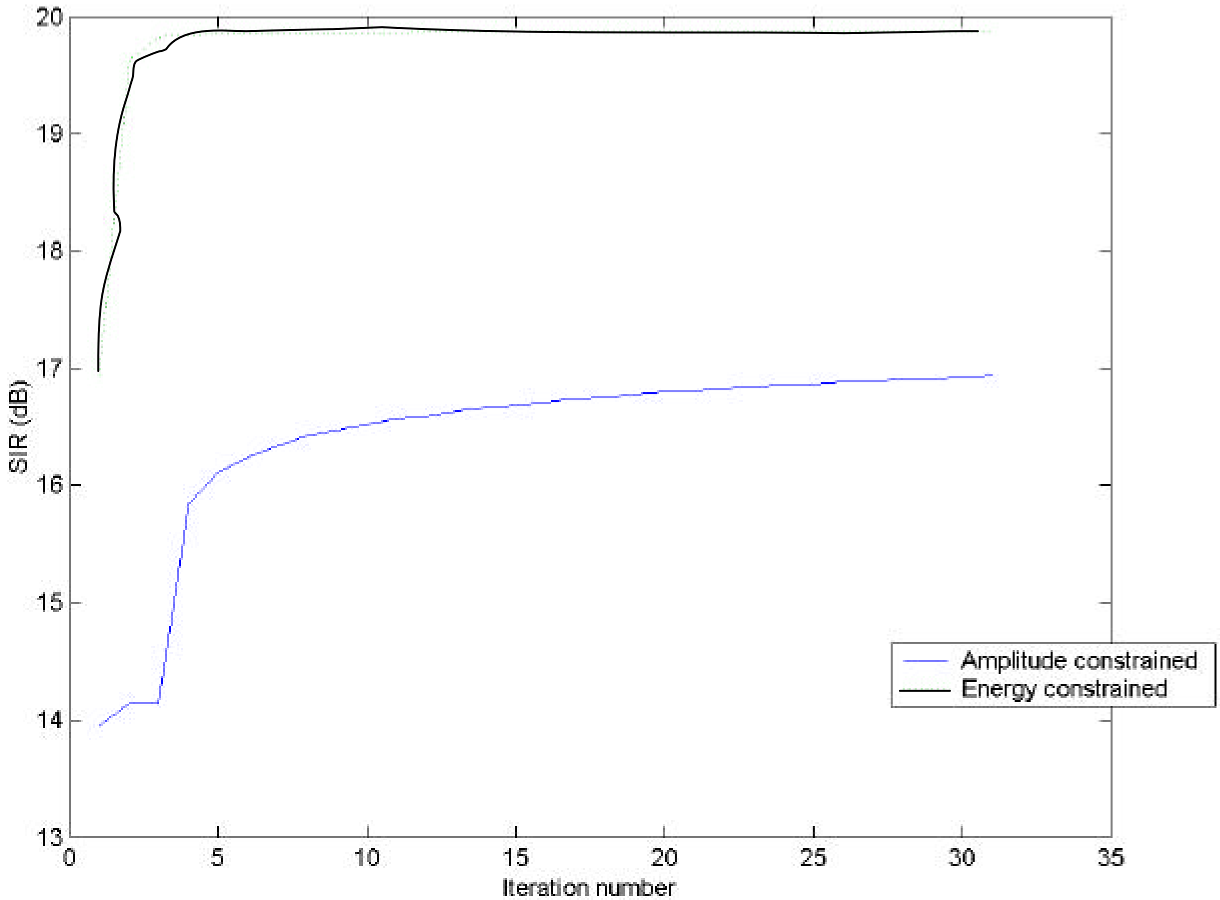

Figure 6 SIR values obtained by amplitude-constrained and energy-constrained waveform design with constant amplitude quadratic phase initial burst

Figure 7 and 8 show the final signal obtained with amplitude-constrained optimization and the corresponding optimal weighting vector, respectively. Figure 9 shows the signal obtained with energy-constrained optimization. Note that the amplitudes of the energy-constrained signal are lower than those obtained by the same method with constant amplitude zero phase initial burst (see Figure 5).

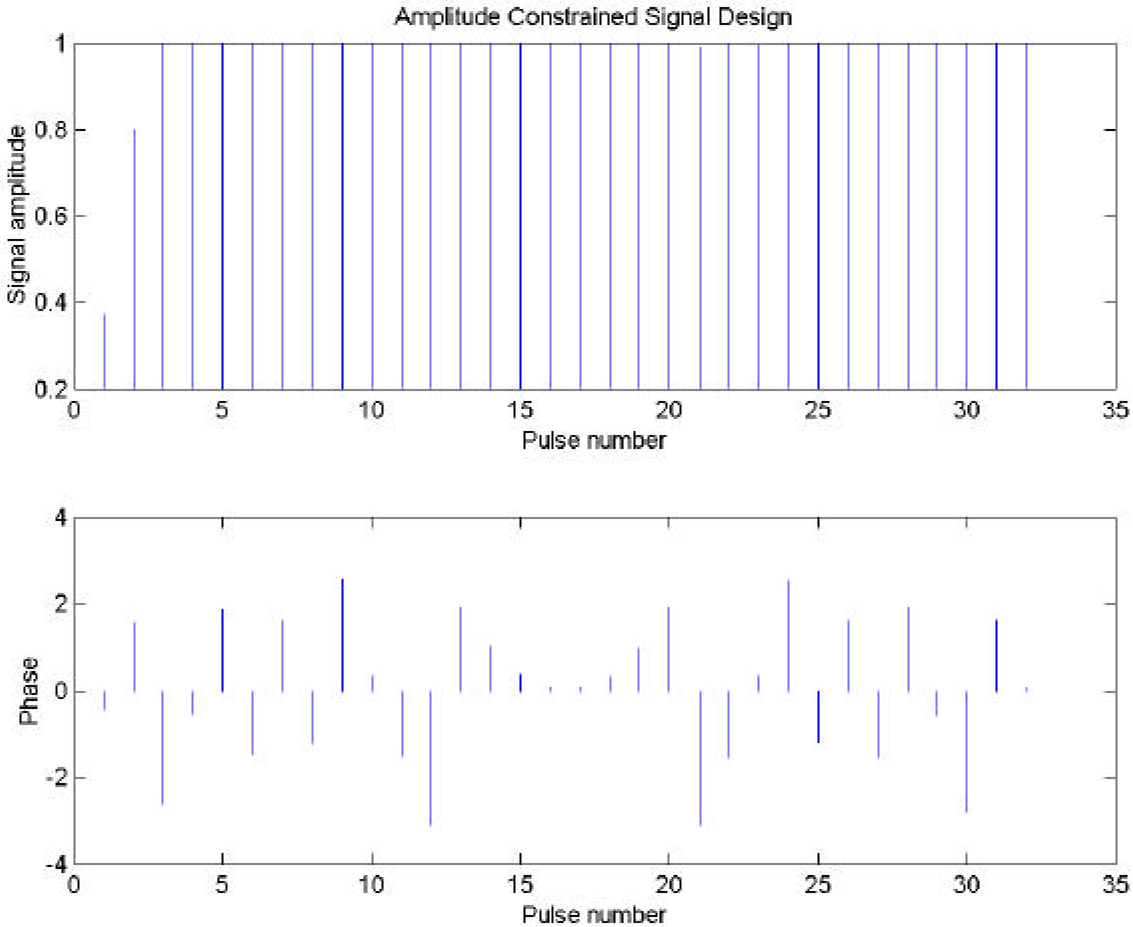

Figure 7 Pulse train at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude quadratic phase initial burst

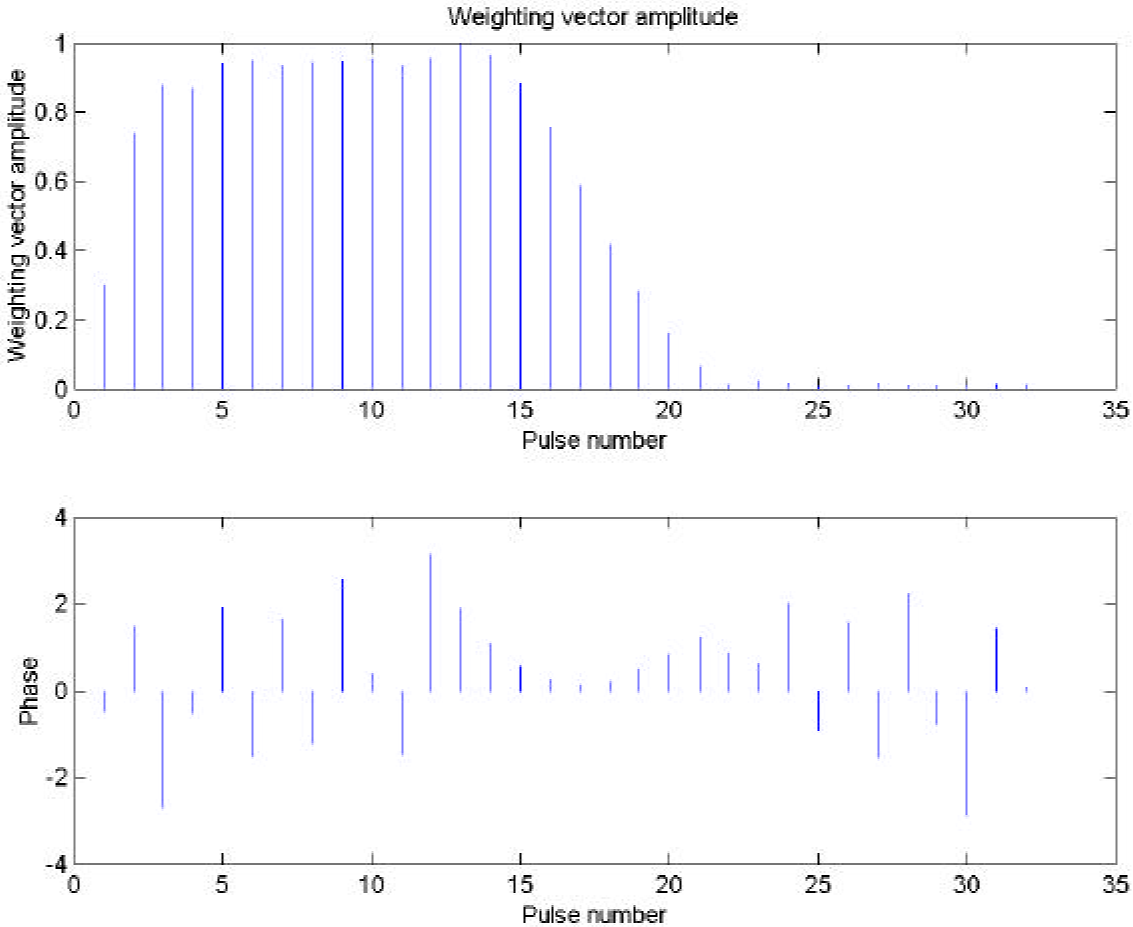

Figure 8 Optimal weighting vector at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude quadratic phase initial burst

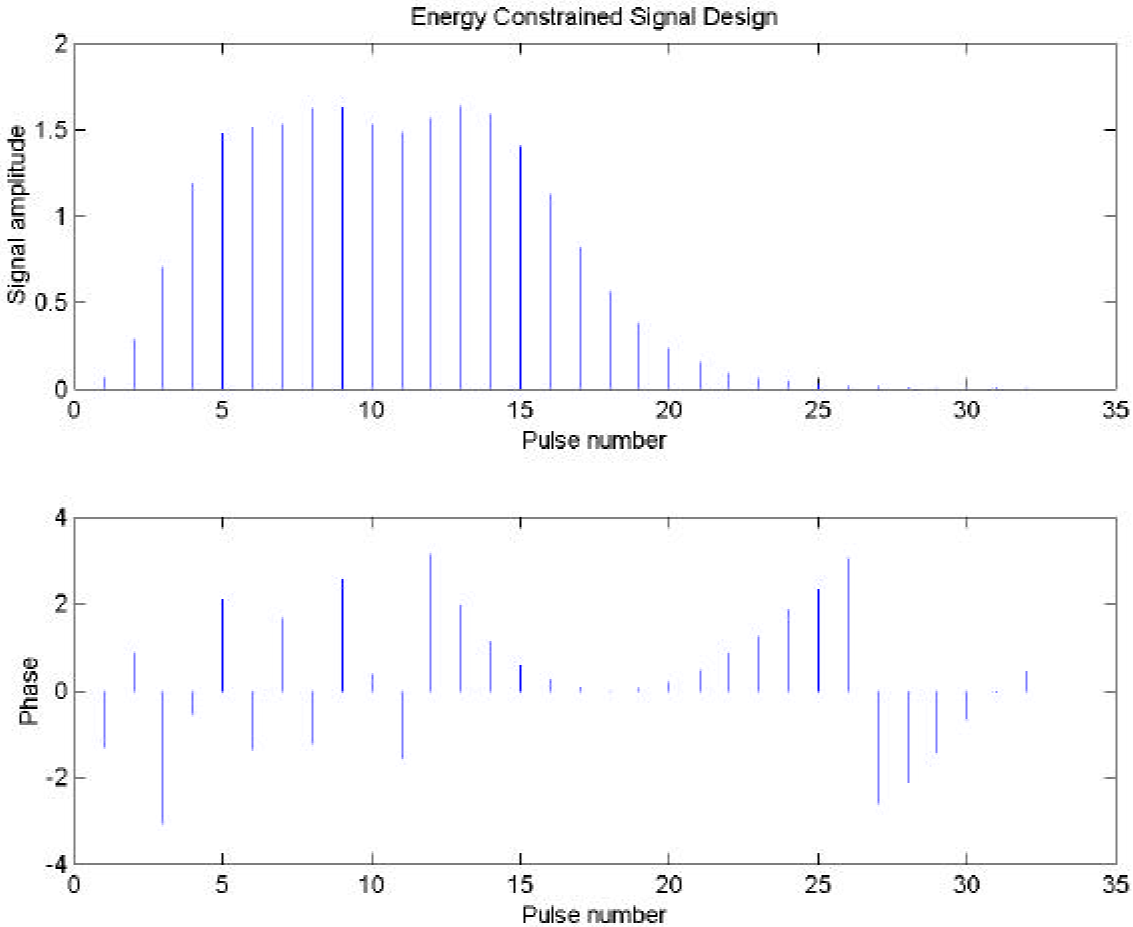

Figure 9 Pulse train at the end of 30 iterations of energy-constrained waveform design with constant amplitude quadratic phase initial burst

4. Experiment 3: Low SNR target

In order to quantify the benefits of waveform adaptation in low SNR scenarios, additional experiments were conducted. Figure 10 shows the SIR values obtained with energy-constrained and amplitude-constrained adaptation when the single-pulse SNR is -5dB, which results in an integrated SIR of 10dB.

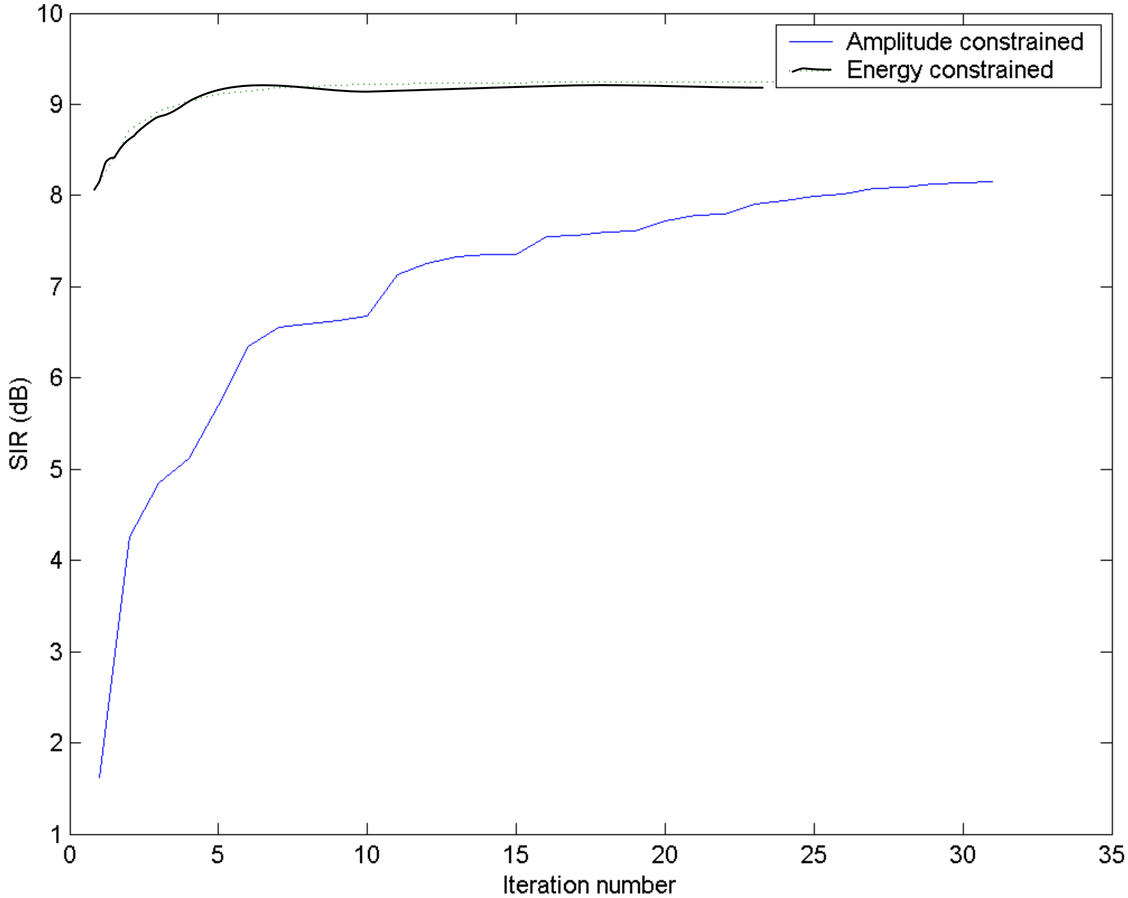

Figure 10 SIR values obtained by amplitude-constrained and energy-constrained waveform design with constant amplitude zero phase initial burst and single-pulse SNR of -5dB.

It can be seen from the figure that even at low SNR values the energy-constrained design approaches the steady-state SIR of 9.1dB (compared with the ideal SIR of 10dB) within 5 iterations. In contrast, the amplitude-constrained design takes longer (as before), but reaches about 7dB within 10 iterations. The SIR value without adaptation is around 1dB.

The waveforms obtained at the end of 30 iterations with the amplitude-constrained design are shown in Figure 11. It can be seen that the amplitude-constrained design is saturated for most of the pulses, which results in lower SIR than with the energy-constrained design. One solution to counter this is to use a higher limit on the amplitude constraint, which is set to 1 at present. This argument is strengthened further by Figure 12, which shows the waveforms obtained with energy-constrained adaptation. Note that in this case all the amplitudes are less than 1.2.

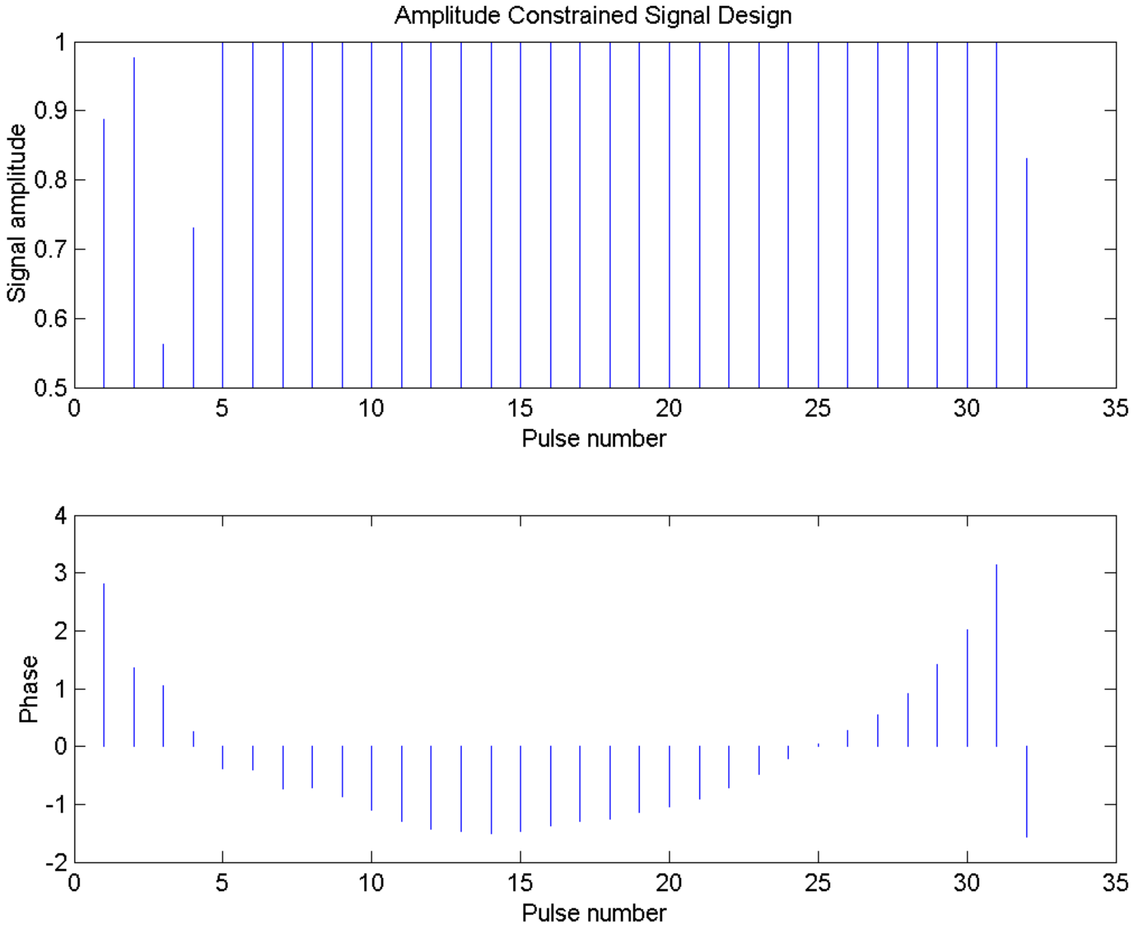

Figure 11 Pulse train at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude zero phase initial burst (single-pulse SNR = -5dB)

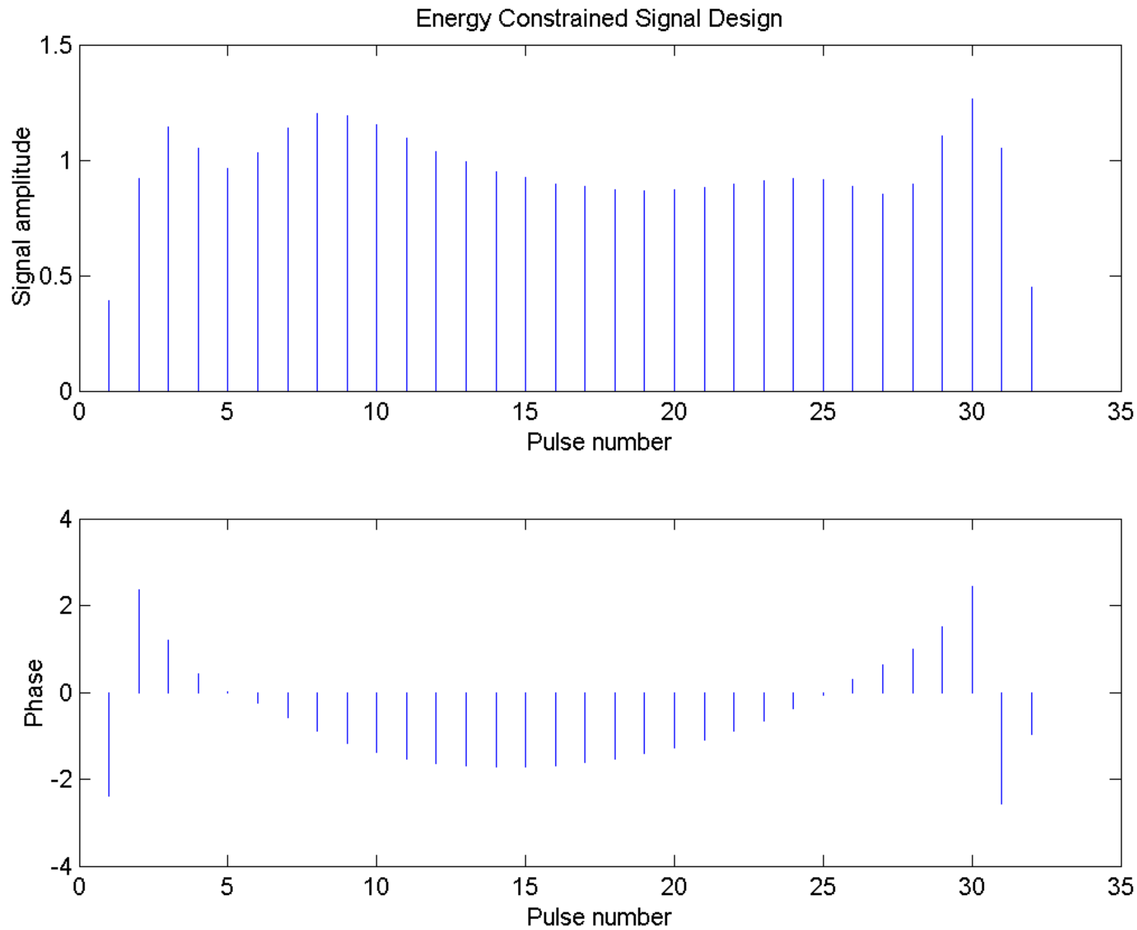

Figure 12 Pulse train at the end of 30 iterations of energy-constrained waveform design with constant amplitude zero phase initial burst (single-pulse SNR = -5dB)

Figure 13 shows the SIR values when the single-pulse SNR is -15dB, which results in an overall SIR of 0dB. Again, the adapted waveforms reach the ideal SIR within a few iterations. In fact, the performance of amplitude-constrained design approaches that of the energy-constrained one. Without adaptation, the SIR values are below -3dB. These results show that adaptation is still beneficial even under very low SNR conditions.

Figure 14 shows the waveforms generated by the amplitude-constrained design. It can be seen that all pulses are saturated in amplitude. This is typically observed under low SNR conditions.

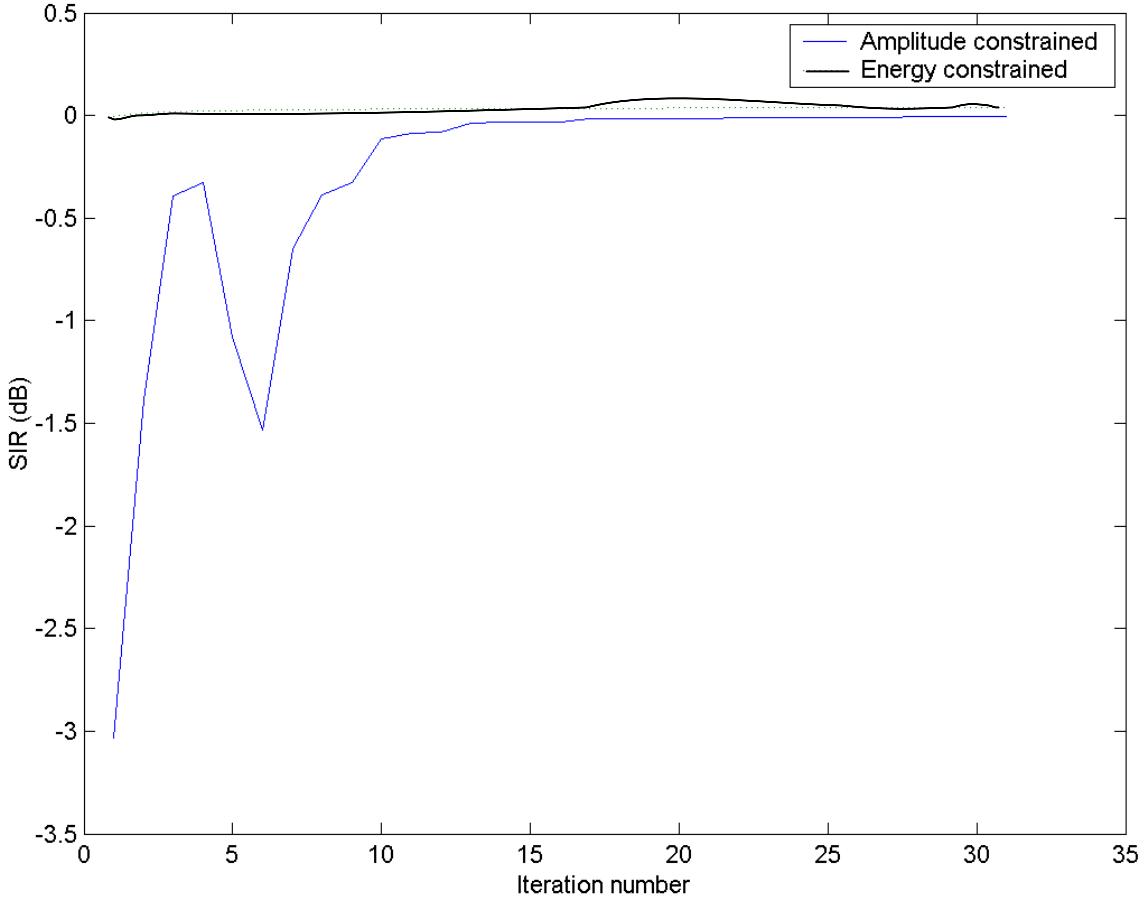

Figure 13 SIR values obtained by amplitude-constrained and energy-constrained waveform design with constant amplitude zero phase initial burst and single-pulse SNR of -15dB.

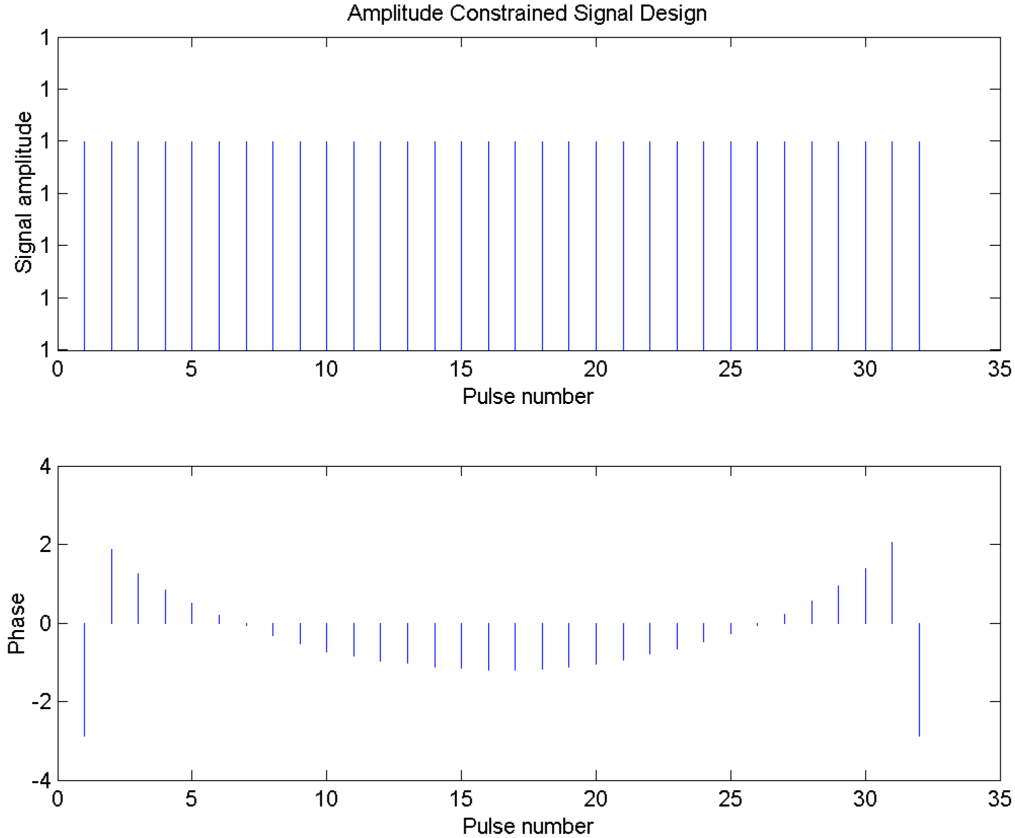

Figure 14 Pulse train at the end of 30 iterations of amplitude-constrained waveform design with constant amplitude zero phase initial burst (single-pulse SNR = -15dB)

5. Experiment 4: Effects of target location

The last experiment was carried out to evaluate the SINR at the output for different target locations. Figure 15 shows the scenario, which is similar to the one shown in Figure 1 except for the location of the target, where the target location in the Doppler-range plane has been moved horizontally so as to lie just outside the clutter distribution. Figure 16 shows the SINR values obtained with amplitude-constrained and energy-constrained waveform designs. It can be seen that the output is the same as the one obtained in Figure 2 with the target being far away from clutter. The reason for this observation is that the clutter matrix calculation in the De Long & Hofstetter method is not affected as long as the *point* target is *outside* the clutter region. Of course, this will not be true if the target is an extended one (i.e., it is not a point target). Another scenario shown in Figure 17 also resulted in an identical plot. In this test scenario, the target is moved downward so as to lie just above the clutter distribution.

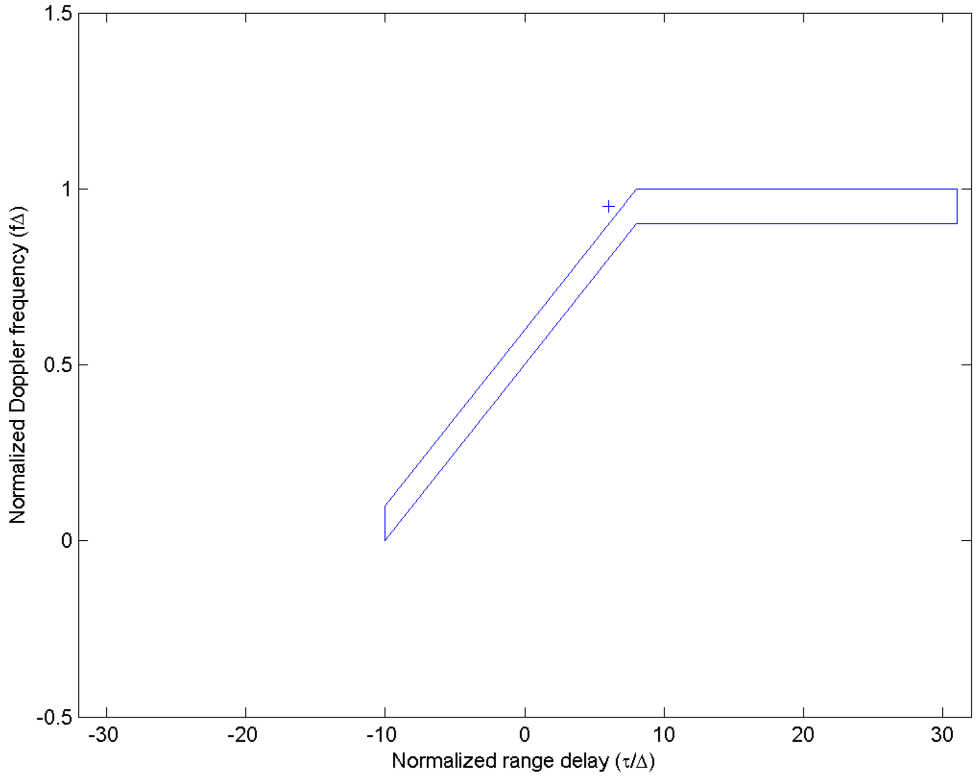

Figure 105 The modified target-clutter environment

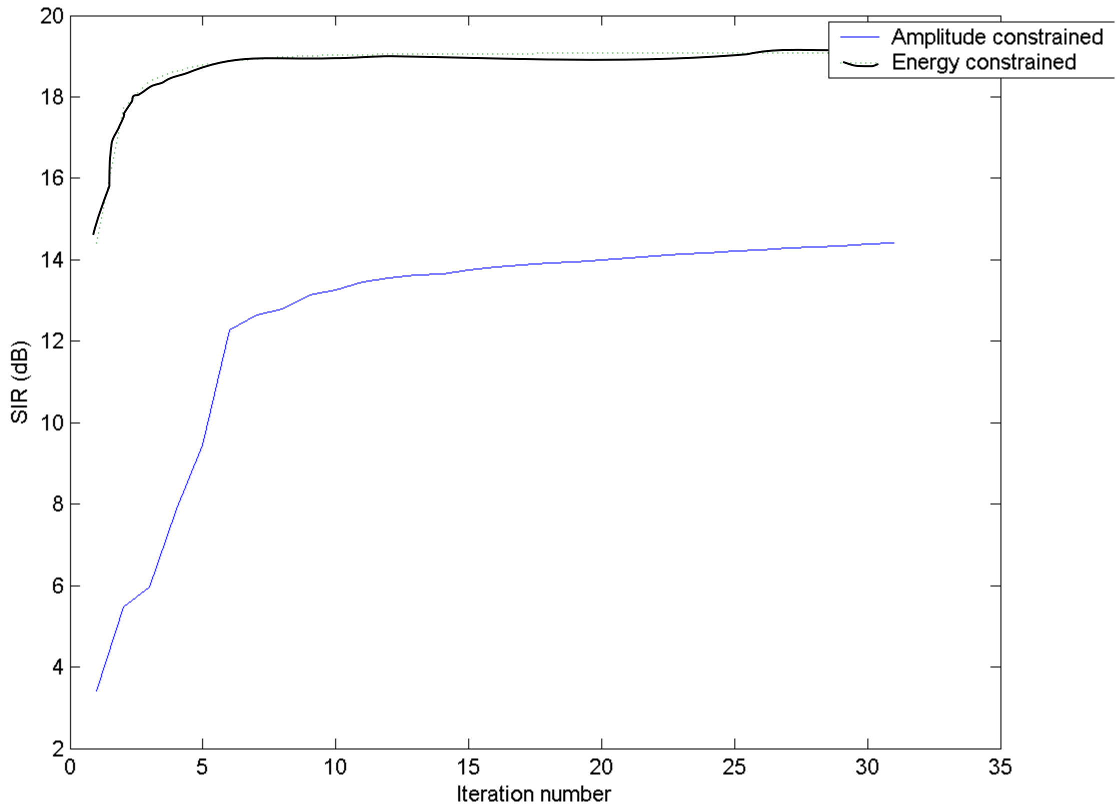

Figure 16 The SINR values obtained with the modified target-clutter environment

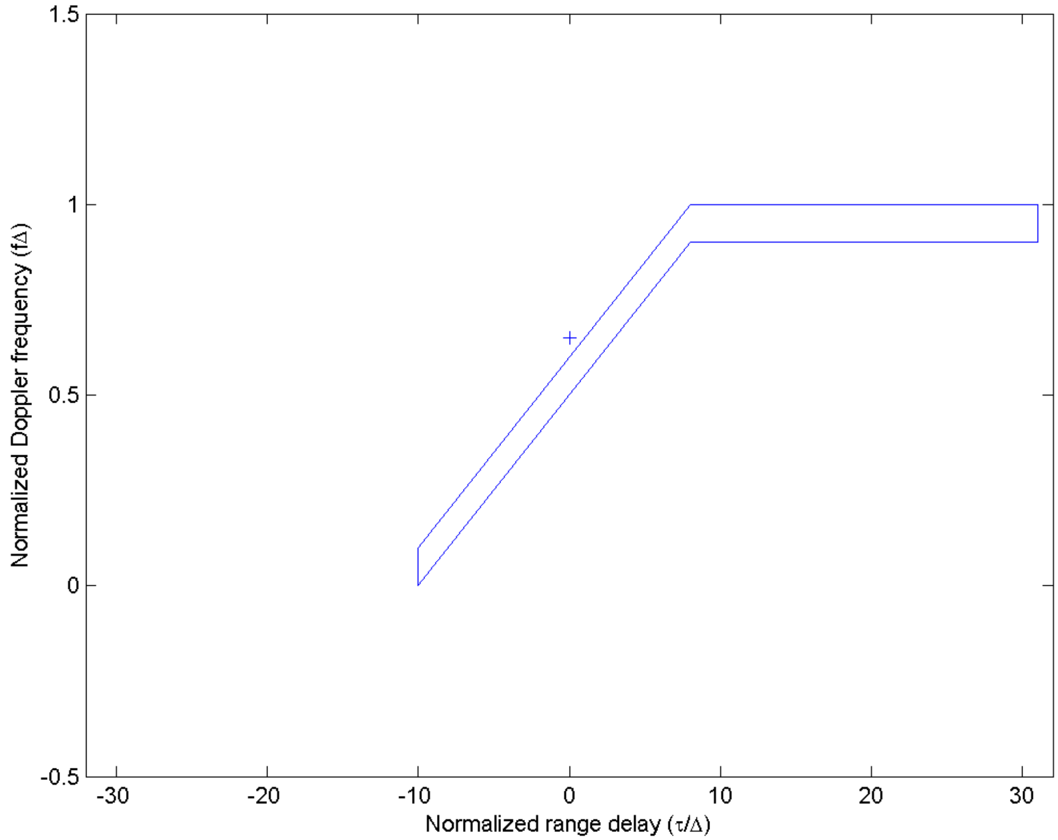

Figure 17 The additional modified target-clutter environment

- -

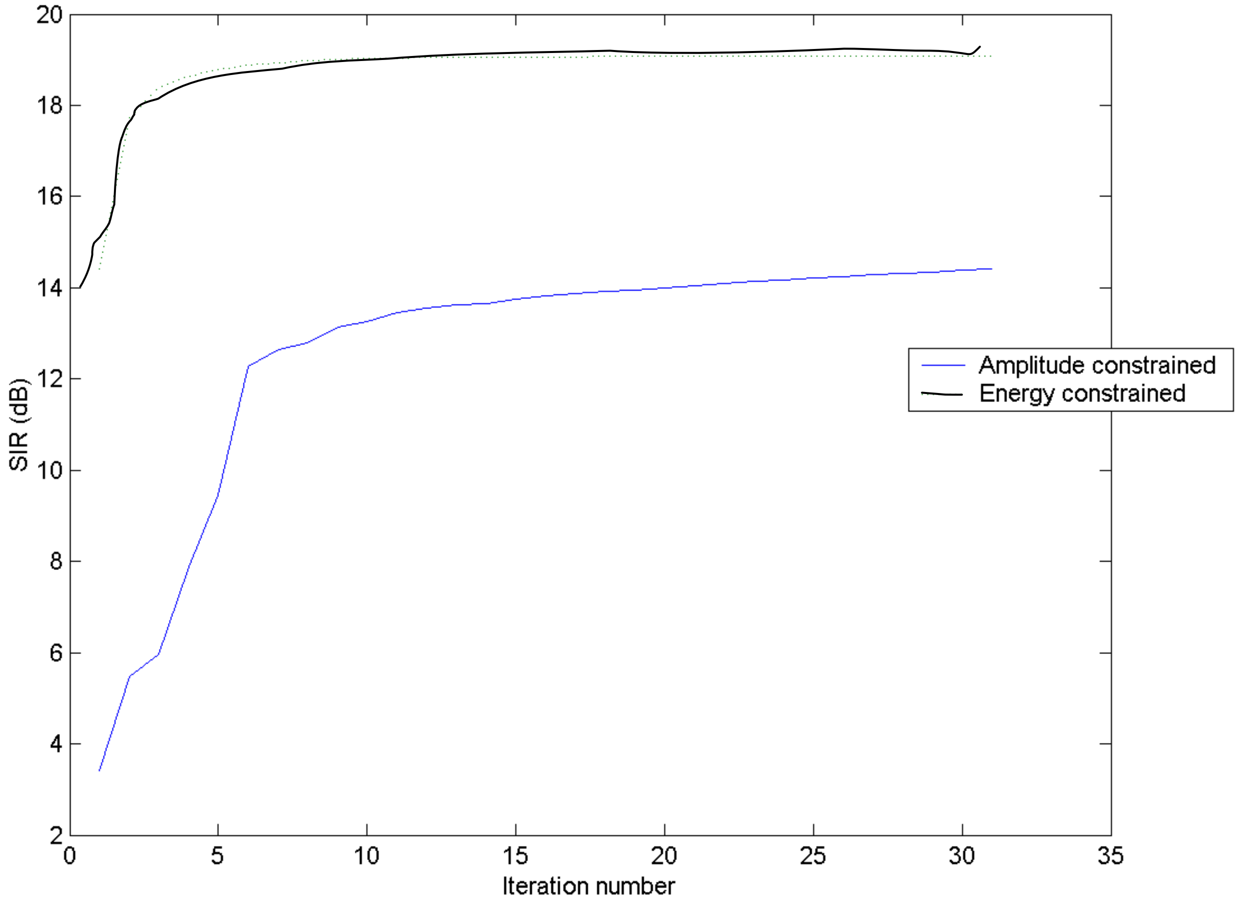

Figure 18 The SINR values obtained with the modified target-clutter environment in Figure 17

6. Benefits of Adaptation

In order to illustrate the benefits of adaptation, the following experiment was conducted: During the first iteration, the optimal waveform was selected for the scenarios corresponding to Experiment 1 and Experiment 2 discussed above. During the subsequent iterations, the waveform selected in the first iteration is used. That is, the waveform is kept constant from second iteration onwards. Figure 19 shows the SIR values obtained with amplitude-constrained and energy constrained waveforms for the scenario described in Experiment 1. Similar to Experiment 1, the energy constrained waveform was selected based on the energy of the last amplitude-constrained waveform. Comparing Figures 2 and 19, the benefit of waveform adaptation is very clear: For the amplitude-constrained waveform, the SIR gain with adaptation is about 10dB and for the energy-constrained waveform it is about 5dB.

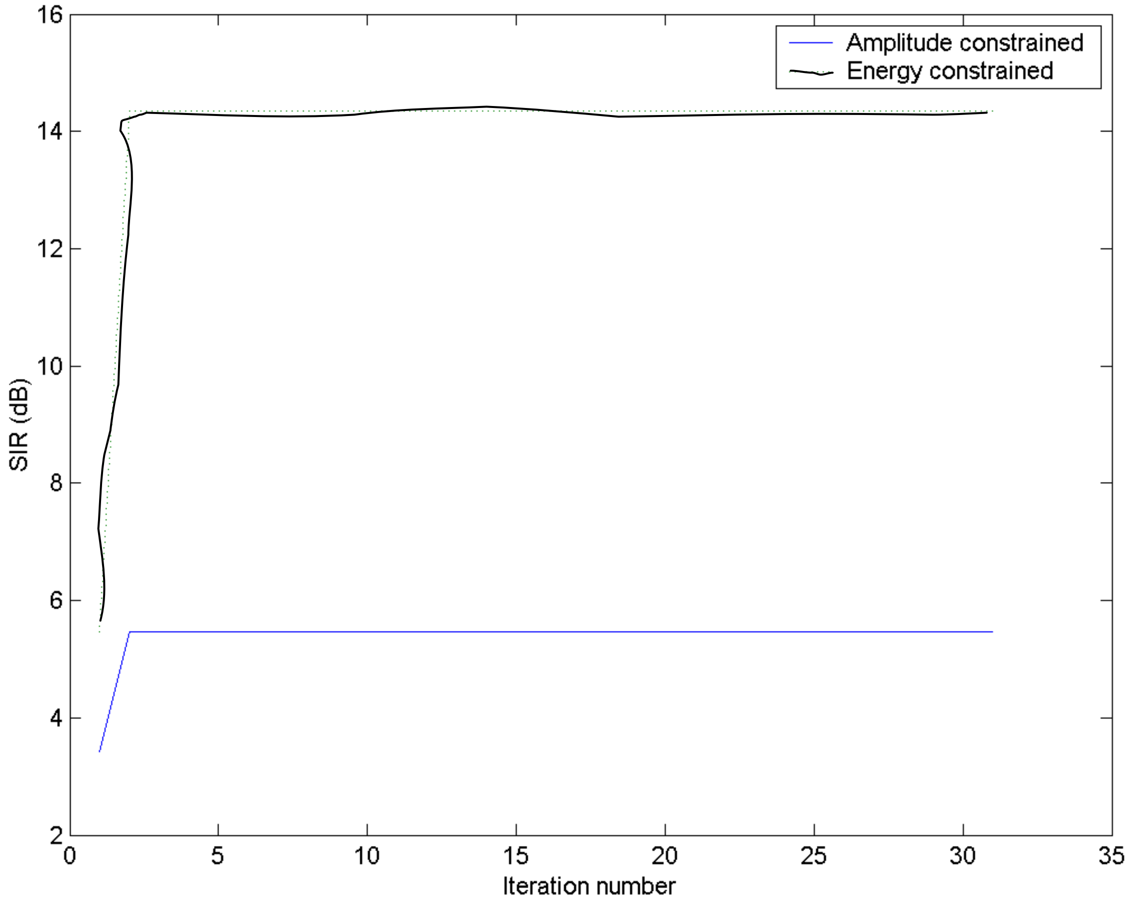

Figure 19 SIR values obtained by amplitude-constrained and energy-constrained waveform design at only iteration 1 with constant amplitude zero phase initial burst

Figure 20 shows the SIR values obtained when the initial signal was constant amplitude quadratic phase burst (similar to Experiment 2). Comparing with Figure 6, it can be seen that the SIR gain with adaptive waveform design is about 3dB and 1dB for amplitude-constrained and energy-constrained waveforms, respectively.

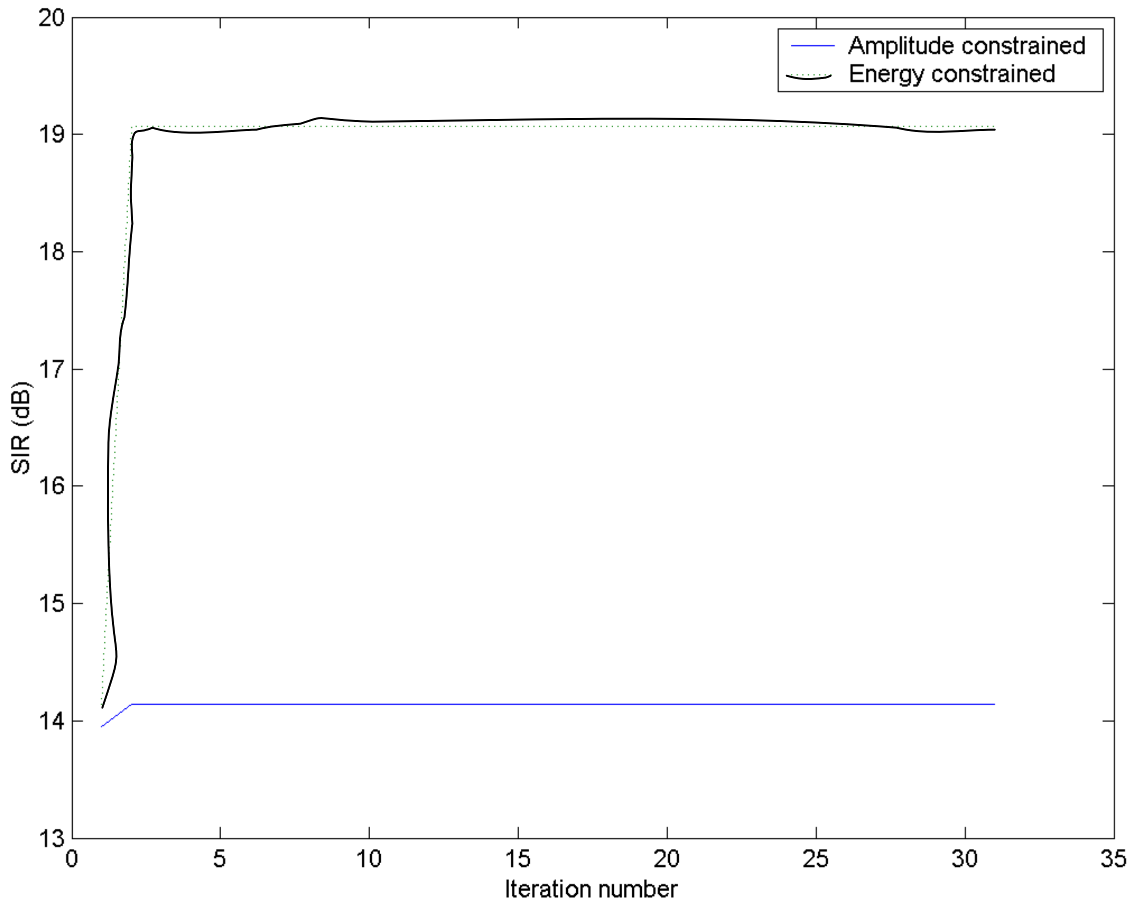

Figure 20 SIR values obtained by amplitude-constrained and energy-constrained waveform design at only iteration 1 with constant amplitude quadratic phase initial burst

Note that the SIR values in Figures 19 and 20 were obtained using optimal waveform design at iteration 1 (although the waveforms were not adaptively updated at subsequent iterations). Selecting the initial waveforms non-optimally (for example, randomly or arbitrarily fixed) resulted in further reduced SIR values.

Another way to quantify the benefits of adaptation is by evaluating the discrete Fourier transform (DFT) coefficients of the received signal. Figures 21 and 22 show the coefficients without and with adaptation, respectively. Note that the low value terms (corresponding to clutter) have lower mean with adaptation, which means a higher SIR. It can be seen that the improvement is around 3dB.

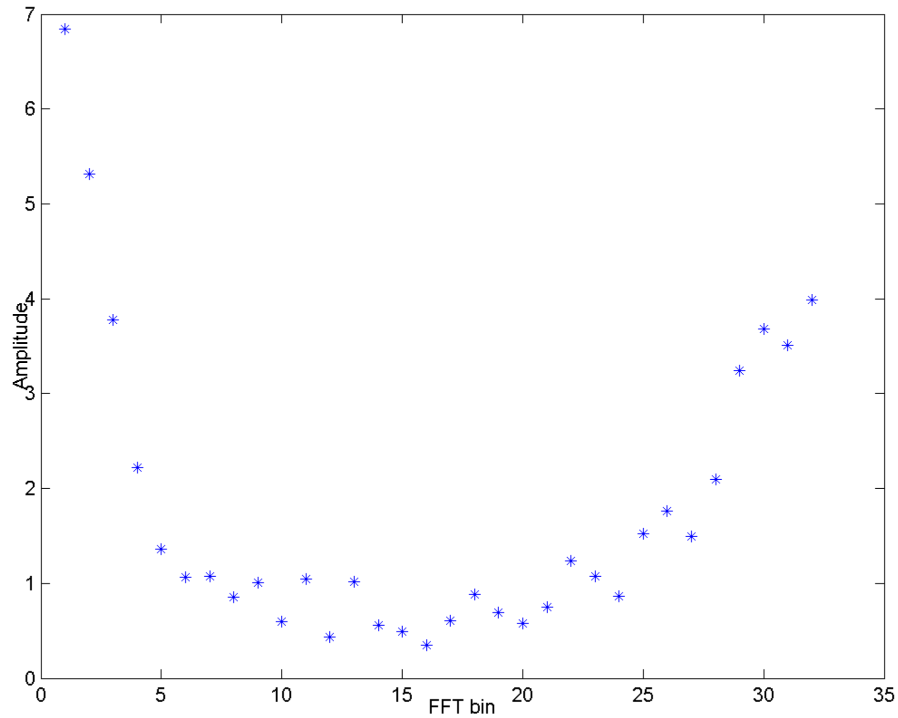

Figure 21 DFT coefficients of the received signal without adaptation

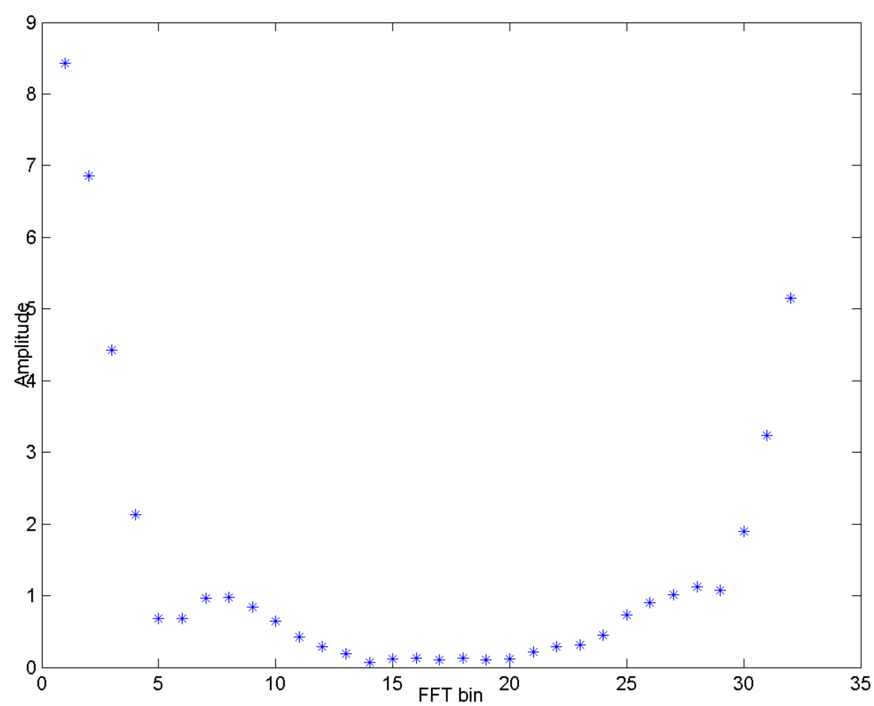

Figure 22 DFT coefficients of the received signal with adaptation

7. Possible Experiments on Adaptive Radar

Based on the simulation results presented herein and the discussions that were held at DRDC, Ottawa, the following items are suggested for further experimentation:

1. The DeLong-Hofstetter algorithm is simple to implement in software or hardware and it can be incorporated into existing radar systems. However, this will require some modification at the transmitter as well as the receiver

2. Real experiments can be conducted on static clutter maps with point targets. In dynamic scenarios, we would have to learn the clutter maps online. In addition, the technique needs to be extended to handle distributed targets or moving targets

3. The adaptation in a real dynamic scenario should include detection, tracking and classification performance. In the current work, we have considered only optimization of the SINR.

8. Summary

The above results show that it is possible to achieve better SIR by the judicious selection of initial burst as well as the subsequent signals. Energy-constrained waveform design yields better SIR as well as better convergence properties, enabling better detection. The results show that it is possible to attain the ideal SIR with energy-constrained waveform design (i.e., nearly 100% clutter suppression). Amplitude-constrained waveform design yields comparable results, but always results in lower SIR and slower rate of convergence. However, in some radar applications, the amplitude constraint is preferred to the energy constraint, given the practical limitations of present-day linear power amplifiers. In addition, the benefit of adaptation was demonstrated by comparing the SIR values obtained with fixed waveforms and adaptively designed waveforms.

With the current simulation program it is possible to experiment with different waveforms and clutter maps and adaptively design improved waveforms for better target detection. The program is written in a modular fashion so that different waveform design techniques can be used within the same framework.

References

DeLong, D.F. and E. M. Hofstetter, 1967, “On the design of optimum radar waveforms for clutter rejection”, IEEE Trans. on Information Theory, IT-13, no. 3, July.

DeLong, D.F. and Hofstetter, 1969, “The design of clutter-resistant radar waveforms with limited dynamic range”, IEEE Trans. on Information Theory, IT-15, no. 5, May.

Appendix 1: Scenario definition file

```
%% simulation parameters

scenario_name = 'DeLongExpr1'

% target clutter environment
% column1: normalized range delay (tau/delta)
% column2: normalized Doppler frequency (f * delta)
target = [       0 0.95];
clutter = [    -10  0
            8  0.9
           31  0.9
           31  1.0
            8  1.0
          -10  0.1];

% number of subpulses in the waveform
N = 32;

% minimum waveform amplitude
a = 0.1;

% target's single pulse SNR in the absence of clutter
rho1 = 10^(5/10);

% one half the ratio of clutter cross section seen by
% waveform to target cross section
lambda = 50;

%% optimization parameters

% step size
epsilon = 0.25;
efactor = 1;

% initial signal amplitude and phase
u=ones(N, 1);

% number of iterations for waveform design
num_iter = 30;

% number of trials for finding optimum step size
max_trial = 10;
```

Appendix 2: List of program files

MATLAB program files:

1. AdaptWave.m – main MATLAB program file (called from MATLAB prompt)
2. scenario1.m, scenario2.m, etc. – sample scenario files (called at the beginning of AdaptWave.m; change this to use different scenario setup files)
3. Optimization routines:
   - findc.m
   - findminmax.m
   - findrhocf.m
   - updatescale.m
   - updatesignal.m
4. Plotting routines:
   - plot_scenario.m
   - plot_results.m
   - plotbars.m
5. Utilities:
   - eyetilde.m

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adaptive transmit radar