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A Kalman Filter Based Registration Approach for Multiple Asynchronous Sensors

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Abstract

In this paper, a Kalman filter based registration approach is proposed for multiple sensors with asynchronous measurements. A time-variant linear sensor measurement model is obtained using a first-order approximation. The observability analysis is carried out and it is shown the system is *uniformly completely observable*. A modified measurement model is formulated which includes the asynchronous sensor models and the two-stage Kalman estimator is applied to estimate the target states along with the sensor bias errors. The proposed method can be implemented using a parallel structure and is computationally efficient. Simulation and real-life sensor data are used to demonstrate the effectiveness of the proposed approach. The results are compared with other existing registration techniques.

Résumé

Dans le présent document, nous proposons une méthode d'enregistrement à filtre de Kalman pour les radars multiples à mesures asynchrones. Un modèle de mesures de capteurs linéaires variables dans le temps est obtenu par une approximation de premier ordre. L'analyse d'observabilité a été effectuée et a démontré que le système est entièrement et uniformément observable. Un modèle de mesures modifié, comprenant les modèles de capteurs asynchrones, est établi, et l'estimateur Kalman à deux étapes est appliqué pour l'estimation des états de cibles et aussi des erreurs systématiques des capteurs. La méthode proposée peut être appliquée à l'aide d'une structure parallèle et assure un traitement efficace. Les simulations et les données de capteurs en situation réelle sont utilisées faun de démontrer l'efficacité de la méthode proposée. Les résultats sont comparés à d'autres techniques d'enregistrement existantes.

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Executive summary

Background: Data fusion has been defined as a process of dealing with association, correlation and combination of data and information from single or multiple sources to achieve refined position and identity estimates. Sensor registration is a part of the level one processing of data fusion which includes association, filtering and identifications. It refers to the process of ensuring the requisite error free coordinate conversion of multiple sensor data, and is a prerequisite process for a data fusion system to accurately estimate and correct systematic errors. In multisensor integration, data from each sensor are transformed into a common reference system for merging. Direct transformation of data usually leads to limited success due to the failure to register adequately The effect of sensor registration errors is to introduce biases into fusion, generating ghost targets for multisensor signal processing. When the sensor registration errors are large, it was found that the integrated performance can be even worse than that of a single sensor application.

Various registration algorithms have been proposed in recent years. However, there are two problems that have not been addressed by most of the algorithms: system observability and asynchronous sensor measurements. System observability plays an important role in system analysis and parameter estimation. It relates to the amount of information contained in the measurement about the state of the system, and decides whether the system state is recoverable from the measurements. It also determines the optimality of the Kalman filtering. In other words, if the system is not observable, its state cannot be completely determined from the measurements, and additional or alternate measurements should be considered, or the system dynamics be reformulated. The other problem of concern is that the sensors may be asynchronous. Most of existing algorithms are based on the assumption that the sensors are synchronized, an assumption that may not be true in practical application, especially when legacy sensors are considered in a fusion system. In reality, sensors often operate asynchronously and deriving a common reference time for the measurement is often difficult. For example, sensor clocks may not be synchronized and they may have time offset between one another. In addition, the scanning process of the radars may not be synchronized and have different rate. For ESM sensors, the track report output depends totally on the illumination timing of the emitters. When sensors are deployed at different locations relative to the emitters, the reports from different sensors are not time-aligned and indeed asynchronous.

Results: A Kalman filter based registration approach is proposed for multiple asynchronous sensors. Constant range and azimuth errors are considered for the sensors. Observability analysis is carried out and it is shown that the system is *uniformly completely observable*. A measurement model is formulated which includes different sensors and their registration errors. The two-stage Kalman estimator is applied to estimate the sensor registration errors along with the target state. The proposed two-stage Kalman approach is computationally efficient and can be implemented using a parallel and distributed structure. It is robust and suitable for real time applications. Simulated and real multiple radar data are used to demonstrate the performance of the proposed approach. Comparisons are made with the popular LS registration method. It is shown that the proposed approach outperforms the LS method in terms of registration performance and the ability to handle different track patterns. When using real-life radar data, the Kalman filter based approach has also demonstrated a better generalization ability than the LS method.

Significance: The proposed sensor registration approach is able to handle asynchronous sensor measurements. This ability is important not only for multiple radars registration, but also for multiple ESM/radar multiple ESM registration applications where the sensor measurements are always asynchronous. Unlike most existing registration algorithms, the proposed approach does not require any computationally sophisticated time alignment procedures and is suitable for real-time applications. In addition, the algorithm can be implemented using a parallel and distributed structure, making it computationally robust and efficient. The mathematical proof of the system observability provided an insight understanding of the sensor registration problem and demonstrated the optimality of the proposed registration approach.

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Sommaire

Contexte : La fusion de données de base a été définie comme un procédé d'association, de corrélation et de combinaison de données et d'informations provenant de sources uniques ou multiples pour effectuer des estimations de position et d'identité perfectionnées. L'enregistrement de capteur fait partie de la première étape du procédé de fusion de données, qui comprend l'association, le filtrage et l'identification. Il fait appel au procédé de conversion requise de coordonnées sans erreur des données de capteurs multiples, et il est un procédé prérequis pour que le système de fusion de données estime et corrige précisément les erreurs systématiques. Dans l'intégration multicapteur, les données de chaque capteur sont transformées en un système de référence commun pour la fusion. La transformation directe des données ne donne généralement pas de bons résultats étant donné l'enregistrement inadéquat. Les erreurs d'enregistrement ont pour effet d'introduire des erreurs systématiques dans la fusion, ce qui génère des cibles fantômes au niveau du traitement multicapteur. Lorsque les erreurs d'enregistrement sont importantes, on a découvert que le rendement intégré peut même être pire que celui de l'application à un seul capteur.

Différents algorithmes d'enregistrement ont été proposés durant les dernières années, toutefois, la plupart ne tiennent pas compte de deux problèmes : l'observabilité du système et les mesures de capteurs asynchrones. L'observabilité du système joue un rôle important dans l'analyse et l'estimation des paramètres de système. Elle fait appel à la quantité d'informations contenues dans les mesures relatives à l'état du système et décide si l'état du système est récupérable à partir de celles-ci. Elle détermine également l'optimalité du filtrage de Kalman. En d'autres mots, si le système n'est pas observable, son état ne peut pas être complètement déterminé à partir des mesures, alors des mesures différentes ou supplémentaires doivent être considérées, ou la dynamique du système doit être reformulée. Le second problème est que les capteurs peuvent être asynchrones. La plupart des algorithmes existants sont basés sur l'hypothèse que les capteurs sont synchrones, hypothèse qui peut être fausse dans les applications pratiques, particulièrement lorsque les capteurs existants sont considérés dans un système de fusion. En réalité, les capteurs fonctionnent souvent de façon asynchrone et il est souvent difficile d'établir un temps de référence commun pour les mesures. Par exemple, les horloges du capteur peuvent ne pas être synchronisées et elles peuvent présenter un décalage dans le temps l'une par rapport à l'autre. De plus, le procédé de balayage des radars peut ne pas être synchronisé et peut présenter différentes vitesses. Pour les capteurs MSE, la sortie de compte rendu de tracé dépend entièrement de la synchronisation de l'illumination des émetteurs. Lorsque des capteurs sont déployés à différents emplacements relativement aux émetteurs, les rapports des différents capteurs ne sont pas alignés dans le temps et sont donc asynchrones.

Résultats : Une méthode d'enregistrement à filtre de Kalman est proposée pour les capteurs multiples asynchrones. Les erreurs de portée et d'azimut sont prises en considération pour les capteurs. L'analyse de l'observabilité a été effectuée et elle a démontré que le système est entièrement observable de façon uniforme. Un modèle de mesure comprenant les différents capteurs et leurs erreurs d'enregistrement est établi. L'estimateur de Kalman à deux étapes est appliqué pour l'estimation des erreurs d'enregistrement des capteurs et de l'état de la cible. La méthode proposée de Kalman en deux étapes est exécutée par ordinateur et est mise en application à l'aide d'une structure parallèle distribuée. Elle est robuste et elle convient aux applications en temps réel. Des données simulées et réelles de radars multiples ont servi à démontrer la performance de la méthode proposée. Des comparaisons ont été établies avec la populaire méthode d'enregistrement LS. Il a été démontré que la méthode proposée surpasse la méthode LS en termes de rendement d'enregistrement et de capacité à traiter différentes configurations de tracés. Lors de l'utilisation de données radar en situation réelle, la méthode d'enregistrement à filtre de Kalman a également démontré une plus grande capacité de généralisation que la méthode LS.

Importance : La méthode d'enregistrement de capteurs proposée peut traiter les mesures de capteurs asynchrones. Cette capacité est importante non seulement pour l'enregistrement des radars multiples, mais aussi pour les applications d'enregistrement MSE multiple/radar MSE multiple où les mesures de capteurs sont toujours asynchrones. Contrairement à la plupart des algorithmes existants, la méthode proposée ne nécessite pas de procédure de synchronisation à traitement complexe et elle convient aux applications en temps réel. De plus, l'algorithme peut être appliqué à l'aide d'une structure parallèle distribuée, ce qui le rend robuste et efficace. La preuve mathématique de l'observabilité du système offre une meilleure compréhension du problème d'enregistrement de capteur et démontre l'optimalité de la méthode d'enregistrement proposée.

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1. INTRODUCTION

Data fusion has been defined as a process of dealing with association, correlation and combination of data and information from single or multiple sources to achieve refined position and identity estimates [1]. Sensor registration is a part of the level one processing of data fusion which includes association, filtering and identifications. It refers to the process of ensuring the requisite error free coordinate conversion of multiple sensor data, and is a prerequisite process for a data fusion system to accurately estimate and correct systematic errors. In multisensor integration, data from each sensor are transformed into a common reference system for merging. Direct transformation of data usually leads to limited success due to the failure to register adequately by the individual sensors. Major source of registration errors includes: 1) sensor position and alignment errors; 2) sensor biases including the azimuth bias error with respect to a common reference and the range offset errors of each sensor; 3) timing errors, i.e., the clocks of each sensor may not be calibrated or synchronized and 4) coordinate conversion errors, errors introduce when transforming sensor measurements from local coordinates to global one. The effect of sensor registration errors is to introduce biases into fusion, generating ghost targets for multisensor signal processing [2]. It was found that, if the sensor registration errors are large, the integrated performance can be even worse than that of a single sensor application.

Various registration algorithms have been proposed in recent years. In general, there are two types of registration algorithms; track-independent and -dependent algorithms. In the track-independent algorithms [2][3][4][5][6], the sensor measurements are treated as plots and no correlations among the measurements are explored. They use discrepancies of measurements from different sensors in the common coordinate system to estimate the registration errors. These algorithms include the least squares (LS) method [7][4][6], the generalized linear least squares estimation (GLSE) techniques [2], the maximum likelihood (ML) estimator [3] and the approach based on the application of the Kalman filter [8]. The track-dependent algorithms [10][7][9], on the other hand, use different track models for target motion dynamics, and apply the Kalman filter technique to estimate the target state as well as sensor registration errors. They often adopt the augmented Kalman filter methodology by appending the sensor registration errors to the target state vector. The augmented Kalman filter may encounter difficulties in practical situations due to the prohibiting computational requirement. In [9], Blom *et al.* proposed an approach in which the augmented Kalman estimator is decoupled into separate filters: maintenance Kalman filters for the target states, and a Kalman-like filter for the sensor registration errors. By decoupling the augmented Kalman filter into two filters of smaller dimensions, the computation can be reduced significantly. Another commonly used approach [10][7] is the application of the two-stage Kalman estimator proposed by Friedland [11] and further developed by Ignagni [12]. The two-stage Kalman estimator is able to decouple the estimation of the target state and the bias parameters, leading to a substantial reduction in computation complexity. Its structure is suitable for parallel implementations. In general, the track-independent algorithms are superior to the track-dependent approaches in terms of robustness because they don't rely on any assumptions about the track data. On the other hand, the track-dependent algorithms can produce more accurate registration error estimates because they tend to explore the underlying correlations among track measurements. However, the performance of the track-dependent algorithms rely on the correct modeling of the target motion, and may deteriorate when inappropriate models are used.

However, there are two problems that were not addressed by the algorithms mentioned in the above: system observability and asynchronous sensor measurements. System observability plays an important role in system analysis and parameter estimation. It relates to the amount of information contained in the measurement about the state of the system, and decides whether the system state is recoverable from the measurements. It also determines the optimality of the Kalman filtering. In other words, if the system is not observable, its state cannot be completely determined from the measurements, and additional or alternate measurements should be considered, or the system dynamics be reformulated. In the past, few algorithms have considered the observability analysis. They often assume that the system is observable by default, an assumption that may not be always valid. In [4], a simple observability analysis was carried for the range and angular biases. In [6], the authors have observed that the performance of the LS registration method depends on the track pattern, or the distribution of the sensor measurements in the system coordinates relative to the sensor sites. Certain track patterns have caused the data matrix to be ill-conditioned, making the algorithm totally ineffective. This phenomenon is indeed closely related the observability of the sensor measurement model. An approach based on the singular value decomposition (SVD) has been proposed to deal with the ill-conditioned data matrix problem. But, the underlying cause of this ill-conditioning has not been further investigated.

The second problem of concern is that the sensors may be asynchronous. Most of the previous mentioned algorithms are based on the assumption that the sensors are synchronized. This assumption may not be true in practical application, especially when legacy sensors are considered in a fusion system. In reality, sensors often operate asynchronously and deriving a common reference time for the measurement is often difficult. For example, radar clocks may not be synchronized and they may have time offset between one another. In addition, the scanning process of the radars may not be synchronized and have different rates. One approach to handle the problem is to use smoothing or interpolation techniques to translate the measurements of all the sensors to the times of a selected sensor. In [13], Helmick *et al.* used the Interactive Multiple Mode (IMM) filtering algorithms for local tracking and time-translating. In the approach, a one-step fixed-lag smoothing algorithm is used to translate the estimates from one sensor to the times of the other. The time-translated estimates are then passed to the alignment algorithm for sensor alignment error estimation. However, the time-translating approach may have limited applications since it has ignored the coupling between the sensor registration errors and the target dynamical states in the local tracking process.

In this paper, a Kalman filter based registration approach is proposed for asynchronous sensor measurements. Constant range and azimuth errors are considered for the sensors. The sensors are assumed to be asynchronous and have different update rates. The near constant velocity model is used to describe the target motion dynamics. At each sensor, a linear time-varying measurement model is obtained by using a first-order approximation. Observability analysis is carried out and it is shown that the linear time-varying system is *uniformly completely observable*. A measurement model is formulated which includes different sensors and their registration errors. The two-stage Kalman estimator is applied to estimate the sensor registration errors along with the target state. At each sensor site, a bias-free estimate of the target state is computed using a local Kalman filter when a sensor measurement arrives, and the sensor registration errors are estimated by another Kalman filter cycle based on the measurement residual of the bias-free target state estimate. The *a posteriori* estimates of the bias-free target states and the sensor registration errors as well as their corresponding covariance matrices are propagated to the sensor site where the next measurement is received. The algorithm iterates between the sensors to provide the registration error estimates as time evolves. The two-stage Kalman filter is equivalent to but computationally more efficient than the augmented Kalman estimator because it involves state vectors of smaller dimensions. Since the proposed two-stage Kalman approach can be implemented in a parallel and distributed way, it is robust and suitable for real time applications. In this study, the orientation and location errors in the reference frame of the sensors are not considered and it is assumed that the sensors have zero time offset errors to the common electronic time reference. The paper is organized as follows. In Section 2, the near constant velocity model is discussed for target motion dynamics. The linearized sensor measurement model is formulated based on a first-order approximation. Section 3 is devoted to system observability analysis. The application of the two-stage Kalman estimator for sensor registration is developed in Section 4. Some stability properties are also discussed in this section. In Section 5, simulated and real multiple radar data are used to evaluate the performance of the proposed approach. Comparisons are made with the popular LS registration method.

2. PROBLEM FORMULATION

Consider two sensors A and B in a common plane. Without loss of generality, we assume that sensor A is located at the origin of the system coordinate and sensor B at coordinates (u, v). Assume that sensors A and B measure the range and azimuth of a target.

2.1 Target Motion Dynamics

The standard near constant velocity model is used for target motion

(1) $$\underline{x}(k+1) = \Phi(k+1,k)\underline{x}(k) + \Gamma(k+1,k)\nu(k), \quad k = 1,2,3,\ldots,$$

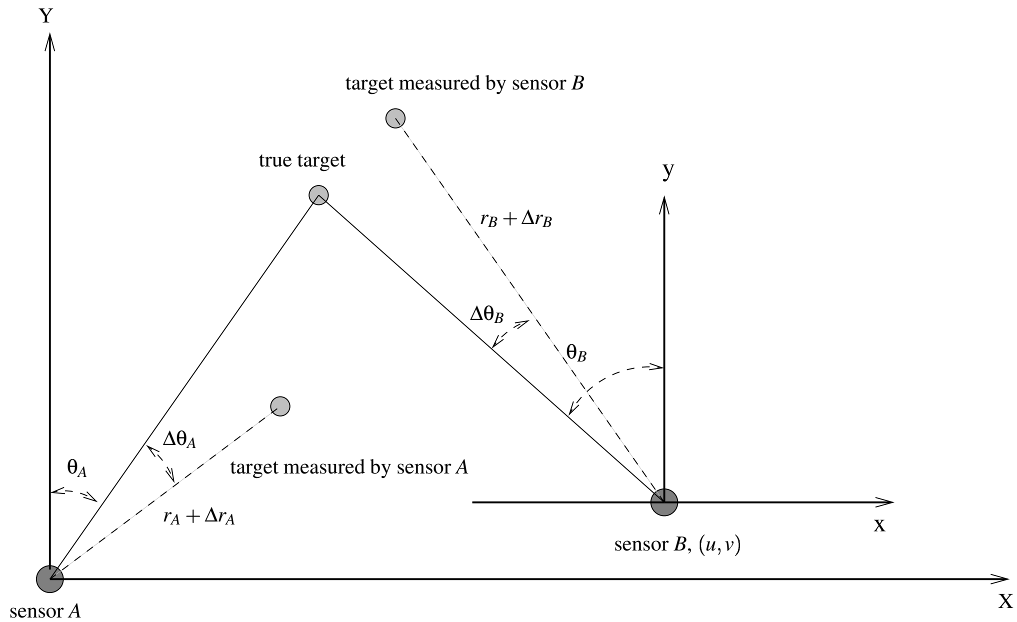

Figure 1: *The geometry of registration errors.*

where $\underline{x}(k) = [x(k), v_x(k), y(k), v_y(k)]^T$ denotes the state vector at the kth time instance where $x(k)$ and $y(k)$ are the actual target coordinates in the system plane, and $v_x(k)$ and $v_y(k)$ denote the speed in x and y coordinate, respectively; $\Phi(k+1,k)$ and Γ are the state and noise transition matrix, respectively, given by

$$(2)\,\Phi(k+1,k) = \begin{bmatrix} 1 & \Delta t_{k+1} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t_{k+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \Gamma(k+1,k) = \begin{bmatrix} \Delta t_{k+1}^2/2 & 0 \\ \Delta t_{k+1} & 0 \\ 0 & \Delta t_{k+1}^2/2 \\ 0 & \Delta t_{k+1} \end{bmatrix},$$

where Δt_{k+1} denotes the difference between the $(k+1)$th and the kth time instances. The process noise $\underline{\nu}(k)$ is assumed to be an independent, identically distributed (*i.i.d*) Gaussian process with zero-mean and covariance $R_\nu = \text{diag}[\sigma_{\nu x}^2, \sigma_{\nu y}^2]$, where diag denotes a diagonal matrix.

2.2 Sensor measurement model

The sensor registration geometry is shown in Figure 1. Let $\{\bar{r}_A(k), \bar{\theta}_A(k)\}$ and $\{\bar{r}_B(k), \bar{\theta}_B(k)\}$ denote the polar coordinates of the target at the kth time instance relative to sensors A and B, respectively. We use $\{\Delta r_A, \Delta\theta_A\}$ and $\{\Delta r_B, \Delta\theta_B\}$ to denote the range and azimuth biases of sensor A and B, respectively. When using $\{x_A(k), y_A(k)\}$

and $\{x_B(k), y_B(k)\}$ to denote the Cartesian coordinates of the target reported by sensors A and B, respectively, in the system plane, they can be written as

$$
\begin{aligned}
x_A(k) &= [\bar{r}_A(k) + \Delta r_A + w_{rA}(k)]\sin[\bar{\theta}_A(k) + \Delta\theta_A + w_{\theta A}(k)] \\
y_A(k) &= [\bar{r}_A(k) + \Delta r_A + w_{rA}(k)]\cos[\bar{\theta}_A(k) + \Delta\theta_A + w_{\theta A}(k)] \\
x_B(k) &= [\bar{r}_B(k) + \Delta r_B + w_{rB}(k)]\sin[\bar{\theta}_B(k) + \Delta\theta_B + w_{\theta B}(k)] + u \\
y_B(k) &= [\bar{r}_B(k) + \Delta r_B + w_{rB}(k)]\cos[\bar{\theta}_B(k) + \Delta\theta_B + w_{\theta B}(k)] + v,
\end{aligned} \tag{3}
$$

where $\{w_{rA}(k), w_{\theta A}(k)\}$, $\{w_{rB}(k), w_{\theta rB}(k)\}$ are the range and azimuth measurement noise of sensor A and B, respectively. They are assumed to be *i.i.d* Gaussian processes with zero-mean and variances σ^2_{rA}. $\sigma^2_{\theta A}$, σ^2_{rB} and $\sigma^2_{\theta B}$, respectively. Let $\{r_A(k), \theta_A(k)\}$ and $\{r_B(k), \theta_B(k)\}$ denote the polar coordinates of the target reported by sensor A and B, respectively. In the system coordinate, the sensor measurements can be approximated by

$$
\begin{aligned}
x_A(k) &= x(k) + \sin\theta_A(k)\Delta r_A + r_A(k)\cos\theta_A(k)\Delta\theta_A + \sin\theta_A(k)w_{rA}(k) + r_A(k)\cos\theta_A(k)w_{\theta A}(k) \\
y_A(k) &= y(k) + \cos\theta_A(k)\Delta r_A - r_A(k)\sin\theta_A(k)\Delta\theta_A + \cos\theta_A(k)w_{rA}(k) - r_A(k)\cos\theta_A(k)w_{\theta A}(k) \\
x_B(k) &= x(k) + \sin\theta_B(k)\Delta r_B + r_B(k)\cos\theta_B(k)\Delta\theta_B + \sin\theta_B(k)w_{rB}(k) + r_B(k)\cos\theta_B(k)w_{\theta B}(k) \\
y_B(k) &= y(k) + \cos\theta_B(k)\Delta r_B - r_B(k)\sin\theta_B(k)\Delta\theta_B + \cos\theta_B(k)w_{rB}(k) - r_B(k)\cos\theta_B(k)w_{\theta B}(k),
\end{aligned}
$$

where

$$
\begin{aligned}
x(k) &= \bar{r}_A(k)\sin\bar{\theta}_A(k) = \bar{r}_B(k)\sin\bar{\theta}_B(k) + u \\
y(k) &= \bar{r}_A(k)\cos\bar{\theta}_A(k) = \bar{r}_B(k)\cos\bar{\theta}_B(k) + v.
\end{aligned} \tag{4}
$$

The approximation is to the first order of sensor registration errors and noise terms. Define

$$
\Sigma_A(k) = \begin{bmatrix} \sin\theta_A(k) & r_A(k)\cos\theta_A(k) \\ \cos\theta_A(k) & -r_A(k)\sin\theta_A(k) \end{bmatrix}, \quad \Sigma_B(k) = \begin{bmatrix} \sin\theta_B(k) & r_B(k)\cos\theta_B(k) \\ \cos\theta_B(k) & -r_B(k)\sin\theta_B(k) \end{bmatrix} \tag{5}
$$

and

$$
L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{6}
$$

In matrix form, the above approximate measurement model can be written as

$$\underline{z}_A(k) = L\underline{x}(k) + \Sigma_A(k)\underline{\delta}_A + \Sigma_A(k)\underline{w}_A(k) \quad (7)$$

$$\underline{z}_B(k) = L\underline{x}(k) + \Sigma_B(k)\underline{\delta}_B + \Sigma_B(k)\underline{w}_B(k), \quad (8)$$

where

$$\begin{aligned} \underline{z}_A(k) &= [x_A(k), y_A(k)]^T \\ \underline{z}_B(k) &= [x_B(k), y_B(k)]^T \\ \underline{w}_A(k) &= [w_{rA}(k), w_{\theta A}(k)]^T \\ \underline{w}_B(k) &= [w_{rB}(k), w_{\theta B}(k)]^T \\ \underline{\delta}_A &= [\Delta r_A, \Delta\theta_A]^T \\ \underline{\delta}_B &= [\Delta r_B, \Delta\theta_B]^T. \end{aligned} \quad (9)$$

Define the augmented state vector by appending the sensor biases to the target state vector

$$\begin{aligned} \underline{\eta}_A(k) &= [x(k), v_x(k), y(k), v_y(k), \Delta r_A, \Delta\theta_A]^T \\ \underline{\eta}_B(k) &= [x(k), v_x(k), y(k), v_y(k), \Delta r_B, \Delta\theta_B]^T. \end{aligned} \quad (10)$$

Then, the measurement model can also be written in terms of the the augmented state vector as

$$\underline{z}_A(k) = H_A(k)\underline{\eta}_A + \Sigma_A(k)\underline{w}_A(k) \quad (11)$$

$$\underline{z}_B(k) = H_B(k)\underline{\eta}_B + \Sigma_B(k)\underline{w}_B(k), \quad (12)$$

where

$$H_A(k) = [L \mid \Sigma_A(k)], \;\; H_B(k) = [L \mid \Sigma_B(k)]. \quad (13)$$

The dynamical equation for the augmented state vector can be written as

$$\underline{\eta}_A(k+1) = \Phi_a(k+1,k)\underline{\eta}_A(k) + \Gamma_a\nu(k) \quad (14)$$

$$\underline{\eta}_B(k+1) = \Phi_a(k+1,k)\underline{\eta}_B(k) + \Gamma_a\nu(k), \quad (15)$$

where

$$\Phi_a(k+1,k)=\begin{bmatrix}1 & \Delta t_{k+1} & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & \Delta t_{k+1} & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \text{ and } \Gamma_a(k+1,k)=\begin{bmatrix}\Delta t_{k+1}^2/2 & 0\\ \Delta t_{k+1} & 0\\ 0 & \Delta t_{k+1}^2/2\\ 0 & \Delta t_{k+1}\\ 0 & 0\\ 0 & 0\end{bmatrix}, \tag{16}$$

2.3 Observability analysis

Observability is an important property for a dynamical system. It relates to the amount of information contained in the measurement about the state of the system. In general, if a system is observable, the mapping from the state space to the measurement space should be one-to-one and the state space can be reconstructed from the measurement space. However, if a system is not observable, its state cannot be determined completely from the measurements, i.e., sufficient measurement data may be obtained to ascertain the state of interest over one specific domain but not over another domain. If little information can be gained from the measurements, additional or alternate measurements should be considered, or the system dynamics be reformulated.

In the following, we discuss the observability conditions for dynamical system (1), (11) and (12). The system for sensor A will be analyzed in detail. For sensor B, the same conclusions can be drawn similarly. System (1) and (11) for sensor A is a linear time-varying system. A system is *completely observable* if its state is observable everywhere on its defined state space. The information matrix $\mathcal{J}(k+S,k)$ for the system (1) and (10) can be defined as [14]

$$\mathcal{J}(k+S,k)=\sum_{i=k}^{k+S}[\Phi_a(i,k)]^T H_A^T(i)R_A^{-1}(i)\Phi_a(i,k)H_A(i),$$

where $R_A(i)=\Sigma_A(i)Q_{wA}\Sigma_A^T(i)$ and

$$Q_{wA}=\begin{bmatrix}\sigma_{Ar}^2 & 0\\ 0 & \sigma_{A\theta}^2\end{bmatrix}. \tag{17}$$

The system is said to be *completely observable* if and only if

$$\mathcal{J}(S,0)>0 \text{ for some } S>0. \tag{18}$$

Another observability of interest is the *uniformly completely observability*. In practice, the *uniformly completely observability* is more useful because it is also related to the system detectability and stability. A system is said to be *uniformly completely observable* if there exists a positive integer S and positive ξ_1 and ξ_2 such that

$$\xi_1 I \leq \mathcal{J}(k+S,k) \leq \xi_2 I, \tag{19}$$

for all $k > 0$, where I denotes an identity matrix. It can be seen that the information matrix involves the inverse of $R_A(k)$.

The information matrix can be decomposed into $\mathcal{J}(k+S,k) = U^T \Omega^T \Omega U$, where Ω is a block diagonal matrix with the ith sub-matrix given by $Q_{wA}^{-1/2} \Sigma_A^{-1}(k+i-1)$, and

$$U = \begin{bmatrix} H_A(k) \\ H_A(k+1)\Phi_a(k+1,k) \\ \vdots \\ H_A(k+S-1)\Phi_a(S+k,k) \end{bmatrix}. \tag{20}$$

The matrix U is dependent on H_A and Φ_a and is called the observability matrix [15]. Let $S = 3$. The observability matrix can be obtained as

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 & \sin\theta_A(k) & r_A(k)\cos\theta_A(k) \\ 0 & 0 & 1 & 0 & \cos\theta_A(k) & -r_A(k)\sin\theta_A(k) \\ 1 & \Delta t_{k+1} & 0 & 0 & \sin\theta_A(k+1) & r_A(k+1)\cos\theta_A(k+1) \\ 1 & 0 & 1 & \Delta t_{k+1} & \cos\theta_A(k+1) & -r_A(k+1)\sin\theta_A(k+1) \\ 1 & \Delta t_{k+2} & 0 & 0 & \sin\theta_A(k+2) & r_A(k+2)\cos\theta_A(k+2) \\ 1 & 0 & 1 & \Delta t_{k+2} & \cos\theta_A(k+2) & -r_A(k+2)\sin\theta_A(k+2) \end{bmatrix}. \tag{21}$$

In Appendix A, it is shown that U has a full column rank, i.e., its columns are all linearly independent of each other. Then, the observability matrix U is said to satisfy the observability rank condition. It is known that both $\Sigma_A(k)$ and Ω are non-singular. Since U has a full column rank and Ω is non-singular, it is sufficient that ΩU has a full column rank. According to a preposition by Song and Grizzle in [16], since ΩU has a full column rank for all $\theta_A(k)$ (a compact set), there must exist positive constant ξ_1 and ξ_2 such that (19) holds, i.e., the dynamical system (1) and (10) is *uniformly complete observable*. In other words, the system states can be uniquely determined from the sensor measurements.

3. KALMAN FILTER APPROACH FOR SENSOR REGISTRATION

In the previous section, the dynamical system for sensor registration problem was formulated and its observability shown. Now we consider the problem of estimating the sensor biases from the asynchronous sensor measurements. The Kalman filter is a natural candidate for such a task. For systems with unknown biases, one commonly used approach is to use both the augmented sensor measurement and the augmented state vector. The augmented measurement vector is formed by putting measurements from different sensors together into one vector while the augmented state vector is obtained by appending the constant sensor biases to the target state. The sensor biases are then estimated along with the target state using the Kalman filter procedures [7][9][10]. Different approach have been explored to decouple the estimation processes of the target state and the sensor registration errors to reduce the computational complexity. However, for asynchronous sensor measurements, the augmented approach encounters difficulties and is not directly applicable. In the following, we propose an approach based on the application of the two-stage Kalman filter to handle the asynchronous sensor measurements.

3.1 Two-stage Kalman estimator for sensor registration

The two-stage Kalman estimator is proposed for estimating the states of a linear system in the presence of an unknown constant bias vector [11][12]. It decouples the estimation of the bias and the system states, and estimates them in a recursive way. The estimator consists of two steps. In the first step, it estimates the bias-free system states using the standard Kalman procedures under the assumption that the system biases do not exist. In the second step, the estimator estimates the system biases based on the measurement residuals of the bias-free state estimates. The bias estimates are in turn used to update the bias-free target state estimates to provide the final system state estimates. One of the advantages of the two-stage Kalman estimator is that it is computationally efficient because it involves state vectors of smaller dimension than that of the augmented state vector.

To handle the asynchronous sensor measurement, we reformulate the measurement model to include both the sensor A and B as

$$\underline{z}(k) = L\underline{x}(k) + \Sigma(k)\underline{\delta} + \underline{w}(k), \tag{22}$$

where $\underline{\delta} = [\Delta r_A, \Delta\theta_A, \Delta r_B, \Delta\theta_B]^T$, $\underline{z}(k)$, $\Sigma(k)$ and $\underline{w}(k)$ are assigned different values depending on whether the measurement at kth time instance is from sensor A or from sensor B. The index k is the index of sensor A and B measurements ordered in time. When sensor A and B measurements occur at the same time, we place the sensor A measurement in front of that of sensor B only for the sake of processing convenience.

Let $k^{(A)}$ and $k^{(B)}$ denote the measurement indices for sensor A and B, respectively. When the kth measurement is the $k^{(A)}$th measurement from sensor A, $\underline{w}(k) = \Sigma_A(k^{(A)})\underline{w}_A(k^{(A)})$ and

$$\text{(23)} \qquad \underline{z}(k) = \underline{z}_A(k^{(A)}),\ \Sigma(k) = \left[\ \Sigma_A(k^{(A)}) \quad \mathbf{0}\ \right].$$

When the kth measurement is the $k^{(B)}$th measurement from sensor B, then, $\underline{w}(k) = \Sigma_B(k^{(B)})\underline{w}_B(k^{(B)})$ and

$$\text{(24)} \qquad \underline{z}(k) = \underline{z}_B(k^{(B)}),\ \Sigma(k) = \left[\ \mathbf{0} \quad \Sigma_B(k^{(B)})\ \right],$$

where $\mathbf{0}$ denotes a zero matrix. When the sensor A and B measurements coincide in time, since the time difference between the two adjacent measurements is zero, the transition matrix $\Phi(k+1,k)$ reduces to an identity matrix and the target dynamical model becomes

$$\text{(25)} \qquad \underline{x}(k+1) = \underline{x}(k).$$

The proposed registration approach is based on the application of the two-stage Kalman estimator to the reformulated measurement model (22). The two-stage Kalman estimator is due to Ignagni [12]. The computation procedures of the estimator are described in the following.

1. BIAS-FREE STATE ESTIMATION Assume that the *a posterior* estimate $\{\hat{\underline{x}}(k-1), \hat{\underline{\delta}}(k-1)\}$ has been obtained at the $(k-1)$th time instance. The bias-free measurement model can be obtained by letting $\Sigma = \mathbf{0}$ as

$$\text{(26)} \qquad \underline{z}(k) = L\underline{x}(k) + \underline{w}(k).$$

Let $\Gamma(k)$ denote $\Gamma(k,k-1)$ for simplicity. The Kalman filter procedures for estimating the *a posteriori* bias-free target states can be written as

$$\text{(27)} \qquad \left\{ \begin{array}{rcl} \tilde{\underline{x}}^{-}(k) & = & \Phi(k,k-1)\tilde{\underline{x}}(k-1) \\ \tilde{P}_{x^-}(k) & = & \Phi(k,k-1)\tilde{P}_x(k-1)\Phi^T(k,k-1) + \Gamma(k)R_v(k)\Gamma^T(k) \\ \tilde{P}_x(k) & = & [I - \tilde{K}_x(k)L]\tilde{P}_{x^-}(k) \\ \tilde{K}_x(k) & = & \tilde{P}_{x^-}(k)L^T(L\tilde{P}_{x^-}(k)L^T + R_w(k))^{-1} \\ \tilde{\underline{x}}(k) & = & \hat{\underline{x}}^{-}(k) + \tilde{K}_x(k)\{\underline{z}(k) - L\tilde{\underline{x}}^{-}(k)\} \end{array} \right. .$$

where $R_w(k)$ denotes the covariance matrix of $\underline{w}(k)$. Define $Q_{wB} = \mathrm{diag}[\sigma^2_{Br}, \sigma^2_{B\theta}]$. The sensor measurement noise covariance $R_w(k)$ is given by $\Sigma_A(k) Q_{wA} \Sigma_A^T(k)$ or $\Sigma_A(k) Q_{wB} \Sigma_A^T(k)$ depending on whether the measurement is from sensor A or sensor B, respectively, at the kth time instance.

2. SENSOR BIAS ESTIMATION In the second step, we assume that the sensor biases are perfectly known. The measurement model is given by (22), which is identical to (26) within a constant. When applying the Kalman procedures, it can be verified that the Kalman gain is identical to $\tilde{K}_x(k)$ when the initial state estimates and their corresponding covariances are chosen to be the same. It follows that

$$(28) \qquad \begin{cases} \underline{\bar{x}}^-(k) = \Phi(k, k-1)\underline{\bar{x}}(k-1) \\ \underline{\bar{x}}(k) = \underline{\bar{x}}^-(k) + \tilde{K}_x(k)[\underline{z}(k) - L\underline{\bar{x}}^-(k) - \Sigma\underline{\delta}], \end{cases}$$

where $\underline{\bar{x}}^-(k)$ and $\underline{\bar{x}}(k)$ are the *a priori* and the *a posterior* estimate, respectively, assuming that $\underline{\delta}$ is perfectly known. From relationships (27) and (28), we obtain

$$(29) \quad \underline{\tilde{x}}(k) = \Phi(k, k-1)\underline{\tilde{x}}(k-1) + \tilde{K}_x(k)[\underline{z}(k) - L\Phi(k, k-1)\underline{\tilde{x}}(k-1)]$$

$$(30) \quad \underline{\bar{x}}(k) = \Phi(k, k-1)\underline{\bar{x}}(k-1) + \tilde{K}_x(k)[\underline{z}(k) - L\Phi(k, k-1)\underline{\bar{x}}(k-1) - \Sigma(k)\underline{\delta}].$$

and the following equation can be obtained

$$(31) \qquad \begin{aligned} \underline{\bar{x}}^-(k) &= \underline{\tilde{x}}^-(k) + U(k)\underline{\delta} \\ \underline{\bar{x}}(k) &= \underline{\tilde{x}}(k) + V(k)\underline{\delta}, \end{aligned}$$

where $U(k)$ and $V(k)$ are the sensitivity matrices. Since

$$(32) \qquad \begin{aligned} \underline{\bar{x}}^-(k) - \underline{\tilde{x}}^-(k) &= \Phi(k, k-1)[\underline{\bar{x}}(k-1) - \underline{\tilde{x}}(k-1)] = \Phi(k, k-1)V(k-1)\underline{\delta} \\ \underline{\bar{x}}(k) - \underline{\tilde{x}}(k) &= \{U(k) - \tilde{K}_x[LU(k) + L\Sigma(k)]\}\underline{\delta}, \end{aligned}$$

we have

$$(33) \qquad U(k) = \Phi(k, k-1)V(k-1) \quad \text{and} \quad V(k) = U(k) - \tilde{K}_x(k)S(k),$$

where $S(k) = LU(k) + \Sigma(k)$. The measurement residual of the bias-free estimate $\underline{\tilde{x}}(k)$ can be written as

(34) $$\underline{\tilde{r}}(k) = \underline{z}(k) - L\underline{\tilde{x}}^-(k) = S(k)\underline{\delta} + \underline{\rho}(k),$$

where $\underline{\rho}(k) = \underline{z}(k) - L\underline{\bar{x}}^-(k) - \Sigma(k)\underline{\delta}$ is the measurement residual of state estimate assuming perfectly known sensor biases. Since $\underline{\bar{x}}^-(k)$ is the minimum variance estimates, the measurement residual is known to be uncorrelated. It follows that the covariance of $\underline{\rho}(k)$ is given by

(35) $$R_\rho(k) = L\tilde{P}_x^-(k)L^T + R_w(k),$$

provided that identical initial estimates and corresponding covariances are used in (27) and (30). Applying the Kalman filter procedures to (34), we have

(36) $$\begin{cases} \underline{\hat{\delta}}^-(k) &= \underline{\hat{\delta}}(k-1) \\ P_\delta^-(k) &= P_\delta(k-1) \\ K_\delta(k) &= P_\delta^-(k)S^T(k)[S(k)P_\delta^-(k)S^T(k) + R_\rho(k)]^{-1} \\ \underline{\hat{\delta}}(k) &= \underline{\hat{\delta}}^-(k) + K_\delta(k)[\underline{\tilde{r}}(k) - S(k)\underline{\hat{\delta}}-(k)] \\ P_\delta(k) &= [I - K_\delta(k)]P_\delta^-(k) \end{cases} .$$

The sensor bias estimates can then be used to update the bias-free target state estimates as

$$\begin{aligned} \underline{\hat{x}}^-(k) &= \underline{\tilde{x}}^-(k) + U(k)\underline{\hat{\delta}}^-(k) && (37) \\ \underline{\hat{x}}(k) &= \underline{\tilde{x}}(k) + V(k)\underline{\hat{\delta}}(k). && (38) \end{aligned}$$

The estimates $\underline{\hat{\delta}}(k)$ and $\underline{\hat{x}}(k)$ can also be written more directly as

(39) $$\begin{aligned} \underline{\hat{\delta}}(k) &= \underline{\hat{\delta}}^-(k) + K_\delta(k)\underline{\hat{r}}(k) \\ \underline{\hat{x}}(k) &= \underline{\hat{x}}^-(k) + K_x(k)\underline{\hat{r}}(k), \end{aligned}$$

where $\underline{\hat{r}}(k) = \underline{z}(k) - L\underline{x}^-(k) - \Sigma(k)\underline{\hat{\delta}}(k)$, where the intermediate bias-free estimate $\underline{\tilde{x}}(k)$ does not appear explicitly

The block diagram of the proposed Kalman filter registration approach is shown in Figure 2. At each sensor site, the local bias-free estimator computes the bias-free target state estimate whenever it receives a measurement. Another Kalman filter cycle is used

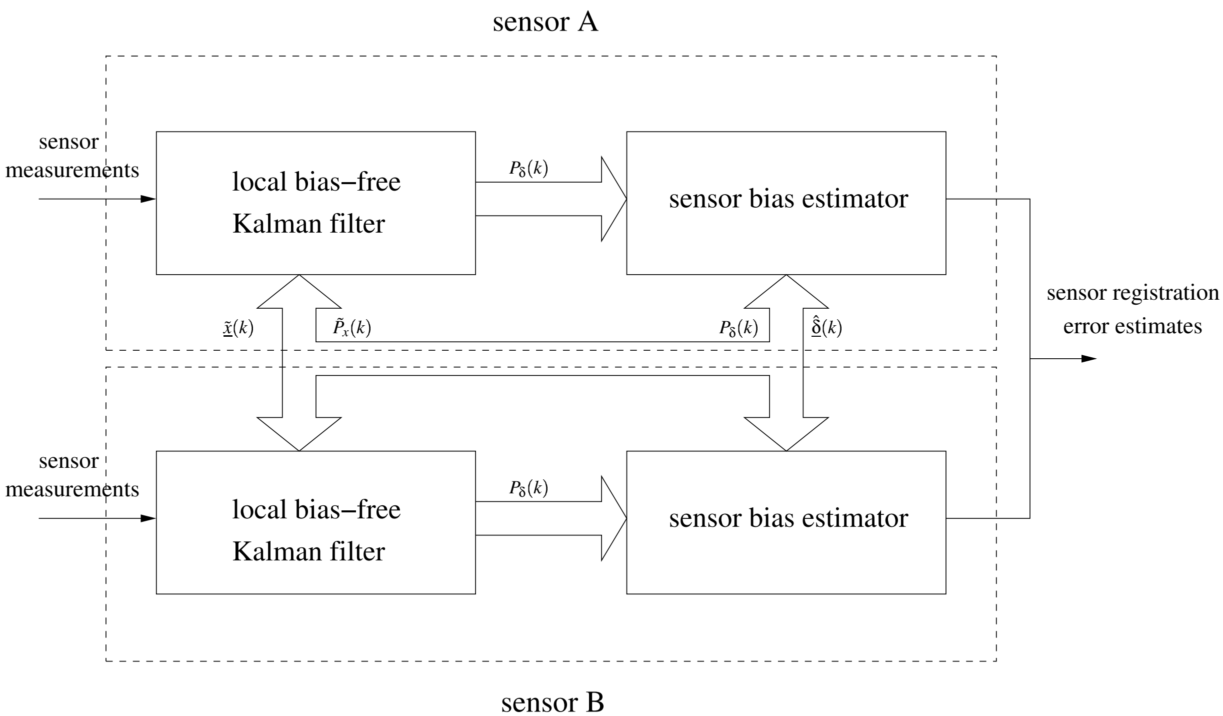

Figure 2: *The two-stage Kalman estimator for sensor registration.*

to estimate the sensor registration errors based on the measurement residual of the bias-free Kalman estimator. The algorithm is implemented in a distributed way. The *a posteriori* estimates of the bias-free target states and the sensor registration errors as well as their corresponding covariance matrices are propagated to the sensor site where the next measurement is received. The algorithm iterates between the sensors and provides accurate sensor registration estimates as sensor measurements arrive.

The two-stage Kalman estimator contains an extra Kalman filter cycle to estimate the sensor biases when compared to the augmented Kalman estimator. However, it does have computational advantages over the augmented Kalman estimator in two-fold. First, the two-stage Kalman estimator involves matrices of smaller dimensions than those required by the augmented Kalman estimator. By doing so, it eliminates the difficulties associated with the computations of large matrices, especially when the application involves a large number of sensors. Secondly, the two-stage Kalman estimator can be implemented using a parallel structure. The algorithm estimates the sensor biases and the bias-free target state estimates can be calculated concurrently, making the algorithm suitable for real-time applications.

The stability is another concern for the Kalman filter approach since the optimality of the Kalman estimator does not imply stability. The stability refers to the condition of the upper and lower bounds on the estimation covariance and its asymptotic properties. In [12], Ignagni has shown that the two-stage Kalman estimator is equivalent to the augmented Kalman filter provided that the initial estimates and covariances are

Table 1: Model Parameters for Different Track Patterns

	$x(0)$ (km)	$y(0)$ (km)	$v_x(0)$ (km/s)	$v_y(0)$ (km/s)	σ_x^2 (km^2/s^4)	σ_y^2 (km^2/s^4)
P1	150.0000	-49.5000	0.0000	0.3300	0.00000	0.00050
P2	114.3300	-35.6690	0.2333	0.2333	0.00035	0.00035
P3	101.2520	-8.5956	0.3250	0.0573	0.00049	0.00001
P4	100.0000	10.00000	0.3300	0.0000	0.00050	0.00000

identical. In Section 2, we have shown that the augmented system at each sensor is *uniformly complete observable*. For a *uniformly complete observable* system, it is sufficient that the system is also detectable [17]. It follows that, since system (1) and (10) is shown to be *uniformly complete observable*, it is detectable. A detectable system has the property that, when the Kalman filter procedures are applied, its *a priori* and the *a posteriori* error covariance of the two-stage Kalman estimator is bounded [17]. In other words, the system represented by (1) and (10) is stable in that it has bounded *a priori* and *a posteriori* estimation covariances.

4. PERFORMANCE EVALUATION

We use computer simulations to study the performance of the proposed registration approach. Two sensors are simulated with sensor A located at the system origin and sensor B at (u, v), where $u = 300$km and $v = 0$km. The standard near constant velocity model is used for simulating the target motion. The standard deviation of the process noise, σ_v is taken to be 0.06 times the mean velocity in a sampling interval.

Four track patterns are simulated as shown in Figure 3. Their parameters are given in Table 1. In the figure, the solid lines represent the actual target trajectories, and the dashed and the dash-dotted ones denote the measurements by sensor A and B, respectively. In (a), the target moves in perpendicular to the line joining the two sensor sites, and the measurements are distributed on both sides of the line. The tracks in (b) and (c) are similar and the measurements reside on both sides of the line joining the sensor sites. The track in (b) has a slope of 1 relative to the site line while the slope of the track in (c) is 0.1736. The track in pattern (d) is parallel to the line connecting the two sites and the measurements are only on one side of the line. The distance between the track and the line connecting the sensor sites is 10 km. The four track patterns are variations of typical flight tracks that could be encountered by any two sensor sites in data fusion scenarios. In the figure, due to sensor registration errors, it can be seen that the tracks reported by the two sensors do not coincide when converted to the system coordinate, generating the so called “ghost” track phenomenon.

The sampling intervals are assumed to be to be $4s$ and $3s$ for sensor A and B, respectively. The time delays at the beginning of the two sensor measurements are assumed to be zero.

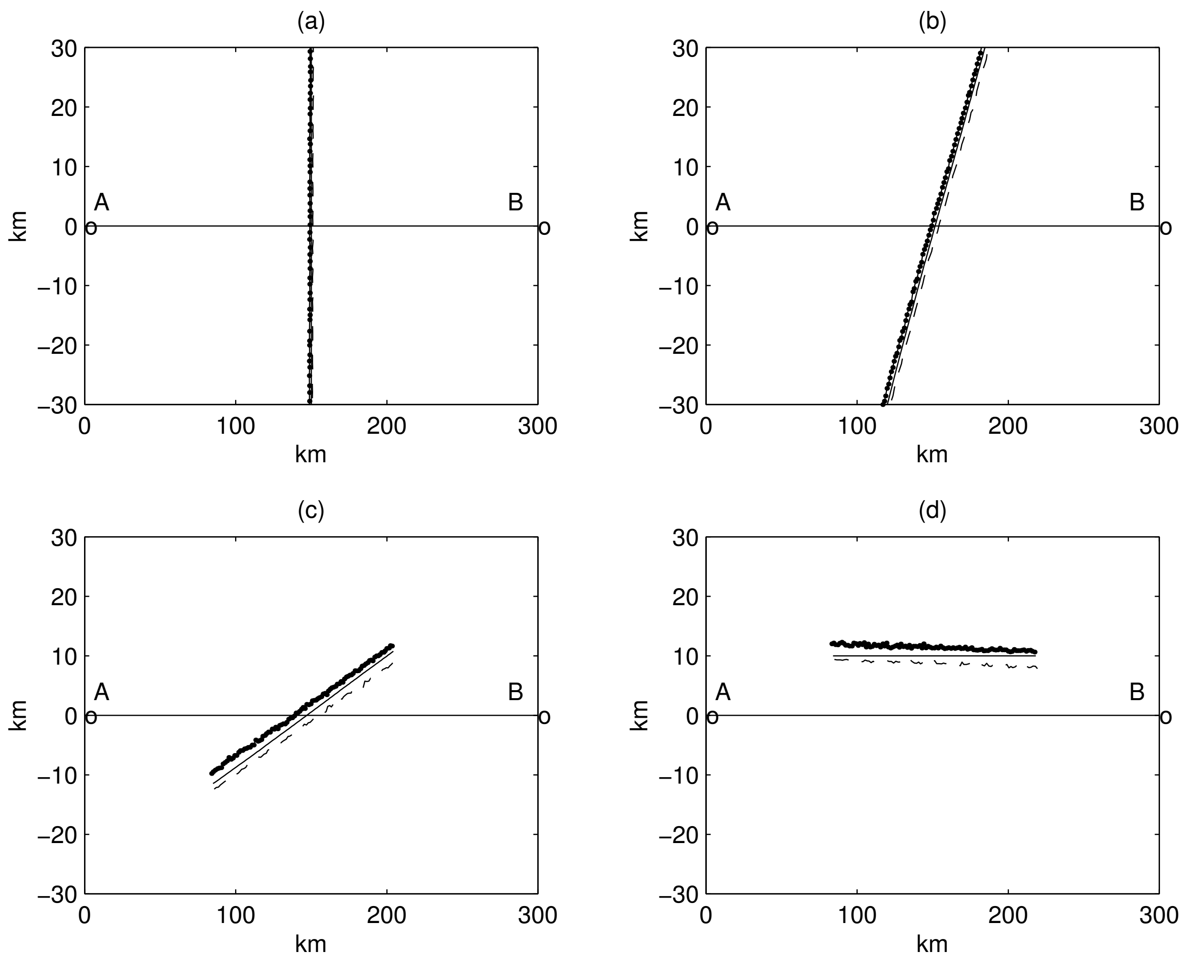

Figure 3: *Different track patterns for sensor registration.*

Table 2: Standard Deviations of Noise in Range and Azimuth Measurement

	case 1	case 2	case 3	case 4	case 5
σ_{rA} (km)	0.05	0.1	0.5	1.0	1.5
$\sigma_{\theta A}$ (radian)	0.0056	0.0011	0.0028	0.0056	0.0111
σ_{rB} (km)	0.05	0.1	0.5	1.0	1.5
$\sigma_{\theta B}$ (radian)	0.0056	0.0011	0.0028	0.0056	0.0111

The LS method [6] is used for comparison. The LS algorithm is a special case of the GLS algorithm [2] in which the measurement covariance is assumed to be an identity matrix. Since the LS method requires synchronized sensor measurements, we use the one-step fixed-lag smoothing approach proposed by Helmick *et al.* [13] to time-translate sensor A measurement to the times of sensor B measurements. For simplicity, a single-model Kalman filter is used for the constant velocity target model. In the smoother, a one-step predictor is first used to predict to the subsequent sensor B measurement time, which is corrected using the next sensor A measurement to produce the smoothed estimates. The smoothed estimates are then passed to the LS method for sensor registration. The local tracking is performed in the Cartesian coordinates. The sensor measurement model used is given by

$$\underline{z}_A(k) = L\underline{x}(k) + \Sigma_A(k)\underline{w}_A(k), \tag{40}$$

where the first-order approximation of $\underline{w}_A(k)$ is used. Model (40) is one of the most commonly used models of converted measurements and is valid for small measurement noise [18].

Figures 4-7 show the variations of the standard deviation (STD) of the sensor registration error estimates for different measurement noise cases. Five measurement noise cases are simulated. The noise is assumed to be *i.i.d* Gaussian processes with zero mean and standard deviations given in Table 2. The sensor registration errors are simulated as $\Delta r_A = 1$km, $\Delta\theta_A = 0.0105$rad, $\Delta r_B = -0.8$km and $\Delta\theta_B = 0.0087$rad. The numbers of sensor measurements are $K_1 = 100$ and $K_2 = 135$ for sensor A and B, respectively. Each test is repeated 200 times to obtain the averaged results. The Kalman filter based registration approach is compared with the LS method. In Figures 4-7, the lines with a diamond legend denote the LS results and lines with 'x' are results obtained by the Kalman filter based approach. For the four track patterns, the Kalman filter based approach demonstrates clear improvements over the LS method. For track patterns 1 and 2, since the tracks are properly placed, the LS method performs well. The Kalman filter based approach outperforms the LS method slightly. For track pattern 3 and 4, since the data matrix in the LS method is ill-conditioned [6], the LS method performs poorly and fails when the measurement noise increases. However, the Kalman filter based approach is still able to maintain a good estimation performance. In general, since the Kalman filter approach takes into account the underlying track

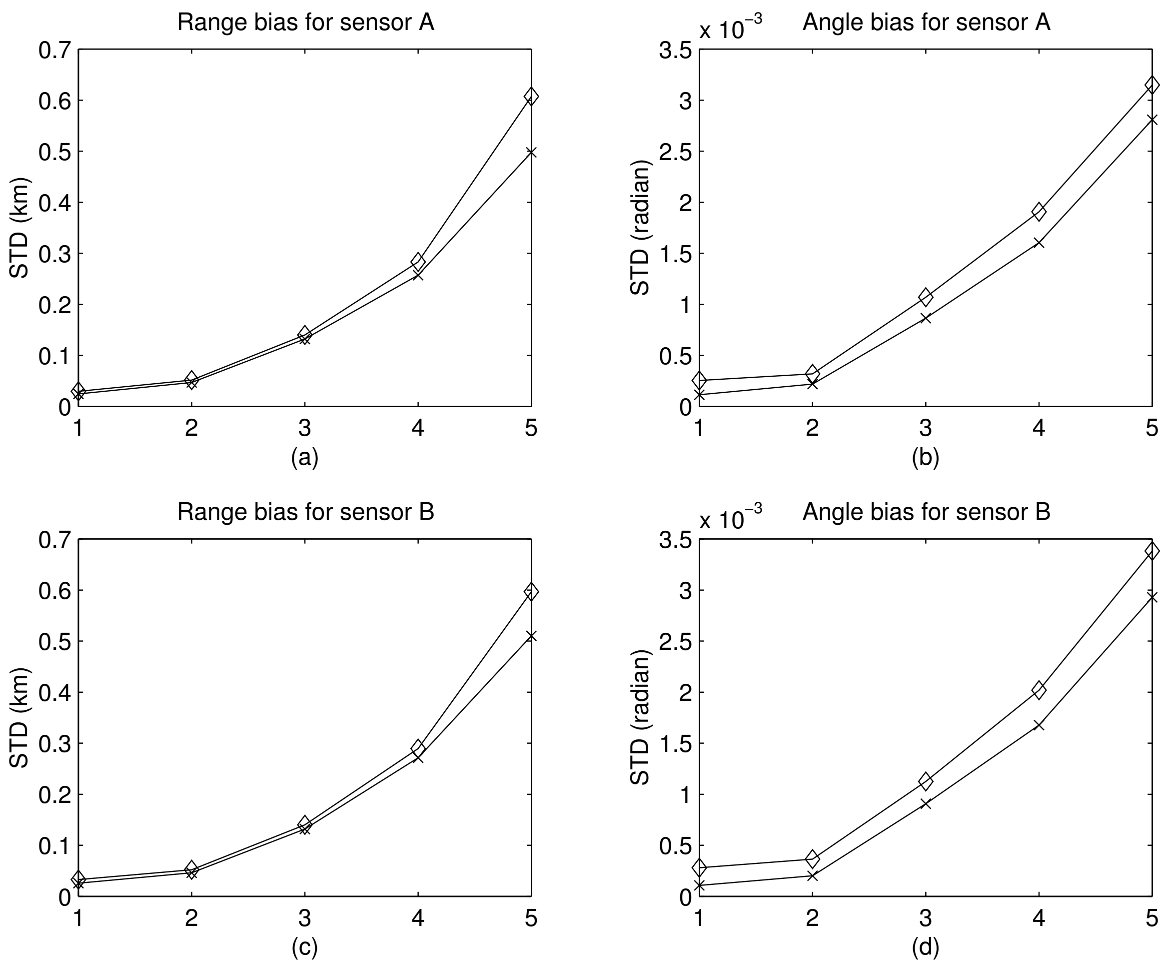

Figure 4: *Variation of the STDs of the sensor registration error estimates via different measurement noise for track pattern one.*

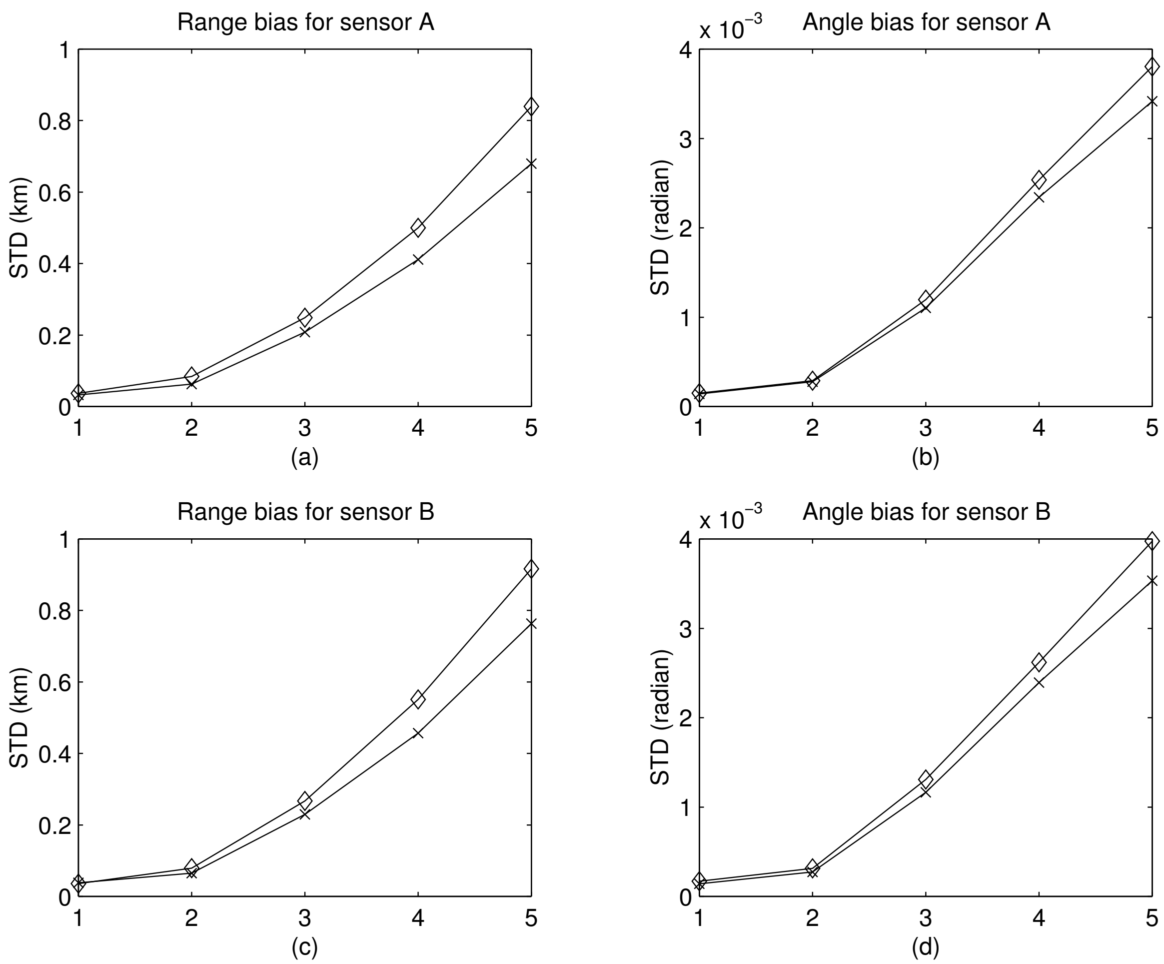

Figure 5: *Variation of the STDs of the sensor registration error estimates via different measurement noise for track pattern two.*

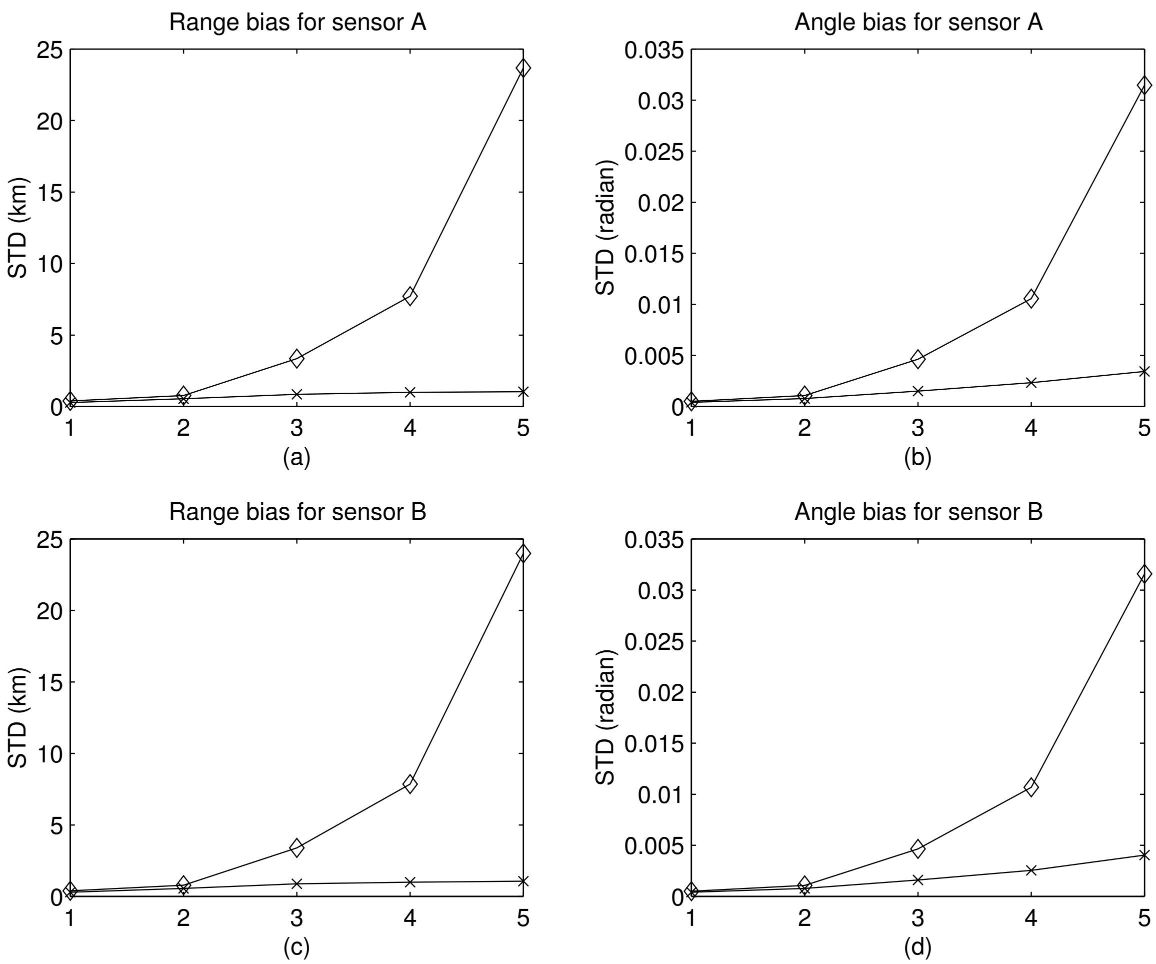

Figure 6: *Variation of the STDs of the sensor registration error estimates via different measurement noise for track pattern three.*

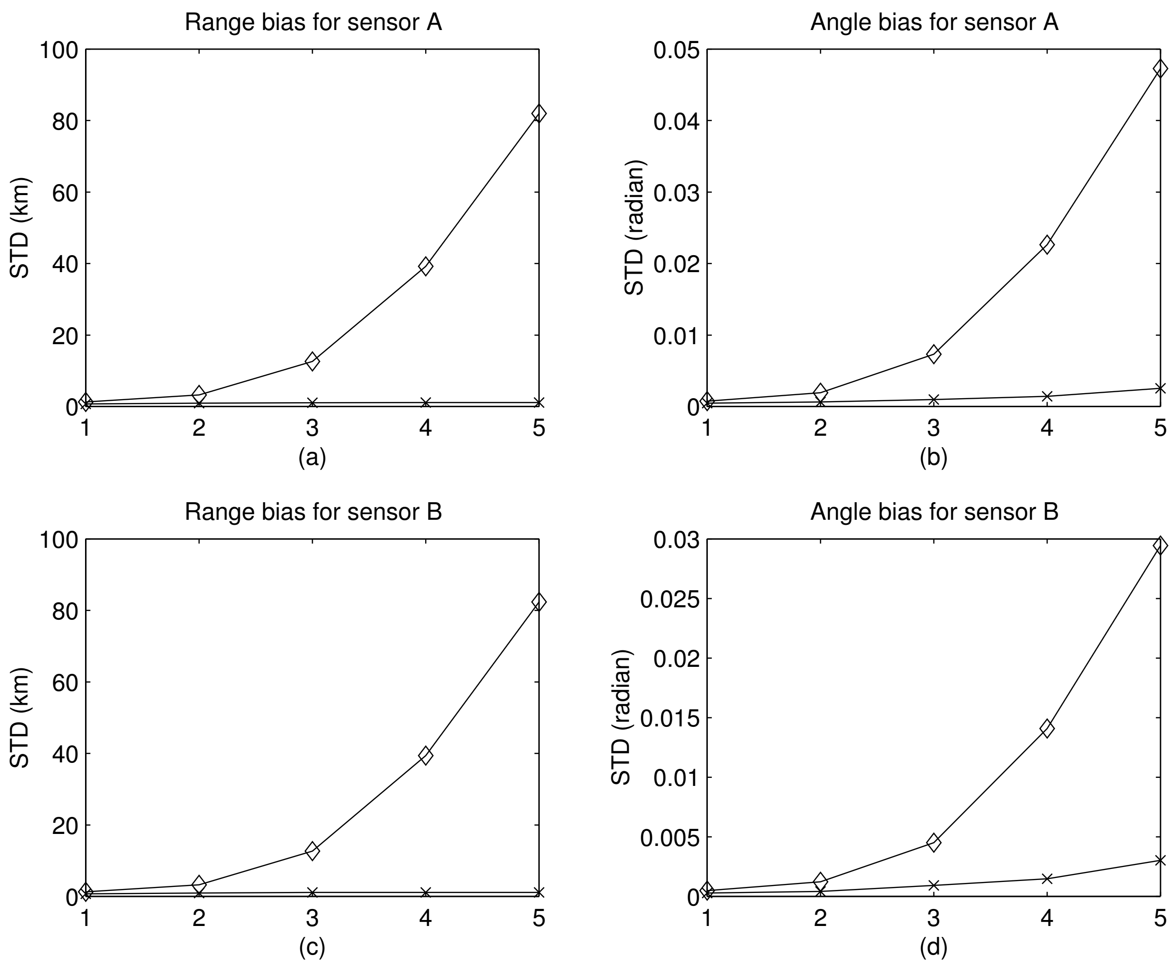

Figure 7: *Variation of the STDs of the sensor registration error estimates via different measurement noise for track pattern four.*

structure, it is able to produce good registration performance consistently for all different track pattern.

The means of the registration estimates for different track patterns and noise cases are also computed and listed in Table 3. It can be seen that for all the track patterns, when the measurement noise is small, the Kalman filter based approach and the LS method are able to produce unbiased estimates (or negligible biases). However, when the sensor measurement noise increases, the estimates become biased due to the first-order approximation used in the sensor measurement model. This indicates that the linearized measurement is a valid model when the sensor measurement noise is small. The results also show that the Kalman filter based approach and the LS method are less susceptible to the measurement noise for track patterns 1 and 2, in terms of estimation biasedness, than for track pattern 3 and 4. For track patterns 3 and 4, the LS method does not work well and fails when the sensor measurement noise increases. The Kalman filter based approach, however, is still able to produce unbiased and accurate azimuth error estimates though its range error estimates are relatively poor when the noise increases.

The modified LS method [6] is also examined in the study. The modified LS method was proposed to handle the difficulties encountered by LS method in dealing with various track patterns. It was found that certain track patterns such as track pattern 3 and 4 may cause the data matrix in the LS method ill-conditioned, making the algorithm ineffective. The modified LS method uses the singular value decomposition (SVD) computes the pseudo-inverse of the data matrix. It ignores the insignificant singular values and approximates the data matrix by one with a reduced rank. The modified LS method was claimed to have an significant improvement over the LS method when dealing with ill-conditioned data matrix [6]. However, in our simulation studies, the modified LS method fails to demonstrate the improvement. Instead, its results are rather irregular depending on the track patterns and values of the sensor registration errors. This can be explained by the fact that the modified LS solution does not have any meaningful interpretations by approximating the ill-conditioned data matrix by low-rank matrix. The ill-conditioning of the data matrix is caused by the observability condition of the sensor registration errors. Thus, the modified LS method, which treats the sensor measurements as isolated points, may not be a proper approach for countering the ill-conditioning problem caused by different track patterns.

To understand the efficiency of the Kalman filter based approach in a practical environment, we also apply the Kalman filter based registration approach to real-life multiple radar data collected from an air surveillance network. The radar network is composed of seventeen 24-by-24 foot phased array L-band long range radars located along the coastline of Canada. With a range of 200 nautical miles, these long range radars measure the range, azimuth as well as the elevation of the targets. Each radar also carries an IFF for target identification. Tracks of air targets arise from commercial airlines. The operating specifications of the radars are summarized as follows :

- operating frequency : 1215 - 1400 MHz

Table 3: *Sensor Registration Error Means by the Kalman Based Approach*

		Kalman filter based approach					LS method				
		noise 1	noise 2	noise 3	noise 4	noise 5	noise 1	noise 2	noise 3	noise 4	noise 5
T1	Δr_A	1.0069	1.0082	0.9698	0.8776	0.6664	0.9815	0.9870	1.0087	0.9501	0.8662
	Δr_B	-0.7919	-0.7903	-0.7656	-0.6473	-0.4356	-7778	-0.7806	-0.8137	-0.7336	-0.6702
	$\Delta\theta_A$	0.0104	0.0104	0.0105	0.0103	0.0102	0.0102	0.0102	0.0104	0.0104	0.0107
	$\Delta\theta_B$	0.0088	0.0088	0.0087	0.0087	0.0085	0.0090	0.0090	0.0089	0.0091	0.0091
T2	Δr_A	0.9981	1.0115	0.9385	0.7324	0.4059	1.0148	1.0502	1.0987	1.1640	1.2346
	Δr_B	-0.7848	-0.7946	-0.7082	-0.4902	-0.1281	-0.8019	-0.8360	-0.8989	-0.9836	-1.0796
	$\Delta\theta_A$	0.0104	0.0104	0.0101	0.0092	0.0089	0.0104	0.0104	0.0103	0.0102	0.0108
	$\Delta\theta_B$	0.0088	0.0088	0.0091	0.0099	0.0100	0.0088	0.0089	0.0090	0.0092	0.0086
T3	Δr_A	0.8514	0.5642	0.1590	0.0183	-0.0312	1.1598	1.3451	2.8029	6.9216	22.9942
	Δr_B	-0.6360	-0.3477	0.0637	0.1852	0.2607	-0.9485	-1.1396	-2.6413	-6.8599	-23.0928
	$\Delta\theta_A$	0.0102	0.0099	0.0092	0.0087	0.0083	0.0107	0.0110	0.0130	0.0184	0.0394
	$\Delta\theta_B$	0.0089	0.0093	0.0101	0.0106	0.0111	0.0085	0.0082	0.0062	0.0008	-0.0202
T4	Δr_A	0.3768	0.0681	-0.0780	-0.0991	-0.0991	1.2956	2.8803	11.6292	38.2935	80.8278
	Δr_B	-0.1564	0.1488	0.2986	0.3134	0.3210	-1.0868	-2.6799	-11.4824	-38.2782	-80.9992
	$\Delta\theta_A$	0.0101	0.0099	0.0098	0.0098	0.0097	0.0106	0.0115	0.0166	0.0318	0.0559
	$\Delta\theta_B$	0.0085	0.0084	0.0083	0.0084	0.0082	0.0088	0.0094	0.0124	0.0220	0.0372

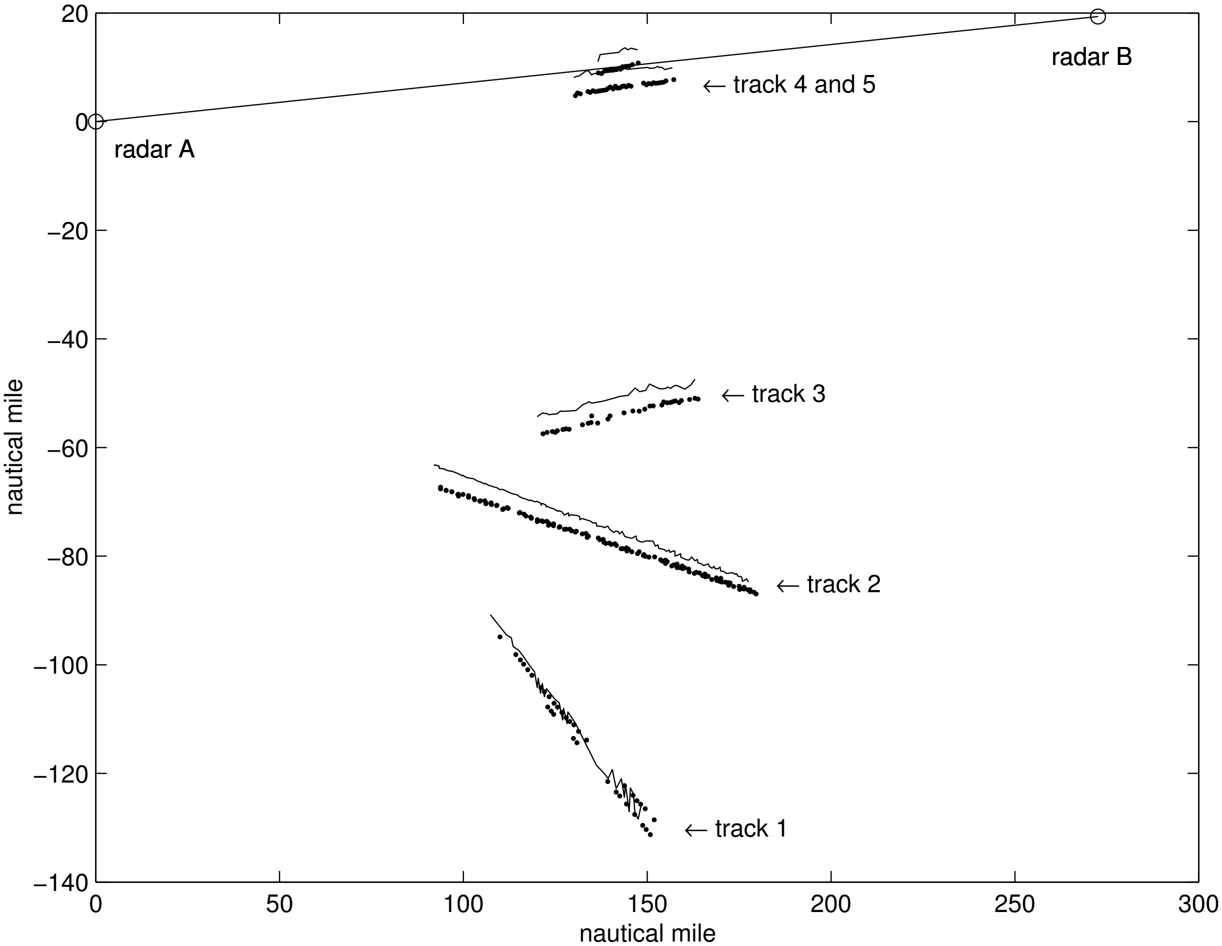

Figure 8: *Real target track measurements collected by two radars of the air surveillance network.*

- instrumental range : 5 - 200 nautical miles
- azimuth coverage : 360^o in 12 seconds.
- range resolution : 300 meters.
- azimuth beamwidth : 2.2^o.
- probability of detection : 0.75

To remove the effects of the false targets (clutters), target ID's provided by the identification of friend and foe (IFF) beacon are used to extract the true aircraft tracks from radar returns for registration calculation. Each site in the system has two satellite dishes for communications, and the communications between the radars are via the Anik communications satellite. This radar network employs the stereographic projection [19] to map the elliptic earth on a plane to get the stereographic ground range from the slant range of a target. The target azimuth is measured relative to the true north at the radar location and is adjusted so that it is relative to the true north at the origin of the common coordinate system. Figure 8 shows the target track measurements collected by two radars of the air surveillance network. In the figure, the solid lines are tracks observed by radar A and the dotted tracks are by radar B. The measurements have been converted to the common system coordinate with the location of radar A as the origin and radar B is located at $(u, v) = (272.6406, 19.3287)$ nautical miles.

In Figure 8, five tracks are reported by radar A and B. We use a segment of track 2 data for registration. The number of data points is 45. Since the track data contains incorrect measurements (clutter, false alarm and multipath), an elliptical gates is used in the local Kalman filter procedures to eliminate the unlikely observation-to-track pairings [20]. A measurement vector $\underline{z}_A(k+1)$ (in the case of sensor A) is considered to be a valid or correct measurement if it satisfies

$$(41) \qquad d^2 = [L\underline{\hat{x}}^-(k+1) - \underline{z}_A(k+1)]^T P_{x^-}^{-1}(k+1)[L\underline{\hat{x}}^-(k+1) - \underline{z}_A(k+1)] \leq G,$$

where $P_{x^-}(k+1)$ is the *a priori* covariance of the state estimate and G is elliptical gate. The value of the gate G is determined by an allowable probability of a correct measurement falling outside the gate. We choose the allowable probability of a correct measurement falling outside the gate to be 0.1. It can be verified that the elliptical gate is given by $G = 4.6$ [20]. The standard deviation of the target dynamical system is selected to be approximately equal to the maximum acceleration observed. In this case, they are given by $\sigma_{vx} = 0.0023$ nautical miles/s^2 and $\sigma_{vy} = 0.0059$ nautical miles/s^2. The variances of sensor range and azimuth measurements are estimated by fitting a polynomial function to the measurements in a least squares sense. The differences between the measurements and data points on the polynomial are used to estimate the variances. An order 3 polynomial is used and the variances are estimated to be $\sigma_{rA} = \sigma_{rB} = 0.05$ nautical miles and $\sigma_{\theta A} = \sigma_{\theta B} = 0.0015$ radians. Applying the Kalman filter based registration approach and the LS method to the track selected, the sensor registration error estimates can be obtained as

$$(42) \qquad \hat{\delta}_{Kalman} = \begin{bmatrix} 0.052 \\ 0.0001 \\ 0.2156 \\ -0.0228 \end{bmatrix} \text{ and } \hat{\delta}_{LS} = \begin{bmatrix} 0.0956 \\ 0.0016 \\ 0.1250 \\ -0.0213 \end{bmatrix}$$

To illustrate the effect of registration, we use the registration error estimates to update the sensor measurements. Figure 9 and 10 show the tracks by sensor A and B after registration using the sensor registration error estimates by the Kalman filter based approach and the LS method, respectively. In the figures, the solid lines represent the radar A measurements and the dotted lines are the measurements by radar B. Comparing to Figure 8, it can be seen that both methods are able to bring the tracks by the two sensors in the system closer than before to represent a single target. In other

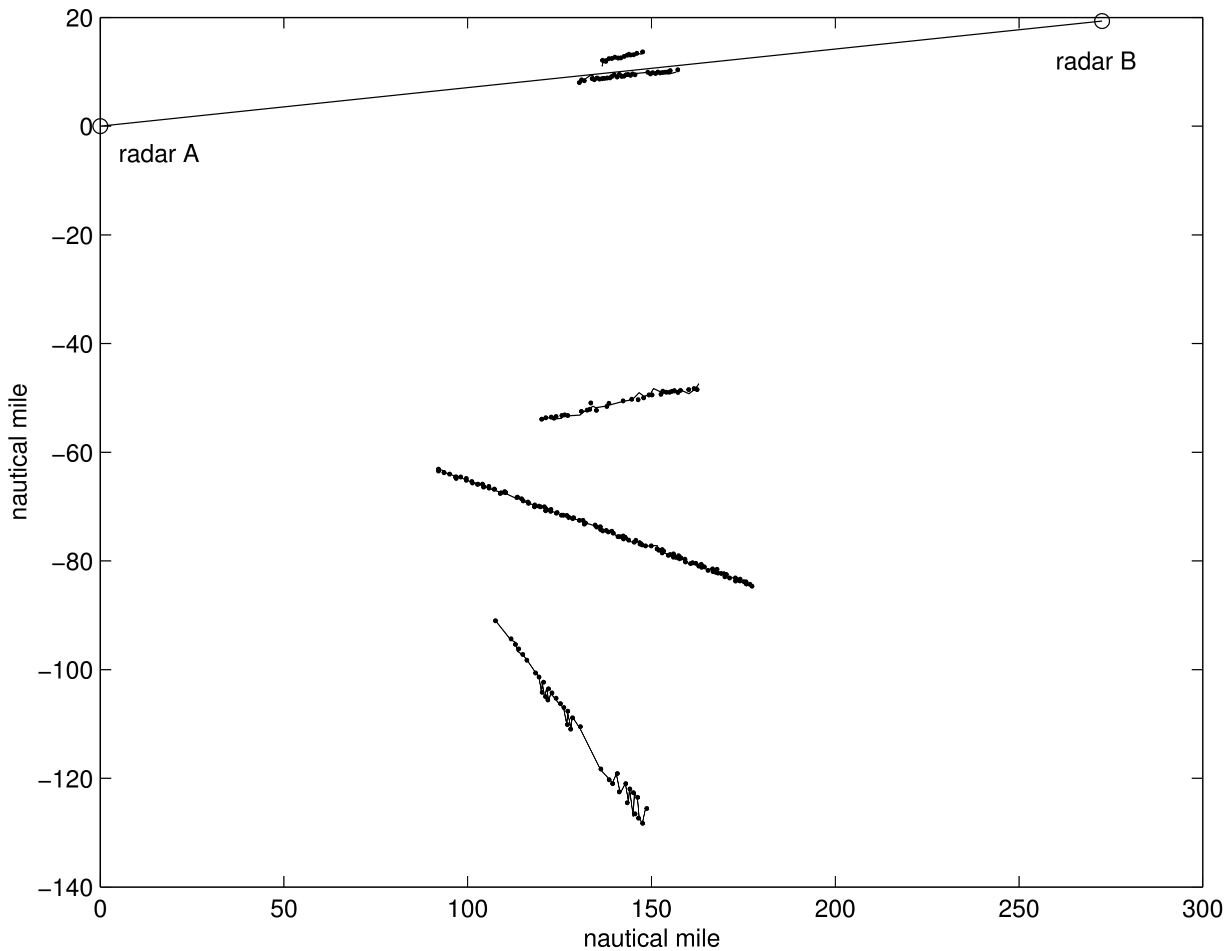

Figure 9: *Track measurements by the two radars after registration by the Kalman filter based approach.*

words, the “ghost” tracks have been successfully eliminated by the two registration methods.

Since there is no ground truth about the sensor registration error, we use the distance between the track measurements by the two sensors before and after the registration as an performance index to measure the quality of the registration algorithm. The distances between the tracks that are used for estimating the sensor registration errors before registration are given by 3.1704 nautical miles. After registration, the distance reduces to 0.2731 and 0.5055, for the Kalman filter based approach and the LS method, respectively. Both methods are able to successfully reduce the track distance after registration to a great extent. However, the Kalman filter based approach outperforms the LS method in that it produces a track distance that is only about half of that by the LS method. We also apply the sensor registration error estimates to other tracks that are not used in the registration estimation process to examine the generalization ability of the algorithms. In Table 4, we list the calculated track distances for different tracks

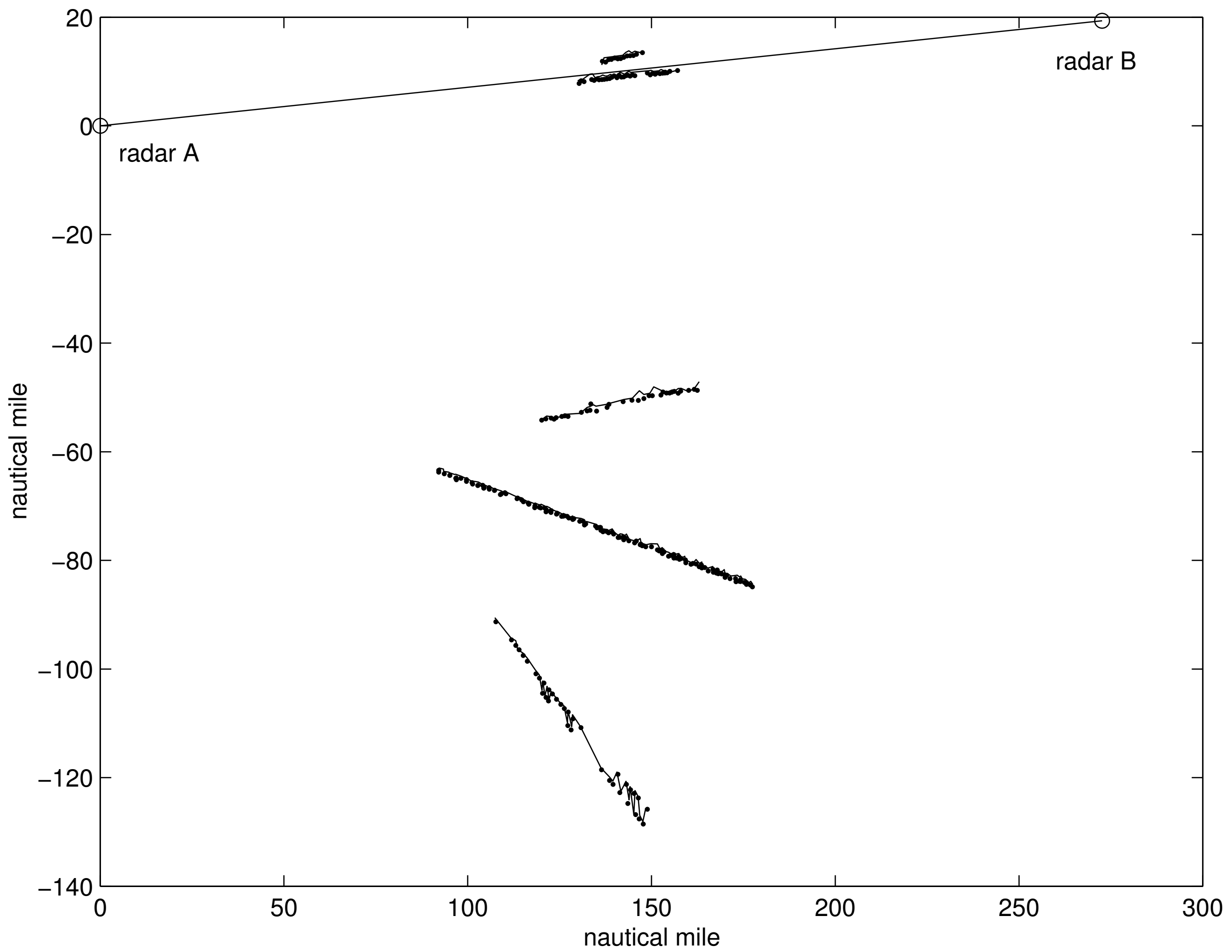

Figure 10: *Track measurements by the two radars after registration by the LS method.*

Table 4: Track Distances Before and After Registration for Different Tracks

	before reg. (nautical miles)	LS method (nautical miles)	Kalman filter based approach (nautical miles)
track 1	3.7795	0.5548	0.2816
track 2	3.3720	0.7618	0.5406
track 3	4.4511	0.5666	0.3159
track 4	3.0976	0.7014	0.5150
track 5	3.0174	0.6491	0.5240

before and after registration using the Kalman filter based approach and the LS method. It can be seen from the table that both the Kalman filter based approach and the LS method can reduce the track distance for tracks that are not used in the registration. However, the generalization errors by the Kalman filter based approach are smaller than those by the LS method. In particular, for track 1, a segment of which is used in registration process, the generalization error by the Kalman filter based approach is only half of that by the LS method. This indicates that the Kalman filter based approach has a better generalization ability than the LS method.

5. CONCLUSIONS

In this paper, the Kalman filter based registration approach has been presented for multiple sensors with asynchronous measurements. The approach is based on a linear time-variant measurement model and the application of the two-stage Kalman filter procedures and is able to handle asynchronous sensor measurements. Unlike most existing registration algorithms, it does not require any computationally sophisticated time alignment procedures for asynchronous sensor measurements. The proposed method is recursive and computationally efficient. Simulation and real-life data are used to demonstrate the effectiveness and performance of the proposed approach. The results were compared with the popular LS method. The Kalman filter based approach was shown to outperform the LS method in terms of registration performance and the ability to handle different track patterns. When using real-life radar data, the Kalman filter based approach has demonstrated a better generalization ability than the LS method.

The observability and stability of the system model has been investigated and it was shown that the dynamical system (1) and (10) is *uniformly complete observable*. In other words, the system states which includes the target state and the sensor registration errors can be uniquely determined from the sensor measurements. It should be pointed out that the observability analysis was carried out for the range and azimuth offset errors of the sensors. The orientation and location errors in the reference frame of the sensors were not considered. In the simulation studies, four typical track patterns were used. It was shown that the proposed Kalman filter based approach performed well for all the track patterns while the LS method was found to be ineffective for track pattern 3 and 4, where the sensor measurements are distributed close to the line joining the sensor sites.

The simulation results also showed that when the sensor measurement noise is small, the linearized measurement model can be considered to be a good approximation of the original nonlinear model, and the Kalman filter based approach is able to produce accurate sensor registration error estimates with negligible biases. However, when sensor noise increases, the biasedness of the estimates becomes apparent, indicating that the linearized model may not be an appropriate model for large sensor measurement noise.

Annex A
Proof of Observability Rank Condition for U

To prove that the observability matrix U has a full column rank, we need to show that the columns of U are linearly independent of each other [21]. Denote $\{\underline{u}_i;\ i = 1, 2, \ldots, 6\}$ as the columns of U. It can be verified that the first four columns $\{\underline{u}_i;\ i = 1, 2, 3, 4\}$ are linearly independent. To show that $\underline{u}_5$ is independent of $\{\underline{u}_i;\ i = 1, 2, 3, 4\}$, assume that there exists a set of coefficients $\{\alpha_i;\ i = 1, 2, 3, 4, 5\}$ such that

$$\sum_{i=1}^{5} \alpha_i \underline{u}_i = 0. \tag{A.1}$$

From (A.1), we must have

$$\begin{aligned} \alpha_1 + \alpha_5 \sin\theta_A(k) &= 0 \\ \alpha_1 + \alpha_2 \Delta t_{k+1} + \alpha_5 \sin\theta_A(k+1) &= 0 \\ \alpha_1 + \alpha_2 \Delta t_{k+2} + \alpha_5 \sin\theta_A(k+2) &= 0, \end{aligned} \tag{A.2}$$

which has a solution of $\alpha_i = 0$ for $i = 1, 2, 5$ provided that $\theta_A(k+2) \neq \theta_A(k+1)$. Substituting the solution $\alpha_i = 0$ for $i = 1, 2, 5$ back into (A.1) yields $\alpha_3 = 0$ and $\alpha_4 = 0$. Thus, there does not exist a set of non-zero coefficients $\{\alpha_i, i = 1, 2, \ldots, 5\}$ that satisfies (A.1). In other words, $\{\underline{u}_i;\ i = 1, 2, \ldots, 5\}$ are independent of each other. Similarly, it can be proved that $\underline{u}_6$ is linearly independent of $\underline{u}_i;\ i = 1, 2, 3, 4\}$. Consider the relationships between $\underline{u}_5$ and $\underline{u}_6$. Let

$$\alpha_5 \underline{u}_5 + \alpha_6 \underline{u}_6 = 0, \tag{A.3}$$

It can be verified that $\alpha_6 = 0$ and

$$\begin{aligned} \alpha_5 \sin\theta(k+1) &= 0 \\ \alpha_5 \cos\theta(k+1) &= 0, \end{aligned} \tag{A.4}$$

and which gives $\alpha_5 = 0$, meaning that $\underline{u}_5$ and $\underline{u}_6$ are linearly independent. With all the independent relationships among the columns of U, it can be concluded that the columns of U are linearly independent, and the observability matrix has a full column rank.

References

1. D. L. Hall, *Mathematical techniques in multisensor data fusion*, Artech House, Norwood, MA, 1992

2. M. P. Dana, "Registration : A prerequisite for multiple sensor tracking", In Y. Bar-Shalom (ed.), *Multitarget-Multisensor Tracking : Advanced Applications*, Artech House, 1990

3. Y. Zhou, H. Leung and P. Yip, "An exact maximum likelihood registration for data fusion", *IEEE Trans. Signal Processing*, vol. 45, no. 6, pp. 1560-1573, June 1997

4. R. E. Helmick and T. R. Rice, "Removal of alignment errors in an integrated system of two 3-D sensors", *IEEE Trans. AES*, vol. 29, no. 4, pp. 1333-1343, October 1993

5. W. Fischer, C. Muche and A. Cameron, "Registration errors in a netted air surveillance system", MIT Lincoln Labs, Technical Notes 40, 1980

6. H. Leung, M. Blanchette and C. Harrison, "A least squares fusion of multiple radar data", *Proc. International Conf. Radar'94*, pp. 364-369, Paris, May 1994

7. S. Dhar, "Application of a recursive method for registration error correction in tracking with multiple sensors", *Proc. American Control Conference*, pp. 869-874, San Francisco, CA, June 1993

8. E, J. Dela Cruz, A. T. Alouani, T. R. Rice and W. D. Blair, "Estimation of sensor bias in multisensor systems", *Proc. IEEE Southeast Con'92*, pp. 210-214, Birmingham, Alabama, 1992

9. H. A. P. Blom, R. A. Hogendoorn and B. A. van Doorn, "Design of a multisensor tracking system for advanced air traffic control", in Y. Bar-Shalom *ed.*, *Multitarget-Multisensor Tracking: Applications and Advances*, pp. 31-64, volume II, Artech House, Norwood, MA, 1992

10. Y. Zhou and H. Leung, "A two-step extended Kalman filter fusion approach for misaligned sensors", *Proc. the First International Conference on Multisource Multisensor Fusion*, Las Vegas, July 1998

11. B. Friedland, "Treatment of bias in recursive filtering", *IEEE Trans. Automat. Contr.*, vol. AC-14, pp 359-367, August 1969

12. M. B. Ignagni, "An alternative derivation and extension of Friedland's two-stage Kalman estimator", *IEEE Trans. Automat. Contr.*, vol. AC-26, no. 3, pp 746-750, June 1981

13. R. E. Helmick, J. E. Conte, S. A. Hoffman and W. D. Blair, "One-step fixed-lag IMM smoothing for alignment of asynchronous sensors", *Proc. 1994 SPIE Aerospace Sensing Symposium - Signal and Data Processing of Small Targets*, vol. 2235, pp. 507-518, Orlando, FL, April 1994

14. A. H. Jazwinski, *Stochastic processes and filtering theory*, New York : Academic, 1970

15. M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice*, Englewood Cliffs, NJ: Prentice-Hall, 1993

16. Y. Song and J. W. Grizzle, “The extended Kalman filter as a local asymptotic observer for nonlinear discrete-time systems”, *J. Math. Syst. Estim. Contr.*, vol. 5,no. 1, pp. 59-78, 1995

17. B. D. O. Anderson and J. B. Moore, “Detectability and stabilization of time-varying discrete-time linear systems”, *SIAM J. Control and Optimization*, vol. 19, no. 1, pp. 20-32, January 1981

18. X. R. Li and V. P. Jilkov, “A survey of maneuvering target tracking - Part III: Measurement models”, *Proc. 2001 SPIE Aerospace Sensing Symposium - Signal and Data Processing of Small Targets*, vol. 5204, pp. 423-446, San Diego, CA, July-August 2001

19. K. H. Kim and P. A. Smyton, “Stereographic projection in netted radar system”, MITRE Corporation Technical Report, No. 10296, May 1988

20. S. S. Blackman, *Multiple Target Tracking with Radar Applications*, Artech House, Dedham, MA, 1986

21. G. H. Golub and C. F. Van Loan, *Matrix computation*, Baltimore, MD, John Hopkins University Press, 1984

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In this paper, a Kalman filter based registration approach is proposed for multiple sensors with asynchronous measurements. A time-variant linear sensor measurement model is obtained using a first-order approximation. The observability analysis is carried out and it is shown the system is *uniformly completely observable*. A modified measurement model is formulated which includes the asynchronous sensor models and the two-stage Kalman estimator is applied to estimate the target states along with the sensor bias errors. The proposed method can be implemented using a parallel structure and is computationally efficient. Simulation and real-life sensor data are used to demonstrate the effectiveness of the proposed approach. The results are compared with other existing registration techniques.

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