Modeling the turbulent trailing ship wake in the infrared

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The sea surface turbulent trailing wake of a ship, which can be rather easily observed in the infrared by airborne surveillance systems, is a consequence of the difference in roughness and temperature between the wake and the sea background. We have developed a phenomenological model for the infrared radiance of the turbulent wake by assuming that the sea surface roughness is dependent upon the turbulent intensity near the sea surface. Describing the sea surface roughness with a Cox and Munk probability distribution function of slopes, we distinguish on the sea surface between the sea background and the turbulent wake by the variance of sea surface slopes, \( \sigma_{CM}^2 \) and \( \sigma_{TW}^2 \). The latter dependence is assumed to be inversely proportional to the turbulent intensity of the wake, \( U_{rms}(x,y) \).

Given the incident solar, atmospheric, and sky infrared radiances, we calculate the reflected and emitted sea surface radiance from both the wake and the background. We compare the infrared contrast of the wake with infrared image data obtained in an airborne trial. Our predictions and the measurements agree very well in trend over a significant range of observer zenith angles. Our calculations reveal the strong dependence of the wake radiance on the observer zenith angle, allowing for positive and negative contrasts with the background.

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1. Introduction

Images, and the data extractable from them, are an indispensable component of a modern defense analysis capability. In the maritime domain the sea surface, which is seldom stationary, has a broad range of appearances in the visible and infrared optical bands. The optical appearance of the sea surface is influenced by wind, sky, and clouds so much so that the basic measurables, such as brightness or contrast, vary significantly with prevailing and changing conditions. This sensitive dependence may be exacerbated in the infrared, where the sea surface and the atmosphere are themselves thermal sources.

The sea surface is complex. The surface is not constant, it is not always mathematically smooth having cusps and singularities when it is rough, it extends to the limits of visibility, its appearance depends on the position of the observer (the viewing angle from the average normal to the sea surface) reflecting different parts of the sky, which can be clear, cloudy, etc. There are several approaches to modeling the sea surface and its optical appearance visually and in the infrared [1–5]; however, given the perpetual motion and inherent stochasticity of the sea surface, the measurables are invariably temporal or spatial averages. Yet, there are often deterministic or long-lived coherent features on the sea surface, such as ship wakes, swells, and gravity waves. Ship wakes, in particular, are important features in airborne and satellite-based surveillance (see, e.g., [6]) and are perceived to be a vulnerability for defense purposes.

Ship wakes consist of a physically complex disturbance of the air–sea interface and subsurface water. In infrared imagery of ship wakes, there are three distinct components: the Kelvin wake, the turbulent wake, and the white water wake. The Kelvin wake
consists of a modulation of sea surface elevation imprinted on a background of the wind-driven sea surface. The white water wake consists of air–water interfaces that are strongly scattering and evolve rapidly, breaking up and being reabsorbed into the sea. The turbulent wake is the relatively smooth centerline wake starting just after the bubbly turbulent zone behind the ship. This zone is relatively smooth because of the attenuation of the short waves by ship-generated turbulence from upwelling flows and surface tension.

The above features are equally well-observed in the infrared as in the other types of imagery. The variation in the sea surface elevation of the Kelvin wake highly affects the orientation of the sea surface facets and consequently its infrared radiance. The white water wake is a strong scattering medium that is created by the bow and propeller. As the sea temperature often varies significantly with depth, the turbulent wake created by the mixing effect of the propulsion system may have a different temperature than the ambient sea surface. Furthermore, the turbulent wake is smoother than the sea surface background. This variation in the roughness and temperature results in an infrared radiance contrast between the turbulent wake and the sea surface background that can be easily detectable under most conditions.

Here, we have developed a model for simulating the turbulent wake and the ambient sea surface radiance in the infrared. We also developed a sea surface radiance simulator (SSRS) based on this model. The simulator calculates the contrast between the turbulent wake and the sea surface background using the sky spectral radiance and the atmosphere spectral transmittance outputs from ShipIR [7]. To validate our model, we have compared the results generated by the SSRS with data collected from airborne imaging trials performed in October 2006. Our predictions and the measurements agree very well in trend over a significant range of observer zenith angles. Our calculations reveal the strong dependence of the wake radiance on the observer zenith angle, allowing for positive and negative contrasts with the background. This paper is organized as follows: Section 2 describes the overall model of the SSRS; Section 3 details the turbulent wake radiance model and the contrast calculation; Section 4 presents our results and compares the results with field data; and Section 5 concludes the paper.

2. Overall Model

Our approach to modeling the sea surface radiance in the infrared is designed to include both a coherent ship wake structure and an averaged background component. The ship wake structure consists, geometrically, of a Kelvin gravity wave field, a white water wake, and a turbulent trailing wake. The background component is derived from the probabilistic description of slopes of the sea surface driven by the wind. Combining the sky and atmospheric radiation that is reflected by these features on the sea surface with the emitted radiation from the water and the radiation along the path, we can model the total radiance received by the observer. While this general "summation of different parts" approach appears simple in principle, its implementation is rife with intricacies.

The geographical space of the sea surface is divided into four parts: background zone, turbulent wake zone, Kelvin wake zone, and white wake zone. The main difference between the first three zones is the roughness function that describes the variance of the probability density function of the facet slopes. In the background zone this variance is constant, while it varies in the turbulent wake and in the Kelvin wake. In this paper, we have described our modeling of the turbulent wake. In a subsequent paper, we will describe our modeling of the Kelvin wake.

Figure 1 shows the general architecture of the SSRS. The IR radiance of each zone as seen by one pixel of the receiver is simulated. Knowing the receiver position and orientation, as well as the target information, the geographical area seen by each camera pixel is determined, as well as the path toward the receiver. Once the geographical space seen by one pixel is identified, the appropriate roughness function corresponding to the background, or turbulent, wake is determined. Then, this area is considered as an
ensemble of facets that acts like an intrinsic thermal source and as a mirror that reflects the sky radiance. To simulate the sea surface background we used a model based on Cox and Munk’s probability density function (PDF) [8] for the sea surface slopes. To model the turbulent wake we modify Cox and Munk’s PDF in the turbulent zone, as developed in Section 3.

For each point of the discretized space a PDF of the different possible facets is generated. For each possible facet and relative receiver position, the geometrical related parameters such as the sky orientation seen by it, the normal orientation, and all the parameters necessary to find the probability density function for the slopes are calculated. The orientation of the sky radiance to be reflected on each possible facet, as well as the optical properties, the spectral sea surface intrinsic radiance, and the spectral sky radiance reflected on the sea surface are computed. Finally, the total radiance of the discretized space is computed for the whole wavelength band. Figure 2 shows the high level algorithm for this calculation.

A. Sea Surface Background

Each zone from the ambient sea surface detected by one pixel of the receiver can be seen as an ensemble of facets. Each facet has its orientation defined by its upwind, \( z_x = \partial z / \partial x \), slope and crosswind slope, \( z_y = \partial z / \partial y \), when the sea surface is driven by a wind speed, \( w \), m/s. Each facet acts simultaneously as a source that emits its own infrared radiance, \( L_e(\theta_r, \phi_r, z_x, z_y) \), toward a receiver located at \( R(\theta_r, \phi_r, h_r) \) and as a mirror with reflected sky radiance, \( L_r(\theta_r, \phi_r, z_x, z_y) \). Figure 3 shows the different parameters of a sea surface facet geometry. The emitted radiance can be found with the law of blackbody radiation, \( P(T, \lambda) \), at temperature, \( T \), and wavelength, \( \lambda \). The reflected radiance can be found with the geometric reflection of sky radiance, \( L_s(\lambda, \theta_s, \phi_s) \). Hence,

\[
L(\lambda, T, \theta_r, \phi_r, z_x, z_y) = L_e(\lambda, T, \theta_r, \phi_r, z_x, z_y) + L_r(\lambda, \theta_r, \phi_r, z_x, z_y) \\
= [1 - \rho(\omega, \lambda)] \times P(T, \lambda) \\
+ \rho(\omega) \times L_s(\lambda, \theta_s, \phi_s),
\]

where \( \rho(\omega, \lambda) \) is the spectral reflectivity of the sea surface in the direction of the receiver. \( L_e(\lambda, \theta_e, \phi_e) \) is the sky radiance arriving at the facet from the direction \((\theta_e, \phi_e)\) where \( \theta_e \) and \( \phi_e \) are the coordinates of the part of sky reflected on the facet and they are dependent on \( \theta_r, \phi_r, z_x \) and \( z_y \). Also, as shown in Fig. 3, \( \omega = \omega_r(\theta_r, \phi_r, z_x, z_y) = \omega_s(\theta_r, \phi_r, z_x, z_y) \), the angle between the receiver and the normal to the facet, \( \hat{n}(\theta_n, \phi_n, z_x, z_y) \). Expressions for \( \omega, \theta_s, \) and \( \phi_s \) are found by simple geometry and are given in Appendix A.
The total spectral radiance, \( L(\lambda, T, \theta_r, \phi_r) \), outgoing from an area seen by one pixel is the sum of the radiance emitted and reflected from all the facets in this area pointing toward the receiver. The contribution of each facet to the radiance received by the detector corresponds to its relative, or fractional, area toward the receiver, \((dA_r(\theta_r, \phi_r, z_x, z_y)/A_r(\theta_r, \phi_r))\), so

\[
L(\lambda, T, \theta_r, \phi_r) = \int \int_{\omega \leq \omega} \frac{dA_r(\theta_r, \phi_r, z_x, z_y)}{A_r(\theta_r, \phi_r)} L(\lambda, T, \theta_r, \phi_r, z_x, z_y) \, dz_x \, dz_y
\]

(2)

with \( A_r \), the total area projected toward the receiver, and \( dA_r(\theta_r, \phi_r, z_x, z_y) \) is the projection of the facet with slopes \( z_x, z_y \), toward the receiver. We show in Fig. 3 the coordinate system and the geometry of a sea surface facet and the different geometric parameters for a geographic area seen by one pixel in Fig. 4.

The relative area of each facet toward the receiver can be found from the relative horizontal area given by the Cox and Munk probability distribution density function, using a simple geometric projection,

\[
dA_r(\theta_r, \phi_r, z_x, z_y) = \cos \omega(\theta_r, \phi_r, z_x, z_y) \cos \theta(\theta_r, \phi_r, z_x, z_y)
\]

\[
\times dA_h(\theta_r, \phi_r, z_x, z_y).
\]

(3)

The relative horizontal area is given by the following Cox and Munk probability distribution density function [1,5]

\[
dA_h(\theta_r, \phi_r, z_x, z_y) = p_{cm}(z_x, z_y, w) \, dz_x \, dz_y,
\]

(4)

where \( A_h \) is the horizontally projected total area seen by one pixel of the receiver, \( dA_h(\theta_r, \phi_r, z_x, z_y) \) is the horizontal projection of the facet with slopes \( z_x, z_y \), and \( p_{cm} \) given by

\[
p_{cm}(z_x, z_y, w) \approx \frac{1}{2\pi \sigma_u \sigma_c} \exp \left[ -\frac{1}{2} \left( \frac{z_x^2}{\sigma_u^2} + \frac{z_y^2}{\sigma_c^2} \right) \right].
\]

(5)

In this equation, the variances in the upwind and crosswind directions, respectively, are

\[
\sigma_u^2 = 3.16 \times 10^{-3} \, w,
\]

\[
\sigma_c^2 = 0.003 + 1.92 \times 10^{-3} \, w.
\]

(6)

The total projected area toward the receiver is constant for given receiver characteristics and position, unlike the total horizontal projected area, which is dependent on the roughness. Since some of the facets do not point toward the receiver, \( A_r \neq A_h \cos(\omega) \), but

\[
A_r = \int \int_{\omega \leq \omega} \frac{dA_r(\theta_r, \phi_r, z_x, z_y)}{A_r(\theta_r, \phi_r)} \, dz_x \, dz_y
\]

if we neglect the shadowing problem. This implies that

\[
\frac{dA_r(\theta_r, \phi_r, z_x, z_y)}{A_r(\theta_r, \phi_r)} = \frac{\int \int_{\omega \leq \omega} dA_r(\theta_r, \phi_r, z_x, z_y) \, dz_x \, dz_y}{A_r(\theta_r, \phi_r)}.
\]

(7)

and then the contribution toward the receiver from each facet becomes

\[
\frac{dA_r(\theta_r, \phi_r, z_x, z_y)}{A_r(\theta_r, \phi_r)} = \frac{\cos \omega(\theta_r, \phi_r, z_x, z_y)}{\cos \theta(\theta_r, \phi_r, z_x, z_y)} p_{cm}(z_x, z_y, w) \, dz_x \, dz_y
\]

(8)

Using this result in Eq. (2), the spectral radiance of the background becomes

\[
L(\lambda, \theta_r, \phi_r, x, y) = \int \int_{\omega \leq \omega} \xi p_{cm}(z_x, z_y, w, x, y) \, dz_x \, dz_y
\]

(9)

with \( \xi = aL(\lambda, T, \theta_r, \phi_r, z_x, z_y) \), \( a = (\cos \omega(\theta_r, \phi_r, z_x, z_y))/\cos \theta(\theta_r, \phi_r, z_x, z_y) \), and the total midwave radiance outgoing toward a receiver from a geographical area seen by a pixel is found by
\[ L(T, \theta_r, \phi_r, x, y) = \int_{\lambda_1}^{\lambda_2} L(\lambda, T, \theta_r, \phi_r, x, y) d\lambda. \] (10)

The total inband wavelength is between \( \lambda_1 \) and \( \lambda_2 \).

### 3. Turbulent Wake Modeling

In October 2006, during a cooperative surveillance trial (Q300), the Canadian forces maritime patrol aircraft (MPA), Aurora, made multiple passes over the Canadian forces auxiliary vessel (CFAV) Quest, and collected midwave IR (3–5 \( \mu \)m) data. These data were analyzed in previous publications [10] and the peak wake contrast of the turbulent wake behind Quest was measured, as a function of distance astern, and found to decrease downstream. The decay observed was initially a gentle decrease, crossing over to a steeper power-law decay, \( \propto x^{-(4/5)} \), with distance.

Following this previous study, since we assume the damping of short waves proportional to the turbulence, we propose the roughness of the sea surface in the trailing wake to be inversely proportional to the turbulence intensity. We consider the turbulence intensity, \( U_{\text{rms}} \), in the turbulent wake to be the turbulent field from a hydrodynamical self-propelled cylinder [10–13],

\[ U_{\text{rms max}} \propto x^{-\frac{4}{5}}, \] (11)

where \( x \) is the downstream distance. The dependence of the turbulence intensity in the cross stream direction (\( y \)) is given by

\[ U_{\text{rmsy}} \propto \left( 1 - \frac{1}{2} \frac{y}{l(x)} \right) e^{-\frac{y^2}{l(x)}}. \] (12)

where

\[ \zeta = \frac{y}{l(x)} \] (13)

and

\[ l \propto x^{\frac{1}{5}}. \] (14)

Hence, we build a roughness function for the turbulent wake with cross stream and downstream variations that are inversely proportional to the turbulence intensity and approach the background roughness far from the ship. We also constrained the roughness function to be valid for \( |y| < l \), the cross stream width of the turbulence intensity function.

The resulting turbulent wake roughness function is given by

\[ \sigma_{\text{TW}}^2 = \left[ \frac{1}{U_A} \frac{1}{U_{\text{rmsy}}} + \frac{1}{U_B} \right] * \Gamma + (1 - \Gamma) \frac{1}{U_B}. \] (15)

with

\[ \Gamma = \begin{cases} 0 & \text{if } y > l, \\ 1 & \text{if } y \leq l. \end{cases} \] (16)

where \( U_A \) is the maximum turbulence intensity that corresponds to the minimum roughness with variance of sea surface slopes, \( \sigma_A^2 \). \( U_B \) is the turbulence intensity of the background with variance, \( \sigma_B^2 = \sigma_{\text{CM}}^2 \).

For an isotropic Cox and Munk probability distribution [8], the roughness is represented by the variance of the probability density function of slopes and is given by

\[ \sigma_{\text{CM}}^2 = \frac{0.003 + 0.00512w}{2}. \] (17)

From Eq. (17) we can see that the minimum roughness corresponding to zero wind speed is \( \sigma_A^2 = 0.0015 \). This allows us to determine the limits of the constants, \( U_A = (1/\sigma_A^2) \) and \( U_B = (1/\sigma_{\text{CM}}^2) \).

We used the trailing wake roughness function in Eq. (15) to find a new probability distribution density function, \( p_{\text{tw}}(z_x, z_y, w, x) \), for the trailing wake by substituting it for the slope variance in the Cox and Munk PDF:

\[ p_{\text{tw}}(z_x, z_y, w, x) \approx \left[ \frac{\sigma_{\text{CM}}^2}{\sigma_A^2} x^{-4/5} + 1 \right] p_{\text{CM}}(z_x, z_y, w) \]

\[ * e^{-\left( \frac{y^2 + z^2}{\sigma^2} \right)} \], (18)

where \( p_{\text{CM}}(z_x, z_y, w) \) is the isotropic Cox and Munk probability distribution,

\[ p_{\text{CM}}(z_x, z_y, w) \approx \frac{1}{2\pi\sigma_{\text{CM}}^2} e^{-\frac{z^2 + w^2}{2\sigma_{\text{CM}}^2}}. \] (19)

The inband, \( L_{\text{tw}}(\lambda, \theta_r, \phi_r, x, y) \), and spectral, \( L_{\text{tw}}(\theta_r, \phi_r, x, y) \), radiance outgoing from an area of the trailing wake seen by a pixel are found using the same modeling and algorithms used to find the background radiance, but using the trailing wake PDF of slope facets. Hence, the expression for the trailing wake radiance becomes

\[ \iint \xi_{\text{tw}} p_{\text{tw}}(z_x, z_y, w, x, y) dz_x dz_y \]

\[ L_{\text{tw}}(\lambda, \theta_r, \phi_r, x, y) = \frac{\iint \xi_{\text{tw}} p_{\text{tw}}(z_x, z_y, w, x, y) dz_x dz_y}{\iint \xi_{\text{tw}} p_{\text{tw}}(z_x, z_y, w, x, y) dz_x dz_y}. \] (20)
with \( \zeta_{tw} = aL_{tw}(\lambda, T, \theta_r, \phi_r, z_x, z_y) \), where

\[
L_{tw}(\theta_r, \phi_r, x, y) = \int_{\lambda_1}^{\lambda_2} L_{tw}(\lambda, \theta_r, \phi_r, x, y) d\lambda. \tag{21}
\]

We should note that, as \( \theta_r \) and \( \phi_r \) are dependent on \( x \) and \( y \), the radiance of the background is space-dependent for a uniform wind speed. In the case of radiance of the turbulent wake, the roughness and the temperature are space-dependent as well, which increases the dependence on \( x \) and \( y \). We show in Fig. 5 the distribution of the roughness and turbulence in space as a function of cross stream and downstream distance from the ship.

Figure 6 shows the Cox and Munk PDF for two different wind speeds. As expected, we can see that high wind speeds have a greater probability of occurrence at higher wind speeds corresponding to a rough sea surface. Figure 7 shows the trailing wake PDF as a function of the slopes, \( z_x \) and \( z_y \), at a wind speed of 4 m/s for two different positions: just behind the ship at 30 m and far from the ship at 500 m. As expected, at 500 m, higher slopes have a greater probability of occurrence to reach the roughness of the background, as the sea surface roughness increases far away from the ship. In this case, \( \sigma_{TW}(w = 4 \text{ m/s}, x = 500 \text{ m}) = 0.0116 \) is closer to \( \sigma_{CM}(w = 4 \text{ m/s}) = 0.0117 \) than \( \sigma_{TW}(w = 4 \text{ m/s}, x = 30 \text{ m}) = 0.0107 \), where the sea surface is smoother.

A. Contrast Modeling

The contrast between the radiance of the trailing wake and the radiance of the sea surface background is defined by the difference between these two radiances. In our modeling, the contrast becomes

\[
\text{Contrast} (\lambda, x, y) = L_{tw}(\lambda, T_{tw}, \theta_r, \phi_r, x, y) - L_{bck}(\lambda, T_{bck}, \theta_r, \phi_r, x, y)
\]

\[
\approx \int_{\frac{x-2z}{2}}^{\frac{x+2z}{2}} \int_{\frac{y-2z}{2}}^{\frac{y+2z}{2}} p_{tw}(z_x, z_y, w, x, y) dz_x dx_y
\]

\[-\int_{\frac{x-2z}{2}}^{\frac{x+2z}{2}} \int_{\frac{y-2z}{2}}^{\frac{y+2z}{2}} p_{cm}(z_x, z_y, w, x, y) dz_x dz_y, \tag{22}
\]

where \( \zeta_{bck} = aL_{bck}(\lambda, T_{bck}, \theta_r, \phi_r, z_x, z_y) \), \( \zeta_{bck} \) and \( a \) are as defined previously. For the central line of the trailing wake, \( U_{rms} = 1 \), which implies that

\[
p_{tw}(z_x, z_y, w, x, y)
\]

\[
\approx \left[ \frac{\sigma_{cm}^2}{\sigma_A^2} x^{-4/5} + 1 \right] \left[ \frac{\sigma_{cm}^2}{\sigma_A^2} x^{-4/5} \right]^2 e^{-\frac{1}{2} \left( \frac{w}{\sigma_A^2} \right)^4} \tag{23}
\]

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Hence, the contrast of the central line of the trailing wake becomes

\[
\text{Contrast}(\lambda, x, y) = \left\{ \begin{array}{l}
\int \int_{\xi_{1a} \leq \xi \leq \xi_{2a}} \xi_{bw} e^{-\frac{\xi_{1a}^2 + \xi_{2a}^2}{2\lambda}} p_{cm}(z_x, z_y, w, x, y)dz_x dz_y \\
\int \int_{\xi_{1b} \leq \xi \leq \xi_{2b}} \xi_{bck} p_{cm}(z_x, z_y, w, x, y)dz_x dz_y \\
\int \int_{\xi_{1b} \leq \xi \leq \xi_{2b}} \xi_{bck} p_{cm}(z_x, z_y, w, x, y)dz_x dz_y \\
\int \int_{\xi_{1b} \leq \xi \leq \xi_{2b}} ap_{cm}(z_x, z_y, w, x, y)dz_x dz_y,
\end{array} \right.
\]

which, after expanding in a Maclaurin series, gives

\[
\text{Contrast}(\lambda, x, y) = \sum_{n=0}^{\infty} \left( \frac{-x\xi_{1a}^2}{2\sigma_x^2} \right)^n \frac{1}{n!} y_{1a}^n - \frac{\gamma_{1b}}{\gamma_{1a}} y_{2a},
\]

where

\[
\gamma_{1a}^k = \int \int_{\omega_{1a} \leq \omega \leq \omega_{2a}} \xi(z_x^2 + z_y^2) p_{cm}(z_x, z_y, w, x, y)dz_x dz_y
\]

and

\[
\gamma_{1a}^k = \int \int_{\omega_{1a} \leq \omega \leq \omega_{2a}} \alpha(z_x^2 + z_y^2) p_{cm}(z_x, z_y, w, x, y)dz_x dz_y.
\]

The Maclaurin series expansions in Eq. (25) are convergent. Approximating this sum by taking the first two terms results in

\[
\text{Contrast}(\lambda, x, y) \approx \frac{2\sigma_x^2 \gamma_{1a}^0 \gamma_{1b}^0 - \gamma_{1a}^0 \gamma_{1b}^0}{2\sigma_x^2 (\gamma_{1a}^0)^2} + \frac{\gamma_{1b}^0 \gamma_{1a}^0 - \gamma_{1b}^0 \gamma_{1a}^0}{2\sigma_x^2 (\gamma_{1a}^0)^2} x^{-4/5}.
\]

The higher order terms are negligibly small for a downstream distance, \( x > \left[ \gamma_{1b}^0 / 2\sigma_x^2 (\gamma_{1a}^0)^2 \right]^{3/4} \).

For negligible temperature variations between the trailing wake and the background, \( \xi_{1b} = \xi_{bck} = \xi \). This case occurs when the sea temperature is constant in the trailing wake. With constant temperature, the trailing wake contrast is only dependent on roughness:

\[
\text{Contrast}(\lambda, x, y) \approx \left[ \gamma_{1a}^0 - \gamma_{1b}^0 \right] x^{-4/5}.
\]

The inband contrast can be found by

\[
\text{Contrast}(x, y) = \int_{\lambda_{\text{a}}}^{\lambda_{\text{b}}} \text{Contrast}(\lambda, x, y) d\lambda.
\]

For a relatively far broadside observer, where the variation of \( \theta_r \) and \( \phi_r \) is negligible along the length of the trailing wake, the term \( (\gamma_{1a}^0 - \gamma_{1b}^0) / 2\sigma_x^2 (\gamma_{1a}^0)^2 \) becomes independent of spatial position in the trailing wake. Thus, the trailing wake contrast outgoing from an area with negligible temperature variation seen by one pixel in the central line becomes

\[
\text{Contrast}(x, y) \approx Cx^{-4/5},
\]

where

\[
C = \int_{\lambda_{\text{a}}}^{\lambda_{\text{b}}} \left[ \gamma_{1a}^0 - \gamma_{1b}^0 \right] d\lambda.
\]
B. Contrast of the Trailing Wake with Atmospheric Propagation

To determine the reflected and emitted radiance of the sea surface seen by a receiver pixel, the intrinsic atmosphere scattering radiance and the atmospheric attenuation have to be taken into account. As the contrast is the difference between the radiance of the trailing wake and the radiance outgoing from the background around the trailing wake, the intrinsic radiance of the atmosphere along the path between the sea surface and the receiver vanish in the subtraction, but the attenuation effect of the atmosphere has to be considered. The attenuation of the atmosphere is wavelength dependent with the spectral atmosphere transmission coefficient of the atmosphere, $\tau$. The resulting contrast is given by

$$\text{Contrast}(\lambda, x, y) = \tau(\lambda)L_{\text{tw}}(\lambda, T_{\text{tw}}, \theta_r, \phi_r, x, y) - \tau(\lambda)L_{\text{bck}}(\lambda, T_{\text{bck}}, \theta_r, \phi_r, x, y). \quad (30)$$

For the case of the relatively far broadside observer discussed above, we get an approximate inband contrast given by

$$\text{Contrast}(x, y) \approx Cx^{-4/5}, \quad (31)$$

where

$$C = \int_{\lambda_1}^{\lambda_2} \tau(\lambda) \left[ \frac{\sin^2 \theta - \sin^2 \theta_0}{2\sigma_A^2(\sin \theta_0)^2} \right] d\lambda.$$

4. Results and Discussion

We used ShipIR\cite{7} to generate the sky radiance, $L_s(\lambda, \theta, \phi_r)$, data using MODTRAN4\cite{14}. In Table 1 we list the different parameters of the atmosphere simulated with ShipIR for the atmospheric conditions during Q300. Geographical and temporal information, such as position and time, as well as climatic information, such as wind speed and direction and sea and air temperatures, were specified in ShipIR in order to generate the sky radiance data. These data were exported to csv files containing the spectral thermal emission of the atmosphere, $L_{\text{atm}}$, the spectral indirect solar radiance, $L_{\text{scat}}$, and the spectral direct Sun radiance, $L_{\text{sun}}$. $L_{\text{atm}}$ is a function of the air temperature and the zenith direction. $L_{\text{scat}}$ is scattered mostly by water molecules and the dust particles in the atmosphere. $L_{\text{scat}}$ is dependent on the air mass parameter, the relative humidity, the zenith angle, and the azimuth angle. $L_{\text{sun}}$ is the direct Sun radiance, so it is dependent on the solar zenith and azimuth angles. From these three contributions, $L_s(\lambda, \theta, \phi_r)$ was found and used in our simulator to calculate the reflected sky radiance on the sea surface. Furthermore, the emitted sea surface radiance was calculated and added to the reflected sky radiance for both the background and the trailing wake.

For a receiver at an altitude of 300 m and at a range of 6500 m cross stream distance from the trailing wake, the variation of the relative position for a trailing wake of length 300 m is less than 0.03° for $\theta_r$ and less than 3° for $\phi_r$. Considering a constant sea surface temperature in the trailing wake, we have simulated the radiance of the central line of the trailing wake and the radiance of the background and, consequently, we have found the contrast with $L_{\text{tw}}(T, \theta_r, \phi_r, x, y) - L_{\text{bck}}(T, \theta_r, \phi_r, x, y)$. This is the exact contrast given by Eq. (22). We also calculated numerically the constant, $C$, as defined in Eq. (29) to find the approximated contrast. We show in Fig. 8 the exact and the approximated contrasts obtained from Eq. (22) and Eq. (29), in blue (dashed) and red (solid), respectively. In this calculation we used $\sigma_A^2 = 0.0078$, which corresponds to $2\sigma_{cm}^2/3$. For this $\sigma_A^2$, the approximation given by Eq. (26) is valid for $x > 5$ m. Note the very good agreement between the exact and approximate contrast in Fig. 8, for $x \geq 10$ m.

We simulated the contrast of the trailing wake for different receiver angles and we found that the contrast was strongly dependent on the receiver’s zenith position. The trailing wake contrast was dependent on the receiver range, as well. In Fig. 9 we show the variation of the parameter, $C$, with the receiver position [Eq. (29)]. The variation in the receiver

<table>
<thead>
<tr>
<th>Table 1. Background Parameters for the Trial Q300</th>
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<tbody>
<tr>
<td>Sky type</td>
</tr>
<tr>
<td>Wind direction</td>
</tr>
<tr>
<td>Wind speed</td>
</tr>
<tr>
<td>Relative humidity</td>
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<tr>
<td>Ambient temperature</td>
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<td>Sea temperature</td>
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<tr>
<td>Location</td>
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<tr>
<td>Time</td>
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<tr>
<td>Sun position</td>
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![Fig. 8](image) Contrast radiance of the central line of the trailing wake as a function of downstream distance, (top) in a linear scale and (bottom) in a log–log scale. |
range results in a modification of the propagation path length. This change in the path length affects the atmospheric propagation, thus modifying the contrast of the trailing wake. The variation of the zenith angle of the receiver has a very strong effect on the contrast of the trailing wake. Depending on the sea surface roughness and temperature, the variation of the zenith angle of the receiver can result in negative and positive contrast radiances.

Observing the sea from a high receiver zenith angle results in an infrared radiance that is mainly reflected, due to the large reflectivity coefficient and small emissivity coefficient of the smooth sea surface (Fig. 10). When the sea surface roughness increases, the probability of occurrence of sea surface facets with high slopes increases as well. In fact, for a receiver with a high zenith angle, the angle, $\omega_R$, between the receiver and the normal to those facets is more likely to decrease compared to a horizontal facet (Fig. 11). Hence, the emissivity coefficient increases and, consequently, the thermal emitted radiance grows while the reflectivity coefficient decreases and, thus, the sky reflected radiance diminishes. At the same time, the zenith angle of the sky reflected on those facets is smaller than the zenith angle of the sky reflected on a flat sea surface, which further decreases the reflected sky radiance as the sky radiance generally increases with its zenith angle. As a result of the roughness variation in the trailing wake, the total radiance of the trailing wake decreases when the roughness increases in the downstream direction, neglecting direct solar reflection.

The variation of the temperature in the trailing wake as well as the roughness also has an effect on the contrast of the trailing wake for a high receiver angles. With increasing temperature in the trailing wake, the emitted radiance of the sea surface increases but the reflected contribution is not affected. Hence, the total radiance of the trailing wake increases in the downstream direction. We show this temperature variation effect on the total emitted and reflected radiance in Fig. 12.

For a high receiver angle, where the reflectivity coefficient is high and emissivity coefficient is low, a variation in the reflected radiance due to increasing roughness in the wake can overcome the variation in the emitted radiance due to the relaxation of the temperature in the trailing wake. Fig. 12 shows the roughness and temperature variation effects where we can see that the roughness variation overcomes...
the temperature variation. In this case, the temperature variation in the trailing wake has the same trend as the roughness, with a 4°C maximum difference. The increase in the total radiance of the trailing wake compared to the background has a positive contrast that decreases downstream until it reaches the radiance of the background.

In the case of a low zenith receiver angle, the reflectivity coefficient is small and the emissivity coefficient is large when the sea surface is flat (Fig. 10). Unlike the case of a high receiver angle, as the roughness increases for a low receiver angle, the angle, $\omega_R$, between the receiver and the normal to those facets is more likely to increase compared to a flat facet. Hence, the emissivity coefficient decreases and the reflectivity coefficient increases with increasing sea roughness. Consequently, the thermal emitted radiance decreases and the sky reflected radiance increases. At the same time, the zenith angle of the sky reflected on those facets is more likely to increase compared to the zenith angle of the sky reflected on a flat sea surface. This further increases the reflected sky radiance because the sky radiance increases with zenith angle (Fig. 13). As a result of the roughness variation in the trailing wake, the total radiance of the trailing wake increases with roughness. We show this roughness variation effect on the total, emitted, and reflected radiance in Fig. 14.

Similar to the case with a high receiver angle, the emitted radiance increases with the temperature and the reflected part is not affected. Hence, the total radiance of the trailing wake increases in the downstream direction. We show this temperature variation effect on the total, emitted, and reflected radiance in Fig. 14, with the plus symbol. As the emissivity coefficient is high for a low zenith receiver angle, the variation in the emitted radiance can dominate the variation in the reflected radiance, which can explain the negative contrast in the trailing wakes.

We compared the results of the SSRS with data collected from an airborne imaging trial of a ship trailing wake obtained by an MPA (Aurora) that made multiple passes over CFAV Quest in October 2006. In brief, the trial was in open ocean, approximately 25 nautical miles southwest of Halifax Harbor. Quest cruised at a nearly constant speed of 10 knots. Her heading was chosen so that the Sun was directly astern. Data were collected with the Wescam MX − 20 camera system with a spectral band of 3.4–5.2 µm and a field of view of 21.7° × 16.4°. The 8-bit digital video converted to mpeg video with a 640 × 512 pixel resolution.

Figure 12. Total radiance (top), the emitted radiance (middle), and the reflected radiance (bottom) of the background (blue line), the trailing wake with a variable roughness and constant temperature (°), the trailing wake with a variable temperature and constant roughness (+), the trailing wake with a variable temperature and roughness (red line) toward a low altitude broadside receiver (high zenith angle) located at (0, −1150, 100).

Figure 13. Examples of the geometry for a receiver at a low zenith angle: (left) with no roughness; (middle) example of a high roughness: $\omega$ decreases but $\theta_s$ increases; (right) example of a high roughness: $\omega$ and $\theta_s$ increase.

Figure 14. Total radiance (top), the emitted radiance (middle), and the reflected radiance (bottom) of the background (blue line), the trailing wake with a variable roughness and constant temperature (°), the trailing wake with a variable temperature and constant roughness (+), the trailing wake with a variable temperature and roughness (red line) toward a high altitude broadside receiver (low zenith angle) located at (0 − 1000 1000).
Data collected from four passes were analyzed to study the trailing wake, two passes for each flight altitude of 30 and 300 m. From each pass, the video streams were decomposed into a sequence of image frames. Each image was cropped and rotated so the trailing wake is horizontal and the coordinate system is defined with the downstream distance, \( x = 0 \), at the stern of Quest. Figure 15 shows a typical image frame of Quest and a prominent trailing wake [10]. At the stern, \( n \) vertical columns of data were averaged, with \( n \) chosen to span 8 m. The peak intensity contrast is obtained from the cross stream variation by subtracting out the sea background intensity, which is obtained from a fit to a polynomial using only the data outside the wake, as illustrated in Fig. 16. Hence, each image was analyzed with a sequence of: rotation, cropping, length scale determination, column averaging, sea background fit, and extraction of peak wake intensity. In total, 1300 images were processed for the passes at altitude 30 m between ranges of 750–2250 m. At the higher altitude of 300 m, 3100 images were processed for ranges between 500 and 6500 m. The peak wake contrasts were averaged over range bins of 250 m width, assuming that the speed of approach of the MPA and the altitude are constant through the pass. We show in Fig. 17 the curves of the extracted contrast of the trailing wake; each curve is for a different averaged range bin as a function of the distance astern from the ship. Immediately after the ship, there is a gentle decay of the contrast caused by the foamy white water wake. Beyond this, the decay of the contrast is proportional to \( x^{-4/5} \). We show in the same plot (Fig. 17) the fits to constant \( x^{-4/5} \) of the different curves corresponding to ranges in which there was sufficient downstream data.

For the same atmospheric and geographical conditions given in Table 1, we have simulated the contrast of the trailing wake with the SSRS, assuming that the sea temperature was constant. We show in Fig. 18 the simulated exact contrast radiance from the SSRS calculation [Eq. (22)] and approximation [Eq. (31)] with the \( Cx^{-4/5} \) function for the MPA flights. The variation in the parameter, \( C \), is due to the different receiver positions that were chosen to correspond with the average MPA camera positions in the Q300 data.

In order to validate the simulation results with the trial data, we have to compare the constant from the
fits for the experimental results (constant $x^{-(4/5)}$) to the parameter, $C$, calculated for the different ranges from the ship with Eq. (31). Note that the fit function is constrained to downstream distances sufficiently greater than the smooth decay associated with the white water wake. We show in Fig. 19 the different normalized values of $C$ for each range for the flight at 300 m from the Q300 data and SSRS with atmospheric propagation. Since the measurements are not calibrated, it is necessary to normalize the data in order to compare the power detected by the receiver to the radiance at the receiver predicted by SSRS. After multiplication by the range squared, the Q300 data collected at altitude 300 m were scaled by factor $k_{300} = 7.1 \times 10^{-11}$. Lacking a proper calibration, we can only reasonably compare trends between data and prediction. The agreement in the trend of the two curves in Fig. 19 suggests that our modeling for ranges $>1375$ m is valid. The Q300 data does not contain sufficient images with $x > 80$ m for ranges less than 1375 m to test our simulator. We show in Fig. 20 the same results for the flights at 30 m. The Q300 data collected at altitude 30 m were normalized and scaled by a factor $k_{30} = 5.6 \times 10^{-10}$.

Fig. 18. Peak wake contrast intensity from SSRS for various approach ranges for the MPA flights at (top) 300 m and (bottom) 30 m. Squares with dotted lines denote exact contrast radiance from the SSRS calculation [Eq. (22)] and the solid lines refer to the approximated results [Eq. (31)] with $Cx^{-4/5}$.

Fig. 19. Different $C$ values corresponding to each range for the flight at 300 m, from the Q300 data and SSRS.

Fig. 20. Different $C$ values corresponding to each range for the flight at 30 m from the Q300 data and SSRS.

Fig. 21. Different $C$ values corresponding to each range for the flight at 30 and 300 m from the Q300 data and from SSRS, with and without atmospheric propagation as a function of the zenith angle.
In Fig. 21 we show the parameter, \( C \), corresponding to each range, for the flights at 30 and 300 m, simulated with SSRS for contrast, calculated at the sea surface (green solid lines) and at the receiver (blue dashed lines). The continuity between the values of \( C \) at 30 and 300 m demonstrates the strong effect of the variation in the zenith angle of the receiver. Comparing the green and blue lines, we can confirm that the effect of the atmospheric propagation is greater for the runs at 300 m, where the propagation path length is longer. Also, in this figure we show the different normalized values of the coefficients of the wake contrast from Q300, for the flights at 30 and 300 m.

5. Conclusions

Although the Q300 data were not optimized for the study of the trailing wake, but rather the ship itself, we can conclude from Fig. 21 that the trends of the Q300 wake contrast coefficient constant are in agreement with the simulated results from SSRS. The temperature variation modeling capabilities of SSRS will be tested in future trials designed to study and validate our simulator.

We now describe the richness in the behavior of the turbulent wake contrast. We found that the contrast between the radiances of the trailing wakes and that of the background is highly dependent on the receiver position. In the turbulent wake, decreasing the receiver zenith angle decreases the contribution of the sky radiance by decreasing the reflectivity of the sea surface. Increasing of the receiver zenith angle decreases the thermal emitted sea surface radiance by the decreasing the sea surface emissivity. Therefore, the effect of the roughness variation of the turbulent wake can be better observed from a high receiver zenith angle, while the effect of temperature variation of the turbulent wake can be better observed at a low receiver zenith angle.

While our phenomenological model correctly reproduces trends in the trailing wake infrared radiances that are observed in experimental data, the underlying model is dependent on two hypotheses. First, we have proposed that the roughness of the sea surface in the trailing wake is inversely proportional to the intensity of the turbulent flow; and second, we have assumed the probability density function of slopes in the trailing wake to be an isotropic Cox and Munk PDF with a variance given by the roughness.

We arrived at these hypotheses due to the underlying complexity of developing a fully mathematical physics model of the time and space dependent sea surface slope distributions in the ship turbulent trailing wake. A moving ship will lay a turbulent trailing wake that will perturb the wind generated background sea surface slope distribution. Exactly how the ship perturbs the sea surface in the trailing wake is dependent on the interaction between the turbulent flow that is generated by the ship and the sea surface. It is further complicated by the interaction of the upwelling flows from rising bubbles and the mixing of film surfactants. These flow interactions result in the damping of the sea surface waves and so reduce sea surface roughness. The damping is scale dependent, and so the statistical distribution of sea surface slopes will be different from the background, but transient. The regeneration of the roughness due to wind and other wave interactions restore the background distribution of slopes far behind the ship, where the wake blends into the background and is no longer observable. This time and space dependence of the sea surface slope distribution in the trailing turbulent ship wake is not generally known. Consequently, we are led to hypothesize that the sea surface slopes in the trailing wake are slaved, spatially and temporally, to the underlying turbulence, and that the distribution of sea surface slopes in the trailing wake is distributed similarly to the background, but with the principal distinction that the slope variance has the same spatial and temporal dependence as the underlying turbulence.

Both of the foregoing hypotheses require testing experimentally. This would be rather challenging, given the scope of the hypotheses. On the other hand, theoretical development of the mathematical physical model, or direct numerical simulations of the sea surface roughness can, perhaps, be more readily undertaken. Such advances are necessary to better understand and more accurately model ship wakes in the infrared, improving upon our phenomenological model.

In a future study, our modeling of the Kelvin wake component already a capability in our simulator will be tested against experimental data and reported. Sea surface elevation in the Kelvin wake can be calculated and, from this elevation, the orientation of the different facets on the sea surface can be deduced. Hence, the radiance of the Kelvin wake zone can be predicted. If needed, an update of the model with consideration of the multiple reflection and shadowing will be developed. A sea trial should be conducted to collect measured data from different observer angles for different Sun elevations and different sea surface and air temperatures. The trial should vary the ship speed so as to validate the radiance of the Kelvin wake predicted by our simulator for different wave fields. Thereafter, we plan to develop a model to predict the radiance of the white water wake in the infrared.

Appendix A

In our modeling, the direction of the observer or receiver \((\theta_n, \phi_n)\) and the slopes of the facet \((z_x, z_y)\) are known. The directions of the facet normal \((\theta_n, \phi_n)\) and the source \((\theta_s, \phi_s)\) have to be expressed in terms of these known parameters. A little algebra gives

\[
\theta_n = \tan^{-1}\left(\frac{z^2_x + z^2_y}{zc}ight)
\]  

(A1)
and
\[
\phi_n = \begin{cases} 
\text{atan} \left( \frac{z_y}{z_x} \right) & z_x < 0 \text{ and } z_y < 0, \\
\text{atan} \left( \frac{z_y}{z_x} \right) + 2\pi & z_x < 0 \text{ and } z_y > 0, \\
\text{atan} \left( \frac{z_y}{z_x} \right) + \pi & z_x > 0.
\end{cases}
\]

(A2)

In Fig. 22 we show the zenith and azimuth orientation of the correspondent normal vector to the facet \( \mathbf{n} = (\theta_n, \phi_n) \). For a receiver at \( (\theta_r = 85, \phi_r = -90) \), we show in Fig. 23 the zenith orientation of the reflected sky by each facet and, therefore, Fig. 24 shows \( \omega_r \) and the function of the slopes where the condition \( \omega \leq \frac{\pi}{2} \) applies.

The reflected sky radiance orientation is determined by \( \omega_s = \omega_r = \omega \). Note that since \( \mathbf{n}, \mathbf{s}, \text{ and } \mathbf{r} \) (the unit vectors for the facet normal, source and receiver) are in the same plane, the orientation of the sky that is reflected by a facet with slope \( (z_x, z_y) \) toward a receiver at \( (\theta_r, \phi_r) \) becomes

\[
\cos \theta_n = 2\zeta \cos \theta_n - \cos \theta_r \\
\cos \phi_n = 2\zeta \sin \theta_n \cos \phi_n - \sin \theta_r \cos \phi_r \sin \phi_s
\]

(A3)

and

\[
\cos \phi_s = 2\zeta \sin \theta_n \cos \phi_n - \sin \theta_r \cos \phi_r \sin \phi_s
\]

(A4)

where
\[
\zeta = \sin \theta_n \cos \phi_n \sin \theta_r \cos \phi_r \\
+ \sin \theta_n \sin \phi_n \sin \theta_r \sin \phi_r \\
+ \cos \theta_n \cos \theta_r \\
= \hat{n} \cdot \hat{r} = \cos \omega. 
\]  

\text{(A5)}

In Fig. 24 we show an example of the angle, \(\omega\), between the receiver and the normal to the facet as a function of the slopes \(z_x\) and \(z_y\) and as well as the function where the condition \(\omega \leq (\pi/2)\) applies for a receiver at \((\theta_r = 85, \phi_r = -90)\).

References
