Distributed data-aggregation consensus for sensor networks

Global connectivity assessment through local data exchange

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Abstract

In this report, the connectivity of a wireless sensor network with a random information flow graph is studied. A threshold on the transmission range of the sensors is given based on the properties of the underwater acoustic channel. Then, the probability of vertex isolation is derived which addresses connectivity in the local sense. The notion of 1-connectivity is subsequently proposed as a global measure of connectivity, and an upper bound on its probability is provided. This upper bound is sufficiently close to the probability of 1-connectivity for the case where the network size is large enough and the distribution of the vertices is homogenous.

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1 Introduction

Note: *At the time the present contract was awarded, the outcome of the "Adaptive Multi-sensor Biomimetics for Unsupervised Submarine Hunt" (AMBUSH) Technology Investment Fund (TIF) competition was not known. Since then, the TIF AMBUSH project has been awarded. The present contract is thus viewed as an initial effort which will lead into and benefit the extended TIF project. In light of this, the Project Authority has authorized the contractor to write a shorter-than-normal contract report so as to focus on research and staffing efforts which will ultimately benefit the TIF project.*

One of the most popular measures for characterizing the connectivity of a network is the second smallest (first non-zero) eigenvalue of the graph Laplacian, which is known as the Fiedler eigenvalue [1]. The magnitude of this measure shows how well-connected the overall graph is. However, this measure is zero for all disconnected networks, and hence it may not be used for sparse and disconnected graphs [2]. Moreover, the notion of vertex (edge) connectivity is introduced to evaluate the strength of connectivity of a graph as it represents the minimum number of vertices (edges) which need to be eliminated such that the graph loses its connectivity [3]. As another measure, natural connectivity is defined as the average eigenvalue of the graph spectrum, which increases monotonically with the addition of edges [4], and is closely related to the redundancy of the routes between different vertices. Intuitively, the connectivity of a graph with a higher number of alternative routes connecting two vertices is more robust. This measure can also be used for the graphs which are not connected or graphs whose links (between different pairs of vertices) are random. However, there is no reported work in the literature for measuring the connectivity of a random graph using the above technique. Reachability index, on the other hand, is a measure of connectivity which is useful for sparse networks, and is defined as the fraction of the node pairs in the network that are connected [5]. In particular, this measure is more expressive than algebraic connectivity when dealing with sparse networks.

The notion of connectivity in random graphs has been investigated widely in the literature. The most general form of random graphs is called Erdös-Rényi which is composed of n vertices, where each possible edge between any pair of nodes occurs independently with probability p [6]. Also, geometric random graphs are introduced as a special case of random graphs where the probability p depends on the mutual distance of the vertices in a given space. One can use statistical measures to characterize the robustness of random networks [7]. To this end, it is required to define a proper performance index which deteriorates as the network disintegrates. The most commonly used performance indices for this purpose are diameter, the size of the largest connected component, the average path length, the number of reachable vertex pairs, and the tree-width [8].

This report extends some of the existing measures for the connectivity of a graph, to the random graphs. Some background on the relevant results in graph theory is provided first. Then, an acoustic communication channel induced by harsh underwater environment like limited bandwidth, propagation delay, and noise is modeled. A probabilistic connectivity measure is developed for a geometric random graph representing an underwater wireless sensor network. The trade-off between the network size and the transmission range of the sensors is discussed, which needs to be addressed in the network design.

2 Preliminaries

In this section, the geometric random graph $G_l^d(n,r)$ with vertex set V and edge set E, along with some related notions, are defined.

Definition 1 *Let the randomly chosen independent, identically distributed (i.i.d.) points* $\{q_1, q_2, ..., q_n\} \in [0,1]^d$ *correspond to the location of the vertices* $V = \{v_1, v_2, ..., v_n\}$ *of a geometric random graph* $G_l^d(n,r) = (V,E)$ *in the* $\mathbb{R}^d$ *space. Then,* $(i,j) \in E$ *for any two arbitrary vertices* $i,j \in V$ *if and only if* $\|q_i - q_j\|_l \leq r$*, where* $\|.\|_l$ *represents the* l*-norm.*

In this work, it is assumed that the agents belong to the 2D space and the distances are measured by the Euclidean norm. Thus, $d = l = 2$.

Definition 2 *From the definition of the geometric random graph* $G_2^2(n,r)$*, the neighbor set of an arbitrary agent* i *is defined as:*

$$N_i = \{j \in V | \ \|q_i - q_j\|_2 \leq r\} \tag{1}$$

Also, the degree of vertex i*, denoted by* d_i*, is defined as the number of its neighbors, i.e.* $d_i = Card(N_i)$*.*

A path between two distinct vertices in graph G is a subgraph of G defined by a set of consecutive edges connecting two vertices. Two paths in G are said to be independent if any vertex common to both paths is an end vertex of both paths. The notion of k-connectivity which is defined below, is considered as one of the preferred connectivity measures for the case of underwater sensor network [9].

Definition 3 *A graph* G *is said to be* k*-connected* ($k \in \mathbb{N}$) *if for each pair of vertices, there exist at least* k *mutually independent paths connecting them. Equivalently,* G *is* k*-connected if and only if there is no set of* $(k-1)$ *vertices whose removal would disconnect the graph. In other words, if any set of* $(k-1)$ *vertices fail, the graph is guaranteed to be still connected. Obviously, a* k*-connected graph is also* $(k-i)$*-connected for* $i = 1, 2, ..., k-1$*.*

3 Acoustic Channel Characterization

The path loss for a signal of frequency f in an underwater acoustic channel over a distance r is denoted by $A(r,f)$, and is given by [10]:

$$A(r,f) = A_0 r^{\kappa} a(f)^r \tag{2}$$

where A_0 is a normalizing constant, κ is the spreading factor, and $a(f)$ is the absorption coefficient. The spreading factor κ represents the propagation geometry and is typically chosen as $\kappa = 2$ for spherical spreading, $\kappa = 1$ for cylindrical spreading, and $\kappa = 1.5$ for the practical spreading cases. In the case of radio channels, κ is usually between 2 an 4. Also, the absorption coefficient $a(f)$ can be obtained experimentally. The path loss describes the attenuation of an acoustic signal on a single unobstructed propagation path. Therefore, if an acoustic signal of frequency f and power p_t is transmitted over a path, the power of the received signal, denoted by p_r, is as follows [11]:

$$p_r = \frac{p_t}{A(r, f)N(f)B(f)} \tag{3}$$

where $N(f)$ shows the power spectral density of the channel noise at frequency f, and $B(f)$ is the usable bandwidth around the center frequency f.

Assume that two sensors form a wireless link if the received power p_r is larger than or equal to a threshold power, and let all sensors have the same received threshold power $p_{r,th}$. Then, two sensors establish a communication channel if the following inequality holds:

$$A(r, f)N(f)B(f) \leq \frac{p_t}{p_{r,th}} \tag{4}$$

Since the acoustic path loss is both frequency-dependent and distance-dependent, it causes bandwidth limitation in underwater communication systems, such that a greater bandwidth is available for shorter distances. Thus, the network throughput can be improved by using the multihop communication channels in which each sensor relies on its neighbors to relay its transmission to the desired destination.

By choosing an optimal fixed frequency f_0 which maximizes the channel bandwidth, r_{th} can be found as the threshold transmission range for successful inter-agent communication, described as:

$$r_{th} = g\left(f_0, \frac{p_t}{p_{r,th}}\right) \tag{5}$$

where $g(.)$ is a function obtained by solving equation (4) numerically. Therefore, two arbitrary sensors with mutual distance r can form a communication link if $r \leq r_{th}$.

4 Proposed Connectivity Measure

Consider an underwater sensor network consisting of n sensors distributed randomly in a 2D space $\mathcal{A}$ of area A and denote the location of an arbitrary sensor by q. The sensing area of this sensor is a circle of radius r_{th} centered at q. Let a second vertex be randomly placed at $q' \in \mathcal{A}$. Considering a uniform distribution of vertices, the probability of having a communication link between q and q' is equal to $\pi r_{th}^2/A$.

Since the notion of k-connectivity highly depends on the degree of vertices in the graph, it is assumed that the degree of vertices is represented by a random variable $D \in \mathbb{N}_0$. For a large

n, the probability that the degree of the vertex associated with q is d can be approximated with the Poisson distribution given below [9]:

$$Pr\{D = d|\ q\} \cong \frac{(\mu_0(q))^d}{d!} e^{-\mu_0(q)} \tag{6}$$

where $\mu_0(q) = E\{D|\ q\}$ is the expected value of the degree of the vertex. By considering a homogenous uniform distribution for the vertices, the expected degree can be simplified as follows:

$$\mu_0(q) = \mu_0 = \frac{n}{A}\pi r_{th}^2 = \rho\pi r_{th}^2 \tag{7}$$

where ρ is the density of the network, defined as the number of vertices per unit area. In the next two sections, this simplified formulation is used to find a measure for connectivity.

4.1 Local Connectivity

The concept of *local connectivity* is concerned with the probability of the existence of a link between an arbitrary vertex and its neighbors. The *isolation probability* of a single vertex, denoted by $Pr\{\text{iso}\}$, is defined as the probability that a vertex of a geometric random graph has no connection to any other vertices, meaning that its corresponding degree is zero. Using the results of [12]:

$$Pr\{\text{iso}\} = Pr\{D = 0\} \cong e^{-E\{D\}} \tag{8}$$

Considering a homogenous uniform Poisson distribution of density ρ for the sensors as described in (7), the isolation probability of each vertex can be rewritten as:

$$Pr\{\text{iso}\} = e^{-\frac{n}{A}\pi r_{th}^2} = e^{-\rho\pi r_{th}^2} \tag{9}$$

(note that in a homogeneous distribution, the density of the network $\rho = n/A$ remains constant even if both n and A tend to infinity).

4.2 Global Connectivity

The notion of 1-*connectivity*, denoted by $Pr\{\text{1-con}\}$, is defined as the probability that every pair of vertices in the network are connected by at least one path. Since the considered underwater sensor network is time-varying, $Pr\{\text{1-con}\}$ can be interpreted as the fraction of time that the random network is 1-connected for a given number of sensors n with a sensing range threshold r_{th}. The choice of a proper threshold r_{th} involves a trade-off: on the one hand, the value of r_{th} should be large enough to keep the entire network connected; on the other hand, it should be small enough for optimal power consumption and minimum channel interference. In general, the notion of 1-connectivity provides a relatively strong measure for the overall network connectivity, and an upper bound on this measure is given as follows [12]:

$$Pr\{\text{1-con}\} \leq Pr\{\text{no-iso}\} \tag{10}$$

where $Pr\{\text{no-iso}\}$ is the probability that there is no isolated vertex in the network. It is to be noted that having no isolated vertex in a graph is necessary but not sufficient condition for the 1-connectivity of the network. Note also that and $Pr\{\text{no-iso}\}$ can be written in terms of $Pr\{\text{iso}\}$ as:

$$Pr\{\text{no-iso}\} = (1 - Pr\{\text{iso}\})^n \tag{11}$$

(the isolation of the vertices are independent of each other). According to [9], the upper bound on $Pr\{\text{1-con}\}$ in (10) becomes tight for high probability of no isolation in (10). In other words,

$$Pr\{\text{1-con}\} = Pr\{\text{no-iso}\} - \varepsilon \tag{12}$$

for some $\varepsilon \geq 0$ and $\varepsilon \to 0$ as $Pr\{\text{no-iso}\} \to 1$. Considering the Poisson distribution as noted earlier, (11) can be written as:

$$Pr\{\text{no-iso}\} = e^{-nPr\{\text{iso}\}} = e^{\left(-\rho A\, e^{\left(-\rho\pi r_{th}^2\right)}\right)} \tag{13}$$

By considering a sufficiently high probability for $Pr\{\text{no-iso}\}$, the 1-connectivity of the network is implied. Equation (13) can be used to find an appropriate threshold r_{th} for a given network size n, such that the network connectivity is preserved.

5 Future Plan

The previous results for 1-connectivity was based on the assumption that the network size is sufficiently large, and that the sensors are placed according to a Poisson point process with certain finite density. This can limit the applicability of the proposed method in a practical network. For the future work, it is desired to introduce the path probability as a connectivity measure to address the above-mentioned limitation and make the result more applicable for practical sensor networks. Path probability is defined as the probability that two arbitrary nodes are connected via either a direct link or a multihop path. Also, the acoustic channel described in this work can be modified properly to characterize a more realistic underwater communication channel.

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