Economic Forecasting with Optimal Replacement for the CP-140 Fleet Under the Aurora Incremental Modernization Program

David W. Maybury
Materiel Group Operational Research

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Defence R&D Canada
Centre for Operational Research and Analysis

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Abstract

The CP-140 Aurora fleet of long range maritime patrol aircraft presents the Department of National Defence (DND) with a complicated repair-replace problem. In 1998 DND initiated the fleet-wide renewal Aurora Incremental Modernization Program (AIMP), only to suspend and then re-tool the program in 2007 with the further decision to reduce the fleet from eighteen to ten aircraft. DND plans to use the size-reduced CP-140 Aurora fleet as a surveillance platform until 2020. We apply a stochastic optimal replacement model with forecasting capabilities to historic fleet data to gain insight into the total ownership cost and performance of the fleet. We find that the effects of uncertainty in both costs and operational availability play an important role in predicting the optimal replacement time, and in predicting the future operation and maintenance costs. The application of the stochastic model yields a probability envelope for the future of the CP-140 Aurora fleet in operation and maintenance costs per operational availability. Our model demonstrates that only recently (2005) has the Aurora fleet surpassed the optimal fleet replacement time, but by 2020 the Aurora fleet is expected to exceed the optimal replacement barrier in dollars per operational availability by an order of magnitude.

Résumé

La flotte des avions de patrouille maritime à long rayon d’action CP140 Aurora pose au ministère de la Défense nationale (MDN) un problème complexe en ce qui a trait à la réparation et au remplacement. En 1998, le MDN a entamé le Programme de modernisation progressive de l’Aurora (PMPA), dont la portée s’étendait au renouvellement de toute la flotte, programme que l’on a suspendu puis relancé en 2007 lorsqu’on a pris la décision de réduire la flotte de dix-huit à dix appareils. Le MDN entend utiliser la flotte réduite des CP140 Aurora comme plateforme de surveillance jusqu’en 2020. Nous appliquons un modèle stochastique de remplacement optimal et des capacités de prévision aux données historiques concernant ces aéronefs, afin d’obtenir un aperçu du coût global de la propriété et du rendement de la flotte. Nous constatons que les effets de l’incertitude, du point de vue des coûts et de la disponibilité opérationnelle, sont déterminants pour la prévision du délai optimal de remplacement des aéronefs et la prévision des frais éventuels de fonctionnement et d’entretien. En appliquant le modèle stochastique, nous obtenons une gamme de probabilités pour l’avenir de la flotte des CP140 Aurora en coûts de fonctionnement et d’entretien par rapport à la disponibilité opérationnelle. Notre modèle montre que la flotte des Aurora vient tout juste (2005) de dépasser le délai optimal de remplacement, mais que l’on peut s’attendre qu’elle dépassera par un multiple de dix d’ici 2020 la limite du remplacement optimal en dollars par rapport à la disponibilité opérationnelle.

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Executive summary

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David W. Maybury; DRDC CORA TM 2009–023; Defence R&D Canada – CORA; June 2009.

The 27 year operational history of the CP-140 Aurora fleet presents the Department of National Defence (DND) with a challenging repair-replace problem. The practice of retaining fleets for unprecedented long service lives represents a particularly difficult problem for today's Canadian Forces. In order to gain insight into the CP-140 Aurora optimal fleet replacement problem, we apply a stochastic model [1] that generates the expected replacement time with a future costing envelope. The model we use is based on the underlying assumption that random events occur independently of fleet age but fleet resiliency diminishes with time, leading to proportionally evolving operation and maintenance (O&M) costs in a random background. We capture platform performance in the model by discounting the O&M costs by operational availability (Ao). We refer to dividing O&M costs by Ao as a linear utility discount. Thus, the model determines the optimal replacement time in O&M cost per utility space.

In order to evaluate the optimal replacement time of the CP-140 Aurora fleet, the stochastic model provides a critical region for replacement based on estimates of model parameters from the costing data, including the Aurora Incremental Modernization Program (AIMP). Once the data sample path reaches the critical region, the model signals replacement, modernization or a "reset-the-clock" overhaul. Only once the sample path surpasses the top boundary of the critical region does the model indicate that the optimal replacement time has passed. Application of the stochastic model reveals:

- The CP-140 Aurora fleet sample path surpassed the top boundary of the critical region for replacement in 2005. The utility discount always penalizes the fleet for Ao below 100% and thus the result represents a lower bound on all possible linear utility functions. Figure 1 demonstrates the important features of the model's predictions. The lack of quality data prior to 1995 requires us to evolve the sample path backward in time which contributes to the width of the critical region. We have indicated the backward envelope in figure 1.

- If we ignore utility, the CP-140 has an expectation time to surpass the upper boundary of the critical region in 2014.

- A 23% chance remains that the fleet will re-enter the critical region by 2020. The

> budget year dollar range per utility at the 95 % confidence level in 2020 will be
>
> $$\$5 \times 10^7/\mathrm{Ao} < m_{2020} < \$9 \times 10^9/\mathrm{Ao}. \tag{1}$$
>
> In figure 2, we show future expectation along with the 95% confidence contour. Note that the figure indicates that by 2020 the expectation of the CP-140 Aurora sample path exceeds the critical region by an order of magnitude. Again, we stress that the result rests on the linear utility function.

The stochastic approach implicitly attaches a value to the capacity to delay the replacement decision. We must always bear in mind that the fleet might improve in the future, and we must ensure that we properly value the probability of this potential improvement. In addition to accounting for potential random favourable fluctuations, the opportunity for decision-makers to receive more information also provides value. In stochastic environments, we must evaluate all aspects of the project in order to make the best possible decision.

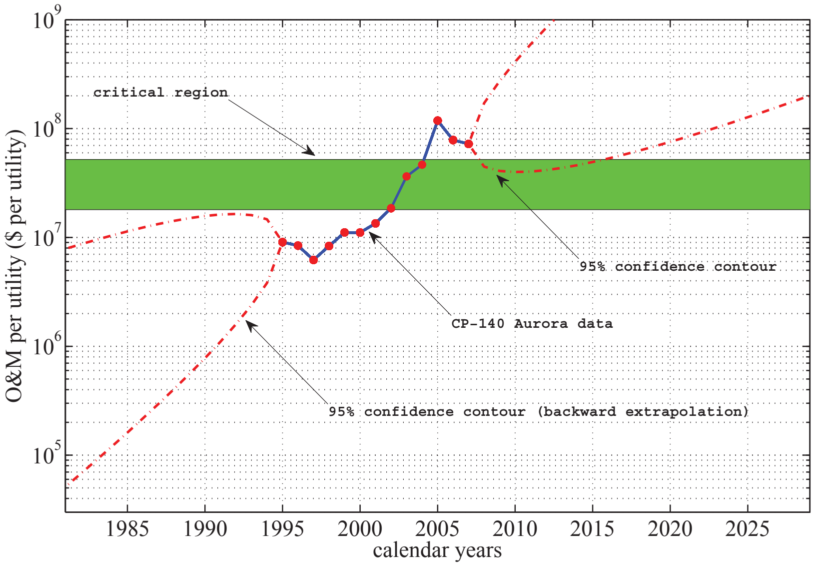

Figure 1: *The Aurora data with confidence contours backward and forward in time. The backward contour allows an estimate of ownership costs during the first 13 years of service. The Aurora fleet fully exited the critical region in 2005.*

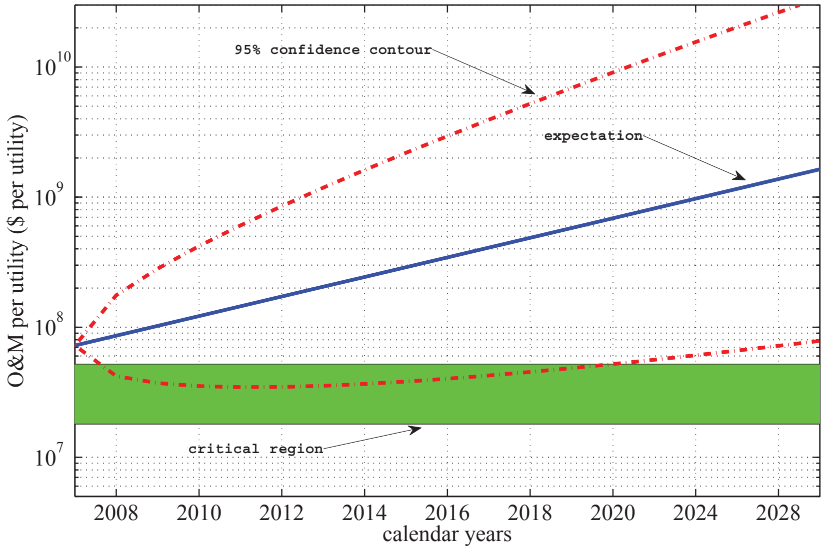

Figure 2: *The forward time CP-140 Aurora fleet prediction until 2030 with the critical region. There is a 23% chance that the sample path will re-enter the critical region prior to 2020.*

Sommaire

Economic Forecasting with Optimal Replacement for the CP-140 Fleet Under the Aurora Incremental Modernization Program

David W. Maybury ; DRDC CORA TM 2009–023 ; R & D pour la défense Canada – CARO ; juin 2009.

Les 27 ans d'utilisation opérationnelle de la flotte de CP140 Aurora apportent au ministère de la Défense nationale (MDN) des défis de taille en matière de réparation-remplacement. La pratique de conserver des flottes de navires durant des périodes de service sans précédent pose un problème particulièrement délicat aux Forces canadiennes d'aujourd'hui. Pour avoir une idée du délai optimal de remplacement de la flotte des CP140 Aurora, nous employons un modèle stochastique [1] qui génère le délai de remplacement prévu ainsi qu'une gamme des coûts éventuels. Le modèle que nous utilisons se fonde sur l'hypothèse sous-jacente que des événements aléatoires surviennent peu importe l'âge de l'appareil, mais que la résilience de la flotte s'affaiblit au fil du temps, ce qui entraîne des dépenses de fonctionnement et d'entretien (F& E) qui augmentent proportionnellement dans un contexte aléatoire. Nous obtenons le rendement de la plateforme dans le modèle en actualisant les coûts de F& E selon la disponibilité opérationnelle (Do). Nous appelons la division des coûts de F& E par la Do : l'actualisation de l'utilité linéaire. Ainsi, le modèle détermine le délai optimal de remplacement en coûts de F& E par rapport à l'utilité.

Afin d'évaluer le délai optimal de remplacement de la flotte des CP140 Aurora, le modèle stochastique établit une zone critique de remplacement basée sur des estimations de paramètres établies à partir des données sur le calcul des coûts, y compris le Programme de modernisation progressive de l'Aurora (PMPA). Lorsque la trajectoire des données de l'échantillon atteint la zone critique, le modèle signale un remplacement, la modernisation ou une remise à neuf. Ce n'est qu'une fois que la trajectoire de l'échantillon dépasse la limite supérieure de la zone critique que le modèle signale le dépassement du délai optimal de remplacement. L'application du modèle stochastique révèle ce qui suit :

– La trajectoire de l'échantillon de la flotte des CP140 Aurora a dépassé la limite supérieure de la zone critique du remplacement en 2005. L'actualisation de l'utilité pénalise toujours la flotte pour une Do inférieure à 100% et le résultat représente donc une limite inférieure pour toutes les fonctions possibles de l'utilité linéaire. La figure 3 indique les caractéristiques importantes des prévisions du modèle. L'absence de données de qualité avant 1995 nous oblige à reculer la trajectoire de l'échantillon dans le temps, ce qui contribue à la largeur de la zone critique. Nous avons indiqué la fourchette vers le passé à la figure 3.

- Si nous ne tenons pas compte de l'utilité, l'appareil CP140 devrait dépasser la limite supérieure de la zone critique en 2014.
- Il reste une probabilité de 23% que la flotte entre de nouveau dans la zone critique d'ici 2020. La fourchette (en dollars de l'année budgétaire) selon l'utilité au niveau de confiance de 95% en 2020 sera de 5

$$5\$ \times 10^7/\text{Do} < m_{2020} < 9\$ \times 10^9/\text{Do}. \tag{2}$$

La figure 4 indique les prévisions d'avenir ainsi que la courbe de confiance de 95%. Il faut noter que la figure montre que d'ici 2020, on s'attend que la trajectoire de l'échantillon des CP140 Aurora dépasse la zone critique par un multiple de dix. Encore une fois, nous soulignons que le résultat repose sur la fonction de l'utilité linéaire.

La méthode stochastique accorde implicitement une valeur à la capacité de retarder la décision de remplacement. Nous devons toujours garder à l'esprit que la flotte pourrait s'améliorer, et nous devons veiller à attribuer une valeur convenable à la probabilité de cette amélioration éventuelle. En plus de tenir compte de fluctuations aléatoires favorables possibles, la possibilité pour les décideurs d'obtenir de plus amples renseignements a aussi une valeur. Dans les contextes stochastiques, nous devons évaluer tous les aspects du projet afin de prendre la meilleure décision possible.

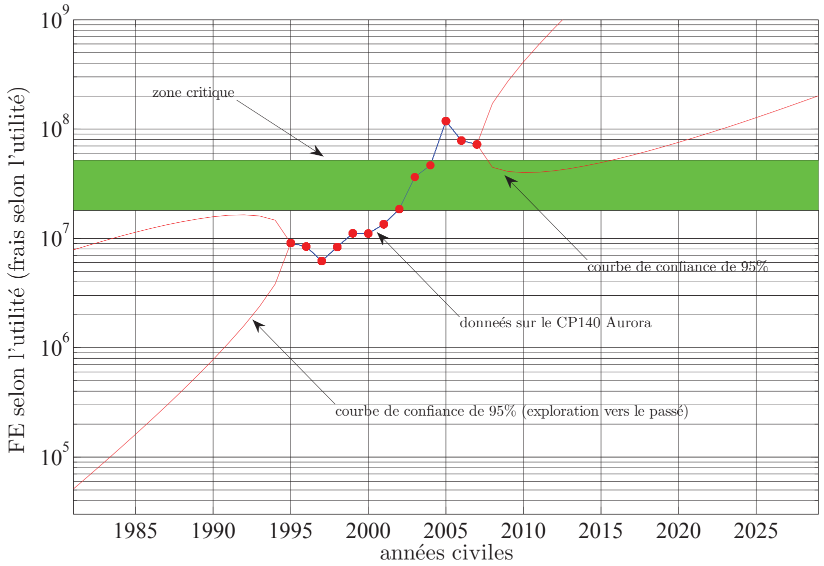

Figure 3: *Données sur l'Aurora et courbes de confiance vers le passé et vers l'avenir. La courbe vers le passé permet d'estimer les coûts de propriété des 13 premières années de service. La flotte des Aurora a quitté totalement la zone critique en 2005.*

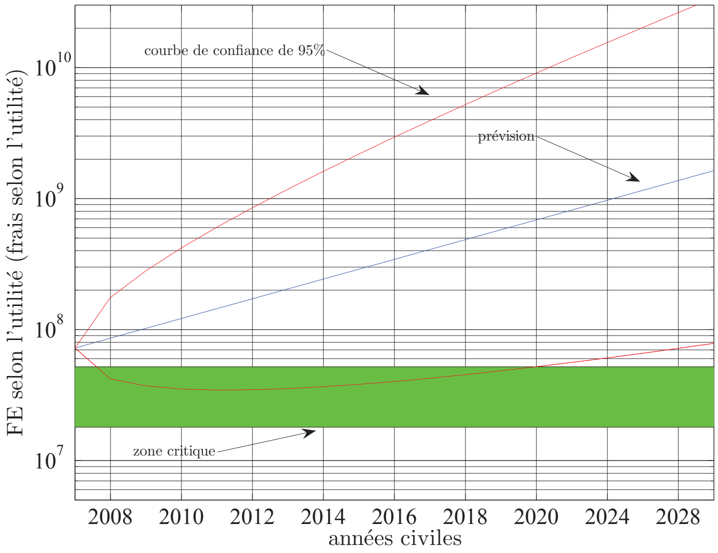

Figure 4: *Prévisions concernant la flotte des CP140 Aurora jusqu'en 2030 et indication de la zone critique. Il existe une probabilité de 23% que la trajectoire de l'échantillon atteigne à nouveau la zone critique avant 2020.*

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Acknowledgements

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1 Introduction

Most important, many of the problems associated with aging material have emerged with little or no warning. This raises the concern that an unexpected phenomenon may suddenly jeopardize an entire fleet's flight safety, mission readiness, or support costs ... — Raymond Pyles, Senior Information System Scientist, RAND (1999)

1.1 Background

While we intuitively expect higher operation and maintenance (O&M) costs and lower productivity from military platforms over the course of service life, extracting age specific effects from other inflationary pressures presents a highly challenging problem. Researchers for the United States Air Force (USAF) and the United States Navy (USN) have struggled to understand aging effects in aviation since the 1960s [2]–[14]. While researchers have discovered a positive relationship between maintenance requirements – in nearly all activities – with airframe age, investigators warn [13] that changing accounting practices, budget sluggishness, and relatively fixed maintenance-personnel requirements plague USAF and USN studies, creating a difficult environment in which to extract age effects from other pressures.

Today's Canadian Forces operate increasingly aging military platforms, often retaining fleets for unprecedented long service lives. The Department of National Defence (DND) therefore faces the difficult task of deciding the most appropriate time to replace a fleet of aging vehicles rather than to continue an ongoing maintenance regime. The CP-140 Aurora large range patrol aircraft represents a specific fleet aging case.

Acquired from Lockheed Aircraft Corporation in 1980/1981, the CP-140 Aurora fleet, composed of 18 aircraft, performs maritime surveillance along Canada's coasts. The Canadian Forces operate thirteen aircraft with 14 Wing Greenwood, Nova Scotia and five other aircraft with 19 Wing Comox, British Columbia. With a range of 5000 nautical miles (9260 km) and with a flight duration of 14 hours, the Aurora provides the Canadian Forces with capabilities that include or enable anti-submarine warfare, drug and human trafficking interdiction, fisheries enforcement, search-and-rescue, and sovereignty patrols.

In 1998, DND initiated a $1.67 billion modernization program – the Aurora Incremental Modernization Program (AIMP) – in order to give the Aurora fleet new life as a surveillance platform. Originally composed of 23 separate projects grouped into 4 chronologically consecutive block upgrades, the discovery of worse-than-expected airframe problems prompted the Minister of National Defence (MND)to suspend AIMP in September of 2007 for a strategic review. In December of 2007, DND decided to continue with AIMP in a new form by committing to outfit ten CP-140 airframes with AIMP block III equipment (sensors and communications gear) and to include core structural upgrades. Upon conclusion of the re-initiated modernization program, DND expects the size-reduced CP-140 Aurora fleet to continue operating until 2020.

If we combine AIMP costs and Cost Factors Manual (CFM) data to create an aggregate O&M cost for the CP-140 Aurora, the fleet has seen O&M costs per vehicle in budget year dollars evolve from $3.7 million in 1995 to $14.1 million in 2007. As the result of budget practice and engineering record keeping changes over the lifetime of the CP-140 Aurora fleet, the O&M and operational availability (Ao) data prior to 1995 does not possess sufficient quality for life cycle costing studies. Given the growing evidence from the literature that suggests age impacts maintenance and hence O&M costs, we can apply the methodologies of [1] to the CP-140 Aurora fleet.

The model developed in [1] assumes that fleet resiliency diminishes with time while random events, such as cost fluctuations, occur independently of airframe age. By seeking tradeoffs between costs, availability, and uncertainty, the model captures the stochastic nature of the problem and leads to replacement criteria that reflect the complexities of the random processes at work.

1.2 Scope

In this report, we model the CP-140 Aurora O&M costs, discounting by aircraft operational availability (Ao), using the methodologies in [1]. The DMGOR response to the request [15] by COS DGAEPM for a study of the O&M evolution and optimal replacement time with forecasting for the CP-140 Aurora is:

- to determine the O&M future evolution envelope
- to determine the optimal replacement time

We develop our model based on 13 years (1995-2007) of costing and Ao data for the CP-140 Aurora. We obtained the data from the Cost Factors Manual (CFM) and the PERFORMA database via [16], an earlier study that examines a similar problem.

We organize this paper into four parts. In Section 1, we introduce the problem and discuss background issues. Section 2 of the paper presents the basics of the mathematical model and section 3 details the results of the study. In Section 4, we discuss the main conclusions.

2 The model

Empirical evidence suggests a positive relationship between the age of military platforms and their O&M costs [2]–[14]. From a modelling point of view, the most important feature emerging from literature suggests that “many of the problems associated with aging material have emerged with little or no warning”[13]. Thus, the research studies lead us to consider stochastic uncertainty in addition to constant growth in O&M costs. We apply the methodology of [1] which augments the repair-replace analysis of [17] and [18]. The model we apply is based on the idea of an infinite horizon generational replacement model (see [19] for contextual examples) with stochastic noise. In this section we present a bare description of the model features.

Replacing a military platform represents a significant investment decision and thus we require a methodology that assesses all aspects of the sunk investment costs. We imagine that the military maintains a fleet of vehicles which yield a capability for a period of time. At some date after initial procurement, the military will exchange the fleet for a new platform in order to retain a needed capability. This situation is not too dissimilar from the decision of a forester to re-harvest a stand of trees after replanting and subsequent maturity. In each case, we can imagine a sequence of replacement decisions, suggesting that we can model the problem through an infinite number of replacements – the infinite horizon generational replacement model. The model requires us to find the optimal replacement interval in the sequence that the model generates. The infinite horizon generational replacement model is based on the net present-value of a given fleet, namely

$$\mathrm{NPV} = p + \int_0^s dt\, m(t) \exp(-rt), \tag{3}$$

where r, s, and p denote the interest rate discount, the replacement time, and the acquisition costs respectively, and $m(t)$ represents the budget year O&M costs. Since we imagine that the replacement cycle repeats ad infinitum, thus requiring the determination of the optimal replacement interval, the total ownership cost becomes a converging geometric series, yielding the relation

$$c(s) = \frac{p + \int_0^s dt\, m(t) \exp(-rt)}{1 - \exp(-rs)}. \tag{4}$$

By minimizing the total ownership cost, we can obtain the optimal time to replace the fleet. Unfortunately, the framework of the infinite horizon generational replacement model represented by eq.(4) requires the knowledge of the O&M cost as a function of time. Furthermore, the required O&M cost function enters the model deterministically, implying knowledge of the entire fleet’s past and future performance. In reality, we expect that the O&M costs evolve randomly with an underlying upward drift.

The model in [1] extends the deterministic infinite horizon generational replacement model by including stochastic noise. In treating the problem stochastically, the determination of the optimal replacement becomes similar to pricing financial derivatives. Relations such as eq.(4) no longer have a straight-forward interpretation as any given O&M cost sample path is no longer differentiable. The required analysis uses the techniques of stochastic calculus and we refer the reader to [1] for model details. An important feature of the stochastic model centers on its implicit evaluation of the capacity to delay a decision. Investment models based solely on deterministic net present-value arguments implicitly assume that the investment choice is either reversible, in that the investor can cancel the investment if circumstances become unfavourable, or if irreversible, the investment choice occurs as a "now or never proposition". In this respect, the stochastic model recognizes the waiting value attached to the sunk investment costs. Few investments occur under these conditions, and the decision to procure a new military platform certainly does not fit the assumptions underlying a deterministic net present-value approach.

The main feature of the stochastic model rests on the stochastic differential equation (SDE),

$$dm = \alpha m dt + \sigma m dW(t), \tag{5}$$

where α and σ label the annual growth rate and annual volatility of the O&M costs respectively, and $dW(t)$ denotes the Weiner process (again we refer the reader to [1] for further details). The stochastic model yields a barrier in O&M costs which in turn yields an expectation time for fleet replacement. Application of the stochastic model reveals the essential difference relative to the deterministic net-present value approach. The deterministic case balances the marginal costs and the known savings from delay through a small increment in time. On the other hand, the stochastic case balances the marginal costs and savings from delaying until the O&M costs increase by a small increment. In the stochastic case, the time interval required to increment the O&M costs by a given amount is random.

In addition to O&M costs, we must ensure that the replacement decision also includes a measure of platform performance. Simply minimizing total ownership cost – even within the stochastic framework – over-looks important military indicators such as mission readiness, operational availability (Ao), and the ability to perform a quality military task. In light of military requirements, the stochastic model discounts the O&M costs by a utility function based on Ao. The utility function within the model can be used to reflect planners' preferences regarding operational standards based on performance expectations. In the analysis presented in the remainder of this paper, we assume a utility function which discounts solely by Ao[1]. More sophisticated approaches are presented in [1].

The stochastic model determines a critical region in the O&M per utility space which indicates optimal replacement. Once the O&M per utility path enters the critical region, the

---

[1] We consider Ao to be a top-level performance indicator. Aircraft utilization as well as personnel experience level, facilities, sparing, change in publications, policy changes, and test equipment all have a strong influence on the resulting Ao. Since many different factors influence Ao (including errors), we treat Ao fluctuations as random events within the stochastic model.

model signals fleet replacement, modernization, or a “reset-the-clock” overhaul. Uncertainty in the model parameters dictate the size of the critical region. In addition to establishing a critical region for fleet replacement, the stochastic model provides the probability of entering the critical region from any point in the O&M per utility space in addition to yielding the expected sojourn time within the critical region itself.

3 Results

3.1 Data and model assumptions

Based on the data obtained through the CFM [20] and the PERFORMA [21] database, and data provided by [16], we analyze the CP-140 Aurora for optimal replacement in the presence of stochastic noise. The data contain the annual O&M costs (including AIMP) and the average yearly fleet Ao as listed in table 1. We see that the data set contains 13 years of data (1995 - 2007), binned on a yearly basis per aircraft. The acquisition cost was $37.9 million (1981) per aircraft.

The O&M costs include petroleum, oil, lubricants, engineering services, repairs, overhauls, and sparing costs. Since any amortization costs connected to the fleet do not involve the disbursement of funds, and since usual military activity in the form of salaries or unit support will require funding independent of vehicle age, we do not include costs associated with these effects in O&M. We also assume that the CFM accurately reports the total O&M cost each year.

In order to provide a fair playing field, we do not consider a replacement vehicle. The economic model we apply balances tradeoffs among uncertainty, costs, and platform availability, and determines the optimal time to replace the vehicle given the cash flow required to sustain it. While the model uses a hypothetical sequence of infinite replacements that involve the same vehicle, the model does not assume this situation reflects reality. The model simply determines the point at which the specific vehicle under consideration reaches its “best-before-date”. The problem associated with determining the replacement platform should involve factors, such as new capability requirements, that go beyond economic considerations.

We assume a constant discount rate of 5.70%, which provides a good approximation to a military specific inflation measure in Canada over the period in consideration [22]. Earlier studies [16] showed through a sensitivity analysis that deterministic net present-value calculations remain largely insensitive to moderate changes in the discount rate. Thus, we keep the discount rate fixed throughout the study.

To summarize our model assumptions, we have:

- replacement choice not considered;
- salaries supporting DND civilian or military personnel not included in O&M;
- direct squadron support not included in O&M;
- amortization costs not included;
- time value of money matched to military inflator, 5.70% fixed;

CP-140 Aurora Data (fleet size: 18 aircraft)		
Year	O&M per Aircraft ($000)	Fleet Ao (%)
1995	3704	40.93
1996	3774	44.85
1997	2983	48.02
1998	4029	48.29
1999	4701	42.26
2000	4462	40.23
2001	4917	36.52
2002	5964	33.04
2003	9487	26.06
2004	10257	21.99
2005	19616	16.59
2006	14095	17.95
2007	14088	19.47

Table 1: *CP-140 Aurora yearly O&M costs per aircraft and fleet Ao data (AIMP included). Acquisition cost: $37.9M per aircraft (1981)*

- fleet becomes less resilient to random events with time;
- the utility measure cannot reach unity below 100% Ao; and
- random events occur independently of vehicle life.

3.2 Expected fleet replacement with utility

For utility discounting purposes, we apply the Ao utility function depicted in figure 5. The utility function is neither concave nor convex indicating risk indifference in regards to Ao. More succinctly, the linear utility function implies that planners expect the average annual fleet Ao to lie uniformly between 0% and 100%. In light of changing mission objectives of the CP-140 Aurora over its service life (from tasks such as Cold War anti-submarine warfare in the 1980s to human trafficking interdiction today), we feel that our utility function provides a base measure of platform usefulness for discounting. In addition, the linear utility function can only reach unity at 100% Ao. In reality, the fleet Ao will be bounded above from mandatory maintenance activities. Thus, our resulting optimal replacement time will represent a lower bound on all linear utility functions. We note that added difficulties exist in creating an appropriate utility measure for the CP-140 Aurora fleet. The December 2007 decision to reduce the Aurora fleet from eighteen to ten aircraft will affect military objectives. Thus, we could imagine a utility discount that would penalize the fleet for size reduction. At this time, we do not include the effect of fleet reduction in the utility measure. As the role of the CP-140 Aurora continues to evolve, we feel that the base linear utility function captures the main features of fleet performance.

Applying the model of [1], the geometric random walk with drift takes the form of the stochastic differential equation (SDE)

$$dm_u = \alpha m_u \, dt + \sigma m_u \, dW, \tag{6}$$

where m_u denotes O&M cost per utility, and the parameters α and σ label the annual growth rate and annual volatility respectively. We will find it fruitful to transform eq.(6) by

$$\tilde{m_u} = \log(m_u), \tag{7}$$

which yields the log transformed SDE

$$d\tilde{m_u} = \frac{dm_u}{m_u} - \frac{1}{2}\frac{(dm_u)^2}{m_u^2} = (\alpha - (1/2)\sigma^2)dt + \sigma dW. \tag{8}$$

By applying statistical tests for normality to the data, we find no compelling evidence at the 95% confidence level to reject our hypothesis of a geometric random walk. Estimating the model parameters from the data, we find

$$\alpha = 0.2 \pm 0.1 \quad \sigma = 0.4 \pm 0.1. \tag{9}$$

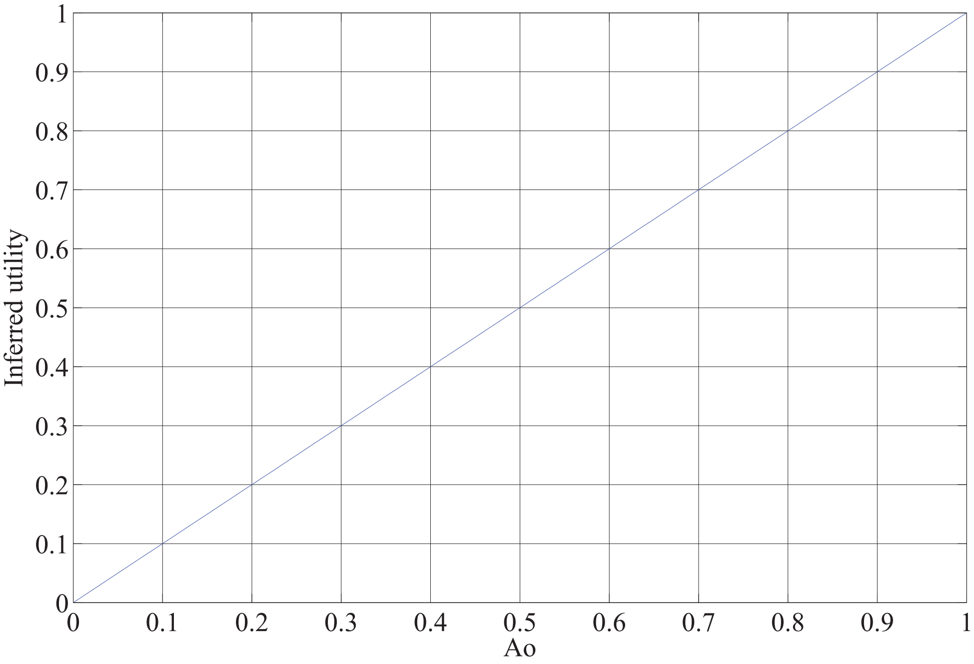

Figure 5: *Utility function derived from an uniform expectation on Ao.*

In order to apply the model in [1], we require the O&M costs per utility for each year of ownership. Recall that the net-present value rests on the inter-generational cost function

$$\mathrm{NPV} = p + \int_0^s dt\, m_u(t) \exp(-rt), \tag{10}$$

where p denotes the initial procurement costs and s labels the replacement time. Eq.(10) requires integration from the date of initial procurement. Unfortunately, we do not have reliable data that would yield the O&M per utility curve from 1981 through 1995. The total ownership cost since procurement feeds into the stochastic model, thus requiring us to estimate the effect of the missing data. Splitting eq.(10) into two parts yields

$$\mathrm{NPV} = p + \int_{t_{1981}}^{t_{1994}} dt\, m_u(t) \exp(-rt) + \int_{t_{1994}}^{t_{2007}} dt\, m_u(t) \exp(-rt), \tag{11}$$

with data lacking for the first integral in eq.(11). The 13 years data between 1995 and 2007 provides us with the ability to use the stochastic model in reverse by extrapolating a confidence contour back to 1981. The extrapolated contour permits an estimate on the first integral in eq.(11) with error

$$\int_{t_{1981}}^{t_{1994}} dt\, m_u(t) \exp(-rt) = \$(20 \times 10^6 \pm 10 \times 10^6)/\mathrm{Ao}. \tag{12}$$

Thus, we can make the model parameter transformation,

$$p \rightarrow p + \$(20 \times 10^6 \pm 10 \times 10^6)/\mathrm{Ao} = p' \tag{13}$$

in [1]. In making the identification in eq.(13), we included the effect of ownership from past O&M per utility costs in the stochastic determination of the optimal replacement time. The uncertainty in p' leads to an extra source of broadening (60% effect) in the critical region. Using the uncertainty in all model parameters at the 95% confidence level, the model yields the critical region (denoted by m_u^*)

$$\$18 \times 10^6/\mathrm{Ao} < m_u^* < \$52 \times 10^6/\mathrm{Ao}. \tag{14}$$

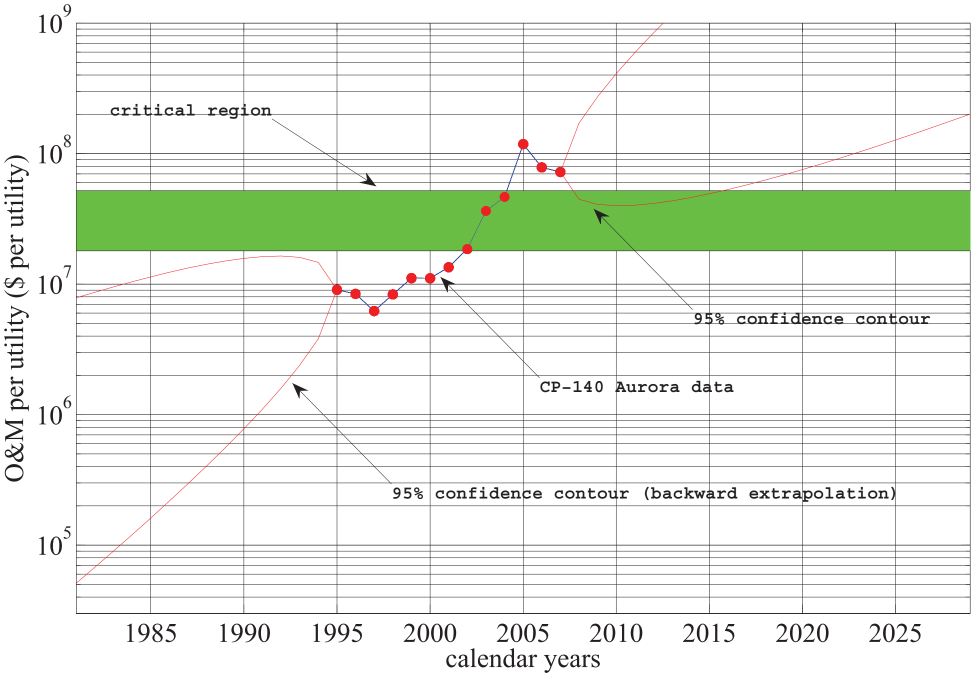

Figure 6: *The Aurora data with confidence contours backward and forward in time. The backward contour allows an estimate of ownership costs during the first 13 years of service. The Aurora fleet fully exited the critical region in 2005.*

In figure 6, we display the CP-140 Aurora sample path in O&M per utility space as a function of calendar year. Note the backward extrapolated 95% confidence contour initiated from 1995. This contour allows an estimate of the total ownership cost prior to 1995 and contributes to the width of the critical region. We see that in 2005 the sample path fully exited through the top of the critical region. Only once the sample path surpasses the upper boundary of the critical region can we confidently claim that the model has called for replacement under the assumption of the base utility measure. In addition to the backward extrapolation, we see the forward 95% contour through the calendar year 2030. Recall that DND expects to operate the size-reduced CP-140 Aurora fleet until 2020. We see that in 2020, the O&M sample path per utility will exceed the critical region by approximately one order of magnitude. The 2020 forecast at the 95% confidence level, in budget year dollars, reads

$$\$5 \times 10^7/\mathrm{Ao} < m_{2020} < \$9 \times 10^9/\mathrm{Ao}. \tag{15}$$

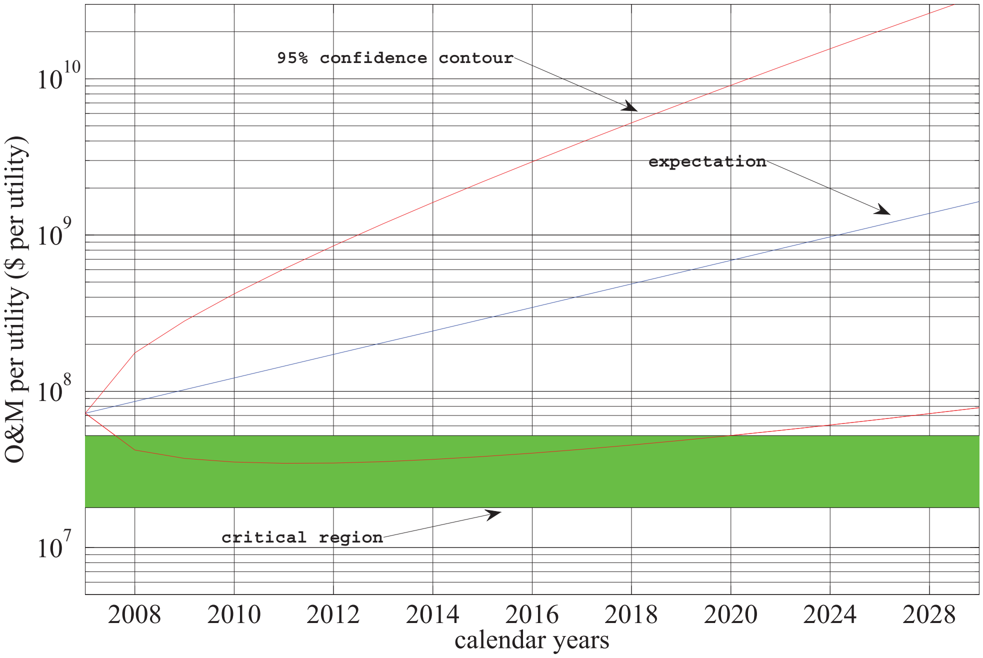

Figure 7: *The forward time CP-140 Aurora fleet prediction until 2030 with the critical region. There is a 23% chance that the sample path will re-enter the critical region prior to 2020.*

Figure 7 displays the expectation of the CP-140 Aurora fleet as a function of calendar year beginning in 2007. We clearly see that by 2020 the expected O&M per utility lies well above the critical region for replacement. While the expectation implies that the optimal replacement time has passed, the stochastic model forecasts a 23% chance that the sample path will drift back into the critical region prior to 2020. The moderately large probability (slightly better than 1 chance in 5) that the sample path will re-enter the critical region complements the high O&M costs per utility expectation in 2020 with the large annual volatility. We must also emphasize that the forecasted results use the base utility discount as shown in figure 5 which represents a lower bound on all linear utility functions. On the other extreme, if we ignore utility completely (i.e., we do not include a measure of performance), the Aurora sample path has an expectation to cross the upper boundary of the critical region in 2014. We provide a sensitivity analysis using different utility functions based on observed data in the Annex.

In addition to the analysis of future performance, we can examine the model projections from the first year of reliable data. Starting from the data point in 1995 – which appears below the critical region – the model yields an expectation of 10 years (2005) for the CP-140 Aurora fleet to exit the critical region through the upper boundary. Again, if we ignore utility, this expectation becomes 19 years (2014).

4 Discussion and conclusions

Over the last two years, the CP-140 Aurora fleet has seen many decisions regarding the future use of the fleet. From AIMP suspension for strategic review to modernization re-initialization with fleet reduction, the CP-140 Aurora fleet represents a perfect case study for stochastic analysis. The stochastic model we apply assumes that fleet resiliency diminishes with time and that random events occur independently of airframe age. As the fleet slowly deteriorates, random events – including new discoveries – compound, leading to a geometric random walk with drift. The discovery of worse-than-expected structural problems with the CP-140 Aurora fleet during AIMP provides a concrete example of this compounding and random effect. We should also note that random events, such as the decrease in costs from market fluctuations, can have a favourable effect.

The model we present tells us that under the assumption of a linear utility function:

- The CP-140 Aurora fleet sample path surpassed the upper boundary of the critical region for replacement in 2005. The linear utility discount always penalizes the fleet for Ao below 100% and thus represents a lower bound on all possible linear utility functions.

- If we ignore utility, the CP-140 has an expectation to surpass the upper boundary of the critical region in 2014.

- Starting from the 2007 data point, there is a 23% chance that the fleet will re-enter the critical region by 2020. The budget year dollar range per utility at the 95 % confidence level in 2020 will be

  $$\$5 \times 10^7/\mathrm{Ao} < m_{2020} < \$9 \times 10^9/\mathrm{Ao}. \tag{16}$$

  Again, we stress that the result rests on the linear utility function (Ao).

The stochastic approach implicitly attaches a value to the capacity to delay the replacement decision. We must always bear in mind that the fleet might improve in the future, and we must ensure that we properly value the probability of this potential improvement. In addition to accounting for potential random favourable fluctuations, the opportunity for decision-makers to receive more information also provides value. In stochastic environments, we must evaluate all aspects of the project in order to make the best possible decision.

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Annex A: Utility sensitivity analysis

An effective fleet replacement strategy includes a measure of platform performance and we satisfy this requirement in the main analysis of this paper by using fleet Ao as a discount factor in the O&M costs. Thus, the optimal replacement time signaled by the model includes O&M costs and Ao as inputs. As demonstrated in [1], we can expand the concept of an effectiveness measure by creating an Ao utility function.

The Ao utility function presented in [1] uses empirical fleet Ao data with an expected Ao range. We imagine that on initial procurement, military planners have based operational standards for the fleet around an expected average fleet Ao. If the fleet Ao falls below the expected range, the Canadian Forces will become increasingly dissatisfied with the fleet's performance as military activities become increasingly disrupted. On the other hand, if the fleet exceeds expectation, the fleet will provide the Canadian Forces with a marginal increase in effectiveness. In this context, the expected Ao range forms the basis of a probability distribution with a trapezoid profile centered on the expected Ao range (see figure A.1), and we interpret the cumulant of the distribution as the fleet's utility. The expected Ao range represents DND's best guess as to the fleet's performance.

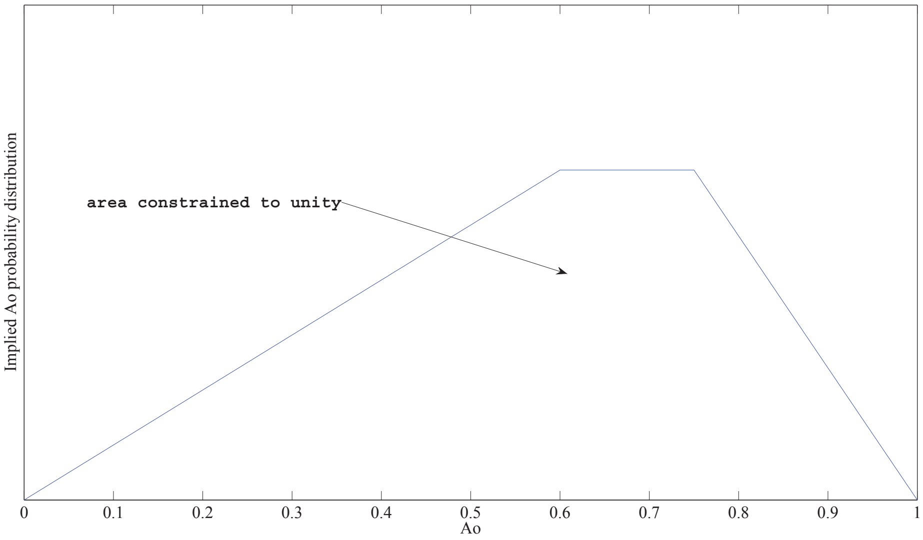

Figure A.1: *Trapezoid profile with an expected Ao in the range 60% - 75%. The utility function results from the cumulant of the trapezoid.*

In light of the different expected Ao ranges that DND might have regarding the CP-140 Aurora fleet, the analysis presented in this paper uses a linear utility function derived from an expected average fleet Ao between 0% and 100%. Thus, while the utility measure penalizes the fleet for low Ao, the level of penalization in any interval of Ao remains constant. In discussion with DGAEPM staff [2], we have decided to investigate the sensitivity of the results to other choices of utility (and hence of expected Ao.).

In table 1, we learn that the highest and lowest annually averaged recorded fleet Ao are 48.29% and 17.95% respectively. We can therefore imagine three limiting cases that will test the robustness of the results obtained using the linear utility discount:

- Limit 1: Expected Ao range, 17.95% to 100%
- Limit 2: Expected Ao range, 48.29% to 100%
- Limit 3: Expected Ao range, 17.95% to 48.29%

Each limiting case uses the highest and lowest recorded Ao in different fashions. In Limit 1, we imagine that military planners expected the fleet Ao to lie above the lowest recorded value with each interval above the lowest recorded value occurring with equal probability. The range of Ao in Limit 1 indicates that only if Ao falls below 17.95% (the lowest recorded Ao value) will DND become increasingly dissatisfied with fleet performance. Limit 2 indicates that the DND expected CP-140 fleet's performance to lie in an Ao range above the highest recorded value with each interval above the highest recorded value occurring with equal probability. Finally, in Limit 3 we see that the CP-140 fleet has always sat in the expected Ao range. Roughly, we can interpret Limit 1 as an optimistic case, Limit 2 as a pessimistic case, and Limit 3 as a realistic case. The top panel figures of tables A.1 – A.4 displays the utility curves resulting from the limiting cases.

With the utility curves establishing limiting cases, we can perform an analysis using each utility curve for discounting purposes. Recall from the main analysis in the paper that the expectation time for the CP-140 sample path to exit completely the critical region was 10 years. The limiting cases yield:

- Limit 1: expectation time, 8.8 years (2003)
- Limit 2: expectation time, 6.8 years (2001)
- Limit 3: expectation time, 9.8 years (2004)

In the bottom panel figures of tables A.1 – A.4 we see the distribution of the expectation time for completely exiting the critical region.

Notice that the expectation time varies by a maximum of 3.2 years between the limiting cases and the linear utility discount used in the main analysis. This variation represents approximately a 30% shift from the linear utility discount result. Given the level of noise

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[2] Private communications with Maj. P. Appell DAEBM 2-4, 5 January 2009, and 13 January 2009

in the system and the unknown O&M cost curve from fleet procurement to 1995, the result between the limiting cases demonstrates robustness. Note that while the limiting cases arise from reasonable limits on the expected Ao, if the actual expected Ao range sat well above the maximum of the fleet's performance, we would obtain a much lower expectation time. In a case where the fleet expectation sits in an interval close to 100%, the CP-140 fleet would have performed drastically below expectation and thus the model would indicate replacement under 10 years (2005). If the trapezoid profile became infinitely peaked and infinitely narrow (the Dirac-delta function) at 0% Ao, the resulting cumlant yields a constant utility of 1 for all Ao. The constant unit utility completely ignores Ao (thus ignoring any indication of platform usefulness) and, analysis with the unit utility yields a large increase in the expected replacement time. Applying the unit utility curve, we find that the CP-140 fleet's sample path does not exit the critical region until nearly 2015 – a decade increase over the result using the linear utility measure. In order to apply the model rigorously, a solid understanding of expected fleet performance is required. Since the CP-140 Aurora fleet has seen many changes – including fleet size reduction and new mission objectives, we feel that the above limiting cases reflect a robust test.

Finally, we should note that we can improve on the utility measure outlined in [1]. The trapezoid profile runs over the interval 0% to 100% implying that the utility curve only reaches 1 at 100% Ao. While this construction yields a useful approximation, we could tailor the approximation to the maximum Ao obtainable by the fleet. Regular scheduled maintenance activity or warranty prescribed activities will set an upper boundary in Ao even if the fleet performs perfectly. Adjusting the trapezoid to reflect this reality has the potential to give a more robust optimal replacement time. The effect of trapezoid tailoring will be examined in a forthcoming note.

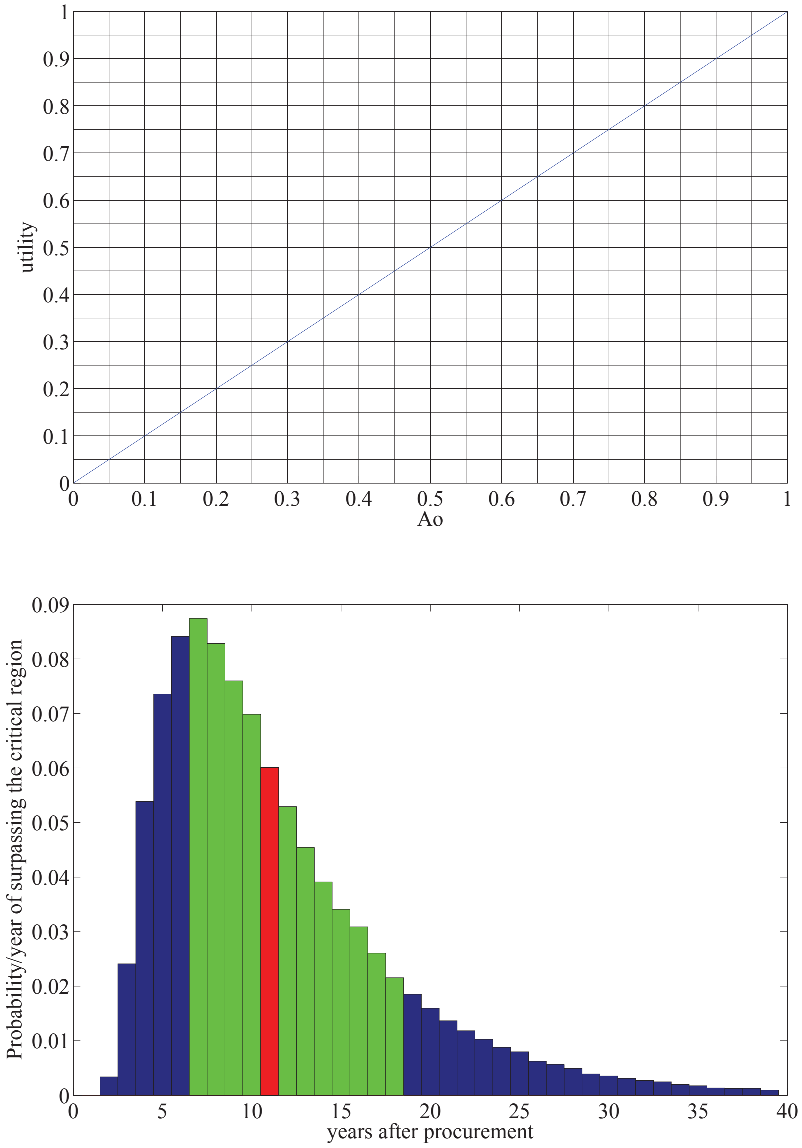


Table A.1: *Top panel: utility curve based on an expected Ao of 0% to 100% – the linear discount. Bottom panel: distribution of the expectation time to exit fully the critical region based on the linear utility function. The red bar indicates the expectation and the green bars give the 60% confidence region around the expectation.*

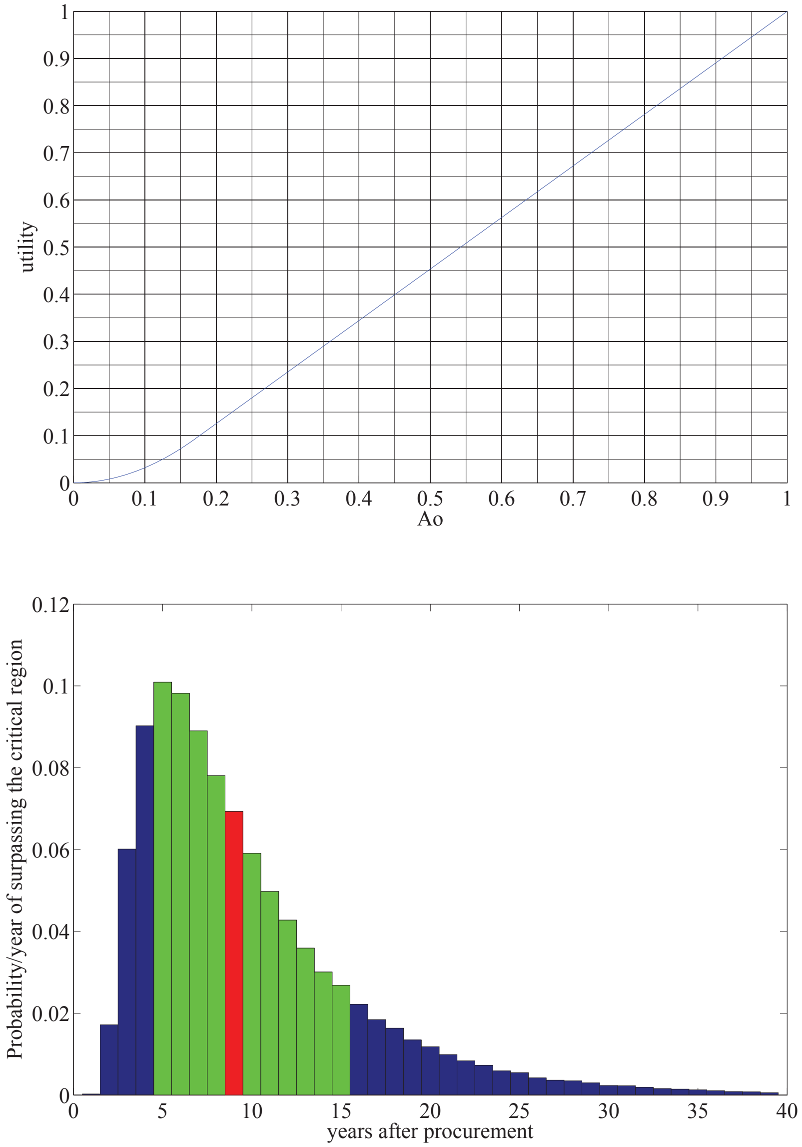


Table A.2: *Top panel: utility curve based on an expected Ao of 17% to 100%. Bottom panel: distribution of the expectation time to exit fully the critical region based on the utility curve from Limit 1. The red bar indicates the expectation and the green bars give the 60% confidence region around the expectation.*

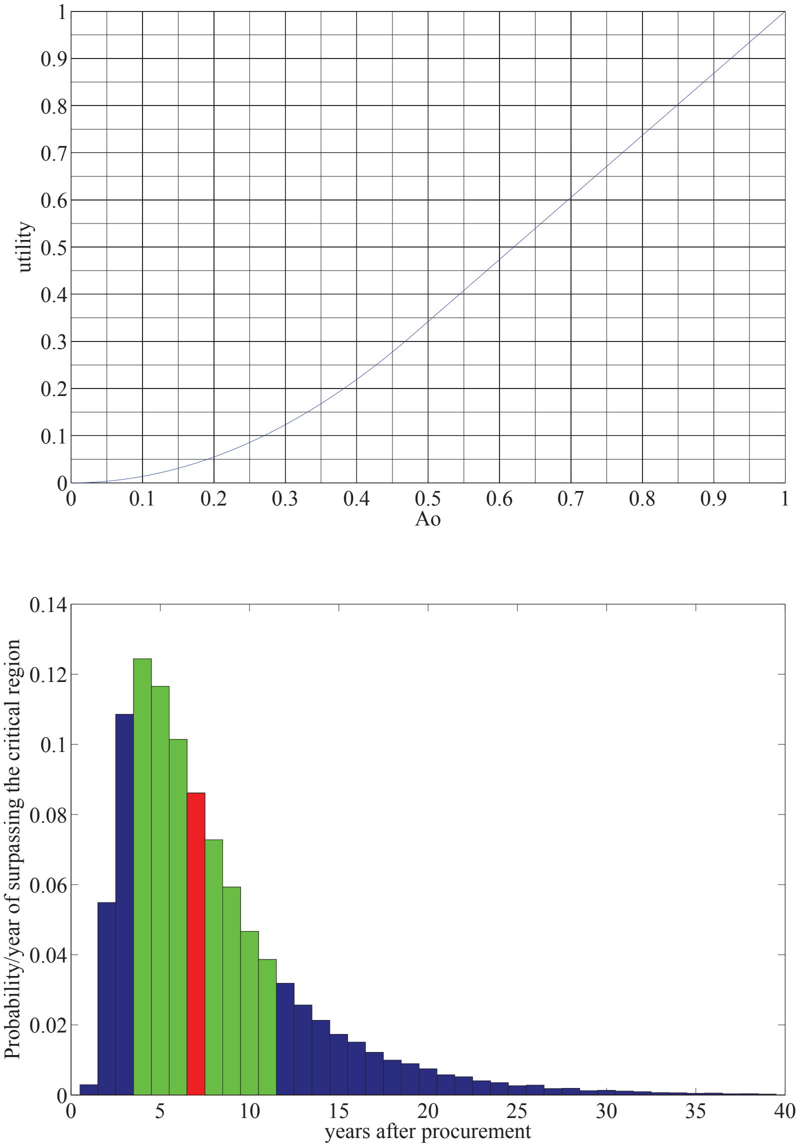
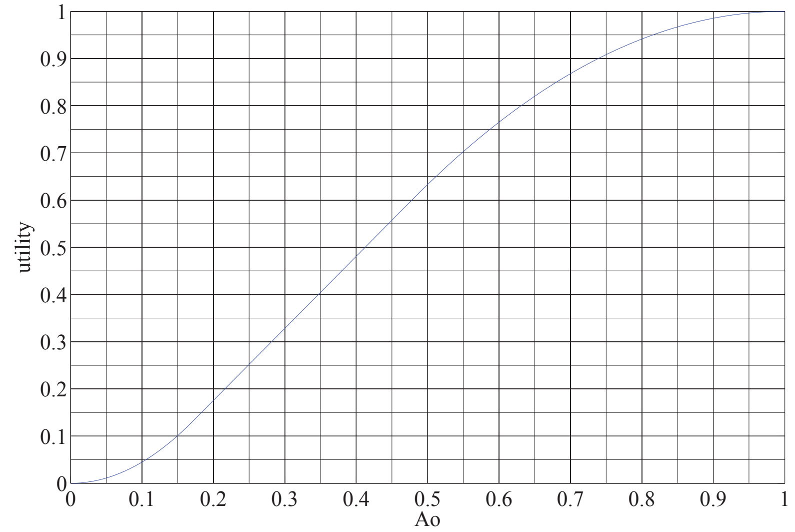


Table A.3: *Top panel: utility curve based on an expected Ao of 48% to 100%. Bottom panel: distribution of the expectation time to exit fully the critical region based on the linear utility function from Limit 2. The red bar indicates the expectation and the green bars give the 60% confidence region around the expectation.*



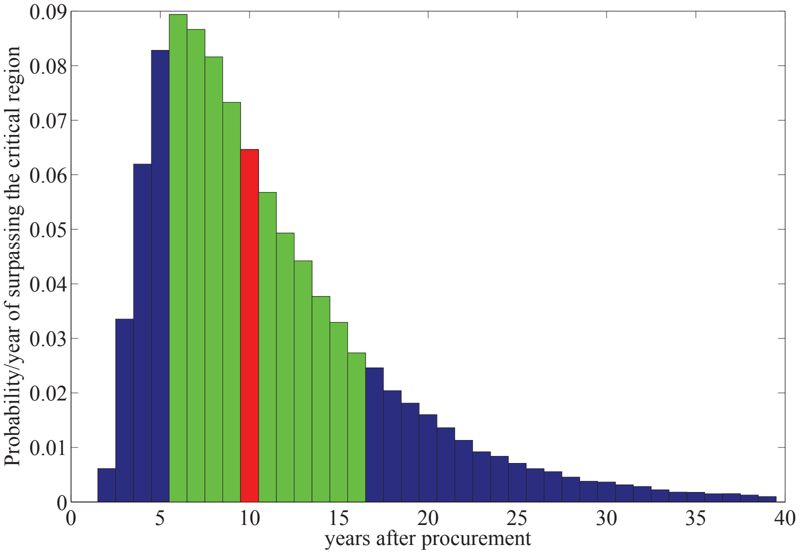


Table A.4: *Top panel: utility curve based on an expected Ao of 17% to 48%. Bottom panel: distribution of the expectation time to exit fully the critical region based on the utility curve from Limit 3. The red bar indicates the expectation and the green bars give the 60% confidence region around the expectation.*

List of Acronyms

Ao	Operational Availability
CFM	Cost Factors Manual
CORA	Centre for Operational Research and Analysis
DAEBM	Director Aerospace Equipment Business Management
DGAEPM	Director General Aerospace Equipment Program Management
DMGOR	Directorate Materiel Group Operational Research
DND	Department of National Defence
DRDC	Defence Research and Development Canada
MND	Minister of National Defence
O&M	Operation and Maintenance
SDE	Stochastic Differential Equation
USAF	United States Air Force
USN	United States Navy

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The CP-140 Aurora fleet of long range maritime patrol aircraft presents the Department of National Defence (DND) with a complicated repair-replace problem. In 1998 DND initiated the fleet-wide renewal Aurora Incremental Modernization Program (AIMP), only to suspend and then re-tool the program in 2007 with the further decision to reduce the fleet from eighteen to ten aircraft. DND plans to use the size-reduced CP-140 Aurora fleet as a surveillance platform until 2020. We apply a stochastic optimal replacement model with forecasting capabilities to historic fleet data to gain insight into the total ownership cost and performance of the fleet. We find that the effects of uncertainty in both costs and operational availability play an important role in predicting the optimal replacement time, and in predicting the future operation and maintenance costs. The application of the stochastic model yields a probability envelope for the future of the CP-140 Aurora fleet in operation and maintenance costs per operational availability. Our model demonstrates that only recently (2005) has the Aurora fleet surpassed the optimal fleet replacement time, but by 2020 the Aurora fleet is expected to exceed the optimal replacement barrier in dollars per operational availability by an order of magnitude.

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