An Integrated Level of Repair Analysis and Spare Parts Stocking Model

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Abstract

Although the level of repair analysis (LORA) and spare parts stocking (SPS) problems are interrelated, the two sets of problems are usually considered sequentially or even independently in practice. The joint problem is typically difficult to solve due to the non-linearity of the constraint associated with system availability. This report illustrates how the two problems can be solved simultaneously by formulating a single mixed integer non-linear optimization model that explicitly captures the interdependency between the repair network (location of resources and allocation of decisions) and sparing inventory decisions (stock levels at each location). The resulting integrated LORA-SPS optimization model is solved by utilizing a decomposition optimization approach, a simulation model, and a genetic programming-based symbolic regression methodology. Preliminary computational results show that the proposed methodology has the capacity to tackle the integrated problem.

Résumé

Bien que les problèmes d'analyse du niveau de réparation (ANR) et de stockage des pièces de rechange (SPR) soient intimement liés, ils sont habituellement considérés de façon séquentielle, voire indépendante, dans la pratique. En général, le problème conjoint est difficile à résoudre à cause de la non-linéarité de la contrainte associée à la disponibilité du système. Le présent rapport montre comment les deux problèmes peuvent être résolus simultanément grâce à l'élaboration d'un seul modèle d'optimisation non linéaire en numérotation mixte qui exprime explicitement l'interdépendance du réseau de réparation (emplacement des ressources et répartition des décisions) et des décisions concernant les stocks de pièces de rechange (niveaux de stocks à chaque emplacement). On obtient un modèle intégré d'optimisation ANR–SPR en utilisant une approche d'optimisation de la décomposition, un modèle de simulation et une méthode de régression symbolique fondée sur la programmation génétique. Les résultats des calculs préliminaires indiquent que la méthode proposée peut permettre d'aborder le problème intégré.

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Executive summary

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Bissan Ghaddar, Yaw Asiedu; DRDC CORA TM 2012-248; Defence R&D Canada – CORA; October 2012.

Background: The use of expensive capital equipment (e.g., weapon systems) is quite common in defence establishments. The downtime of a weapon system impacts not only the operational cost but also the unit readiness. Consequently, the quick recovery of these systems is of utmost importance. However, given their high cost, capital systems cannot be discarded and replaced when they fail but are rather maintained using a "repair by replacement" policy. That is, the failed component is replaced with a functioning spare. The failed component is then repaired if required or discarded otherwise. Level of repair analysis (LORA) is used by most establishments to determine an optimal provision of maintenance facilities and repair decisions to minimize the overall system in-service support cost. The complementary spare parts stocking (SPS) problem seeks to allocate spare parts inventory in the repair/maintenance network such that a certain level of system availability is achieved. Although these two problems are interdependent, they are seldom solved simultaneously due to the complicating nature of the relationships between spare levels and system availability.

Solution Methodology: The solution methodology is composed of three basic modules: a genetic programming approach, a discrete-event simulation, and a non-linear optimization modelling framework to solve medium to large-scale instances of the integrated LORA-SPS problem. In order to solve the joint problem efficiently, it is essential to handle the non-linearity due to system availability or equivalently, the expected backorder (EBO). To approximate the non-linear function, a multi-gene symbolic regression approach is combined with a simulation framework to develop EBO expressions that truly reflect the system under consideration. The EBO equation is then integrated with the LORA model to obtain a mixed integer non-linear program that allows the simultaneous solution of the two problems.

Principal results: Three sets of sparing problems are presented to study the feasibility of the proposed genetic programming approach: a single-echelon single-indenture problem, a single-echelon multi-indenture problem and a multi-echelon multi-indenture problem. Preliminary results revealed that the genetic programming approach is promising as the obtained mathematical expressions produced the same optimal results as the VARI-METRIC/marginal analysis approach presented in the literature. Further, a test instance of

the joint LORA-SPS problem is solved for a multi-echelon multi-indenture problem which shows that the proposed methodology has the potential to tackle the integrated problem.

Future work: In order to solve large-scale instances of LORA-SPS, Lagrangian relaxation is adopted. However, the problems addressed in this report do not fall in this category. Future research would study how to utilize the proposed Lagrangian approach along with simulation and genetic programming to solve real-world large-scale LORA-SPS instances.

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Contexte : L'utilisation de biens d'équipement coûteux (comme des systèmes d'arme) est très courante dans les établissements de défense. Le temps d'indisponibilité d'un système d'arme influe non seulement sur le coût opérationnel mais aussi sur l'état de préparation de l'unité. Le rétablissement rapide de ces systèmes est donc d'une importance capitale. Toutefois, étant donné leur coût élevé, les systèmes de biens d'équipement ne peuvent pas être jetés et remplacés lorsqu'ils tombent en panne ; ils sont plutôt entretenus selon une ≪politique de réparation par remplacement≫, c'est-à-dire que le composant défectueux est remplacé par une pièce de rechange en bon état de marche. Ce composant est ensuite réparé au besoin ou jeté. La plupart des établissements ont recours à l'analyse du niveau de réparation (ANR) pour déterminer la fourniture optimale d'installations de maintenance et les décisions de réparation en vue de réduire au minimum le coût global de soutien en service du système. La solution au problème complémentaire lié au stockage des pièces de rechange (SPR) cherche à répartir les stocks de pièces de rechange dans le réseau de réparation/maintenance de manière à atteindre un certain niveau de disponibilité du système. Bien que ces deux problèmes soient interdépendants, ils sont rarement résolus simultanément en raison de la complexité des relations entre les niveaux de pièces de rechange et la disponibilité du système.

Méthode de solution : La méthode de solution est composée de trois modules de base : une approche de programmation génétique, une simulation par événements discrets et un cadre de modélisation d'optimisation non-linéaire pour résoudre les occurrences de moyenne à grande échelle du problème intégré ANR–SPR. Afin de régler le problème conjoint de manière efficiente, il est essentiel de traiter la non linéarité attribuable à la disponibilité du système ou, de façon équivalente, les arriérés de commande prévus (ACP). L'approximation de la fonction non linéaire s'effectue en combinant une approche de régression symbolique multigénique avec un cadre de simulation pour exprimer des ACP qui reflètent réellement le système à l'étude. On intègre ensuite l'équation ACP au modèle ANR afin d'obtenir un programme non linéaire en numérotation mixte qui permet de résoudre simultanément les deux problèmes.

Principaux résultats : Trois séries de problèmes liés aux pièces de rechange sont présentées en vue d'étudier la faisabilité de l'approche de programmation génétique proposée : un problème à échelon unique et à niveau d'intégration unique, un problème à échelon unique

et à niveaux d'intégration multiples et un problème à échelons multiples et à niveaux d'intégration multiples. Les résultats préliminaires ont révélé que l'approche de programmation génétique est prometteuse, car les expressions mathématiques obtenues ont produit les mêmes résultats optimaux que le modèle VARI-METRIC / la méthode d'analyse différentielle figurant dans la littérature. En outre, une occurrence d'essai du problème conjoint ANR–SPR est résolue dans le cas d'un problème à échelons multiples et à niveaux d'intégration multiples, ce qui montre que la méthode proposée peut permettre d'aborder le problème intégré.

Recherches futures : La relaxation lagrangienne est adoptée afin de résoudre les occurrences ANR–SPR à grande échelle. Toutefois, les problèmes traités dans le présent rapport n'entrent pas dans cette catégorie. Les futurs travaux de recherche porteraient sur la façon d'utiliser l'approche lagrangienne proposée avec la simulation et la programmation génétique pour résoudre des occurrences ANR–SPR à grande échelle dans le monde réel.

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1 Introduction

Manufactured products fail due to different processes such as corrosion, wear and tear, and fatigue. Inexpensive products, such as consumer electronics and appliances, are generally discarded and replaced upon failure. On the other hand, capital equipment such as ships, manufacturing equipment, and airplanes, are characterized by high acquisition costs, high downtime costs, technical complexity, low failure rate, and a long life cycle. Consequently, they are maintained using a "repair by replacement" policy. Failed components are removed and replaced by a functioning spare, if available. If no functioning spare is available, then the replacement is delayed and the system becomes unavailable. A defective component removed from the system can either be discarded or repaired. If it is repaired, then a subcomponent would need to be replaced by a functioning one, possibly at a different location.

In the defense industry, components and subcomponents are commonly referred to as *line replaceable units* (LRUs) and *shop replaceable units* (SRUs), respectively [1]. As the names implies, an LRU is a modular component of a system that is designed to be replaced quickly at an operating location while an SRU is designed to be repaired in a shop. An example of an LRU is a truck's fuel pump with a fuel pressure sensor in the pump being an example of its SRUs. A system may contain tens or hundreds of assemblies, and probably hundreds or thousands of sub-assemblies, components, parts etc., organized into multiple levels of indenture. In Figure 1, an example of a *multi-indenture* product structure is shown, where components and subcomponents are assigned to different indenture levels. Theoretically, there is no limit on the number of indenture levels, however, a two-level indenture structure is commonly considered when modelling most systems in practical applications.

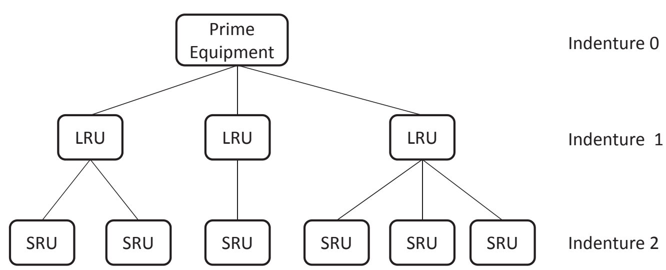

Figure 1: *Depiction of a multi-indenture product structure.*

Various levels of maintenance may be carried out at locations that are geographically dispersed with the capabilities of the repair facilities increasing the farther it is from the system operating site (or base). A repair network may therefore consist of multiple echelon levels as shown in Figure 2. In other words, real world systems are characterized by a multi-indenture product structure and maintained by a multi-echelon repair network.

Level of Repair Analysis (LORA) is used in defence logistics planning to determine where in the repair network a component (or subcomponent) should be repaired or discarded

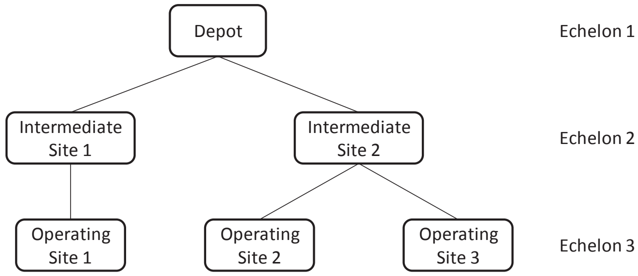

Figure 2: *Depiction of a multi-echelon repair network.*

upon its failure. To enable certain types of repairs, resources have to be located in the repair network as well. For a complex system containing multiple LRUs and SRUs, and with a number of possible repair decisions, LORA seeks to determine an optimal provision of repair and maintenance facilities to minimize overall life-cycle costs. The *spare parts stocking* (SPS) problem on the other hand, seeks to allocate spare parts inventory in the repair/maintenance network such that a certain (high) level of system availability is achieved or equivalently, a certain level of the expected backorder (EBO) at the operating sites is not exceeded. A backorder occurs when a failed component cannot be replaced immediately with a functioning spare.

Although these two problems are interdependent, they are seldom solved simultaneously due to the complicating nature of the relationships between spare levels and system availability [2]. Most previous research considered the LORA and SPS problems separately: first solving a LORA problem, and then setting stock levels based on the LORA input. This often leads to a LORA output with most of the repairs performed at a central location since it tends to be cheaper to locate repair equipment at only one location. Unfortunately this leads to the operating sites facing long resupply lead times, which requires higher levels of spare parts inventory to ensure that a high system availability is realized. On the other hand, if a lot of repairs are performed at the operating sites, then more repair equipment, but less spares inventory are needed. In an integrated model as depicted in Figure 3, the costs of resources can be balanced against the costs of spares to obtain the most cost effective solutions for the desired system availability.

1.1 Research Objective

The research objective is to develop an optimization model that can be used to analyze the integrated LORA-SPS problem. The solution of the integrated problem will prescribe for a given system and repair/maintenance network:

1. which components to repair upon failure and which to discard;
2. where in the repair network to perform the repairs;

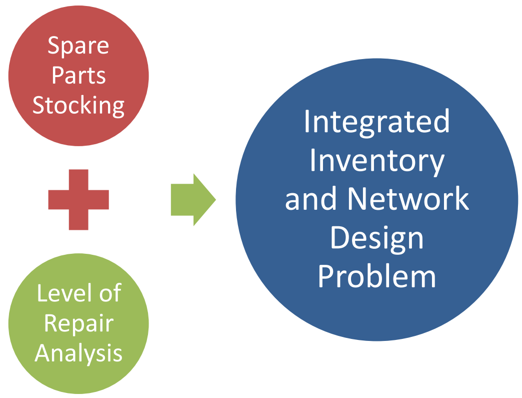

Figure 3: *Integrated LORA and SPS model.*

3. where to install the required resources in the network; and
4. how many spare parts to stock at each location in the repair network,

such that a certain target availability (expected backorder) is achieved against the lowest possible life cycle costs.

The objective is therefore to find a network design that will not only minimize resource and transportation costs but also minimize inventory costs while achieving the system availability (number of backorders).

1.2 Report Outline

The remainder of the report is organized as follows. Section 2 provides a brief literature review on the LORA and SPS problems. Section 3 introduces the assumptions, the notation, and the model for the LORA problem, which is formulated as a capacitated facility location problem. Section 4 introduces the notations used and the logic for understanding the inventory component of the problem, illustrates how to calculate the EBO using the VARI-METRIC approach, and proposes a genetic programming-based approach for approximating the EBO using simple mathematical expressions. The complete formulation for the joint LORA-SPS problem is given in Section 5. The proposed solution methodology utilizing Lagrangian relaxation for solving the optimization model and an integrated simulation and genetic programming-based approach to obtain better approximations for the EBO function is also presented in Section 5. Finally, Section 6 provides a brief summary and highlights future research directions.

2 Literature Review

In this section, an overview of the literature of the level-of-repair analysis problem is presented. A general overview of the most popular methods in the literature that solve the spare parts stocking problem is also discussed, focusing on the popular METRIC-based models. Finally, models that solve the LORA and SPS problems simultaneously are introduced. A more detailed review can be found in [3].

2.1 The LORA Problem

Barros and Riley [4], Saranga and Kumar [5], Gutin et al. [6] and Basten et al. [2] modelled LORA as an integer program in which all repair locations at the same echelon are aggregated and the model assumes infinite capacity of resources. Brick and Uchoa [7] formulated a more general LORA model taking into account the capacity of resources.

Barros and Riley [4] proposed an integer programming model for LORA, as well as branch and bound algorithms to find exact solutions. However, Barros and Riley made a number of simplifying assumptions, resulting in a formulation that does not fit very well in real situations. For instance, the issues of the geographical distribution of the operation sites and the capacity of the maintenance facilities are not considered. In addition, it is assumed that if a facility is capable of performing the repair of a component belonging to a certain indenture level, then it is capable of repairing all other components belonging to that same level. Gutin et al. [6] formulated the LORA problem as an optimization homomorphism problem on bipartite graphs and proved that the LORA problem with two indenture levels and two echelons can be solved in polynomial time using an algorithm for network flow over a bipartite graph. Saranga and Kumar [5], adopted the same integer programming model as Barros and Riley but assumed that each component bore a specific fixed cost whereas in Barros and Riley's model, components at the same indenture share the same fixed costs. To overcome the computational limitations of solving a mixed integer program, Saranga and Kumar used genetic algorithms to solve the resulting problem. Basten et al. [2] also proposed an integer programming model and formulated the LORA problem as a minimum cost flow problem.

2.2 The Spare Parts Stocking Problem

Various approaches to solve the spare parts stocking problem for multi-echelon, multi-indenture systems have been described in the literature. Sherbrooke's 1968 paper [8] is generally considered the seminal work on spare parts inventory management. He introduced the famous Multi-Echelon Technique for Recoverable Item Control (METRIC) scheme for managing repairable spare part inventory levels in a multi-echelon system. Later work by Graves [9] improved on METRIC and provided a better understanding of the distribution of inventory levels in a repairable spare parts supply chain network. In particular, Graves found a closed form expression for the expectation and distribution of the number of backordered items at each echelon of the network, which has been used frequently in subsequent research. The METRIC approach and its extensions (e.g., VARI-METRIC [10]) employ simple assumptions that make it very easy to use. One of these assumptions is that the repair capacity is infinite, hence there is no queuing of components waiting for a repair resource. Consequently, the replenishment lead times can be considered as statistically independent. However, the infinite capacity assumption may not be applicable in practice. This influences the calculation of the performance metrics (EBO) as well as the optimization procedure. De Haas and Verrijdt [11] and Sleptchenko et al. [12] examine capacity effects in the spare parts stocking problem. Further, Caggiano et al. [13] study generalized time-based service levels and determine base stock levels for multiple parts in a multi-echelon spare parts distribution network. The recent book by Muckstadt [14] is now considered the primary reference for service parts inventory management.

2.3 The LORA and SPS Problem

To the best of our knowledge, only the work by Basten [2] and Alfredsson [15, 16] address, in a limited fashion, the joint LORA-SPS problem.

Alfredsson [15] proposed a model to address this overall problem, using marginal analysis, but acknowledged that many simplifying assumptions were made in order to circumvent convexity problems. It was also assumed that the maintenance structure was pre-defined and consisted of two echelons. Basten [2] addressed the same problem. In [2], a modification of the VARI-METRIC procedure [10], which also relies on marginal analysis, was used to obtain near-optimal solutions. Several approximations also had to be made to avoid problems with non-convex constraints in the model.

A single mixed integer non-linear optimization model that explicitly captures the interdependency between the LORA (location of resources and allocation of decisions) and sparing decisions (stock levels at each location) under more general assumptions is proposed in this report. The key to the development of this model is the approximation of the non-linear EOB function by linear combinations of low order mathematical expressions. The approach is composed of three basic models: a simulation model to generate the data sample, a Genetic Programming (GP) model to approximate the EBO function using the data sample and an optimization modelling framework to solve medium to large-scale instances of the integrated LORA-SPS problem.

3 The LORA Model

The decisions of where to repair components, locate resources, and what inventory levels each location should maintain are intertwined in a complex manner. The primary decisions in such a system are grouped into two sets, LORA and SPS decisions. The first set of decisions are LORA decisions, which encompass a broad class of facility location and allocation problems. The main tradeoffs in LORA decisions are between transportation, operating, and capital costs versus satisfying the demand (i.e., making decisions for the faulty components). The LORA model is formulated in this section and utilized in Section 5 in an integrated LORA-SPS problem.

An equipment may have many different types of components; each with a different function. These components differ in terms of their failure frequencies (which generate demands for service parts) and their criticality levels. When a system fails, it is due to a failure in one of its LRUs. The defective LRU, removed from the system, generates a demand for maintenance actions by the repair system. This demand originates at the operating sites (or operating bases) were the equipment are deployed. There are many ways in which an LRU may fail. These are called failure modes, and it is assumed that a specific SRU is responsible for each failure mode. The first decision in LORA is between LRU repair and disposal, depending on the failure mode. In the case of repair, the faulty SRU contained in the LRU is either repaired or discarded. LORA also prescribes to which maintenance facility a failed SRU should be sent for service which may depend on the failure mode. Different types of resources are required to perform each of the possible maintenance actions. The LORA model determines:

- for each component c, whether to repair or discard it upon failure;
- at which location l in the repair network to repair or discard it; and
- at which location l in the repair network to install the required resource r,

such that the lowest possible life cycle cost is achieved. Life cycle cost consists of both fixed costs and variable costs. Fixed costs include costs for installing resources such as test equipment while variable costs which depend on the number of failures, include costs of hiring service engineers (for repair) and transportation of components.

3.1 Mathematical Model

The main assumptions of the LORA model in this report are:

1. components fail at the operating base according to a Poisson process with constant rate,

2. the system fails due to a failure in one of the LRUs which can be discarded or repaired by replacing the failed SRU,

3. a failure in an LRU is caused by a failure in at most one SRU,

4. components that fail may be moved in the repair network to different locations to be repaired,

5. all components have equal priorities,

6. repairs are always successful,

Let C be the set of all components, with $C_1 \subseteq C$ being the set of all LRUs and $C_2 \subset C$ being the set of all SRUs. $\Gamma_c \subseteq C_2$ is the set of SRUs of LRU $c \in C_1$. Let n_c be the number of LRUs in the system and $n_{c_2,c}$ be the number of SRUs, $c_2 \in \Gamma_c$, contained in LRU c. The structure of the repair network is defined by specifying the locations and transportation links between locations. Let G be the set of locations and A, the set of directed transportation links. Define $B \subset G$ as the set of system operating bases (demand generating sites) and the set $L \subseteq G$, as locations with the capacity to host facilities for repair and disposal. The locations l that can receive components from operating base b constitute the set Φ_b and the operating bases that can supply components to location l form the set B_l. Let F be the set of all failure modes in the system. There is a surjective mapping between F and C_2 so that each failure mode f is associated with only one component $c \in C_2$. Let $F(c) \in F$ be the set of failure modes f, associated with a component $c \in C_2$ and θ_f be the failure rate for the failure mode f. Each occurrence of failure mode f requires a decision $d \in D$ to be taken. The set $D = \{1,2,3,4\}$ consists of the possible decisions that can be made for each failure mode $f \in F$ (repair or discard):

- d=1: LRU c is repaired by repairing one of its SRUs $c_2 \in \Gamma_c$, LRU and SRU are transported together,
- d=2: LRU c is repaired by repairing one of its SRUs $c_2 \in \Gamma_c$, LRU and SRU are split and only the SRU is transported for repair,
- d=3: LRU c is repaired by discarding and replacing one of its SRUs $c_2 \in \Gamma_c$,
- d=4: LRU c is discarded.

Let R be the set of the resources and Ω_r is the set of tuples (f,d) such that $(f,d) \in \Omega_r$ if and only if failure mode $f \in F$ requires resource $r \in R$ in order to enable decision $d \in D$.

The parameter $u_{r,f,d}$ is the required capacity of the resource $r \in R$ used to service failure f by the execution of decision d where $(f,d) \in \Omega_r$. The installation of one unit of resource r at location l costs $rc_{r,l}$ and each installed unit provides a capacity $M_{r,l}$. Installing resources at a particular location has a fixed cost of $fc_{r,l}$. The maximum number of units of resource

r that can be installed at location l is $N_{r,l}$. The variable cost, $vc_{f,b,d,l}$, consists of transportation cost ($tc_{c,b,l}$) and either maintenance cost ($mc_{c,l}$) or discard cost ($dc_{c,l}$) where $f \in F(c)$ (see Table 1 for details).

Table 1: *Variable costs for the set of possible LORA decisions.*

Decision	Description	Unit Cost $vc_{f,b,d,l}$
$d=1$	repair LRU c by repairing SRU $c_2 \in \Gamma_c$ (LRU with SRU is transported to the repair location)	$tc_{c,b,l}+mc_{c_2,l}$
$d=2$	repair LRU c by repairing SRU $c_2 \in \Gamma_c$ (only the SRU is transported to the repair location)	$tc_{c_2,b,l}+mc_{c_2,l}$
$d=3$	repair LRU c by discarding SRU $c_2 \in \Gamma_c$	$\frac{tc_{c_2,b,l}}{2}+dc_{c_2,l}$
$d=4$	discard LRU c	$\frac{tc_{c,b,l}}{2}+dc_{c,l}$

Let the number of units of the equipment installed at an operating base, b, be n_b and let γ_b be the annual operating time of the system at operating base b. Then $\theta_{f,b}$, the demand rate for failure mode f at operating base $b \in B$, can be calculated as follows:

$$\theta_{f,b} = n_b \gamma_b n_c n_{c_2,c} \theta_f. \tag{1}$$

Consider the following decisions variables:

- $X_{f,b,d,l}$ = fraction of the demand due to failure mode f generated at operating base $b \in B$ that are attended to by the execution of the decision d at location $l \in \Phi_b$.
- $Y_{r,l}$ is the number of units of resource r installed at location $l \in L$.
- $Z_{r,l}$ is a binary variable, with a value of 1 if resource r is installed at location $l \in L$ and 0 otherwise.

The model is presented as follows:

$$\textbf{(LORA-MIP)}\ \min \sum_{f\in F}\sum_{b\in B}\sum_{d\in D}\sum_{l\in\Phi_b} vc_{f,b,d,l}\theta_{f,b}X_{f,b,d,l} + \sum_{r\in R}\sum_{l\in L} rc_{r,l}Y_{r,l} + \sum_{r\in R}\sum_{l\in L} fc_{r,l}Z_{r,l}$$

$$\text{s.t.} \sum_{d\in D}\sum_{l\in\Phi_b} X_{f,b,d,l} = 1, \qquad \forall f\in F, \forall b\in B \tag{2}$$

$$\sum_{(f,d)\in\Omega_r}\sum_{b\in B_l} \theta_{f,b}u_{r,f,d}X_{f,b,d,l} \le M_{r,l}Y_{r,l}, \qquad \forall r\in R, \forall l\in L\backslash B \tag{3}$$

$$\sum_{(f,2)\in\Omega_r}\sum_{l\in\Phi_b} \theta_{f,b}u_{r,f,2}X_{f,b,2,l} + \sum_{(f,d)\in\Omega_r} \theta_{f,b}u_{r,f,d}X_{f,b,d,b} \le M_{r,b}Y_{r,b}, \qquad \forall r\in R, \forall b\in B \tag{4}$$

$$0 \le X_{f,b,d,l} \le 1, \qquad \forall f\in F, \forall b\in B, \forall l\in\Phi_b, \forall d\in D \tag{5}$$

$$0 \le Y_{r,l} \le N_{r,l}Z_{r,l} \qquad Y_{r,l}\in\mathbb{Z}, \forall r\in R, \forall l\in L \tag{6}$$

$$Z_{r,l}\in\{0,1\}, \qquad \forall r\in R, \forall l\in L. \tag{7}$$

where $\mathbb{Z}$ is the set of integers. The objective function is given by the sum of the costs to perform the required decision, which consists of variable costs (transportation, maintenance, and/or discard costs), per unit costs for resources, and fixed costs for installing resources. Constraints (2) guarantee that for each generated demand at the operating site a decision is made. Constraints (3) impose a limitation on the resource capacities at all locations except the operating sites. Constraints (4) enforce a limitation on the resource capacities at the operating sites and ensure that in the case where the LRU and SRU need to be split, a resource is available at the operating site for this task. Constraints (6) control the number of resources that can be installed at each location. Note that if it is desirable to take only one action per failure mode originating from a specific operating base then $X_{f,b,d,l}$ can be treated as a binary variable.

3.2 Solution Methodology

LORA-MIP can be considered as a Capacitated Facility Location Problem (CFLP). The CFLP problem is well known and has many real world applications. The literature in this area contains a wide range of variants and extensions. Interested readers may refer to Aikens [17] and Brandeau and Chiu [18] for an overview. In general, a set of demand points (demands generated by failure modes) and a set of proposed locations (for **LORA-MIP**, this is the set of locations where resources can be installed) are given. Each resource has a fixed operating cost, and each transportation link between a location and an operating base point has a shipping cost per unit of demand. In **LORA-MIP** setup, the facility location problem is concerned with choosing the best location for resources from a given set of potential sites so as to minimize the total operating, fixed, and transportation costs while servicing the demand. The resources are assumed to have a finite supply capacity and can only accommodate a limited amount of demand. The CFLP is known to be NP-Hard, as such **LORA-MIP** is also NP-Hard. However, CPLEX, a mathematical programming solver that utilizes branch-and-cut techniques to solve mixed integer programs, may be used to solve medium-sized instances of **LORA-MIP**. Lagrangian relaxation has been widely used to solve several variants of large scale capacitated (see Cornuejols et al. [19] and Beasley [20]) and uncapacitated (see Galvão and Raggi [21]) facility location problems.

3.3 Preliminary Results

In this section, a small instance of the LORA problem is solved using the formulation presented in Section 3.1. The case study consists of the following input parameters:

- one LRU consisting of ten SRUs;
- ten failure modes;
- the repair network consists of 11 locations, four of which are operating bases (see Figure 4);

- three types of resources;
- a maximum of 5 of each resource per location is allowed;
- vc, rc, and fc are random integers taken from the intervals [1,7], [1,20], and [1,50] respectively, due to the large number of cost parameters, the values are not shown; and
- the resource capacity and capacity limit are also generated randomly.

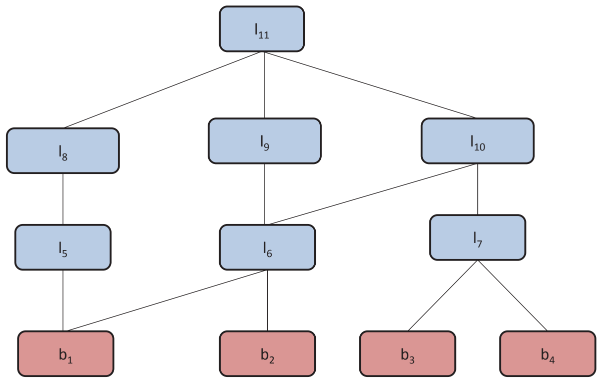

Figure 4: *The network structure for the sample LORA problem.*

Matlab was used to implement **LORA-MIP** and CPLEX was used to solve the resulting optimization problem. As seen in Table 2, the total computational time was 0.23 sec with a resulting objective function value of $1,394. Branch-and-cut algorithm was used to solve the mixed integer programming problem and the total number of nodes in the branch-and-cut tree was 280 and a total of 87 cuts were added.

Table 2: *Performance characteristics for the CPLEX mixed integer programming algorithm.*

Root Lower Bound ($)	1384
Root Upper Bound ($)	1411
Optimal Objective ($)	1394
Time (sec)	0.23
Nodes	280
Cuts	87

The optimal solution of the problem is given in Table 3 and Figure 5. Table 3 shows the locations where the decision taken for each failure mode is carried out. A check mark in the table refers to a repair or to a discard decision being taken at location l for failure mode f (refer to Table 4 for a more detailed solution). In Figure 5, the shaded locations are assigned resources and perform some repair/discard tasks while the blank locations are not allocated any resources and are not assigned any maintenance tasks. Due to the large number of variables (1826 in all), only the decisions taken for each of the failure modes are given whereas the number of resources of each type per location are not shown. As the LORA model is a special case of CFLP, solving small to medium-sized instances of the LORA model can be done efficiently using mixed integer programming techniques [19]. Hence, utilizing the CFLP structure of the LORA model is key to solving the joint LORA-SPS problem, as will be shown in Section 5.3.1.

Table 3: *Location of the repair/discard decisions for each failure mode for the sample LORA problem.*

f \ l	b_1	b_2	b_3	b_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}
f_1										✓	✓
f_2	✓			✓							✓
f_3										✓	✓
f_4	✓			✓		✓	✓	✓	✓	✓	
f_5								✓	✓	✓	✓
f_6	✓						✓				✓
f_7	✓			✓							✓
f_8								✓	✓	✓	
f_9	✓			✓		✓	✓				✓
f_{10}	✓							✓	✓	✓	

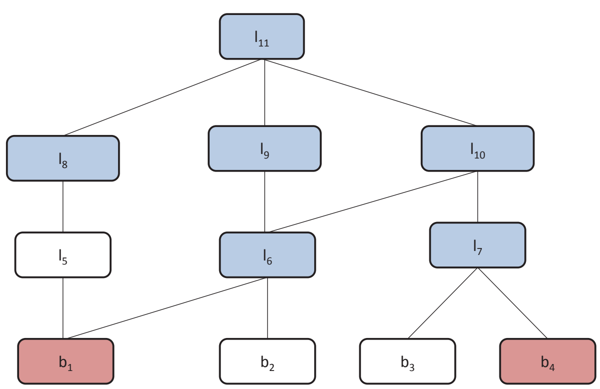

Figure 5: *Optimal allocation of resources for the sample LORA problem.*

Table 4: The optimal repair/discard decisions for the sample LORA problem.

f	b	d	l	$X_{f,b,d,l}$	f	b	d	l	$X_{f,b,d,l}$
1	1	3	10	1	5	4	3	10	0.250
1	2	3	11	1	6	1	1	1	1
1	3	3	10	1	6	2	1	11	1
1	4	3	11	1	6	3	1	7	1
2	1	1	1	1	6	4	1	11	1
2	2	3	11	1	7	1	3	1	1
2	3	1	11	0.750	7	2	3	11	1
2	3	3	11	0.250	7	3	1	11	1
2	4	1	4	0.750	7	4	3	4	0.875
2	4	3	11	0.250	7	4	3	11	0.125
3	1	1	11	1	8	1	3	8	0.750
3	2	1	10	1	8	1	4	8	0.250
3	3	1	10	1	8	2	3	9	1
3	4	1	10	1	8	3	3	10	1
4	1	4	1	0.750	8	4	3	10	1
4	1	4	8	0.042	9	1	1	1	1
4	1	4	10	0.208	9	2	4	6	1
4	2	4	6	0.500	9	3	1	11	0.625
4	2	4	9	0.300	9	3	4	7	0.375
4	2	4	10	0.200	9	4	4	4	0.950
4	3	4	7	0.250	9	4	4	7	0.050
4	3	4	10	0.750	10	1	1	8	0.625
4	4	4	4	0.500	10	1	3	1	0.375
4	4	4	10	0.500	10	2	1	9	1
5	1	3	8	1	10	3	1	10	0.500
5	2	3	9	1	10	3	4	10	0.500
5	3	3	10	1	10	4	4	10	1
5	4	2	11	0.750					

4 The Spare Parts Stocking Problem

In Section 3.1, a model for solving the LORA problem was introduced. Missing from this model was the consideration of the amount of the spare parts to acquire. The second set of decisions required to solve the integrated LORA-SPS problem is related to setting inventory levels. The main trade-off in the spare parts stocking problem is between inventory holding costs and target availability and hence EBO.

Generally, the problem of computing the number of spare parts to allocate at the different locations in the repair network such that a target system availability with the lowest possible costs is treated separately from the LORA problem. This section provides a brief overview of the most popular method currently available in the literature for solving the spare parts stocking problem; that is, the VARI-METRIC approach. Although popular, the VARI-METRIC method requires elaborate procedures/computations to ensure the EBO-cost curves, which are used for the optimization procedure, are convex. Additionally, the form of the EBO equations does not permit their use in standard mathematical programming models (see Annex B). To overcome the complexity of calculating the EBO, a GP-based symbolic regression methodology is proposed in Section 4.3 to approximate the EBO function using simpler mathematical expressions that can be integrated in the LORA problem.

4.1 METRIC-based Approaches

The METRIC model developed by Sherbrooke [8], is generally considered in the literature as the first multi-echelon spare parts inventory model. The objective of the classical METRIC-based models is to determine the optimal stock levels of spare parts at each location of the multi-echelon repair network such that the EBO at the operating sites is minimized (which is equivalent to maximizing the availability of the system) subject to a budget constraint.

4.1.1 Assumptions

The most important assumptions made in the spare parts stocking model of METRIC-based approaches are as follows:

- components fail at the operating base according to a Poisson process with constant rate;
- there is no lateral resupply between operating bases;
- for each component at each location, an $(s-1,s)$ inventory control policy is used, i.e., an order for the resupply of a spare is placed immediately the number of spares, s, is reduced by one;

- unsatisfied demand due to a stock out at a particular location is backordered;
- there is no commonality, so an SRU may not be part of two different LRUs;
- replacement of a failed component by a functioning component (if a spare exists) takes zero time;
- a failure of one type of component is statistically independent of those that occur for any other type of component;
- all components have equal priorities;
- the repair lead time includes the time for transporting the failed component to the repair location and for diagnosing the cause of the failure; and
- all repair lead times, replenishment lead times, and move lead times are independent random variables.

4.1.2 EBO Calculation

The optimal economic order quantity for high-cost and low demand items like those found in capital equipment is one, hence the assumed $(s-1, s)$ inventory control policy. For this economic order quantity, the stock balance equation which is fundamental to METRIC-based methodologies is given as:

$$s = OH + DI - BO. \tag{8}$$

Equation (8) shows that for any given location in the maintenance network, the stock level, s, is equal to the on-hand inventory (OH) plus the inventory due in from repair and resupply (DI) minus any existing backorders (BO). DI is the number of items in repair and in resupply from another location and is also referred to as the pipeline. A backorder only occurs when $DI > s$. Consequently, the EBOs can be calculated as:

$$EBO(s) = \sum_{x=s+1}^{\infty} (x-s)P(DI = x) \tag{9}$$

where $P(\cdot)$ denotes the steady-state probabilities for the number of units due in. The calculation of $P(DI = x)$ is what distinguishes METRIC[1] from VARI-METRIC. METRIC assumes that the number of items in the pipeline is Poisson distributed with a mean equal to the product of the item demand rate (λ) and mean repair time (T) as shown in Equation (10), this follows from Palm's theorem [22].

$$p(x) = (\lambda T)^x e^{-\lambda T}/x! \tag{10}$$

[1]Similar to METRIC, MOD-METRIC also uses a Poisson distribution.

However, in most multi-echelon and multi-indenture (MEMI) systems, the number of items in the pipeline can be better approximated using the negative binomial distribution which can be defined as:

$$p(x) = \binom{a+x-1}{x} b^x (1-b)^a \tag{11}$$

where $a = \frac{\mu}{V-1}$ and $b = \frac{V-1}{V}$, with μ being the pipeline mean and V the pipeline variance-to-mean ratio.

For a given inventory budget, METRIC-based approaches often use a marginal analysis procedure to select at each iteration, the component for which an additional stock should be purchased and the location of the spare to yield the greatest reduction in EBO per dollar [23]. To compute the EBO expression, the procedure takes into account: component demands, repair times, lead times, and costs (refer to Annex B for detailed equations for calculating the EBO expression). The output of marginal analysis is a set of curves from which for a given cost, one can obtain the minimum EBO that can be achieved and the corresponding spares stock levels.

4.2 A Mathematical Model Approach

For the development of the mathematical model the following notation is used:

- $\lambda_{c,b}$ is the demand rate of component $c \in C$ at operating base $b \in B$
- $hc_{c,l}$ is the holding cost per spare of component $c \in C$ at location $l \in L$
- K is the total budget
- E is the target EBO
- $s_{c,l}$ is the spare stock level for each component $c \in C$ at location $l \in L$.

With the notation defined above and the EBO equation provided in Annex B, the optimization model for the spare parts stocking problem can be stated as follows:

$$\min \sum_{c \in C} \sum_{l \in L} hc_{c,l} s_{c,l} \tag{12}$$

$$\text{s.t.} \sum_{c \in C_1} \sum_{b \in B} EBO(s_{c,l}) \leq E \tag{13}$$

$$s_{c,l} \geq 0 \text{ and integer} \qquad \forall c \in C, \forall l \in L. \tag{14}$$

The objective is to minimize the total holding cost, $hc_{c,l}$, of the spares subject to a back-order (or availability) constraint. The value of E is chosen based on the desired system availability level (see [1]). Note that if the number of pieces of equipment is not identical across the operating bases, then a separate EBO constraint is needed for each operating base resulting in $|B|$ constraints similar to constraint (13). Another way is to have a single weighted EBO constraint where a weight, w_b reflects the number of equipment at each operating base b, i.e., replace the left hand side of constraint (13) by

$$\sum_{c \in C_1} \sum_{b \in B} w_b EBO(s_{c,l}).$$

Another way to formulate the problem is to consider minimizing EBO subject to a budget constraint:

$$\textbf{(SPS-MEMI)} \quad \min \sum_{c \in C_1} \sum_{b \in B} EBO_{c,b}(s_{c,l}) \tag{15}$$

$$\text{s.t.} \sum_{c \in c} \sum_{l \in L} hc_{c,l} s_{c,l} \leq K \tag{16}$$

$$s_{c,l} \geq 0 \text{ and integer} \qquad \forall c \in C, \forall l \in L. \tag{17}$$

As mentioned earlier, the EBO equation is highly non-linear in general. As a result, mostly heuristics and marginal analysis techniques have been used to solve these optimization models but none of these approaches have quality guarantees or provide a proof of optimality. Additionally, Lagrangian multipliers and dynamic programming have also been tried in the past but resulted in limited benefits. The optimization problems may become tractable if equation (13) (or (15)) is approximated with a suitable function. In the following section, a genetic programming algorithm is proposed to determine such approximate functions. These functions are used to solve the spare parts stocking problem using an optimization approach as opposed to the traditional marginal analysis method.

4.3 Solution Methodology

Genetic Programming is an evolutionary computation technique that was originally developed by Koza [24] in 1992 with the attempt to automatically solve problems through the evolution of computer programs. The primary advantage of GP lies in its systematic and domain-independent formulation which makes it easily applicable to a variety of problems. In particular, GP algorithms are commonly exploited to solve symbolic regression problems. A significant benefit of GP-based symbolic regression over traditional methods lies in its capability to automatically evolve both the structure and the parameters of a desired symbolic input-output model, i.e., given a set of input-output data and a user defined set of *functions*, e.g., $F = \{+, -, \times, /, exp, cos, sin\}$, a GP algorithm will automatically discover explicit mathematical equations that best describe the functional relationships between the independent (input) and the dependent (output) variables. The five major steps for the use of genetic programming requires:

1. the set of independent and dependent variables of the problem;
2. the set of functions for each branch of the program to be evolved;
3. the fitness measure (for measuring the fitness of individuals in the population);
4. the termination criterion; and
5. method for designating the result of the run.

It has been shown in the literature that a multi-gene symbolic regression approach is generally more accurate and computationally more efficient than other classical and more common GP-based methods [25]. Multi-gene symbolic regression is performed by evolving a population of trees, where each tree is referred to as a *gene*. Specifically, the generated symbolic model is a weighted linear combination of a number of mathematical expressions represented by different GP trees (hence, the name multi-gene).

An example of a multi-gene symbolic regression solution is shown in Figure 6. It models a problem with one output (y) and two inputs (x_1 and x_2), where the variable d_i, $i \in \{1,2\}$ denotes the weighting coefficient of the ith gene and d_0 is a bias/offset term. These coefficients are estimated using ordinary least squares techniques. It can be observed that the model is a weighted linear combination of different genes, each of which may be represented using non-linear terms (e.g., exponential function) specified in a user-defined set of arithmetic operations, i.e., functions.

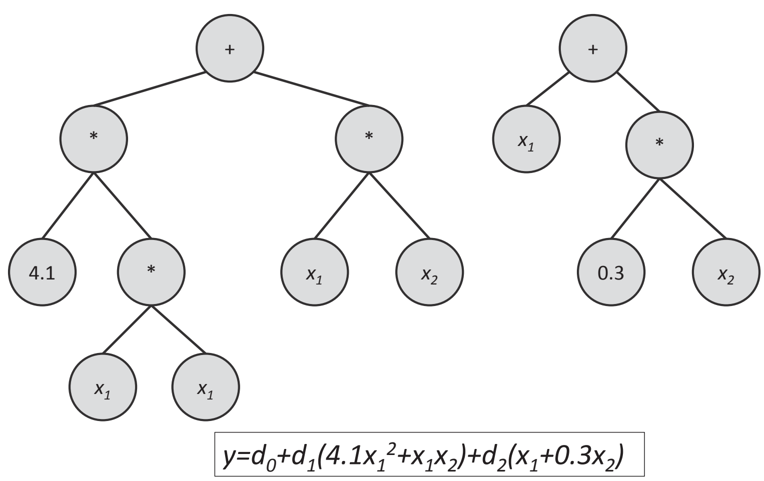

Figure 6: *A simple example of a multi-gene model.*

The multi-gene symbolic regression implementation exploited in this work is that of Searson *et al.*, called GPTIPS [26]. GPTIPS is a genetic programming tool used with Matlab.

Initially, a random population of GP trees is created. The number of genes per tree (G_{max}) and the maximum tree depth (D_{max}) are two of a number of user defined parameters. This allows for some control over the complexity of the evolved models. Restricting the values of D_{max} allows for the evolution of relatively simple and compact models that are linear combinations of low order non-linear transformations of the input attributes [26].

A GP-based multi-gene symbolic regression algorithm is used in this work due to its inherent capability to model the objective function of **SPS-MEMI** using low-order mathematical expressions. This is particularly useful since a low-order mathematical equation can be integrated with a LORA model to solve the joint LORA-SPS optimization problem.

4.4 Preliminary Results

Three sparing problem instances are used to study the feasibility of the proposed approach. The data set used in developing the regression model in each case is generated using the corresponding Equation (B.1) presented in Annex B. Each data set was randomly sampled and split into training, validating, and test data sets. The training and validation data sets are exploited to generate a model that best fits the data using the GP-based multi-gene algorithm. The training data set is used to generate the model while the validation data set is used to assess how well the model performs against unseen data and also to check against overtraining. The test data set is used to evaluate the accuracy and the generalization capabilities of the obtained solution. For all the case studies, the fitness function used is based on the root-mean squared error (RMSE) between the measured and predicted output of the evolved expressions.

Once the mathematical expression of the EBO is derived using GP, the optimization problem, **SPS-MEMI**, defined earlier, is solved using Matlab global optimization toolbox. For the purposes of comparison, the METRIC and the VARI-METRIC approaches are also implemented and solved using Matlab. Recall that for a given inventory budget, VARI-METRIC and METRIC use a marginal analysis procedure to select spares that yield the lowest EBO.

4.4.1 Case Study 1 – Single Echelon Single Indenture (SESI)

In this example, it is assumed that the equipment is made up of two LRUs and no SRUs, and the repair infrastructure is located at the operating base. The GP parameters used are shown in Table 5.

Table 5: *Subset of the GP parameters used for Case Study 1.*

GP Parameter	Value, Range or Set
Population size	200
Number of generations	500
Tournament size	3
D_{max}	3
G_{max}	15
Function nodes set	$\{+, -, \times\}$
Constant nodes range	[-10, 10]

The best approximation of the EBO was determined as:[2]

$$\begin{aligned} EBO_{GP} &= 0.4014 - 0.01507\,\mathrm{s_2}^3 + 0.1069\,\mathrm{s_1}\,(\mathrm{s_1}+1) \\ &- 0.4810\,(\mathrm{s_1}+\mathrm{s_2}) + 0.6264\,\lambda_2\,\mathrm{T_2} - 0.6264\,\mathrm{s_2}^2 \\ &- 0.2740\,\mathrm{s_1}\,\lambda_1\,\mathrm{T_1} - 0.006100\,\mathrm{s_2}\,{\lambda_2}^2\,\mathrm{T_2} \\ &- 0.07678\,\mathrm{s_2}\,\lambda_2\,\mathrm{T_2}^2 + 0.07371\,{\lambda_1}^2\,\mathrm{T_1}^2 \\ &+ 0.01485\,\mathrm{s_1}^2\,\lambda_1\,\mathrm{T_1} + 0.7740\left(\mathrm{s_2}^2 + \lambda_1\,\mathrm{T_1}\right) \\ &- 0.1801\,\mathrm{s_2}\,\lambda_2\,\mathrm{T_2} - 0.006401\,\mathrm{s_1}^3 \\ &+ 0.01676\,\mathrm{s_2}^2\,\lambda_2\,\mathrm{T_2} + 0.1038\,{\lambda_2}^2\,\mathrm{T_2}^2 \\ &+ 0.0004730\mathrm{T_2}^4, \end{aligned} \tag{18}$$

where $\lambda_1, \lambda_2, T_1, T_2$ are the respective demand rates and the repair times for the two LRUs. The stock levels are the variables s_1 and s_2. Note that the subscripts corresponding to the locations have been dropped. Comparisons of the predicted versus the actual EBO values are shown in Figure 7. As can been seen from the values of the root-mean squared error between the measured and predicted output and the variance coefficient, R^2, the GP model is able to predict the EBO values with a high degree of accuracy. The RMSE values are 0.0744, 0.0775, and 0.0763 for the training, test, and validation data sets, respectively, and the corresponding R^2 values are 99.25%, 99.09%, and 99.24%.

For given demand rates and repair times, **SPS-MEMI** reduces to **SPS-SESI**, and can be solved to obtain the optimal spare levels of the components:

$$\textbf{(SPS-SESI)} \quad \min \quad EBO_{GP}(s_1, s_2)$$
$$\text{s.t. } hc_1 s_1 + hc_2 s_2 \leq K$$
$$s_1, s_2 \geq 0 \text{ and integer.}$$

[2] A summary of the data sets used in the development of the GP regression models in this report is provided in Annex C.

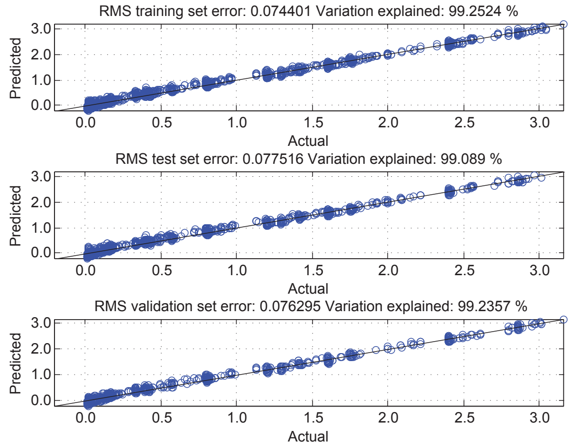

Figure 7: *Prediction scatter plot for the GPTIPS multi-gene regression model for Case Study 1.*

The objective function refers to Equation (18) for a given set of demand rates and repair times, i.e., $\lambda_1, \lambda_2, T_1$, and T_2 are fixed. Figure 8 is a plot of the objective function using the input data of Instance 1 of Table 6. This shows that EBO_{GP} is a convex function of the stock level and hence **SPS-SESI** can be solved efficiently to optimality.

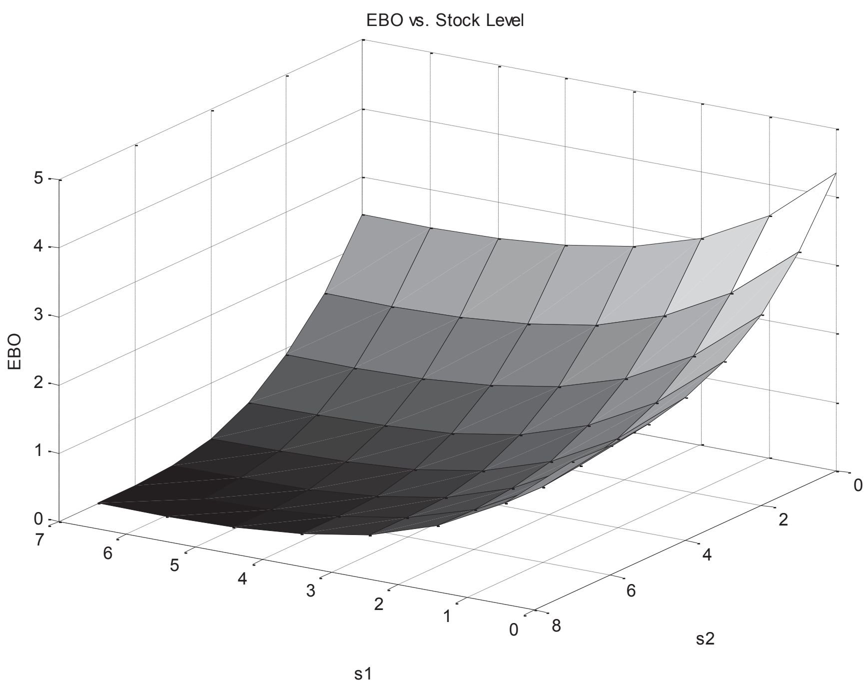

Figure 8: *Expected backorder as a function of the stock level for Case Study 1.*

The approximate model was tested on a set of four different instances. Table 6 provides a comparison between the METRIC approach and the presented GP approach for various input parameters. The spare levels of both the GP-optimization (GP-OPT) approach and the METRIC approach are identical whereas the EBO values vary slightly. Since the METRIC approach provides the optimal spare parts inventory (under the assumptions given in [1]), obtaining the same results as the METRIC approach provides evidence that the GP-based approach is promising in solving SPS problems.

Table 6: *Test results for Case Study 1.*

Instance	Input Parameters							GP-OPT			METRIC		
	λ_1	λ_2	T_1	T_2	c_1	c_2	K	s_1	s_2	EBO_{GP}	s_1	s_2	EBO_{VM}
1	6	5	0.4	0.4	330	440	1100	2	1	2.0098	2	1	1.9345
2	6	5	0.4	0.4	330	440	3500	5	4	0.1548	5	4	0.1269
3	2	4	0.4	0.4	330	440	1100	1	2	0.7241	1	2	0.5762
4	6	5	0.3	0.4	330	440	1100	2	1	1.6529	2	1	1.5635

4.4.2 Case Study 2 – Single Echelon Multi-Indenture (SEMI)

In this case, the system consists of a single LRU which in turn contains a single SRU. The repair infrastructure is located at the operating base. The GP parameters used in this case are presented in Table 7.

Table 7: *Subset of the GP parameters used for Case Study 2.*

GP Parameter	Value, Range or Set
Population size	100
Number of generations	400
Tournament size	3
D_{max}	4
G_{max}	15
Function nodes set	$\{+, -, \times\}$
Constant nodes range	[-10, 10]

Similar to the previous section, the EBO is approximated by the function given below.

$$
\begin{aligned}
EBO_{GP} \;=\;& 0.1467-0.002678\,\mathrm{s}_1\,(2\,\mathrm{s}_1+\lambda_0)-0.01189\,\mathrm{s}_0\,\lambda_0\,\mathrm{T}_0\\
-\;& 0.09075\,\mathrm{s}_1+0.01434\,\mathrm{T}_1-0.01434\,\mathrm{s}_1\,\mathrm{T}_0+0.01434\,\lambda_0\,\mathrm{T}_0\\
+\;& 0.007594\,\mathrm{s}_1\,(\mathrm{T}_0+7.497)+0.007594\,\mathrm{T}_1\,(\mathrm{s}_1-\lambda_0)\\
-\;& 9.319\,\mathrm{s}_0-0.002784\,(6.867\,\lambda_0-\lambda_0\,\mathrm{T}_0)\,(\lambda_0+\mathrm{T}_1+\lambda_0\,\mathrm{T}_0)\\
+\;& 0.0003517\,\mathrm{s}_1\,\mathrm{s}_0{}^2-0.0003517\,\mathrm{T}_0{}^2-0.0003517\,\mathrm{T}_0\\
+\;& 0.002602\,(\lambda_0+\mathrm{T}_1)\,(\mathrm{s}_0-\mathrm{T}_1)+0.2230\,\mathrm{s}_0+1.2593\,\lambda_0 \qquad (19)\\
-\;& 0.0002039\,\mathrm{s}_0{}^2\,\lambda_0\,\mathrm{T}_1+0.03213\,\mathrm{T}_1\left(\lambda_0{}^2+\lambda_0\right)\\
-\;& (0.1627\,\mathrm{T}_1+1.22)\,\lambda_0+0.008327\,(\mathrm{s}_1-\lambda_0\,\mathrm{T}_1)\,(\mathrm{s}_1+\lambda_0+\mathrm{T}_0-\mathrm{T}_1)\\
+\;& 0.1853\,\lambda_0\,(\mathrm{T}_0+\mathrm{T}_1)+0.003590\,\mathrm{s}_1+0.003590\,(\mathrm{s}_0+\lambda_0)\,(\mathrm{s}_0-\mathrm{s}_1)
\end{aligned}
$$

where λ_0 is the demand rate for the LRU and the SRU (they are equal in this case) and T_0 and T_1 are the repair times for the LRU and the SRU respectively. Note that the subscript for the location has been dropped. Comparisons of the predicted versus the actual EBO values are shown in Figure 9. The RMSE values are 0.0362, 0.0362, and 0.0371 for the training, test, and validation data sets, respectively, and the corresponding R^2 values are 80.18%, 80.15%, and 79.63%.

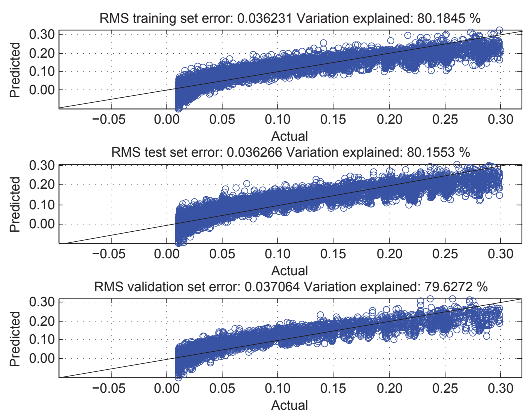

Figure 9: *Prediction scatter plot for the GPTIPS multi-gene regression model for Case Study 2.*

An optimization problem similar to **SPS-MEMI** is solved where s_0 and s_1 are the variables corresponding to the spare levels of the LRU and the SRU respectively. Figure 10 shows a plot of the EBO against sparing levels for a set of demand rates and repair times (Instance 1 of Table 8). As can be seen, the resulting EBO function from the GP is convex and can be solved to optimality efficiently.

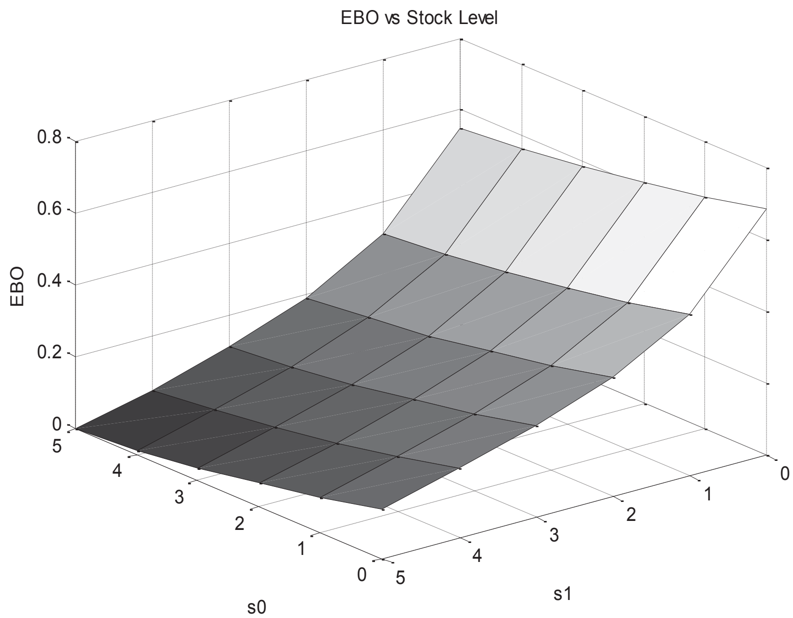

Figure 10: *Expected backorder as a function of the stock level for Case Study 2.*

The approach is tested on four different sets of instances. From Table 8, the GP-optimization approach and the VARI-METRIC approach gave the same spare level values for the LRU and for the SRU with a comparable EBO values.

Table 8: *Test Results for Case Study 2.*

	Input Parameters							GP-OPT			VARI-METRIC		
Instance	λ_0	λ_1	T_0	T_1	c_0	c_1	K	s_0	s_1	EBO_{GP}	s_0	s_1	EBO_{VM}
1	1.1	1.1	1.5	1.5	200	100	1000	4	2	0.1199	4	2	0.0947
2	1.1	1.1	2	1	200	100	1000	4	2	0.1599	4	2	0.1395
3	1.5	1.5	1	1	200	100	1000	4	2	0.0921	4	2	0.0612
4	1.5	1.5	1	1	200	200	1000	5	0	0.1077	5	0	0.1346

4.4.3 Case Study 3 – Multi-Echelon Multi-Indenture

For the final case study, a system with a single LRU and two SRUs is considered (i.e., $C = \{0,1,2\}$, $C_1 = \{0\}$). The repair network consists of a single depot and a single operating base (i.e., $L = \{0,1\}$ and $B = \{1\}$). The LRU repair times for the operating base and depot are 5 and 15 days and the SRU repair times are 10 and 30 days. The resupply time, O_b, is 15 days for all components. The LRU mean daily demand rate at the operating base is 0.1.

The LRU is assumed to cost \$200 and the SRU \$100 and the total budget is \$1100. The GP parameters used in this case are presented in Table 9.

Table 9: *Subset of the GP parameters used for Case Study 3.*

GP Parameter	Value, Range or Set
Population size	100
Number of generations	400
Tournament size	3
D_{max}	4
G_{max}	10
Function nodes set	$\{+, -, \times\}$
Constant nodes range	[-10, 10]

For a given problem, most of the variables except the demand rates and the stock levels at each location would be known. As a result, in this model, the demand rate and the level of spares are the independent variables, i.e., $s_{c,l}$ and $\lambda_{c,l}$, totaling 12 variables in all. The model for the sample problem considered is shown in Equation 20.

$$
\begin{aligned}
EBO_{GP} &= 2.499\,\lambda_{2,1}\,(s_{0,1}+\lambda_{1,0}+\lambda_{2,0})+1.58 \\
&+ 0.0,1692\,(\lambda_{0,0}+s_{2,1}-2\,\lambda_{2,1})\,(s_{0,1}-\lambda_{2,0}+s_{2,1}-\lambda_{2,1}) \\
&- 379.6\,\lambda_{1,1}\,(\lambda_{0,1}-\lambda_{1,1})+0.4792\,\lambda_{2,0}-0.4792\,s_{0,1} \\
&+ 0.5052\,(\lambda_{0,0}+\lambda_{2,0})\,(s_{1,1}-8.127\,s_{0,0})\,(\lambda_{0,0}-\lambda_{1,1}) \\
&- 0.1949\,(\lambda_{0,0}-\lambda_{1,0}+s_{2,1}+2\,s_{2,1}\,\lambda_{2,1}) \\
&+ .05742\,(\lambda_{0,1}+\lambda_{1,1})\,(s_{2,0}-\lambda_{2,1})\,(s_{0,1}+s_{2,1}-s_{0,1}\,\lambda_{2,0}) \\
&- 0.0752\,(2\,\lambda_{0,0}-s_{2,1}+s_{2,0})+0.0,090,18\,s_{2,0} \\
&+ 0.0405\,(s_{0,1}+2\,\lambda_{1,0})\,(s_{0,1}+\lambda_{2,0}+\lambda_{2,1}) \\
&- 0.0,04509\,s_{1,0}+0.0,04509\,(\lambda_{2,0}-s_{2,0})\,(s_{2,1}-s_{2,0})
\end{aligned} \qquad (20)
$$

where $\lambda_{c,l}$ and $s_{c,l}$ correspond to the demand rate and the stock level respectively.

Figure 11 provides a comparison between the predicted and the actual EBO values. The RMSE values are 0.02276, 0.02294, and 0.02139 for the training, test, and validation data sets, respectively, and the corresponding R^2 values are 90.73%, 90.63%, and 90.86%.

For a given instance of the sparing problem, a number of the variables in Equation (20) would be known, e.g., the demand rates of the components. Let the two SRUs have conditional failure probabilities of $q_{0,1,l}=\frac{1}{3}$ and $q_{0,2,l}=\frac{2}{3}$, respectively. Also it is assumed that the LRU is 90% operating base repairable while SRU1 and SRU2 are 60% and 71% operating base repairable, respectively. Hence, the demand rate for the LRU at the depot is calculated to be 0.01, for SRU1 to be 0.0153, and for SRU2 to be 0.0241. At the operating

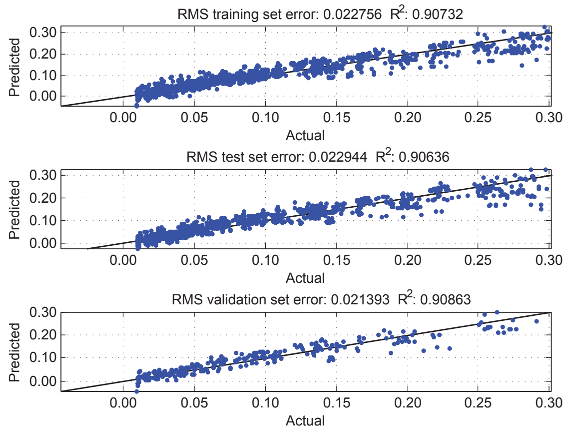

Figure 11: *Prediction scatter plot for the GPTIPS multi-gene regression model for Case Study 3.*

base, the demand rate is found to be 0.03 and 0.06 for SRU1 and SRU2, respectively. Using this information and Equation 20, the optimization problem (EBO-MEMI) can be solved to obtain the optimal amount of spares at each location.

For the scenario considered, Table 10 shows the stock level as given by the GP approach and the VARI-METRIC approach. An optimal stock level of $\bar{s} = \begin{bmatrix} 0 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ is obtained in both approaches, where $\bar{s}$ is a matrix with entries $s_{c,l}$, the stock level of component c at location l. Furthermore, the EBO values of the two approaches are comparable.

Table 10: *Test results for Case Study 3.*

Part	GP-OPT			VARI-METRIC		
	$s_{c,0}$	$s_{c,1}$	EBO_{GP}	$s_{c,0}$	$s_{c,1}$	EBO_{VM}
LRU	0	3	0.0204	0	3	0.0228
SRU	1	1	-	1	1	-
SRU	1	2	-	1	2	-

4.5 Analysis of Computational Results

In this section, it has been shown that using a multi-gene symbolic regression approach, relatively low-order mathematical expressions of the EBO can be generated to permit the solution of the sparing problem without the use of marginal analysis. A comparison of the proposed optimization approach and the marginal analysis method was done using three sample cases. Preliminary results revealed that the proposed approach is promising as the obtained mathematical expressions produced the same results as marginal analysis for a small sample data set.

5 The Combined LORA-SPS Model

5.1 Integrated vs. Sequential Model

It is not difficult to argue that the LORA and SPS problems are interrelated since the repair decisions of the LORA model affect the spare levels required to guarantee the desired system availability. As such, it would be beneficial to integrate the LORA and SPS problems into one mathematical model.

Most previous research considered the network design and inventory problems separately: first solving the LORA problem as a facility location problem, and then setting inventory (spare) levels. However, considering inventory levels in the facility location problem will lead to lower total costs. This was illustrated by Basten [2], but his solution approach, which employed VARI-METRIC, required lengthy computational time.

Although the integrated approach is more complicated and requires significantly longer computation time, the value of finding solutions with lower total costs outweighs the disadvantages of this approach. The objective of this section is to explore techniques to solve large-scale instances of the LORA-SPS problem. To build an understanding of these techniques so that more complex problems can eventually be solved, a small instance with two locations and three components is solved.

5.2 The Mathematical Model

The decisions in the integrated problem are where to locate resources, which repair/discard decision to assign to failure modes and at which locations they should be performed, and how many spares to store at each location. The optimization model can thus be formulated as shown below:

$$\textbf{(LORA-SPS)} \quad \min \sum_{f\in F}\sum_{b\in B}\sum_{d\in D}\sum_{l\in L} vc_{f,b,d,l}\theta_{f,b}X_{f,b,d,l} + \sum_{r\in R}\sum_{l\in L} rc_{r,l}Y_{r,l} + \sum_{r\in R}\sum_{l\in L} fc_{r,l}Z_{r,l} + \sum_{c\in C}\sum_{l\in L} hc_{c,l}s_{c,l}$$

$$\text{s.t.} \quad \sum_{d\in D}\sum_{l\in \Phi_b} X_{f,b,d,l} = 1, \qquad \forall f\in F, \forall b \in B \qquad (21)$$

$$\sum_{(f,d)\in\Omega_r}\sum_{b\in B_l} \theta_{f,b}u_{r,f,d}X_{f,b,d,l} \le M_{r,l}Y_{r,l}, \qquad \forall r\in R, \forall l \in L\backslash B \qquad (22)$$

$$\sum_{(f,2)\in\Omega_r}\sum_{l\in \Phi_b} \theta_{f,b}u_{r,f,2}X_{f,b,2,l} + \sum_{(f,d)\in\Omega_r} \theta_{f,b}u_{r,f,d}X_{f,b,d,b} \le M_{r,b}Y_{r,b}, \qquad \forall r\in R, \forall b\in B \qquad (23)$$

$$EBO(s_{c,l}, X_{f,b,d,l}) \le E, \qquad (24)$$

$$0 \le X_{f,b,d,l} \le 1, \qquad \forall f\in F, \forall b\in B, \forall l\in \Phi_b, \forall d\in D \qquad (25)$$

$$0 \leq Y_{r,l} \leq N_{r,l} Z_{r,l}, \qquad Y_{r,l} \in \mathbb{Z}, \forall r \in R, \forall l \in L \quad (26)$$

$$Z_{r,l} \in \{0,1\}, \qquad \forall r \in R, \forall l \in L \quad (27)$$

$$s_{c,l} \in \mathbb{Z}^{+}, \qquad \forall c \in C, \forall l \in L \quad (28)$$

where $\mathbb{Z}^{+}$ is the set of non-negative integers. The objective function is the sum of the resource, transportation, and inventory costs. Constraints (21) require that for each demand at the operating base, a decision is taken at some location. Constraints (22) manage the capacity consumption of the resources at all locations except the operating sites. Constraints (23) enforce a limitation on the resource capacities taking into account the case where the LRU and SRU need to be split. The EBO constraint is given in Equation (24), and it depends on the stock levels $s_{c,l}$ and the fraction of the demand $X_{f,b,d,l}$. Note that **LORA-SPS** is a combination of the **LORA-MIP** and **SPS-MEMI**. Equations (21), (22), and (23) are identical to Equations (2), (3), and (4) respectively of the LORA model. Similarly, Equation (24) is identical to Equation (13) of the sparing model.

The commonly used expression for calculating the EBO is given in Equation (B.1). If this expression is substituted for the left hand side of the EBO expression (constraint (24)), then the EBO constraint would be non-linear. This makes **LORA-SPS** more difficult to solve in general. Instead of replacing the EBO expression with equation (B.1), genetic programming (as presented in Section 4.3) is utilized to generate low-order expressions for the EBO constraint to make **LORA-SPS** more tractable.

5.2.1 Case Study 4 – LORA-SPS Sample Problem

In this section, a small-scale test instance of **LORA-SPS** is solved. The case study has the following input parameters:

- the model consists of one LRU, c_1, and two SRUs (c_2^1 and c_2^2);
- two failure modes f_1 and f_2 which are associated with c_2^1 and c_2^2 respectively;
- the repair network consists of a single depot, l_1, and a single operating base, b;
- two types of resources;
- a maximum of two of each resource per location is allowed;
- the resource requirements for each failure mode is two;
- the resource capacity for each resource type is five;
- the LRU repair times for the operating base and depot are 5 and 15 days, respectively and the SRU repair times are 10 and 30 days, respectively;
- the resupply time, O_b, is 15 days for all components;

- the failure rate, $\theta_{f,b}$, is $\frac{1}{30}$ and $\frac{2}{30}$ for failure mode 1 and failure mode 2 respectively;
- the target EBO, E, is 0.1; and
- vc, rc, fc, and hc are given in Table 11.

Table 11: *Cost parameters for Case Study 4.*

Parameter	Value ($)
$vc_{f,b,d,b}$	500
vc_{f,b,d_1,l_1}	700
vc_{f,b,d_2,l_1}	600
$rc_{r,l}$	1,500
$fc_{r,l}$	2,000
$hc_{c_1,l}$	1,000
$hc_{c_2^i,l}$	800

Using GP, the EBO function is approximated as a function of X and s since all of the variables are known except for the stock levels and demand fraction at each location (this is determined using **LORA-SPS**). The EBO equation employed is the same as that used in Section 4.4.3 with a slight modification to take into account the demand fractions instead of the demand rates. The new EBO function is given in the equation below:

$$\begin{aligned}
EBO_{GP} &= 0.05844\,X_{1,1,1,0}{}^2 - 0.05844\,X_{1,1,1,0} - 0.05844\,s_{0,1} - 0.05844\,s_{2,0} \\
&+ 0.05844\,X_{2,1,2,0} + 0.8982 + 0.1206\,X_{1,1,1,0}{}^2 + 0.1206\,s_{0,1} - 0.1206\,s_{0,0} \\
&+ 0.02349\,(X_{1,1,1,1} + X_{1,1,2,0})\,(s_{0,0} - X_{2,1,1,0})\,\left(X_{1,1,2,0}{}^2 + s_{0,1} + X_{2,1,1,1}\right) \\
&+ (0.1899 - 0.1899\,X_{2,1,1,0})\,s_{0,1} + 0.02338\,X_{1,1,1,0}{}^2 + 0.02338\,s_{2,1} \\
&- 0.007987\,X_{1,1,2,0}{}^2 - 0.01597\,s_{0,1} - 0.007987\,(s_{1,1} - s_{0,0} - s_{2,0}) \\
&+ 0.004392\,(s_{2,1} + s_{2,0} - X_{1,1,2,0})\,(s_{0,1} + s_{2,0} + X_{2,1,1,1} - X_{2,1,2,0}) \\
&+ 0.02726\,X_{1,1,2,0} - 0.05452\,s_{2,1} + 0.02726\,(X_{2,1,2,0} - s_{0,1}) \qquad (29) \\
&+ 0.9123\,s_{2,1} - 0.9123\,s_{1,0} + 0.9123\,X_{1,1,1,0} + 0.2033\,s_{0,1}{}^2 \\
&- 0.001473\,(2\,s_{2,1} - s_{2,0})\,(s_{0,0} - s_{2,1} + X_{1,1,1,0}\,X_{2,1,1,0}) \\
&- 0.005034\,(s_{2,1} - 2\,s_{0,0})\,(s_{2,1} + s_{0,0} - X_{1,1,1,0}) - 0.02338\,s_{2,0}\,X_{2,1,1,0} \\
&+ 0.9087\,s_{1,0} - 0.9087\,s_{2,1} - 0.9087\,s_{0,1} - 0.01996\,s_{0,1}{}^3
\end{aligned}$$

Consequently, Equation (29) replaces $EBO(s_{c,l}, X_{f,b,d,l})$ in Constraint (24). Note that since the EBO expression is derived using the VARI-METRIC equations, accommodation should be made for the restrictive VARI-METRIC assumptions. In particular, VARI-METRIC does not account for discarding of items. Consequently, only repair decisions can be taken for failed components and hence only decisions $d = 1$ and $d = 2$ are possible in this example.

GAMS is used to implement **LORA-SPS** and BARON global optimizer is utilized to solve the resulting non-linear mixed integer optimization problem. The total number of variables is 23 with 14 being discrete variables. As seen in Table 12, the total computational time was 0.13 sec with a resulting objective function value of $10,050.

Table 12: *GAMS/BARON results for Case Study 4.*

Root lower bound ($)	10,039
Root upper bound ($)	10,050
Optimal Objective ($)	10,050
Time (sec)	0.13
Nodes	29

The optimal solution of **LORA-SPS** results in repairing both SRUs at the operating base and the installation of one resource of each type at the operating base. The number of spares of the LRU required is three at the operating base and none at the depot (see Table 13). No spares are required for SRU1 and SRU2.

Table 13: *Optimal solution for Case Study 4.*

	LORA-SPS		
Part	s_{c,l_1}	$s_{c,b}$	EBO
LRU	0	3	0.089
SRU1	0	0	-
SRU2	0	0	-

Using the VARI-METRIC equations, it was verified that this level of sparing provides an EBO of 0.089, which is equal to the one given by GP and is less than the target EBO of 0.1.

5.3 Solution Methodology for Large-Scale Problems

From the sample problem above, it is obvious that small problems can be easily solved using direct optimization techniques. However, **LORA-SPS** is a discrete and non-linear model, hence large-scale instances cannot be solved efficiently by direct optimization techniques. Additionally, using heuristics to solve the problem is not always attractive since optimality is not guaranteed. In this section, a decomposition scheme is proposed to simplify the model and uncover a structure that makes the model amenable to be solved efficiently using optimization techniques which lead to tight bounds on the optimal solution value.

5.3.1 LORA-SPS Decomposition

In order to solve large-scale instances of **LORA-SPS**, Lagrangian relaxation is adopted. Lagrangian relaxation is a technique well suited for **LORA-SPS** since the constraints can be divided into two sets:

- easy constraints, which taken on their own constitute a problem that can be solved efficiently
- hard constraints that make the problem very difficult to solve.

The main idea is to relax the problem by converting the hard constraints into weighted terms in the objective function. This weight (Lagrangian multiplier) represents a penalty which is added to the objective function value of a solution that does not satisfy the particular constraint. The first step to solve the large-scale **LORA-SPS** instances is to relax the hard non-linear constraint which is the EBO constraint. By relaxing the EBO constraint of **LORA-SPS**, the remaining problem is easier to solve as it becomes a facility location problem with inventory variables that are only constrained by an integrality condition. This is similar to **LORA-MIP** presented in Section 3.1 which was solved using a branch-and-bound approach within CPLEX.

Consider the following Lagrangian subproblem of **LORA-SPS** where the EBO constraint $EBO(s_{c,l}, X_{f,b,d,l}) \leq E$ is relaxed and its violation is penalized in the objective function via a non-negative Lagrangian multiplier μ:

$$\textbf{(LR)}\ \min \sum_{f\in F}\sum_{b\in B}\sum_{d\in D}\sum_{l\in L} vc_{f,b,d,l}\theta_{f,b}X_{f,b,d,l} + \sum_{r\in R}\sum_{l\in L} rc_{r,l}Y_{r,l} + \sum_{r\in R}\sum_{l\in L} fc_{r,l}Z_{r,l} + \sum_{c\in C}\sum_{l\in L} hc_{c,l}s_{c,l} + \mu(E - EBO(s_{c,l}, X_{f,b,d,l}))$$

$$\text{s.t.} \sum_{d\in D}\sum_{l\in\Phi_b} X_{f,b,d,l} = 1, \qquad \forall f\in F, \forall b\in B \qquad (30)$$

$$\sum_{(f,d)\in\Omega_r}\sum_{b\in B_l} \theta_{f,b}u_{r,f,d}X_{f,b,d,l} \leq M_{r,l}Y_{r,l}, \qquad \forall r\in R, \forall l\in L\backslash B \qquad (31)$$

$$\sum_{(f,2)\in\Omega_r}\sum_{l\in\Phi_b} \theta_{f,b}u_{r,f,2}X_{f,b,2,l} + \sum_{(f,d)\in\Omega_r} \theta_{f,b}u_{r,f,d}X_{f,b,d,b} \leq M_{r,b}Y_{r,b}, \qquad \forall r\in R, \forall b\in B \qquad (32)$$

$$0 \leq X_{f,b,d,l} \leq 1, \qquad \forall f\in F, \forall b\in B, \forall l\in\Phi_b, \forall d\in D \qquad (33)$$

$$0 \leq Y_{r,l} \leq N_{r,l}Z_{r,l}, \qquad Y_{r,l}\in\mathbb{Z}, \forall r\in R, \forall l\in L \qquad (34)$$

$$Z_{r,l}\in\{0,1\}, \qquad \forall r\in R, \forall l\in L \qquad (35)$$

$$s_{c,l}\in\mathbb{Z}^+, \qquad \forall c\in C, \forall l\in L \qquad (36)$$

$$\mu \geq 0. \qquad (37)$$

The μ in the last term of the objective function is the dual multiplier corresponding to the EBO constraint. In the dual problem, μ is a non-negative variable. Note that other than the extra term $\mu(E - EBO(s_{c,l}, X_{f,b,d,l}))$ in the objective function, **LR** is the same as **LORA-MIP**. The objective function of **LR** is referred to as the Lagrangian function, and its value is denoted as Λ_{lr}.

Let K represent the set of all feasible solutions to **LR** and $k \in K$. Given a feasible solution $k \in K$, the EBO can be quickly calculated using equation (B.1) in Annex B or the approximated EBO function presented in Section 4.3. Let $EBO(s^k_{c,l}, X^k_{f,b,d,l})$ represent the EBO for solution $k \in K$. Let

$$\delta^k = \begin{cases} 1 \text{ if solution } k = (X^k, Y^k, Z^k, s^k) \text{ is selected,} \\ 0 \text{ otherwise.} \end{cases}$$

The EBO constraint can thus be rewritten as a function of δ^k

$$\sum_{k \in K} \delta^k EBO(s^k_{c,l}, X^k_{f,b,d,l}) \leq \sum_{k \in K} \delta^k E,$$

The optimal solution to the original problem, **LORA-SPS**, is the solution $k \in K$ that also satisfies the above EBO constraint with lowest total cost. Let

$$TC^k = \sum_{f \in F} \sum_{b \in B} \sum_{d \in D} \sum_{l \in L} vc_{f,b,d,l} \theta_{f,b} X^k_{f,b,d,l} + \sum_{r \in R} \sum_{l \in L} rc_{r,l} Y^k_{r,l} + \sum_{r \in R} \sum_{l \in L} fc_{r,l} Z^k_{r,l} + \sum_{c \in C} \sum_{l \in L} hc_{c,l} s^k_{c,l},$$

so that TC^k represents the sum of the resource, transportation, and inventory costs of solution k. The objective function can now be rewritten as a function of δ^k, and the original problem, **LORA-SPS** reformulated in terms of δ^k:

$$\textbf{(MP)} \quad \min \sum_{k \in K} TC^k \delta^k$$

$$\text{s.t.} \quad \sum_{k \in K} \delta^k EBO(s^k_{c,l}, X^k_{f,b,d,l}) \leq \sum_{k \in K} \delta^k E \tag{38}$$

$$\sum_{k \in K} \delta^k = 1 \tag{39}$$

$$\delta^k \in \{0, 1\} \qquad \forall k \in K. \tag{40}$$

This optimization model is used to select the k that minimizes the cost subject to the backorder constraint. The optimal objective function value of **MP**, denoted by Λ_{mp}, is equivalent to that of **LORA-SPS**. The linear relaxation of **MP** is referred to as **MP-LP** with the objective function value denoted as Λ_{mp-lp}. The difficulty with the **MP** formulation is in generating the set of all possible feasible solutions, K. This set will be exponentially large and inefficient to generate and all but one of them will have their associated variable, δ^k, equal to zero at optimality. However, rather than generating the entire set K, the problem can be solved via column generation, which allows the generation of K on an incremental basis as each column corresponds to a solution $k \in K$. That is, columns are left out of **MP** and only a subset $\bar{K} \subseteq K$ of feasible solutions is used. That is, instead of solving **MP**, a more tractable version, **MPR**, is solved.

$$
\begin{aligned}
\textbf{(MPR)}\ \min \sum_{k \in \bar{K}} & TC^k \delta^k \\
\text{s.t.} \sum_{k \in \bar{K}} & \delta^k EBO(s^k_{c,l}, X^k_{f,b,d,l}) \leq \sum_{k \in K} \delta^k E && (41)\\
\sum_{k \in \bar{K}} & \delta^k = 1 && (42)\\
& \delta^k \in \{0,1\} \qquad \forall k \in \bar{K}. && (43)
\end{aligned}
$$

This smaller problem, **MPR**, still has binary variables that may make it difficult to solve. However, bounds on the objective function can be obtained by solving its linear programming relaxation **MPR-LP** defined below:

$$
\begin{aligned}
\textbf{(MPR-LP)}\ \min \sum_{k \in \bar{K}} & TC^k \delta^k \\
\text{s.t.} \sum_{k \in \bar{K}} & \delta^k EBO(s^k_{c,l}, X^k_{f,b,d,l}) \leq \sum_{k \in K} \delta^k E && (44)\\
\sum_{k \in \bar{K}} & \delta^k = 1 && (45)\\
& 0 \leq \delta^k \leq 1 \qquad \forall k \in \bar{K}. && (46)
\end{aligned}
$$

Let Λ_{mpr-lp} represent the optimal objective function value of **MPR-LP** and Λ_{lr}, the objective function of the Lagrangian subproblem **LR**. The relationship between the objective function values of the various problems can be specified as:

$$\Lambda_{lr} \leq \Lambda_{mp-lp} \leq \Lambda_{mpr-lp}.$$

Subsequently, when

$$\Lambda_{lr} - \Lambda_{mpr-lp} \approx 0, \tag{47}$$

then

$$\Lambda_{lr} \approx \Lambda_{mp-lp} \approx \Lambda_{mpr-lp}.$$

The aforementioned models are used to find a solution to **LORA-SPS** using Algorithm 1. At each iteration of the proposed algorithm, the linear relaxation of the **MPR**, i.e., **MPR-LP**, is solved to obtain a dual variable μ. Then the Lagrangian subproblem **LR** is solved to identify "good" columns to add to **MPR-LP**. That is, a solution $k \in K$ to **LR** is converted into a column and added to $\bar{K}$. This process of generating columns and solving **MPR-LP** is repeated until a stopping criteria (Equation (47)) is reached (see Figure 12). Λ_{mp-lp} is then assigned the value Λ_{lr}, which is a lower bound on the original problem, **LORA-SPS**.

The last step is to solve the integer program **MPR** with those columns generated and obtain a feasible solution and hence an upper bound on the original problem, **LORA-SPS**. Note that there is no need to solve **MPR** explicitly as one can choose the column that provides the best objective function value and this is equivalent to the optimal solution of **MPR**. The optimal objective function value of **MPR** is denoted by Λ_{mpr}.

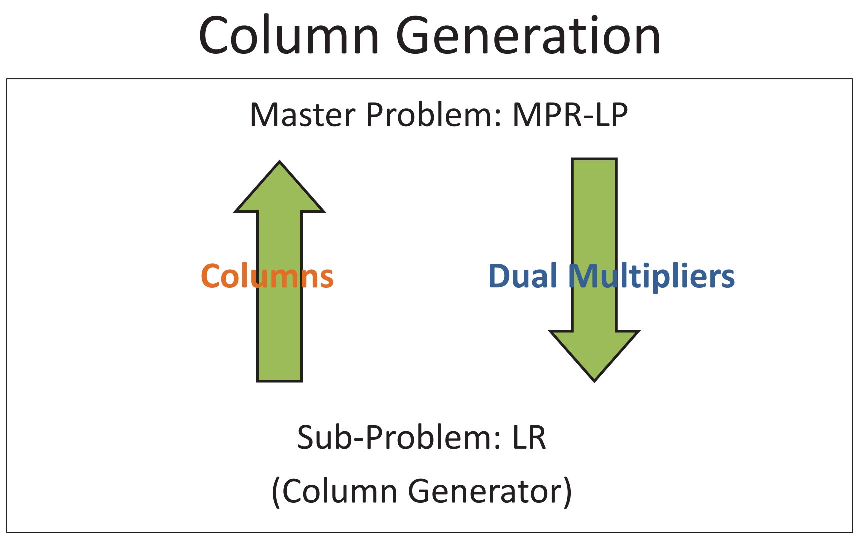

Figure 12: *A Column Generation Approach to* ***LORA-SPS****.*

Algorithm 1: LORA-SPS Decomposition Approach.

Input: $k = 1$
Output: UB and LB

while $\Lambda_{mpr-lp} - \Lambda_{lr} > \varepsilon$ **do**
    Solve **MPR-LP**;
    Obtain μ and Λ_{mpr-lp};
    Solve **LR**;
    Obtain a column δ^k and Λ_{lr};
    Set $k := k + 1$;
end
Set $LB = \Lambda_{mp-lp}$;
Solve **MPR** ;
Set $UB = \Lambda_{mpr}$;

5.3.2 Approximating the EBO Function using Simulation

In order to solve **LORA-SPS** efficiently, it is essential to handle the non-linearity in the objective function due to the EBO equation. To approximate the non-linear EBO function, the GP approach proposed in Section 4.3 is utilized. However, simulation is used in lieu of the predefined VARI-METRIC equations in Annex B to generate the needed data.

The VARI-METRIC approach is limited in its ability to fully describe a repair network. For example, it assumes that the repair capacity at each repair location is infinite so that repair times are independent and there is never any queuing at the repair facilities. In addition, it assumes that failures occur according to a Poisson distribution from an infinite calling population. These assumptions lead to the closed form expressions for the EBO presented in Annex B. However, these expressions tend to overestimate the true EBO since real systems often deviate from the modelled system hence the need to use a simulation model. Additionally, a simulation model can include aspects of the repair system ignored in the VARI-METRIC model. For example, discarding of components cannot be modelled using the VARI-METRIC approach but can be easily included in simulation model.

Let T be the set of sample data drawn from the interval of interest and $|T|$ the size of the sample data. The two key ingredients to perform the algorithm are data sampling (i.e., obtaining the set T) and the simulation model. A sketch of the algorithm is given below:

Algorithm 2: Approximating the EBO function.

Input: $\hat{s}$, $\hat{X}$ sample data vectors, $E\hat{B}O = [\,]$, $t = 0$, and $|T|$
Output: $EBO(s,X)$

while $t \leq |T|$ **do**
    Run Simulation to obtain $E\hat{B}O_t$;
    Set $E\hat{B}O = [E\hat{B}O \quad E\hat{B}O_t]$;
    Set $t := t+1$;
end
Run GP-based multi-gene symbolic regression using $E\hat{B}O$ vector ;
Obtain $EBO(s,X)$ expression ;

Once the EBO mathematical expression is obtained using Algorithm 2, Algorithm 1 is applied to obtain a tight bound on the optimal objective function value of **LORA-SPS**.

5.3.3 Case Study 5 – Simulation-Based LORA-SPS Sample Problem

In this test case, simulation is used to approximate the EBO function instead of the VARI-METRIC equations. The input data is the same as the ones used in Section 5.2.1. Java is used for the simulation model and the genetic-programming based approach presented in

Section 4.3 is used to approximate the resulting EBO function. GAMS is used to implement **LORA-SPS** and BARON global optimizer is utilized to solve the resulting non-linear mixed integer optimization problem.

$$
\begin{aligned}
EBO_{GP} \; = \; & 0.43544 - 0.3382\, s_{2,1} - 0.03546\, \left(s_{0,1}{}^{2} + X_{1,1,2,0} + X_{2,1,2,0} + X_{1,1,1,0}\right) \\
- \; & 0.0009393\, (s_{2,1} + X_{2,1,1,0})\, (X_{1,1,1,1} - 6.877)\, (s_{0,1} + s_{2,1} + X_{2,1,2,0} - 2.82) \\
- \; & 0.01913\, s_{2,0} - 0.402\, X_{1,1,2,0} + 0.003239\, (s_{1,1} - s_{0,0} + X_{2,1,2,0}) \\
+ \; & 0.0006965\, (s_{0,1} + X_{2,1,1,0})\, (s_{2,0} + X_{1,1,1,1}) + 0.3046\, (s_{2,1} + X_{1,1,2,0}) \\
+ \; & 0.03142\, (s_{0,1} + X_{2,1,2,0}) + 0.001638\, (X_{1,1,1,1} - s_{1,0} - s_{0,0}) \qquad (48) \\
+ \; & 0.001538\, s_{2,0}\, (s_{2,0} + 1.179) - 0.001538\, (X_{2,1,1,0} + s_{1,1}) \\
+ \; & 0.0004191\, (s_{2,0} - s_{0,1} - s_{1,1}\, s_{2,0}) - 0.05385\, s_{0,1}\, (X_{1,1,1,0} - s_{0,1} + 3.235) \\
+ \; & 0.002296\, (s_{1,1} - s_{2,0})\, (X_{1,1,1,1} - X_{2,1,1,1}) + 0.133\, (X_{1,1,1,0} - X_{1,1,1,1}) \\
+ \; & 0.0002591\, (s_{0,0} - s_{1,0})\, (X_{2,1,1,1} - X_{2,1,1,0})\, (s_{0,1} - s_{2,0} + s_{1,1}\, s_{2,0})
\end{aligned}
$$

As seen in Table 14, the total computational time was 0.12 sec with a resulting objective function value of $9,850 which is lower than the objective function value of $10,050 obtained using the VARI-METRC approximation given in Section 5.2.1.

Table 14: *GAMS/BARON results for Case Study 5.*

Root lower bound ($)	9,850
Root upper bound ($)	9,840
Optimal Objective ($)	9,850
Time (sec)	0.12
Nodes	92

The optimal solution of **LORA-SPS** results in repairing both SRUs at the operating base and the installation of one resource of each type at the operating base. The number of spares of the LRU required is two at the operating base and none at the depot, and for the SRU2, it is one and zero at the operating base and depot, respectively (see Table 15). No spares are required for SRU1. Note that although the total number of spares is the same for both the VARI-METRIC EBO approximation (Equation (29)) and the simulated EBO approximation (Equation (48)), the allocation of spares is not the same. In Section 5.2.1, the number of spares of the LRU at the operating site is three versus two in this case and for SRU2, the number of spares at the operating base is zero as opposed to one for this case.

Using simulation, it was verified that this level of sparing provides an EBO of 0.0614 which is close to the obtained EBO value of 0.0830 and is less than the target EBO of 0.1. Note that for this level of sparing, an EBO of 0.116 is obtained using the VARI-METRIC

Table 15: *Optimal Solution for Case Study 5.*

Part	**LORA-SPS**		
	s_{c,l_1}	$s_{c,b}$	EBO
LRU	0	2	0.083
SRU1	0	0	-
SRU2	0	1	-

equations, making this solution infeasible as the EBO value is above the target level (i.e., Constraint (24) is violated). That is, this solution would not have been identified had VARI-METRIC been used. However, using the simulation based EBO approximations, this level of sparing is identified as being feasible to Constraint (24) and hence to **LORA-SPS** and results in a total cost lower than the cost based on the level of sparing given by the VARI-METRIC approximation in Section 5.2.1.

6 Conclusion

In this report, a mathematical formulation for the integrated LORA and SPS problem is proposed. This is a non-linear mixed integer optimization model that balances system availability (or EBO) and life cycle costs. A set of techniques based on the approximation of the EBO function alone (for small problems) and together with a Lagrangian relaxation of the EBO constraint (for large problems) were introduced to solve the integrated optimization model.

In developing the methodology, the LORA and SPS problems were first analyzed individually. LORA is modelled as a capacitated facility location problem. A test instance is presented and the resulting mixed integer programming model is solved to optimality. A tractable optimization model of the SPS problem is developed by approximating the non-linear EBO constraint by relatively low-order mathematical expressions generated using multi-gene symbolic regression. To illustrate the performance of the approximating technique, a number of problems were solved. The computational results show that the optimal sparing solution can be obtained using the proposed EBO approximation without the use of METRIC-based greedy approaches. The EBO approximation technique was key to the solution approach of the mixed-integer non-linear formulation of the joint LORA and SPS problem. This contrasts with approaches in the literature that are based on METRIC models which solve the problems either iteratively or sequentially.

A computational study is presented which shows the utility of using the GP-based approximation to solve the integrated optimization problem. In the computational study, two approaches were used to generate data for the EBO approximation: VARI-METRIC and simulation. The simulation provides data that better reflects the actual system and leads to a lower total cost solution since the EBO approximations are better than those obtained from the VARI-METRIC model.

Future research would study how to utilize the presented Lagrangian approach along with simulation and genetic programming to solve real-world LORA-SPS instances.

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Annex A: Notation

Acronyms

CFLP	Capacitated Facility Location Problem
EBO	Expected backorder
GP	Genetic Programming
LORA	Level of Repair Analysis
LR	Lagrangian Relaxation
MEMI	Multi-Echelon, Multi-Indenture
METRIC	Multi-Echelon Technique for Recoverable Item Control
SESI	Single Echelon, Single Indenture
SEMI	Single Echelon, Multi-Indenture
SPS	Spare Parts Stocking Problem

Symbols

A	set of directed transportation links
B	set of system operating bases
C	set of all components
C_1	set of all LRUs
C_2	set of all SRUs
D	set of possible decisions
F	set of all failure modes in the system
$F(c)$	set of failure modes f, associated with a component c
G	set of locations
L	set of locations with the capacity to host facilities for repair and disposal
R	set of the resources
Γ_c	set of SRUs of LRU c
Ω_r	set of tuples (f,d) where failure mode f requires resource r in order to enable decision d
E	target EBO
$fc_{r,l}$	fixed cost of installation of resource r at location l
$hc_{c,l}$	holding cost per spare of component c at location l
K	total available budget
$M_{r,l}$	capacity provided by each installed unit
$n_{c_2,c}$	number of SRUs, $c_2 \in \Gamma_c$, contained in LRU c
$N_{r,l}$	maximum number of units of resource r that can be installed at location l
n_c	number of LRUs in the system
$rc_{r,l}$	cost of installation of one unit of resource r at location l
$s_{c,l}$	spare stock level for each component c at location l
$u_{r,f,d}$	required capacity of the resource r used to service failure f by the execution of decision d
$vc_{f,b,d,l}$	variable cost due to failure mode f generated at operating base b that are attended to by the execution of decision d at location l
$X_{f,b,d,l}$	fraction of the demand due to failure mode f generated at operating base b that are attended to by the execution of decision d at location l
$Y_{r,l}$	number of units of resource r installed at location l
$Z_{r,l}$	indicates if resource r is installed at location l
$\lambda_{c,b}$	demand rate of component c at operating base b
θ_f	failure rate for the failure mode f

Annex B: VARI-METRIC Approach

In this section, Sherbrooke's VARI-METRIC approach [23] is described for a multi-echelon multi-indenture system. First, it is necessary to define the following parameters for a general two-echelon two-indenture system:

- $\lambda_{c,b}$ is the demand rate of component $c \in C$ at operating base $b \in B$.
- $T_{c,l}$ is the repair time for each component $c \in C$ at location $l \in L$.
- $r_{c,l}$ is the probability that the failed component $c \in C$ can be repaired at location $l \in L$.
- $q_{c,c_2,l}$ is the conditional probability that a failure of an LRU c is caused by a SRU $c_2 \in \Gamma_c$ at location $l \in L$.
- $O_{b,l}$ is the order and ship time from location $l \in L$ to any location b where $b \in B_l$.
- $s_{c,l}$ is the spare stock level for each component $c \in C$ at location $l \in L$.

Additionally, the demand rates for the VARI-METRIC approach are calculated as follows:

$$\begin{aligned}
\lambda_{c_2,b} &= \text{the demand of the corresponding SRU } c_2 \in \Gamma_c \text{ at location } b \in B \text{ is } q_{c,c_2,b}\lambda_{c,b}r_{c,b}, \\
\lambda_{c,l} &= \text{the demand of LRU } c \text{ at location } l \in L \backslash B \text{ is } \sum_{b \in B_l} \lambda_{c,b}(1-r_{c,b}), \\
\lambda_{c_2,l} &= \text{the demand of the corresponding SRU } c_2 \in \Gamma_c \text{ at location } L \backslash B \text{ is} \\
&\quad \sum_{b \in B_l} \lambda_{c_2,b}(1-r_{c_2,b}) + q_{c,c_2,l}\lambda_{c,l}.
\end{aligned}$$

Define the stochastic variable $P_{c,l}$ as the pipeline of component c at location l, $EBO_{c,l}$ and $VBO_{c,l}$ as the mean and the variance for backorder of component c at location l. When two parameters are given for the EBO and VBO functions, then a Poisson distribution is assumed; otherwise a negative binomial distribution is assumed [23]. Additionally, $f_{c,l}$ is the fraction of the demand at location l for component c and $E(P_{c,l})$ and $V(P_{c,l})$ is the mean and the variance of $P_{c,l}$.

For $c_2 \in \Gamma_c$ and $l \in L \backslash B$ with $b \in B_l$:

$$\begin{aligned}
f_{c_2,l} =& \frac{\lambda_{c,l}q_{c,c_2,l}}{\lambda_{c_2,l}} \\
E(P_{c,l}) =& \lambda_{c,l}T_{c,l} + \sum_{c_2 \in \Gamma_c} f_{c_2,l}EBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}) \\
V(P_{c,l}) =& \lambda_{c,l}T_{c,l} + \sum_{c_2 \in \Gamma_c} f_{c_2,l}(1-f_{c_2,l})EBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}) + \sum_{c_2 \in \Gamma_c} f_{c_2,l}^2 VBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}).
\end{aligned}$$

For $c_2 \in \Gamma_c$ and $b \in B_l$:

$$\begin{aligned}
f_{c_2,b} =& \frac{\lambda_{c_2,b}(1-r_{c_2,b})}{\lambda_{c_2,l}} \\
E(P_{c_2,b}) =& \lambda_{c_2,b}[(1-r_{c_2,b})O_{b,l} + r_{c_2,b}T_{c_2,b}] + f_{bb}EBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}) \\
V(P_{c_2,b}) =& \lambda_{c_2,b}[(1-r_{c_2,b})O_{b,l} + r_{c_2,b}T_{c_2,b}] + f_{c_2,b}(1-f_{c_2,b})EBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}) \\
& + f_{c_2,b}^2 VBO(s_{c_2,l}, \lambda_{c_2,l}T_{c_2,l}).
\end{aligned}$$

For $c \in C_1$ and $b \in B_l$:

$$\begin{aligned}
f_{c,b} =& \frac{\lambda_{c,b}(1-r_{c,b})}{\lambda_{c,l}} \\
E(P_{c,b}) =& \lambda_{c,b}[(1-r_{c,b})O_{b,l} + r_{c,b}T_{c,b}] + f_{cb}EBO(s_{c,l}, E(P_{c,l}), V(P_{c,l})) \\
& + \sum_{c_2 \in \Gamma_c} EBO(s_{c_2,b}, E(P_{c_2,b}), V(P_{c_2,b})) \\
V(P_{c,b}) =& \lambda_{c,b}[(1-r_{c,b})O_{b,l} + r_{c,b}T_{c,b}] + f_{c,b}(1-f_{c,b})EBO(s_{c,l}, E(P_{c,l}), V(P_{c,l})) \\
& + f_{c,b}^2 VBO(s_{c,l}, E(P_{c,l}), V(P_{c,l})) + \sum_{c_2 \in \Gamma_c} VBO(s_{c_2,b}, E(P_{c_2,b}), V(P_{c_2,b})).
\end{aligned}$$

Finally,

$$EBO(s_{c,l}) = \sum_{c \in C_1} \sum_{b \in B} EBO(s_{c,b}, E(P_{c,b}), V(P_{c,b})). \qquad \text{(B.1)}$$

For a specified budget, the VARI-METRIC approach uses marginal analysis, a greedy approach, to come up with the spare allocation, $s_{c,l}$, that minimizes $EBO(s_{c,l})$.

Annex C: Data Sets used for Regression Models

For each data point in the data sets used in developing the regression models, the value of an independent variable was selected from a predefined interval. The value of the EBO was then determined using the VARI-METRIC model presented in Annex B or through simulation. A data point was discarded if the combination of independent variables produced an EBO value far greater than what is desired in practice. In the interest of saving space, the data sets are not reproduced in this report. Instead, the variable intervals for the data sets are presented in Tables C.1–C.4.

Table C.1: *Summary of data set used for the development of equation 18 (Case Study 1).*

Variable	Mininum Value	Maximum Value
λ_1	2	8
λ_2	2	8
T_1	0.2	0.6
T_2	0.2	0.6
s_1	0	10
s_2	0	10
EBO	0.01	3.2

Table C.2: *Summary of data set used for the development of equation 19 (Case Study 2).*

Variable	Mininum Value	Maximum Value
λ_0	0.1	1.9
λ_1	0.1	1.9
T_0	0.5	5.5
T_1	0.5	5.5
s_0	0	10
s_1	0	10
EBO	0.01	0.3

Table C.3: *Summary of data set used for the development of equation 20 (Case Study 3).*

Variable	Mininum Value	Maximum Value
$s_{0,1}$	0	5
$s_{1,1}$	0	4
$s_{2,1}$	0	4
$s_{0,0}$	0	5
$s_{1,0}$	0	5
$s_{2,0}$	0	8
$X_{1,1,1,1}$	0.1000	0.1000
$X_{2,1,1,1}$	0.0033	0.0300
$X_{1,1,2,0}$	0.0067	0.0600
$X_{2,1,2,0}$	0.0100	0.0900
$X_{1,1,1,0}$	0.0033	0.0330
$X_{2,1,1,0}$	0.0091	0.0659
EBO	0.0100	0.2997

Table C.4: *Summary of data set used for the development of equations 29 and 48 (Case Studies 4 and 5)*

Variable	Mininum Value	Maximum Value
$s_{0,1}$	0	5
$s_{1,1}$	0	4
$s_{2,1}$	0	4
$s_{0,0}$	0	5
$s_{1,0}$	0	5
$s_{2,0}$	0	8
$X_{1,1,1,1}$	0.010	0.900
$X_{2,1,1,1}$	0.011	0.864
$X_{1,1,2,0}$	0.000	0.810
$X_{2,1,2,0}$	0.001	0.810
$X_{1,1,1,0}$	0.100	0.900
$X_{2,1,1,0}$	0.100	0.900
EBO	0.0035	0.1576

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Although the level of repair analysis (LORA) and spare parts stocking (SPS) problems are interrelated, the two sets of problems are usually considered sequentially or even independently in practice. The joint problem is typically difficult to solve due to the non-linearity of the constraint associated with system availability. This report illustrates how the two problems can be solved simultaneously by formulating a single mixed integer non-linear optimization model that explicitly captures the interdependency between the repair network (location of resources and allocation of decisions) and sparing inventory decisions (stock levels at each location). The resulting integrated LORA-SPS optimization model is solved by utilizing a decomposition optimization approach, a simulation model, and a genetic programming-based symbolic regression methodology. Preliminary computational results show that the proposed methodology has the capacity to tackle the integrated problem.

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Level of Repair Analysis
Spare Parts Stocking Problem
Mixed Integer Non-linear Programming
Genetic Programming
Lagrangian Relaxation
Simulation