SAR-GMTI phasor sums and their impact on target velocity measurements

Chuck Livingstone and Marina Dragosevic
SAR-GMTI phasor sums and their impact on target velocity measurements

Chuck Livingstone
DRDC Ottawa

Marina Dragosevic
MD Terrabytes Inc.

Defence R&D Canada – Ottawa
Technical Report
DRDC Ottawa TR 2011-177
November 2011
Abstract

The main focus of this document is the simple two aperture SAR-GMTI radar. A SAR signal model is developed from a phasor sum to illustrate the roles of the major components of a SAR signal including sampling ambiguities. The signal model is expanded to include the case of a two-aperture SAR-GMTI radar. A DPCA (displaced phase center antenna) model is used to describe moving target detection and an ATI (Along-track interferometry) model is used to discuss target motion estimation.

The interactions between the moving target detection and motion estimation processes are described in terms of a simple target and clutter model and then discussions are generalized to more realistic cases to illustrate how the principles deduced from the simple model influence real-world observations. Radar sampling strategies are discussed in terms of sampling ambiguities and their impact on target detection. Physically large moving targets (much larger that the radar impulse response function width) are described as ensembles of clustered but independent, moving point targets.

Some SAR-GMTI measurement examples are introduced to illustrate how the interactions between moving target detection and motion estimation processes influence the measurement outcomes.

An extensive list of references is provided to guide further study.
Résumé

Le présent document porte surtout sur le simple SAR GMTI à deux ouvertures. Un modèle de signal SAR est développé à l’aide d’une somme des phaseurs pour montrer le rôle des principaux éléments d’un signal SAR, y compris les ambiguïtés d’échantillonnage. Ce modèle est étendu pour inclure le cas d’un SAR GMTI à deux ouvertures. De plus, un modèle d’antenne à centre de phase déplacé (DPCA) est utilisé pour décrire la détection de cibles mobiles, et un modèle d’interférométrie longitudinale (ATI) sert à examiner l’estimation des mouvements des cibles.

Les interactions entre les processus de détection de cibles mobiles et d’estimation des mouvements des cibles sont examinées en fonction d’un modèle simple de cibles et de clutters, puis l’analyse est généralisée à des cas plus réalistes pour montrer comment les principes déduits du modèle simple influent sur les observations réelles. On analyse en outre des stratégies d’échantillonnage radar en fonction des ambiguïtés d’échantillonnage et de leur effet sur la détection de cibles. Par ailleurs, on examine de grosses cibles mobiles (beaucoup plus grosses que la largeur de la fonction de réponse impulsionnelle du radar) en tant qu’ensembles de cibles ponctuelles mobiles.

Quelques exemples de mesures faites par le SAR GMTI sont présentés pour montrer comment les interactions entre les processus de détection de cibles mobiles et d’estimation des mouvements influent sur les résultats des mesures.

Une liste exhaustive de références est fournie pour guider une étude ultérieure.
This page intentionally left blank.
Executive summary

SAR-GMTI phasor sums and their impact on target velocity measurements

Chuck Livingstone; Marina Dragosevic; DRDC Ottawa TR 2011-177; Defence R&D Canada – Ottawa; November 2011.

Introduction or background: There is a very extensive literature that addresses the use of radar systems to detect moving objects on the earth’s surface and to measure the motion of the detected objects. Implicit in most of the published work is the assumption that the reader knows the fundamental relationships between the received radar signals, their sources and the radar measurement process. This paper is aimed at readers who have a reasonable mathematical background and who have a basic understanding of synthetic aperture radar but who are unfamiliar with the operation of synthetic aperture radars (SARs) that have a ground moving target indication (GMTI) capability. The discussions are focused on the performance of a simple, two-aperture GMTI instrument that can either detect a moving target or can measure its motion in a single processing step.

Results: This discussion leads the reader through the creation of radar-signal models that are based on the motion properties of the observed terrain and on the sampling processes that are inherent in radar operation. Moving-target detection is discussed in terms of the classical stationary-world cancellation approach, DPCA (Displaced Phase Center Antenna). Target motion estimation is based on the correlation between signals from two spatially-registered receiving antenna apertures, the classical ATI (Along-Track Interferometry) approach. The relationships and interactions between the detection and motion estimation processes are discussed in detail and real-world examples are used to illustrate the points made.

Significance: This paper provides the reader with an in-depth introduction to properties of radar signals that must be processed by SAR-GMTI radar systems and illustrates how the moving target detection process can influence motion estimation outcomes.

Future plans: There are no future plans for additional work on the signal phasor sum topic.
Sommaire

SAR-GMTI phasor sums and their impact on target velocity measurements

Chuck Livingstone; Marina Dragosevic; DRDC Ottawa TR 2011-177; R & D pour la défense Canada – Ottawa; novembre 2011.

Introduction ou contexte: Une vaste documentation porte sur l’utilisation des systèmes radar pour détecter des objets mobiles à la surface de la Terre et pour mesurer le mouvement des objets détectés. De façon implicite, la plupart des travaux publiés supposent que le lecteur connaît les liens fondamentaux entre les signaux radar reçus, leurs sources et le processus de mesure par radar. Le présent document s’adresse aux lecteurs qui ont une formation adéquate en mathématique ainsi que des connaissances de base sur les radars à synthèse d’ouverture (SAR), mais qui ne connaissent pas le fonctionnement des SAR dotés de la capacité d’indication de cibles terrestres mobiles (GMTI). L’étude est axée sur les performances d’un instrument GMTI à deux ouvertures simple qui peut soit détecter une cible mobile, soit mesurer le mouvement de celle-ci en une seule étape de traitement.


Importance: Le présent guide donne au lecteur une introduction très poussée des propriétés des signaux radar à traiter par les systèmes SAR GMTI

Perspectives: Aucune recherche future n’est prévue sur la somme des phaseurs du signal.
# Table of contents

Abstract ........................................................................................................................................................ iii
Résumé ........................................................................................................................................................ v
Executive summary .......................................................................................................................................... vii
Sommaire .................................................................................................................................................... viii
Table of contents .......................................................................................................................................... ix
List of figures ................................................................................................................................................ xi
Acknowledgements ....................................................................................................................................... xiii

1. Introduction ............................................................................................................................................... 1
   1.1 Target velocity measurements using SAR-GMTI techniques .............................................................. 1
   1.2 SAR-GMTI Signal phasor sum background ......................................................................................... 3
      1.2.1 Sampling Effects .............................................................................................................................. 4
      1.2.2 Sampling Ambiguities and GMTI ................................................................................................. 5

2. A signal model for SAR-GMTI analysis ................................................................................................. 9
   2.1 Target model terms ................................................................................................................................ 9
      2.1.1 Moving targets ................................................................................................................................ 9
      2.1.2 Stationary natural terrain .............................................................................................................. 10
      2.1.3 Stationary urban terrain ............................................................................................................. 11
      2.1.4 The sea surface ........................................................................................................................... 11
      2.1.5 Noise samples ............................................................................................................................. 12
      2.1.6 Sampling ambiguities .................................................................................................................. 12
      2.2 Composite signal model .................................................................................................................... 12

3. The moving target detection problem ................................................................................................. 13

4. Phasor sum analysis of ATI velocity estimates .................................................................................... 16
   4.1 Analytic expansion ............................................................................................................................... 18
   4.2 Phasor sum discussion ........................................................................................................................ 19
   4.3 Phasor sum visualization ...................................................................................................................... 21

5. The DPCA target detection rule and its impact on SAR-GMTI measurements .................................. 22
   5.1 The DPCA detection rule model in target velocity-magnitude space .............................................. 22
      5.1.1 DPCA detection model in the ATI amplitude-phase plane ...................................................... 22
      5.1.2 Combined DPCA moving target detection and ATI target-motion estimation for two aperture systems .................................................................................................................. 29
      5.1.3 The impact of the SAR focusing parameters on the ATI motion estimation ....................... 31
   5.2 Clutter Probability Density Functions and their effects on ATI velocity measurements ................ 32

6. Analysis of physically large targets .................................................................................................... 36
   6.1 Ships as large targets .......................................................................................................................... 36
6.2 Trains as large targets ....................................................................................................... 40
6.3 Estimating the bulk motions of large targets ................................................................. 42
7. Summary ........................................................................................................................ ......... 44
8. References ............................................................................................................................ 46
Annex A .. Sampling Ambiguities .......................................................................................... 49
   A.1 Channel registration for GMTI.................................................................................... 52
Annex B... Moving target ambiguities ..................................................................................... 56
Annex C... Phasor diagrams: A Geometrical representation ...................................................... 61
   C.1 Phasor diagrams........................................................................................................... 61
    C.1.1 Clutter stronger than the moving target............................................................... 61
    C.1.2 Clutter and moving target amplitudes are equal................................................. 62
    C.1.3 Clutter is much weaker than the moving target.................................................. 63
Annex D .. ATI, DPCA and matched filter banks ................................................................. 64
List of symbols/abbreviations/acronyms/initialisms .............................................................. 66
—
List of figures

Figure 1: RADARSAT-2 operated as a two-aperture GMTI system .................................................. 3
Figure 2: Radar displacement between two transmitted pulses for DPCA sampling ...................... 6
Figure 3: Moving target spectral wrapping illustration using a power spectrum visualization ...... 7
Figure 4: DPCA residual clutter and noise PDF with a detection threshold ................................. 14
Figure 5: SAR-GMTI scene after DPCA cancellation .................................................................. 15
Figure 6: Land ATI scene with moving vehicle targets ............................................................... 17
Figure 7: Illustration of the joint amplitude-phase PDF for ATI clutter measurements with arbitrary signal dimensions ................................................................................................. 18
Figure 8: ATI magnitude-phase relationship for simulated signals ............................................. 26
Figure 9: Model estimates for a point target with motion phase = 60° ........................................ 27
Figure 10: ATI phase estimates for variable target motion phase ............................................. 29
Figure 11: SCR dependence of target phase estimates for a realistic ATI phase model .......... 33
Figure 12: Phase angle PDFs for selected detection thresholds using a Rayleigh clutter amplitude model ......................................................................................................................... 34
Figure 13: Target phase estimate expected value (solid line) and standard deviation (dashed line) for Rayleigh-distributed Uc when the target amplitudes are estimated from the detection thresholds ................................................................................................. 34
Figure 14: SAR image of a container vessel moving in a calm sea ............................................. 37
Figure 15: ATI phase of a container vessel moving in a calm sea ................................................ 38
Figure 16a: Magnitude image scaled to show the sea surface ................................................ .. 39
Figure 16b: Sea surface and ship in the ATI magnitude-velocity plane ..................................... 39
Figure 17: Target samples from a moving train that have been repositioned from ATI phase estimates ................................................................................................................................. 40
Figure 18a: SAR scene near Trenton Ontario showing repositioned moving targets as symbols .. 41
Figure 18b: Two trains moving in opposite directions through mixed agricultural and wooded terrain as seen in the ATI velocity-magnitude plane ......................................................... 42
Figure A-1: Sampling ambiguity superposition ........................................................................ 54
Figure B-1: Clutter amplitude spectrum envelope for an ideal two-aperture SAR-GMTI radar... 57
Figure B-2: Directionally ambiguous target spectra .................................................................. 58
Figure B-3: Time frequency graph for detected vehicle targets .................................................. 60
Figure C-1: Phasor diagrams for Uc > UT for two equally probable angles φ - θ ....................... 62
Figure C-2: Phasor diagrams for Uc = UT for two equally probable values of φ - θ ................. 63
Figure C-3: Phasor diagram for $U_C < U_T \cos(\varphi_T/2)$ for two equally probable values of $\varphi - \theta$ ....... 63

Figure D-1: Magnitude-speed diagrams for two simulated trains embedded in a realistic clutter background. $U_T$ is at or above the DPCA detection threshold. ....................... 65
Acknowledgements

The authors gratefully acknowledge shared knowledge from many fruitful conversations with members of the DRDC RADARSAT-2 GMTI project team and with collaborating scientists from external institutions. This report is based on group knowledge acquired by the RADARSAT-2 GMTI project team since 1999. Many thanks to Dr. Ishuwa Sikaneta who undertook the review of this report and provided many valuable comments.
This page intentionally left blank.
1. Introduction

It is well known that synthetic aperture radar (SAR) signals returned from objects that move on the earth’s surface have spectra that are shifted in frequency, with respect to the radar spectra of the terrain that they move through, by a system constant \((-2/\lambda)\) times the vehicle velocity component that is projected along the radar observation vector. It is also well known that the moving vehicle positions in a focused SAR image are shifted in the along-track direction (relative direction of radar platform motion) from the terrain image positions that correspond to their physical locations. It is further well known that the radar signal representing a moving object contains the complex sum of the radar returns from the object, the radar returns from the terrain in its displaced image position and the terrain (stationary-world) signal sampling ambiguities at its shifted image position.

This report separates the moving-target extraction and estimation problem for two-aperture SAR-GMTI (SAR-Ground Moving Target Indication) data into two sequential parts: the moving target detection problem and the target motion estimation problem. The target motion estimation problem discussion examines the composition of moving target signals and discusses the contributions of each signal component to target velocity estimation errors for the two-aperture SAR-GMTI case.

There are several analytical approaches that can be used to perform the moving target detection and estimation operations. Each approach has its strengths and weaknesses. In this paper we will discuss a classical approach that uses channel subtraction to separate stationary and moving target signals for detection and uses channel correlation to extract target motion metrics.

The discussion is aimed at readers who are familiar with SAR data and have a reasonable level of mathematical sophistication but who are not expert in the use of multi-aperture SAR-GMTI radars for moving target measurements.

1.1 Target velocity measurements using SAR-GMTI techniques

The fundamental principle underlying GMTI operation is short-time change detection. Dual antenna, two-channel, SAR (Synthetic Aperture Radar), with a suitable along-track baseline between the antenna phase centers, provides a powerful means for detecting moving vehicles by selectively cancelling signals scattered from stationary terrain (stationary world).

A simple and common stationary-world signal cancellation technique subtracts the radar returns captured by the fore antenna aperture from the radar returns captured by the aft antenna at a later time. The signal sample registration of aft aperture data to fore aperture data makes the radar appear stationary for each fore-aft sample pair. Signals that are unchanged over the sample registration time interval are cancelled. Signals that change over the elapsed time between the two registered apertures are not completely cancelled. If the un-cancelled signals are sufficiently large they can be detected and become moving target candidates. This detection process is called the displaced-phase-center-antenna (DPCA) analysis technique.
When the time between radar samples exactly corresponds to the time required for the aft antenna to move to the position formerly occupied by the fore antenna, the sampling rate is said to satisfy the DPCA sampling condition. DPCA is primarily a moving target detection tool.

In most applications the full potential of DPCA moving target detection is achieved only if the location of the detected vehicles can be estimated with sufficient accuracy. The motion-induced shift in the center frequency of the target signal spectrum results in a corresponding shift in the along-track position of the target image from the physical SAR image location of the target. Correction of the target image position requires measurement of the target spectrum shift from the signal data. This frequency estimate is equivalent to measuring the target phase change (the phase difference between fore and aft channels) over the antenna aperture registration time interval.

Along-track interferometry (ATI) is a SAR analysis technique that is very sensitive to scatterer motion in the radar line-of-sight (LOS) direction that uses the phase difference between spatially-registered fore and aft aperture radar data as a velocity estimator. SAR ATI analysis is based on the correlation between the fore and aft channel data to form an ATI interferogram image (computed by multiplying the each point in the fore channel data by the corresponding point in the complex conjugate of the registered aft channel data) and the extraction of the phase of each correlated sample pair as a motion metric.

ATI is well suited for detecting and estimating naturally occurring surface motion [1], [2], [3]. SAR-ATI has also been studied within the concept of ground moving target indication (GMTI) [4], [5]. One application is traffic monitoring [6], where the main objective is detection of moving vehicles and estimation of the LOS component of their velocity (radial speed) and location.

In SAR measurements, moving target signals are superimposed on (summed with) the background clutter signals and clutter sampling ambiguities that are present at their displaced location in the SAR image. The target-clutter interference compromises the accuracy of two-channel SAR-ATI speed estimates. This problem has been discussed [7, 27], and analyzed in recent literature. It is suggested that SAR-ATI estimates of radial speed are biased towards zero [8], [9] by the presence of superimposed, stationary-clutter signals. SAR-ATI performance has been further analyzed via evaluation of the Cramer-Rao lower bound (CRB), starting from the probability density function (PDF) of the ATI phase in the presence of stationary clutter and a moving target [10]. Cramer-Rao analysis is largely based on numerical methods, since closed-form expressions for the PDF are not known for all cases of interest and they are complicated for the known ones [11], [12]. For two-aperture GMTI systems, combined analytical and simulation results indicate that the root-mean-square (rms) error of target velocity estimates increases with increasing radial speed and with decreasing signal to clutter ratio (SCR), while clutter-to-noise ratio (CNR) has an insignificant effect on velocity estimation for all reasonable values. The relationship between rms velocity and target velocity estimation error has been verified experimentally for two aperture SAR-GMTI radar [13].

More complex SAR-GMTI configurations with three and four apertures have been proposed [14], analyzed [15], and some of them experimentally tested [16]. As expected, the added degrees of freedom provided by multi-aperture GMTI radars provide much more precise target velocity estimates than are available from two-aperture systems. For practical implementation reasons (minimization of the number of physical channels by time multiplexing) three and four aperture, space-based, GMTI radars are constrained by their internal data rates and by the required...
sampling rates to trade the number of GMTI apertures for instantaneous radar swath. There are some applications (especially marine target surveillance), however, where two-aperture GMTI measurements are acceptable and allow decreased motion estimation accuracy to be traded for increased radar swath.

SAR-GMTI measurements made with recently available space-borne radars (e.g. TerraSAR-X [17] and RADARSAT-2 [18]) have greatly expanded the number of available SAR GMTI data sets and the variety of observation conditions. The space-based measurements are complementary to earlier airborne results and have provided multiple opportunities to explore the properties of SAR-GMTI observations and to test and validate the outcomes of theoretical investigations. The experimental data from space-based systems use a combination of targets of opportunity and calibration targets (land vehicles or ships carrying recording GPS navigation sensors) [19], or targets of opportunity with independent position and velocity information such as AIS (automatic identification system) transmissions from ship targets [19], or optical tracking of moving vehicles [20], to verify GMTI position and velocity estimates extracted from GMTI radar measurements.

1.2 SAR-GMTI signal phasor sum background

With the previous section in mind, let us cast the elements of the moving-target, terrain interaction problem into a suitable form to build a signal model for further discussion. We will assume that the radar under discussion transmits its signal from the entire antenna and has two antenna receiving apertures that are separated in the radar flight direction by a distance, D. Each of these two apertures is coupled with a physical signal reception system to form a two-channel radar. Figure 1 shows an example of this class of radar system.

Let us further assume that: the two apertures are contiguous with each other, the corresponding radar beams are parallel so that there are no differential squint contributions and that both apertures observe terrain that is illuminated by a radar beam that originates from the two apertures combined into a single antenna.
For simplicity we will assume that the radar antennas are pointed near the ECEF (earth-centered, earth-fixed) coordinate broadside vector to the radar platform flight path so that the Doppler spectra of stationary terrain elements are centered at zero Hertz and that the radar data are focused in the radar range direction prior to subsequent analysis. None of the foregoing assumptions is essential but they do eliminate effects that must be dealt with in the real world and that have no importance to the following discussions.

The signals in each GMTI radar channel contain superimposed: radar returns from the target object, radar returns from the terrain, radar artefact signals and system noise. When more than two GMTI channels are available, signals that are common to all channels can be suppressed [14] and at least two clutter-suppressed channels remain to estimate target velocity and other target properties. When the GMTI radar only has two channels, the target measurement operations must be executed on signals that contain all superimposed elements.

For SAR–GMTI radar sensors SAR images are formed by a linear transformation (matched filter is one way) of time or frequency distributed signal samples that localizes signal returns from a single terrain point to a neighbourhood of a single image point defined by the width of the point target response (PTR). Details of the required signal properties and processing strategies can be found in a number of SAR processing texts such as [21, 22]. GMTI analysis can be performed in the SAR image space or in the signal spectrum space. If the terrain point is moving (moving target), its image point will be shifted from its geometrical location by a distance that is determined by the target velocity and will be superimposed on the other radar signals at this point. In frequency domain analysis the moving target spectrum will be shifted by the motion Doppler frequency and will be superimposed on stationary terrain spectra at that location. This process is discussed in greater detail in Annex B.

1.2.1 Sampling Effects

SARs are inherently sampled systems in which range information is captured from signal samples of radar returns from terrain elements along the radar observation vectors. Following the Shannon Sampling Theorem the signals are sampled, on reception, at a rate that is higher than their transmitted radio-frequency bandwidth (range samples). The radar observation vectors form a manifold whose cross-range width is determined by the cross range antenna pattern. The sampling process outputs the complex sum of all radar returns from the observation-vector manifold at each range-sampling time and this time determines that radar range for that sample. On the surface of the observed terrain, the manifold of constant range observation vectors lies along the intersections of a sphere (whose radius is the radar range of interest) centered at the radar with the set of terrain elements at that range. The sampling direction is nominally taken to be center of the two-way, cross-range antenna pattern and can be later refined in analysis when multiple radar apertures are used [23]. The radar gain along each vector in the manifold, and hence its relative contribution to the complex sum, is determined by the two-way antenna pattern of the radar.

For simple, strip-mapping, SAR-GMTI radars that we are discussing, the formation of a viable radar observation signal-set for SAR processing and image formation, requires the acquisition and storage of coherent radar range observations that span the cross-range, two-way antenna pattern at each range. The maximum size of this area for any radar pulse is often referred to as the real-aperture resolution cell of the radar. The sampling rate (radar pulse repetition frequency or PRF) for the cross-range observations must exceed the constant-range bandwidth of all of the complex
(amplitude and phase) signals in the observation set. For the assumptions made previously in this section, this bandwidth is determined by the rate of change of radar range at the leading and trailing edges of the two-way antenna pattern at any range in the observation set and defines the cross-range Doppler bandwidth of the signal. For all practical radar systems there is no clearly defined leading or trailing edge to the Doppler spectrum due to the angular continuity of real antenna patterns. The conventional determination of the radar PRF uses the -6 dB Doppler bandwidth to set the minimum sampling rate.

Provided that the radar PRF exceeds the desired spectral bandwidth, all signal components, with spectral components within the sampled bandwidth (± PRF/2), are adequately sampled under the sampling theorem and can be interpolated in time or frequency without altering their information content. All signal spectral elements that lie outside of the adequately sampled interval will be mapped into the adequately-sampled interval by a spectral folding process that shifts the incorrectly sampled signal components across the interval and superimposes them on the adequately sampled signals [24]. The sampling ambiguity discussion in Annex A illustrates the ambiguity folding effect in Figure A-1.

The spectrally folded signals are sampling ambiguities and exist at several orders that decline in signal power with increasing separation from the edges of the adequately sampled interval. They are found in all SAR data. If the fore and aft ambiguous signal spectra have identical form and opposite decay direction and are centered in the sampling interval, the combined fore and aft ambiguity spectra are symmetric with respect to the adequately sampled signal spectrum. Since each ambiguity signal spectrum is asymmetric with respect to the sampling interval center, SAR processing will treat the fore and aft ambiguities as moving targets. The (poorly focused) fore and aft ambiguous SAR images will be displaced in opposite along-track directions from the primary SAR image by their apparent motion and will be superimposed on (and thus will degrade) the primary image. These effects are commonly seen in SAR images of land surfaces bordering calm water as “ghost images”. Airborne radars often resolve this problem by increasing the sampling rate significantly so that the ambiguity signal power is very small with respect to the desired, adequately sampled, signals. Space-based radar systems do not often have this luxury for reasons that are beyond the scope of this article.

1.2.2 Sampling Ambiguities and GMTI

For SAR-GMTI systems, sampling ambiguities produce effects that must be dealt with by both radar control and signal processing operations.

The sampling theorem only requires that two signals be adequately sampled for them to be registered with each other by frequency shifting or temporal resampling. For the two-aperture SAR-GMTI systems, that we are discussing here, one of the channels can be resampled to register it with the other to arrive at the “stationary radar” condition needed for moving target detection and motion estimation. Resampling of this type will, indeed, register the adequately sampled signal components but will unbalance (miss-register) the sampling ambiguities [14] between the two radar channels. The ambiguity imbalance will degrade the coherence between the two radar channels, will increase the ambiguity contribution in attempts to perform stationary world signal cancellation (will compromise cancellation) and will create a phase bias in stationary world velocity estimates.
The problem can be resolved by placing an additional constraint on the radar PRF. If the cross-range sampling rate is chosen so that sampling theorem is respected and that the sampling period corresponds to a simple fraction \((1/N)\) of the time required for the aft antenna aperture phase-center to displace into the position formerly occupied by the fore antenna aperture phase-center, channel registration is an integer operation. For integer-shift registration, the sampling ambiguities remain balanced between the two channels and are cancelled with the stationary clutter when the fore and aft aperture signals are subtracted [14]. This is the DPCA sampling constraint as illustrated by Figure 2.

For further discussion, it is assumed that the two-aperture radar data under discussion has been adequately sampled under the DPCA constraint.

Moving targets, without strong glint components\(^1\), have signal spectra that are shaped similarly to the stationary world terrain spectra (they may be broadened by cross range motion and by other motion components) but are displaced in frequency by their LOS motion components (motion along the radar observation vectors). In the frequency domain, the impact of target motion is to shift the target spectrum with respect to the stationary-world spectrum so that the effect of signal sampling is to wrap a portion of the target spectrum around the adequately sampled interval [24]

---

\(^1\) When a coherent scattering structure has physical dimensions greater than the radar antenna and is oriented so that the scattered energy beam-width is narrower that the radar antenna beam-width, a strong radar return whose spectrum is narrower than the radar signal spectrum can be created. This is frequently called a glint return.
as illustrated by the cartoon in Figure 3. In Figure 3, 0 is the center of the stationary-world signal spectrum at the true position of the moving target and \( f_{\text{Dop}} \) is the true Doppler shift of the target due to its motion projection along the radar range vector. Sampling splits the moving target signal spectrum into two components that are wrapped over the sampling interval discontinuity, \(-\text{PRF}/2 < f < \text{PRF}/2\) (Visualize a metal can where one side of the seam is \( \text{PRF}/2 \) and the other side of the seam is \(-\text{PRF}/2 \) and imagine the signal spectrum wrapped around the can) while preserving the total spectral content of the moving target signal. The lines \( f_{\text{Cent}1} \) and \( f_{\text{Cent}2} \) represent the centroids of the wrapped signal components. The composite target plus clutter radar signal will contain evidence for two moving targets travelling in opposite radar range directions and Doppler estimates made from the sampled signal will under-estimate the Doppler frequency \( (f_{\text{Cent}1}) \) and thus will underestimate the speed of the primary spectral component when data are processed using stationary-world criteria to form an image.

![Figure 3: Moving target spectral wrapping illustration using a power spectrum visualization](image)

The effect of stationary-world processing is to generate two moving targets whose SAR images are displaced in opposite cross-range directions from the true target position. The primary target image will be displaced (with respect to the stationary background image at the physical location of the target) by a terrain surface distance \( x_1 = \frac{\lambda f_{\text{Cent}1} R}{2V_s} \) and the secondary target will be displaced by a terrain surface distance \( x_2 = \frac{\lambda f_{\text{Cent}2} R}{2V_s} \) where: \( \lambda \) is
the radar wavelength; \( f_{\text{Cent1}} \) is the centroid of the primary target sampled spectrum; \( f_{\text{Cent2}} \) is the centroid of the secondary target sampled spectrum; \( R \) is the radar range of the target; and \( V_S \) is the radar velocity.

From Figure 3, the relationship between \( f_{\text{Dop}} \) and \( f_{\text{Cent1}} \) indicates a systematic error in the relationship between a target velocity estimated from the target displacement in the image and the true range velocity. There is a similar systematic error for the relationship between \( f_{\text{Dop}} \) and \( f_{\text{Cent2}} \).

The target splitting and displacement problem can be resolved by matching the SAR processing filter to the radial motion of the target which, when correct, places the target in the correct spatial position (as predicted from the radar geometry projections) \([34]\) and maximizes the target image strength. The use of a moving target matched filter degrades the background, stationary-world image. The target signals remain superimposed on relatively-displaced stationary-world signals. Moving target ambiguities are discussed in more detail in Annex B.
2. A signal model for SAR-GMTI analysis

A well designed SAR processor performs linear operations that redistribute and sum complex signals contained in a time history of real-aperture radar returns to create a complex image of the radar scene whose spatial resolution is determined by the range and cross-range bandwidths of the signals. The processing is based on the assumptions that: the time history of the imaging geometry is known, signal coding contained in each radar pulse is known and that the relative motion between the radar and the point being observed is known.

For this discussion, we will assume that: the radar uses a two-channel SAR-GMTI configuration as outlined in section 1.2; the two receiving channels have been amplitude and phase calibrated with respect to each other; and the SAR images from the two channels have been registered to each other so that the spatially registered channels represent two observations of the SAR scene that are separated in time by the time required for the aft antenna aperture to move to the fore aperture position at the previous radar pulse. We will call the signals from the fore and aft aperture channels $S_1$ and $S_2$.

Each complex sample in each image channel contains a sum of signal components that are related to the observed terrain, the electrical noise generated by the radar, and to the radar sampling process used. For this discussion, these are:

1. A moving target component, $S_T = U_T e^{j(\varphi_T + \varphi_T)}$, where the target phase has a geometric component, $\varphi_T$, and a motion component $\varphi_T$. The target amplitude $U_T=0$ when no target is present at the point being considered;
2. A clutter component, $S_C = U_C e^{j(\varphi_C)}$, where the clutter phase may contain both static and motion components, $\varphi_C = \theta + \theta_M$;
3. Sampling ambiguity components, $S_{Af} = U_{Af} e^{j(\xi + \xi_A)}$ and $S_{Aa} = U_{Aa} e^{j(U + U_M)}$ where the subscripts $A_f$ and $A_a$ represent the fore ambiguity (ambiguous signals coming from the terrain that precedes the focused point) and the aft ambiguity (terrain that follows the focused point). The ambiguity amplitudes, $U_{Af}$ and $U_{Aa}$, are normally different and the ambiguity phases, $\varphi_{Af} = \xi + \xi_A$ and $\varphi_{Aa} = \nu + \nu_M$, may contain both static and motion components.
4. System noise at each target point, $S_N = \Re S_N + j \Im S_N = N e^{j\eta}$, is a random, complex number that is uncorrelated between adjacent radar impulse responses and between radar channels.

The subscripts T and M identify the phase components caused by motion. The static phases, $\varphi$, $\theta$, $\xi$ and $\nu$ are uniformly distributed random variables on the interval $\{-\pi, \pi\}$ radians.

2.1 Target model terms

2.1.1 Moving targets

Radar returns from moving targets are captured as clusters of radar samples that represent the radar returns from contiguous portions of the moving objects that have been coherently summed over the radar PTR (also known a impulse-response function). For a target whose physical size
corresponds to the projection of the slant-range impulse response at the target location, the cluster size varies from a single sample point to a block of four adjacent sample points depending on how the radar sampling grid overlays the target position. Although the target lies within one PTR, the target may be complex in that it contains two or more dominant scattering centers. Although the radar system sees this target as a point, the target signal behaviour can deviate from that expected from a single dominant scatterer (true point target). Although it does not affect the discussions in this report, the decomposition of a radar target into dominant scattering centers is important for more detailed examinations of radar behaviour.

Targets that are physically large form clusters of radar sample points. For the purposes of this discussion, moving targets will be treated as rigid bodies (cars, ships, small to medium sized trucks, wind turbine blades, etc.) or as articulated rigid bodies (semi trailers, trains, etc.). The physical motions that generate the moving target phase, \( \varphi_T \), are systematically distributed over the target object and affect the target phases of the samples in the observed target cluster in a systematic fashion. The moving target amplitude, \( U_T \), will vary over the moving target sample set.

Target motion estimates are based on the ensemble properties of the target-sample cluster.

### 2.1.2 Stationary natural terrain

For radar returns from natural, stationary terrain, the spatial correlation distances for radar returns from the scattering centers in the imaged scene are generally smaller than the radar PTR dimensions and the radar image coherence lengths are set by the radar PTR.

In this case, the clutter samples, \([U_C, \theta, \theta_0]\), for adjacent impulse response points are very weakly correlated under the definition of the PTR width and are somewhat more strongly correlated at adjacent sample points under the sample spacing required by the sampling theorem. Points separated by several PTR dimensions are statistically independent of each other. The clutter amplitude statistics can be represented by a textured distribution whose form depends on the terrain cover type. Terrain relief and terrain cover relief will produce radar shadows for most SAR imaging geometries. The clutter amplitude, \( U_C \), in shadowed areas is zero.

Anyone who has stood in a forest on a windy day has noticed that tree branches and tree trunks move in response to the wind. Modest sized trees (~10 cm to ~30 cm trunk diameter) sway with periods of the order of several seconds. The motion phases and directions of adjacent trees appear to be uncorrelated and individual branches and leaves move more quickly but in random directions and at random rates. The natural, stationary world has internal motion.

For SAR systems that have resolution cell dimensions of a few metres (or larger) the motions of scattering centers, whose radar returns sum to form a PTR, result in motion contributions to the clutter phase. These motion terms are random, are uncorrelated over areas represented by a few impulse responses and are seen as weak, stationary-world phase-noise\(^2\). For ATI measurements these motions can somewhat broaden the stationary world phase distribution that is centred on the positive real axis of the complex plane (Figure 6).

\(^2\) The motion displacement over the radar synthetic aperture can be many wavelengths and depends on the synthetic aperture time of the radar. The motion displacement between adjacent radar samples will be relatively small (they correspond to motion speeds smaller than or equal to 1 m/s) and the phase noise contribution will also be small but will be present in the data.
For very high resolution radars, wind-generated foliage motion can behave in a locally coherent fashion and results in the often observed, local, SAR defocusing of the scene in these images.

Terrain elements that are truly stationary over a radar observation interval have no terrain-motion phase component contributions but have radar phase-noise contributions from the radar system. The radar system phase-noise components are uncorrelated over the channel registration time and will appear as weak phase differences in ATI measurements.

### 2.1.3 Stationary urban terrain

For radar returns from terrain containing cultural objects, some terrain elements such as metal flashings, concrete beams, walls and decks, etc. may be coherent in the radar sense over several PTR dimensions because of their orientation, uniformity and dimensions. Radar glint effects (the radar beam width of the reflection surface is smaller than the radar antenna beam width) are common due to the presence of long, linear, building elements (e.g., metal flashings). Radar shadows and radar lay-over (signal superposition of several terrain elements at the same radar range and along-track position) are common. For dense, high relief urban areas multi-path effects (radar signals sequentially reflecting from two or more surfaces) will be common.

In this case there will be local spatial correlations between clutter sample parameters, \([U_C, \theta, \theta_M]\) (equation 2), that correspond to the structure of cultural features. Otherwise, urban and natural terrain returns follow the same signal behaviour but will have different magnitude distribution functions due to the very large dynamic range of the radar returns. Urban trees and moving building components (e.g., swinging signs) can introduce motion effects.

### 2.1.4 The sea surface

Radar returns from the sea surface add a level of complexity to the clutter signal parameters \([U_C, \theta, \theta_M]\) because the sea moves. At the scale of a PTR, sea surface structure with scale lengths that are smaller than the PTR dimensions contribute stochastically to the magnitude and phase of the radar return in the same manner as is seen for stationary clutter. In addition to the small-scale, wind-induced, motions that determine the sea-surface back-scatter efficiency, the sea surface generally moves at spatial scales larger that the radar PTR. Motions of interest are the orbital motion of the water surface associated with waves and bulk motions corresponding to surface currents. The wave orbital motions that project onto the radar observation vectors impact the parameter, \(U_C\), via the velocity-bunching, hydrodynamic modulation and tilt modulation mechanisms discussed in SAR oceanography [25] and impact the parameters, \(\theta_M\), by directly adding a motion phase that is coherent between the two radar apertures. Radial components of surface currents will have a direct impact on the parameter, \(\theta_M\).

In addition to the clutter motion effects, ocean waves whose wavelengths are comparable to or greater than the dimension of a ship (projected in the wave propagation direction) will have a direct effect on the bulk motion of the ship target. This sea motion component of the target motion is embedded in the target sample parameters \([U_T, \varphi_T]\). These effects are discussed in [26].
2.1.5 Noise samples

The noise terms $N_1e^{j\eta_1}$ and $N_2e^{j\eta_2}$, where indices 1 and 2 indicate the aperture and radar channel, represent filtered white noise that originates in the radar system electronics. The noise samples from a radar channel are filtered by the radar system PTR and are locally correlated according to the PTR overlap of the samples considered but are otherwise uncorrelated between samples and between channels.

2.1.6 Sampling ambiguities

The sampling ambiguity terms, $[U_{Af}, \zeta_M]$ and $[U_{Ad}, \nu_M]$ come from terrain surfaces adjacent to and external to the adequately sampled real aperture of the radar as discussed in Section 1.2.2. The ambiguous signals are filtered by the skirts of the two-way radar antenna pattern which gives their spectra asymmetric amplitude weightings and results in degraded cross-range resolution in SAR processing. The same spectral asymmetries result in time-varying phase terms, $\zeta_M$ and $\nu_M$ that will be measured as radial motion by an ATI calculation. Depending on the terrain that is mapped into the sampling ambiguities, real terrain motion effects may be weakly present. A more detailed discussion of sampling ambiguities and the impact of GMTI channel registration is presented in Annex A.

2.2 Composite signal model

SAR signals received by the radar are linear super-positions of all of the signal elements discussed in the last section. Assuming that the SAR data has been linearly processed to create a complex SAR image, each SAR-GMTI scene sample can be expressed in terms of the signal model:

$$S = S_T + S_C + S_{Af} + S_{Ad} + S_N$$

(1)

For a two-aperture SAR-GMTI radar of the form shown in Figures 1 and 2, the composite signal model at each sample point, for each channel, has the form shown in equation 1 and can be expressed as:

$$S_1 = U_1e^{j\zeta_1} = U_Te^{j\left(\frac{\phi + \phi_T}{2}\right)} + U_Ce^{j\left(\frac{\theta + \theta_T}{2}\right)} + U_{Af}e^{j\left(\zeta_M + \frac{\nu_M}{2}\right)} + U_{Ad}e^{j\left(\nu_A + \frac{\nu_M}{2}\right)} + N_1e^{j\eta_1}$$

(2) and

$$S_2 = U_2e^{j\zeta_2} = U_Te^{j\left(\frac{-\phi + \phi_T}{2}\right)} + U_Ce^{j\left(\frac{\theta - \theta_T}{2}\right)} + U_{Af}e^{j\left(\zeta_M - \frac{\nu_M}{2}\right)} + U_{Ad}e^{j\left(\nu_A - \frac{\nu_M}{2}\right)} + N_2e^{j\eta_2}$$

for channels 1 and 2 respectively. To simplify the following discussions, we will assume that the radar pulse repetition frequency satisfies the DPCA conditions, that channels 1 and 2 are registered to each other and that all assumptions in section 1.2 are met.
3. The moving target detection problem

For a SAR-GMTI scene, the radar signals for each sample point can be expressed by equation 2. In the context of the scene, the number of sample points which contain moving target signals is a very small fraction of the total. Moving targets are rare events and a SAR ATI detection algorithm that exploits this fact has been developed for moving target detection [29].

In this and subsequent discussions we will use the fundamental fact that the GMTI target detection process relies on short-time change detection between two radar signal sets. When two data sets are mutually coherent, a radar can detect differential changes in radar range that are smaller than a radar wavelength as changes in signal phase. Let us look at the DPCA-sampled case where the clutter is truly motionless and where the target signal is much larger than the system noise.

If we subtract the registered SAR GMTI scenes we have the complex quantity:

\[
S_1 - S_2 = U_T \left( e^{i \left( \frac{\varphi_1 + \varphi_2}{2} \right)} - e^{i \left( \frac{\varphi_1 - \varphi_2}{2} \right)} \right) + N_1 e^{i \eta_1} - N_2 e^{i \eta_2} \quad \text{and since} \quad U_T \gg N_1 + N_2,
\]

the difference has the magnitude:

\[
U_{DPCA} = |S_1 - S_2| \approx 2U_T \left| \sin \left( \frac{\mu \varphi_T}{2} \right) \right|.
\]

This highly idealized condition is not realistic but does provide one very significant clue. The DPCA clutter cancellation process creates a radial-velocity-dependent notch filter that attenuates targets, clutter and residual ambiguities according to a motion (composite phase) law that varies as \( \sin \left( \frac{\mu \varphi_T}{2} \right) \) where \( \mu \) is any of: \( \varphi_T, \theta_M, \xi_M \) or \( \nu_M \) from equation 2. Taking into account that there are multiple mechanisms that cause channel decorrelation in any real situation (internal motion, phase-noise constraints on the cancellation of strong stationary targets, small beam squints, etc.) , the DPCA notch filter does not have the sharp null suggested by equation 3, but has a "fuzzy" minimum where the notch depth depends on residual, un-cancelled signals. There is some amplitude threshold, \( T \), where the following decision rule applies:

(4) If \( |S_1 - S_2| > T \), a moving target is present.

Else-if \( |S_1 - S_2| \leq T \), there is no moving target present.

The threshold \( T \) is either determined from a statistical model of \( |S_1 - S_2| \) after DPCA clutter cancellation or from an adaptive approach that uses residual DPCA signals from similar, target-free terrain. Associated with the detection threshold is a false-alarm probability that is illustrated by the cartoon in Figure 4.
In Figure 4, if we let the test statistic, \( u = |S_1 - S_2| \) after DPCA cancellation, the false-alarm probability associated with the threshold \( T \) is:

\[
P_{fa} = \int_{T}^{\infty} PDF(u) du
\]

The probability of false alarm, \( P_{fa} \), is often called the false-alarm rate of a detector. From Figure 4 and equation 5, it is obvious that knowledge of the tail of the residual clutter plus noise PDF (probability density function) is critical to correctly estimating the detection threshold, \( T \). The signals that contribute to the residual PDF are: system noise, partially cancelled, strong clutter samples (system phase noise modulation effects), partially cancelled, moving clutter elements and partially cancelled sampling ambiguities when the PRF is not sampling the scene at the DPCA rate.

Figure 5 shows an example from a real GMTI scene in which the clutter and ambiguity terms were not fully cancelled by the DPCA subtraction and add to the system noise in a distribution centered on the origin of the complex plane. In this example, the DPCA difference \( S_1 - S_2 \) is plotted on a polar representation of the complex plane where the positive real axis passes through 0° and
the positive imaginary axis passes through $90^\circ$. In this representation, the detection threshold, $T$, becomes a circle of radius $T$ centered on the complex plane origin and all signals whose amplitude is greater than $T$ are detected as moving targets. All signals that are within the threshold are un-cancelled clutter, ambiguity and noise. When the DPCA detection output is displayed as an image, only the detected target candidates remain.

![Figure 5: SAR-GMTI scene after DPCA cancellation](image)

Noting that the selection of a detection threshold implies the selection of a false-alarm rate, very strong residual clutter or strong clutter ambiguities or system noise spikes that exceed the threshold will create false alarms.
4. Phasor sum analysis of ATI velocity estimates

Equation 2 applies at each data point where there is a target return and combines target, clutter, ambiguity and noise effects at that point in the signal space. For the purpose of this discussion we will consider a physically large moving target to be composed of a cluster of point targets as discussed in Section 2.1.1.

Target motion estimates for the detected target candidates use channel correlation to generate an interferogram of the targets embedded in the scene clutter (along-track interferometry or ATI analysis). Using equation 2 to describe the signals at a target point, the ATI response for the target point is:

\[ S = S_1 S_2^* \]

where \( * \) is the complex conjugate operator.

ATI captures a measure of target motion in terms of the interferometric phase:

\[ \chi = \arg(S_1 S_2^*) = \tan^{-1} \left( \frac{\Im(S_1 S_2^*)}{\Re(S_1 S_2^*)} \right) \]

where the ratio is the imaginary part of the interferogram divided by its real part. In most mathematical libraries, \( \tan^{-1} \) in equation 7 is an ambiguous mapping with the principal values in the interval \((-\pi/2, \pi/2)\). Considering the sign of the real and imaginary parts, we may use the “atan2” function, which extends the interval of principal (unambiguous) values to \((-\pi, \pi)\).

When the ATI interferogram is plotted in a polar representation of the complex plane, Figure 6, stationary scene elements are clustered about the real axis and moving targets appear as radial groups of points at phase angles \( \varphi_T \).

The statistical distribution of the stationary world radar returns about the real axis is the result of system noise (concentrated near the origin of the graph), phase noise effects from the radar system and from internal motion of stationary-world target elements and low-speed clutter motion. A three dimensional view of a stationary clutter scene measurement set is shown in Figure 7.

At a detailed level, the following analysis applies to each individual point in the target point cluster. When analysis results are aggregated, even if the target moves with a constant velocity, the stochastic nature of the clutter, the sampling ambiguities and the noise contributions will result in a statistical spread of the target motion estimates.

A useful metric for analysis and interpretation of ATI data is the signal-to-clutter-plus noise ratio, SCNR.
All symbols in equation 8 are defined in Section 2.0 and in equation 2. When the system noise is negligible with respect to the other variables, the SCNR becomes the signal to clutter ratio, SCR. The SCNR is evaluated at each target sample.
4.1 Analytic expansion

Assuming that two-channel SAR GMTI data are processed by first applying a detection threshold to the difference between channel 1 and channel 2 signals (DPCA processing) and that the detected signals are used to generate a mask that identifies the detected target signals in channel 1 and channel 2. If the mask is applied to the channel output signals, only those points that contribute to the detected moving targets (moving target candidates) are retained. Each retained image point is treated as an independent, moving target.

Now apply the ATI correlation process to the retained data to compute the ATI representation of each point. At each target point we have:
The function $F$ executes all products involving system noise. The ATI correlation at each point has an ATI phase $\arg(\mathcal{S}_1\mathcal{S}_2^*)$, as expressed by equation 7, and an amplitude:

$$(10) \quad U = \sqrt{S_1 S_2^*} = \sqrt{U_1 U_2}. $$

To compactly express the ATI phase, $\chi$, for each point given by equation 2 in terms of the channel correlation expansion shown in equation 9 let us define some indexed variables.

- Define the amplitude variables $W_k \ (k=1:6)$ as:
  - $W_1 = U_T$, $W_2 = U_c$, $W_3 = U_{Af}$, $W_4 = U_{Aa}$, $W_5 = N_1$, $W_6 = N_2$

- Define the static, random phase variables, $\Psi_k$ as:
  - $\Psi_1 = \phi$, $\Psi_2 = \theta$, $\Psi_3 = \zeta$, $\Psi_4 = \nu$, $\Psi_5 = 0$, $\Psi_6 = 0$

- Define the dynamic phase terms, $\Omega_k$ as:
  - $\Omega_1 = \phi_T$, $\Omega_2 = \theta_M$, $\Omega_3 = \zeta_M$, $\Omega_4 = \nu_M$, $\Omega_5 = \eta_1$, $\Omega_6 = \eta_2$

We can now express the ATI phase for a target point as:

$$\chi = \tan^{-1} \left( \frac{\Re(S_1 S_2^*)}{\Im(S_1 S_2^*)} \right) \quad (11)$$

Although equation 11 looks messy and the calculation of its statistical properties is truly nasty, its interpretation is not complex.

4.2 Phasor sum discussion

The phase angle, $\chi$, in equation 11 is used to estimate the radial speed component of the represented point target by the approximation:

$$(12) \quad V_{\text{Targ}} = \frac{\chi \mathcal{A} V_{\text{Sat}}}{4\pi D}. $$

In equation 12:

- $V_{\text{Targ}}$ is the estimated radial velocity of the target in m/s.
• \( \lambda \) is the radar wavelength in m.
• \( V_{Sat} \) is the cross-range velocity of the radar in m/s.
• \( D \) is the two-way antenna aperture baseline separation in m.

A number of comments related to the target velocity estimates arise from the form of equation 11:

1. The terms \( \sum_{k=1}^{4} W_k^2 \sin(\Omega_k) \) and \( \sum_{k=1}^{4} W_k^2 \cos(\Omega_k) \) contain the power-weighted motion components associated with each of the summed signal sources at the target point and the terms containing the product \( W_5 W_6 \) model the single point realization of the system noise power. Since \( W_5 \) and \( W_6 \) are uncorrelated, the ensemble mean of this product is zero but its influence on the variance of \( \chi \) is not negligible in some cases.

In the case where the target contribution dominates the sum, \( U_T^2 >> U_c^2 >> \{U_{Af}^2, U_{Aa}^2\} > N_1 N_2 \), the target terms dominate and all other terms contribute to target-phase estimation bias and to variance of the estimate.

2. In general, the target phase angle estimates, \( \chi \), are biased towards zero degrees by the presence of the clutter power, \( U_c^2 \), and the sampling ambiguity power, \( U_{Af}^2 \) and \( U_{Aa}^2 \), terms in the denominator of equation 12. This is in agreement with simulations and observations of target-clutter interaction effects [27].
   a. When the sampling ambiguities are insignificant with respect to the clutter power at the image point containing the displaced target signal, the clutter power dominates the bias effect
   b. When the clutter power is insignificant with respect to ambiguity signals from near-by scene elements\(^3\), the ambiguity power dominates the bias effect.

3. The cross-product terms, \( \sum_{k=1}^{5} \sum_{m=k+1}^{6} W_k W_m \cos(\Psi_k - \Psi_m) \sin\left(\frac{\Omega_k + \Omega_m}{2}\right) \) and \( \sum_{k=1}^{5} \sum_{m=k+1}^{6} W_k W_m \cos(\Psi_k - \Psi_m) \cos\left(\frac{\Omega_k + \Omega_m}{2}\right) \) contain the inter-modulation terms between all signal components and arise because of the quadratic nonlinearity in \( S_1 S_2 \), \( S_2 \). Radar system non-linearity in the signal reception paths for \( S_1 \) and \( S_2 \) will generate similar cross terms in each of the component signals. For well designed systems the radar non-linearity terms are generally very weak unless signal saturation occurs on the radar electronics.

4. For two-aperture SAR-GMTI radars the most important parts of these sums are the interactions between the moving target contribution and all other terms (the \( k=1 \) case). Using the strong-target signal hierarchy in 1., above, we see that now \( U_T^2 > 2 U_T U_c \cos(\phi - 0) >> U_c^2 >> \{U_{Af}^2, U_{Aa}^2\} > N_1 N_2 \) and the target-clutter interaction can influence the ATI phase estimate, \( \chi \) by adding a small term whose motion phase is the average between the target phase and the clutter phase.

\(^3\) This case can occur when the target is displaced into a low-clutter area or a radar shadow and there are strong sampling ambiguities from adjacent terrain elements (such as buildings).
a. When an ensemble of moving targets (or a single physically large moving structure) are seen in a stochastic background, the \( k=1 \) terms contribute an estimate variance that depends on the target motion. The other non-target terms contribute to the estimate variance in a target-independent manner.

5. The observational evidence suggests that large target condition in 1, above, occurs when the SCNR is greater than 15 dB for target points. Pick a number > 15 that you are comfortable with.

### 4.3 Phasor sum visualization

It is sometimes easier to understand the significant properties of a phasor sum by using a visual representation. The discussions in the previous section look at all major contributors to SAR-GMTI signals to determine the relative influence of each signal type. Given the number of random variables in equations 2, 9 and 11, a diagrammatic representation of the entire signal set would be difficult to read, however, the key points can be visualized by reducing the signal set to two dominant contributors, the target and the clutter.

A discussion of the simplified model and representative phasor diagrams are presented in Annex C.
5. The DPCA target detection rule and its impact on SAR-GMTI measurements

5.1 The DPCA detection rule model in target velocity-magnitude space

The signal processing model discussed in this document assumes that moving target detection and subsequent motion estimation are performed on a sample by sample basis (each radar impulse response is independent of its neighbours) and that any clustering of detection and/or measurement results follow the analysis steps. The model further assumes that the target parameter estimation process is only applied to detected target points.

The detection decision boundary, represented by the rule in equation 4, needs to be optimized in the two-dimensional, complex, signal-space, \( C^2 \), of measurements \( S = [S_1, S_2] \). This optimization is straightforward if the joint PDF, \( p(S) \), is known for both the target-present and no-target hypotheses, and leads to the log-likelihood formulation, [17] and [28], but is very difficult otherwise.

Only in the special case of, uniform clutter, strong coherence (>0.9) between \( \{S_1\} \) and \( \{S_2\} \), sampling constrained by the DPCA condition and simple targets (no glint effects) is the optimal [34] detection test statistic equal to \( U_{DPCA} \) given by equation 3. Although the test statistic, \( U_{DPCA} \), is sub-optimal in all realistic cases it will be used in the rest of this discussion and the detection threshold will be denoted by \( T \).

Instead of \( S \in C^2 \), dual-channel, co-registered measurements can be defined in the three dimensional real signal-space \( R^3 \) using \( \{\zeta, U_1, U_2\}^T \) from equation 2 ( \( \zeta = \zeta_1 - \zeta_2 \) ), as the measurement set, or equivalently, \( \{\zeta, U, \Delta U\} \) where \( U = \sqrt{S_1 S_2^*} = \sqrt{U_1 U_2} \) and \( \Delta U = U_1 - U_2 \), since these values convey the same information. Noting that \( \zeta \) is known at each sample point from its estimate, \( \chi \), (equation 11), it is easy to show that the detection threshold, \( U_{DPCA} = T \), corresponds to a decision boundary surface, \( Z \), in \( R^3 \) given by:

\[
4U^2 \sin^2 \left( \frac{\chi}{2} \right) + \Delta U^2 = T^2.
\]

All terms in equation 13 can be estimated from GMTI data.

It should be clear that equation 13 does not optimize the receiver operating characteristic; it is simply a signal-space equivalent to the DPCA detection rule, which is sub-optimal, under any criterion almost all of the time.

5.1.1 DPCA detection model in the ATI amplitude-phase plane

Any signal processing sequence that constrains parameter estimation to detected moving target candidates imposes a coupling between the detection and estimation process. The sequence used for DPCA/ATI analysis is:
1. Process $S_1$ and $S_2$ data on a point-by-point basis.
2. Detect moving target candidates using the DPCA detection rule.
3. Estimate target motion by calculating the ATI phase for successful detections.

The detection criterion is defined in the three dimensional space $R^3$ of signal measurements \{\mathcal{X}, U, \Delta U\}. The ATI estimates are defined on a two dimensional plane of signal measurements \{\mathcal{X}, U\} which corresponds to the $\Delta U = 0$ plane in $R^3$. The detection threshold surface, $Z$, defined by equation 13, is projected onto the ATI \{\mathcal{X}, U\} plane as the threshold curve $B$ given by

$$U = B(T) = \frac{T}{2 \sin \frac{\pi}{2}}.$$ 

The detection threshold function $B(T)$ has minima at $V_{\text{Targ}} = \pm \frac{n \Lambda V_{\text{Sat}}}{4D}$ corresponding to $\mathcal{X} = n \pi$ where $n$ is an odd positive integer and has poles at $V_{\text{Targ}} = \pm \frac{m \Lambda V_{\text{Sat}}}{2D}$ corresponding to $\mathcal{X} = 2m \pi$ where $m = \text{floor} \left( \frac{|V_{\text{Targ}}|}{2 \Lambda V_{\text{Sat}}} \right)$ is the order of the radar’s blind speed The maxima of $B(T)$ correspond to the directional ambiguity boundary of ATI phase estimates (the negative real axis in ATI amplitude-phase polar plots). The poles of $B(T)$ correspond to the stationary world and to the radar’s blind speeds (the positive real axis in ATI amplitude, phase polar plots).

### 5.1.1.1 A model illustration of DPCA, ATI relationships

It is useful to build a simple model to see how estimates of the ATI phase given by equation 11 map into the ATI \{\mathcal{X}, U\} plane for different selections of target parameters, $U_T$ and $\varphi_T$ and different clutter parameters $U_C$ and $\theta$.

Let us start the process by assuming that we have DPCA-sampled signals, $S_1$ and $S_2$, from a two-aperture SAR-GMTI radar, that are not perfectly coherent with each other. This means that in an area which contains no moving targets, the expected value $\langle |S_1 - S_2| \rangle = \varepsilon$ where $\varepsilon$ is some small number.

Imagining that we have real data, we can compute a histogram of $|S_1 - S_2|$ estimates, match this to a PDF function and estimate a detection threshold, $T$, for a selected false-alarm ratio from equation 5. We can, then, express the threshold as $T = k \varepsilon$ for some factor $k$.

Let us create a very simple signal model that contains only target and clutter samples:

1. Define point targets with ATI motion phases $\varphi_T$ and corresponding amplitudes (under the detection rule) as $U_T = T/(2 \sin(\varphi_T/2))$.
2. Define a set of clutter samples with uniformly distributed amplitudes whose values range from 0 to $10 \varepsilon$ whose geometric phases, $\theta$, are defined with respect to the geometric target phases as a uniform distribution of values in the range $-\pi < \theta \leq \pi$ radians. The mean clutter amplitude, $\langle U_C \rangle$, for this model is $5 \varepsilon$ and its standard deviation is $2.9 \varepsilon$.
3. Allow no noise terms. (Sampling ambiguity terms have already been removed by DPCA sampling.)
4. Allow no clutter internal motion.

This model is very unrealistic (highly improbable in the real world):

- A point target representation for real moving vehicles works some of the time.
- Real clutter amplitudes follow a probability distribution where there is a most probable range of amplitudes. Values outside of this range have declining occurrence rates and possible amplitudes do not have a fixed upper bound. In some ways, our model can be considered as a pathological worst case.
- The uniform distribution of the clutter geometric phase terms is reasonable.
- Noise is always present in real data.

This model can provide insights into the phasor sum interaction between target and clutter samples in realistic situations.

Under our simple model equation 11 reduces to:

\[
\chi = \tan^{-1} \left( \frac{u_T^2 \sin(\varphi_T) + 2u_T u_C \cos(\theta) \sin \left( \frac{\varphi_T}{2} \right)}{u_T^2 \cos(\varphi_T) + u_T^2 + 2u_T \cos(\theta) \cos \left( \frac{\varphi_T}{2} \right)} \right).
\]

From the form of equations 11 and 15, the parameter, \( \varphi \), used to define \( T, U_T, \) and \( U_C \) is not relevant to the calculation outcome. Set \( \varphi = 1 \).

We note that the function \( \tan^{-1} \) in most computation libraries is defined over the interval \(-\pi/2\) to \( \pi/2 \). We will use the function \( \text{atan2} \) which is defined over the interval \(-\pi\) to \( \pi\).

In running the model, each estimate of \( \chi \) corresponds to the specific values of the parameters \( U_T, U_C, \theta, \varphi_T \) that produced that estimate. Each estimate is a single realization of the target-clutter interaction process and a large number of values that span the model parameter space must be run to provide a clear picture of the model output.

To look at the relationship between the detection threshold and the target speed for the fixed clutter model, let us select two detection thresholds \( T_1 \) and \( T_2 \) where \( T_1/\varepsilon \) is 15 dB and \( T_2/\varepsilon \) is 24 dB and let us select two target speeds that are expressed in terms of \( \varphi_T \) as \( \varphi_{T1} = -\pi/3 \) and \( \varphi_{T2} = \pi/6 \). These selections allow us to run four model cases:

**Case 1**: \( T_1 = 15 \text{ dB} = 5.6 \) and \( \varphi_{T1} = -\pi/3 \)

- \( U_C = \{0, 10\} \) and \( \theta = \{-\pi, \pi\} \) where the symbol \( \{ \} \) means the set of values between the enclosed limits.
- \( U_T = T_1/|\sin(-\pi/3)| = 7.9 = 18 \text{ dB}, <U_C> = 5, \max(U_C) = 10 \)
- \( U_T/<U_C> = 1.58 = 3.97 \text{ dB}, U_T/\max(U_C) = 0.56 = -5 \text{ dB} \)

24

DRDC Ottawa TR 2011-177
Case 2: $T_1 = 15 \text{ dB} = 5.6$ and $\varphi_{T_2} = \pi/6$

$U_C = \{0, 10\}$ and $\theta = \{-\pi, \pi\}$ where the symbol $\{ \}$ means the set of values between the enclosed limits.

$U_T = T_1/|\sin(\pi/6)| = 11.2 = 21 \text{ dB}, <U_C> = 5, \max(U_C) = 10$

$U_T/<U_C> = 2.24 = 7 \text{ dB}, U_T/\max(U_C) = 1.12 = 0.98 \text{ dB}$

Case 3: $T_2 = 24 \text{ dB} = 15.8$. and $\varphi_{T_1} = -\pi/3$

$U_C = \{0, 10\}$ and $\theta = \{-\pi, \pi\}$ where the symbol $\{ \}$ means the set of values between the enclosed limits.

$U_T = T_2/|\sin(-\pi/3)| = 18.2 = 25.2 \text{ dB}, <U_C> = 5, \max(U_C) = 10$

$U_T/<U_C> = 3.16 = 10 \text{ dB}, U_T/\max(U_C) = 1.58 = 4 \text{ dB}$

Case 4: $T_2 = 24 \text{ dB} = 15.8$. and $\varphi_{T_1} = \pi/6$

$U_C = \{0, 10\}$ and $\theta = \{-\pi, \pi\}$ where the symbol $\{ \}$ means the set of values between the enclosed limits.

$U_T = T_2/|\sin(\pi/6)| = 31.6 = 30 \text{ dB}, <U_C> = 5, \max(U_C) = 10$

$U_T/<U_C> = 6.32 = 16 \text{ dB}, U_T/\max(U_C) = 3.16 = 10 \text{ dB}$

Run each case by selecting a value of $U_C$ and compute $\chi$ for all allowed values of $\theta$ (choose your increment) then increment the value of $U_C$ and repeat the process.

Figure 8 displays the model calculations for the four cases, plotted with $\chi$ in degrees of ATI phase and $U$ in dB. In Figure 8:

- $B(T_1)$ is the DPCA threshold, $T_1$, projection onto the $\Delta U = 0$ plane and is represented by the solid line.
- $B(T_2)$ is the DPCA threshold, $T_2$, projection onto the $\Delta U = 0$ plane and is represented by the dashed line.
- Case 1 is represented by the red circles and only a sub-set of $U_C$ is plotted to show how the calculated points map into the plane.
- Case 2 is represented by the green circles and all $U_C$ are plotted.
- Case 3 is represented by the red dots and all $U_C$ are plotted.
- Case 4 is represented by the green dots and all $U_C$ are plotted.
Figure 8: ATI magnitude-phase relationship for simulated signals

For each case represented in figure 8, each point on the figure is an ATI \( \{ \chi, U \} \) plane representation of a single model calculation outcome that corresponds to a DPCA target detection. Since, for each case, there is only a single, point target that has been derived from its speed and the detection threshold, all points lie on or to the left of the threshold boundary curves.

The Case 1 representation in Figure 8 is particularly interesting since the target amplitude lies within the clutter amplitude range. For this case, only a subset of points is displayed in the figure so that the trajectory of \( \chi \) estimates for \( U_C < U_T, U_C = U_T \) and \( U_C > U_T \) can be clearly seen as \( \theta \) is varied from \(-\pi\) to \(\pi\) radians. The outcome for \( U_C = 0 \) in Figure 8 is a superimposed set of points at \( \chi = -\pi/3 \) for all values of \( \theta \).

The outcomes for each of the cases are shown in Table 1

---

26 DRDC Ottawa TR 2011-177
Table 1: Model Outcomes for four test cases

<table>
<thead>
<tr>
<th>Case</th>
<th>( U_T )</th>
<th>( \phi_T )</th>
<th>Mean(( \chi ))</th>
<th>SD(( \chi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1, ( T = 15 ) dB</td>
<td>7.9</td>
<td>-60°</td>
<td>-41.9°</td>
<td>47.38°</td>
</tr>
<tr>
<td>Case 2, ( T = 15 ) dB</td>
<td>11.2</td>
<td>30°</td>
<td>30.09°</td>
<td>22.67°</td>
</tr>
<tr>
<td>Case 3, ( T = 24 ) dB</td>
<td>31.6</td>
<td>-60°</td>
<td>-59.8°</td>
<td>14.06°</td>
</tr>
<tr>
<td>Case 4, ( T = 24 ) dB</td>
<td>61</td>
<td>30°</td>
<td>30.02°</td>
<td>4.01°</td>
</tr>
</tbody>
</table>

Looking at Table 1 and Figure 8, we see that the larger DPCA threshold results in a smaller standard deviation for ATI phase estimate, we see that at either threshold the faster targets have larger ATI-phase standard deviations and we see that the standard deviation of the ATI phase estimates decline with the target amplitude. Each of these observations is expected from equation 15.

To explore these results further, let us define the clutter amplitudes in steps of 0.5 to create the vector [0:0.5:10], let us define the clutter phase as the vector [-\( \pi \):0.2:\( \pi \)] and let us run the model for a target motion phase of \( \phi_T = \pi / 3 = 60° \) for a range of DPCA thresholds between 5 dB and 35 dB. The results are captured in Figure 9.

In Figure 9, the DPCA threshold scale in dB is displayed in the boxes on the right hand side of the figure. The red line at target amplitude 10 is the maximum clutter amplitude for the model. The
points to the left of the line represent targets that are whose amplitudes lie in the clutter amplitude range.

Several interesting features appear in figure 9:

1. As the target amplitude, $U_T$, increases from 1.8 at $T = 5$ dB to the maximum clutter amplitude, $U_{C_{Max}}$ at $T = 20$ dB, the mean ATI phase angle, $<\chi>$ increases almost linearly from $10^0$ to $90\%$ of its $60^0$ limiting value, $54^0$. In this domain, equation 15 is dominated by the inter-modulation and bias terms and ATI phase measurements do not provide very good estimates of the target motion phase.

2. As $U_T$ increases beyond $U_{C_{Max}}$, $<\chi>$ rapidly reaches its limiting value, overshoots it and, asymptotically approaches the target motion phase. In this domain, the mean ATI phase estimates rapidly approach the target motion phase and residual errors are small (numerically they are less than $0.8^0$ for $T >25$ dB). This overshoot effect is a secondary bias effect that is related to the proximity of the target signal to the detection threshold boundary curve in the ATI $\{U,\chi\}$ signal space. This effect is discussed for trains in section 6.2.

3. The behaviour of the standard deviation of the ATI phase estimate is more complex than the behaviour of the ATI angle estimate in that it reaches a peak at approximately $T = 17$ dB ($U_T = 7.08$).

4. As $U_T$ increases above 7.08, the ATI phase standard deviation decays monotonically to $4.35^0$ at $T = 35$ dB ($U_T = 56.2$). The standard deviation of the ATI phase estimate drops below $50\%$ of the estimated mean phase when $U_T >12$. Even as the target amplitude becomes increasingly dominant in equation 15, the target-clutter inter-modulation terms play a large role in determining the standard deviation of the ATI phase estimates.

All of these comments are very specific to the simple and unrealistic clutter model, a bounded, uniform distribution, used in this demonstration. For more realistic clutter models, the effects demonstrated in Figure 9 are still present but are less obvious.

Let us now look at the relationship between target speed and the standard deviation of the ATI phase estimates. To do this, we will select an arbitrary DPCA threshold, 24 dB, that was used to generate part of Figure 8, and will use the clutter-model representation that created Figure 9. The results are displayed in Figure 10.
Figure 10: ATI phase estimates for variable target motion phase

In Figure 10, the ATI phase of a target that has been detected with a 24 dB DPCA threshold, has been computed for point targets whose true motion phase was: $\phi_T = [15^0, 30^0, 45^0, 60^0, 90^0, 120^0]$. The ATI phase computed from the model closely matches the true motion phase of the target with the estimation error increasing from 0.04% at a true target phase of $15^0$ to 3.1% at a true target phase of $120^0$. From equation 15, the clutter bias term, $U_C^2$, becomes more significant as the target phase angle approaches the ATI directional ambiguity phase, $\phi_T = \pi = 180^0$. Also from equation 15, the ATI phase bias effect increases with decreasing target amplitude, $U_T$. The standard deviation of the ATI phase estimate increases with the true target motion phase as was anticipated from the results that are displayed in Table 1 due to the relationship between $U_T$ and $\phi_T$ for a fixed DPCA threshold, $T$.

5.1.2 Combined DPCA moving target detection and ATI target-motion estimation for two aperture systems.

The model calculations in the previous section illustrate some general properties of ATI target-velocity estimation when moving target detection is based on DPCA algorithms.

1. From the model calculations and discussions in the previous section, it is evident that the ability to detect a moving target using a DPCA rule does not imply that it is possible to accurately estimate the target motion for two-aperture SAR-GMTI data by applying the
ATI phase estimation algorithm. This is particularly true for weak, fast moving targets in strong, stationary clutter.

2. For a detection threshold, $T$, determined for a selected false-alarm rate given by equation 5, the target signal amplitude $U_T$ and the target motion parameter, $\varphi_T$ are deterministically linked. When the target motion phase is small (the target has a low radial velocity, equation 12), the target amplitude, $U_T$, must be large. As the radial speed of the target increases ($\varphi_T$ increases), the detection threshold, $T$, allows lower amplitude targets to be detected at the same false alarm rate ($U_T$ decreases). When the ATI phase, $\chi$, in equation 11, is used to estimate the radial speed of the target; the coupling between $U_T$ and $\varphi_T$ that is imposed by the DPCA detection condition, results in an increasing variance in the target speed estimate as the target speed increases. This effect is illustrated by the sample clusters on boundary $B_2$ in Figure 8 and by the standard deviation curve in Figure 10. From the form of equation 11, the sensitivity of the variance of $\chi$ to increasing target speed decreases as $\chi$ approaches $\pi$ radians.

3. From equation 14, the model discussions in Section 4.1 and the curves $B_1$ and $B_2$ in Figure 8, the minimum detectable velocity (MDV) for a moving target is not a constant for any detection threshold but is a function. In the simple two-aperture example case, from equations 12 and 13, the function has the form:

$$|MDV| = \frac{\lambda V_{sat}}{2\pi D} \sin^{-1}\left(\frac{\sqrt{T^2 - \Delta U^2}}{2U}\right)$$

When equation 13 is interpreted in the context of equations 11 and 12, it is clear that the minimum detectable target velocity for a two-aperture GMTI radar is not a constant but is a function of the SCNR given by equation 8.

4. Although the model calculation range displayed in Figure 8 is constrained to the target phase estimation range $-\pi < \chi < \pi$, neither the target phase estimation given by equation 11 nor the threshold boundary calculation expressed in equation 14 are constrained to this phase estimate interval. We will adopt the convention that targets moving away from the radar have positive velocity. For targets moving away from the radar, $\chi > 0$ is continuous on the interval $0 \leq \chi < 2\pi$ and for targets towards the radar, $\chi \leq 0$ is continuous on the interval $-2\pi < \chi \leq 0$ and the singularities of $B(T)$ occur at $\chi = 0$ and $\chi = \pm 2\pi$.

When the ATI phase estimates are related to target motion, the angles $\chi = \pm \pi$ from equation 12 are the ATI directional-ambiguity boundaries where the LOS target velocity has magnitude, $V = \pm \frac{\lambda V_{sat}}{4D}$. For targets moving towards the radar, where $\pi \leq \chi < 2\pi$, the direction of target motion determined from estimates of $\chi$ is ambiguous with a target moving away from the radar with motion phase $\chi_1 = \chi - 2\pi$. Provided that there is an independent metric with sufficient sensitivity to resolve the directional ambiguity (SAR focus or range-cell migration over a synthetic aperture), the ATI directional ambiguity region becomes useable [24]. Otherwise, the directional ambiguity imposes a limit on target speed estimation. The ATI directional ambiguity problem is discussed in more detail in Annex B.
With the target direction ambiguity resolved, the inherent ambiguity of circular measures comes into play and the ATI detection boundary curves, $B(T)$, continue beyond $\chi = \pi$ to the singularity of equation 12 at $\chi = 2\pi$. This is the first blind speed of the radar, $V_{\text{Targ}} = \pm \frac{\lambda V_{\text{Sat}}}{2D}$. When seen in a polar representation of the ATI amplitude-phase plane, Figure 6, the angles $\chi = 0$ and $\chi = 2\pi$ both correspond to the positive real axis and the clutter terms clustered about this axis preclude measurement of $\chi$ at both the stationary target and targets moving at the first blind speed since the radar sees zero motion phase at $\chi = 0, 2\pi n$ for any positive or negative integer $n$.

### 5.1.3 The impact of the SAR focusing parameters on the ATI motion estimation

Up to this point in section 5.1, we have implicitly assumed that that we are working with full bandwidth, complex, SAR images that have been correctly focussed and we have ignored the relationship between the radial speed of our targets and SAR focusing. The SAR focussing operation uses prior knowledge (or prior knowledge that has been augmented by an iterative refinement process in the form of an auto-focus algorithm) to focus the signal energy contained in the recorded phase history of each observation point to the theoretical impulse response of the radar.

The range and cross-range phase-histories of strip-map (the simple case considered in this tutorial) radar-return signals are not independent of each other. On the earth’s surface, constant range coordinates are the intersection of a sphere centered at the radar with the earth’s surface (including terrain relief). The cross-range constant Doppler frequency coordinates form a family of hyperbolae centered at the intersection of the antenna bore-sight plane with the earth’s surface. The two coordinate systems are not orthogonal and their relationship must be accommodated in SAR processing algorithms. Details can be found in SAR processing texts such as [21].

For the purposes of our discussion, it is conceptually convenient to describe SAR processing in terms of the range-Doppler signal processing model. In this representation, the radar range signals are encoded by an imposed phase function (often a linear FM (frequency modulation) chirp. The range coding becomes the prior knowledge needed to focus the SAR data in the range direction by a convolution of the SAR phase history with the range code function. In the cross range direction the data are phase encoded by the translational motion of the radar as observed over the adequately sampled portion of the cross-range antenna pattern. The prior knowledge needed to focus the SAR image is constructed from: knowledge of the observation geometry, knowledge of the radar motion and an assumption of the motion of each terrain element. For SAR imaging, the terrain is assumed to be stationary. Moving targets violate the stationary-world assumption.

SAR processing is a linear operation and all radar-return power is preserved. SAR focussing is an energy redistribution process and when the terrain motion assumption is correct, it provides a correct assignment of signal amplitude to each spatial position. When the terrain motion assumption is incorrect, uncompensated motion in the range direction results in an incorrect FM rate and uncompensated motion in the cross-range direction results in an incorrect observation duration. Both effects degrade cross-range SAR focusing.
In section 5.1.1 and Figure 8, comparisons were made between a slow moving target (corresponding to \( \varphi_T = \pi/6 = 30^\circ \)) and a fast moving target (corresponding to \( \varphi_T = -\pi/3 = -60^\circ \)) assuming that the amplitude \( U_T \) is at the detection threshold \( T_1 \) or \( T_2 \). The clutter, \( U_c \), is assumed to be stationary and is identical for all target possibilities. Under such conditions, the minimum necessary ratio \( U_T / < U_c > \) is fixed and depends on the detection threshold and the target radial speed. The underlying assumption of this analysis is that both the targets and the stationary background are focused. However, the SAR processing that focuses the targets is not matched to the terrain motion and the success of the detection assumes a focused target. The implication of matching the processing filter to the target means that the background is defocused somewhat. For the simple clutter case in this model this will not have an impact on \( < U_c > \).

As discussed in the Introduction and illustrated by Figure 3, it is possible to match the SAR processing filter to the radial motion of the target rather than using the standard stationary-world SAR focusing. This is achieved by using a Doppler centroid (DC) offset to center the processed Doppler bandwidth about \( f_{DOP} \) instead of the stationary-world value (assumed 0 in Figure 3). Moreover, the processed Doppler bandwidth may be less than the full available bandwidth limited by PRF. Since target motion is \textit{a priori} unknown, the selection of a best frequency offset implies a search process to maximize the coherent response of the moving target. A good choice of the SAR processing parameters can improve the SCNR, which is favourable for faster targets because they have less overlap with the clutter spectrum when the PRF (and, thus the cross-range over-sampling ratio) is sufficiently large. If a moving target has a sufficiently large radial speed to have a sufficient DC offset with respect to the clutter and if PRF is high enough to neglect clutter ambiguities, then SAR focusing matched to the target radial speed can improve the ratio \( U_T / < U_c > \) (relative to the minimum detectability level discussed earlier). Matched processing will reduce the spreading effect in the \((U, \chi)\) plane. Annex D explores the matched filter map approach to target velocity estimation using a simulated target set.

When PRF is sufficiently large, several Doppler bands can be used in the process of SAR focusing and they can be combined by incoherent multi-looking (interferogram averaging). Interferogram multi-looking decreases the variance of the ATI phase \( \chi \) \[31\] at the expense of reduced image resolution and increased ATI phase bias.

### 5.2 Clutter Probability Density Functions and their effects on ATI velocity measurements

The model discussions in Section 5.1.1 present a very unrealistic, and in some senses pathological, clutter model to explore the fundamental properties of the relationships between DPCA moving-target detection and ATI target motion estimation. Realistic radar clutter is represented in terms of clutter amplitude probability density functions that decrease asymptotically towards zero for large clutter returns. Many years of SAR research have concluded that the K distribution \[30\] provides a good model for moderately heterogeneous SAR-clutter amplitude for land surfaces and is a good representation for sea surfaces at SAR incidence angles less than 70°.

Figure 11 shows the dependence of ATI phase on a modeled, realistic GMTI scene clutter PDF using three looks for interferogram averaging \[32\]. The background radar assumptions for Figure 11 are similar to those used for the simple model in Section 5.1.1 in that the radar is a linear system with perfectly aligned radar beams, no noise floor and the sampling ambiguity terms of
equation 11 are not present. Figure 11 expresses the target in terms of the local SCR and the phase axis is $\chi$ from equation 11. It is evident from Figure 11 that the variance of ATI phase estimates is strongly dependent on the SCR at the detected target points.

The clutter model used to generate Figure 11 is computationally intensive. A somewhat less realistic clutter model that is computationally simple and preserves the salient features of Figure 11 assumes that the clutter amplitude, $U_c$, is Rayleigh distributed and that the clutter phase, $\theta$, is uniformly distributed. The resulting phase distributions for $\chi$ for detection thresholds, $T = \{0\, \text{dB}, 6\, \text{dB}, 12\, \text{dB} \text{ and } 18\, \text{dB}\}$ are shown in Figure 12 for a single look (no interferogram averaging).

![Graph showing phase distributions for varying SCR](image)

*Figure 11: SCR dependence of target phase estimates for a realistic ATI phase model*

When the $T = 18$ dB curves in Figure 12 are related to the target motion phase estimate, $\chi$, it is clearly seen that the variance of $\chi$ increases with increasing radial velocity of the target as was noted previously. Although the shapes of the velocity estimate distributions from the simple clutter model shown in Figure 12 differ from those in Figure 11 for the more exact clutter model, the distribution bias towards zero as $T$ (or SCR) decrease show the same trends in both models and the distribution height decrease plus variance increase with declining $T$ (or SCR) follow similar patterns.

Figure 13 illustrates the relationship between the expected value of the estimated target phase, $\chi$, and the standard deviation of the target phase estimates for detection thresholds between 0 dB and 26 dB and target motion phases between $30^0$ and $150^0$ for the Rayleigh clutter model.
Figure 12: Phase angle PDFs for selected detection thresholds using a Rayleigh clutter amplitude model.

Figure 13: Target phase estimate expected value (solid line) and standard deviation (dashed line) for Rayleigh-distributed $U_C$ when the target amplitudes are estimated from the detection thresholds.
Figure 13 presents a parametric illustration of the relationships between the ATI phase estimates, the standard deviation of these estimates, the detection threshold and the target motion phase. It is instructive to compare the behaviour of the ATI phase and standard deviation estimates in Figure 13 with those shown in Figure 9 for a much simpler clutter model. The relationship between the DPCA threshold, $T$, the target amplitude, $U_T$, and the target motion phase, $\varphi_T$ is particularly important in understanding the behaviour of the plots.

For the slower targets, $\varphi_T = 30^0$ and $\varphi_T = 60^0$, the target amplitude, $U_T$ lies outside of the peak of the clutter distribution used to generate Figure 13 and the standard deviation of the ATI phase is a monotonic function. As the target speed increases, $\varphi_T = 90^0$ to $\varphi_T = 150^0$, the target amplitude decreases for any selected threshold and the standard deviation of the ATI phase develops a peak as the target amplitude falls within the peak of the clutter distribution. The upper corner ($\chi = 90\%$ of $\chi_{\text{Max}}$) of the ATI target speed estimate migrates to larger threshold values for increasing target speed. It can easily be shown that the corner occurs at constant a constant target signal to clutter ratio.
6. Analysis of physically large targets

In section 2.1.1 we noted that physical moving targets are captured by SAR GMTI radars as a cluster of radar samples whose size, amplitude and phase distributions depend on the physical object being imaged and its projection into the image plane. Our analysis in this document treats each detected moving-target sample as an independent moving target. Noting that the physical target moves in three dimensions according to the target structure, the properties of its path and the motion of the surface that it moves on, the target elements imaged by the SAR-GMTI process move with respect to each other in a deterministic manner and the projection of the motion of each element along the radar range vectors is captured by the radar measurement process as the ATI phase of the corresponding detected target point. The bulk motion of the physical target is estimated from the ensemble of point-target motion estimates over the target sample cluster.

In equation 2, the target sample parameters \( U_T, \varphi + \varphi_T / 2 \) and \( \varphi - \varphi_T / 2 \) can have different values at adjacent sample points. For target samples that belong to a physically large, moving structure, the scattered signal amplitudes, \( U_T \), can vary considerably between spatially adjacent radar impulse responses according to the distribution of scattering center strengths over the surface of the target. Typically there will be a small number of strong, dominant scattering centers and a very large number of weak scattering centers. The actual scattering strength distribution and the spatial distribution of the scattering centers over the imaged target surface will depend on the target type and on the detailed configuration of the actual target observed.

The joint amplitude-phase PDFs for physically large targets are not well known and are dependent on the specific target considered. Theoretical evaluation of equation 11 for the statistics of a sample cluster representing a large, physical target is considered to be an intractable problem at present, however, empirical and semi empirical approaches have been useful. Investigations of clutter interactions with point and point-like targets [7, 9, and 32] have been particularly valuable to support conclusions drawn from equation 11.

Depending on the target type and the relative motions of points on the physical structure, the target phase angles, \( \varphi_T \), are either constant (uniform bulk motion) or will vary slowly and systematically over the target object (target flex {trains, for example} or rigid-body rotations {ships for example}). For a large object, moving with uniform radial velocity, the complex plane representation of the target samples forms a line of samples at angle \( \varphi_T \) from the positive, real axis, Figure 13. The highest density of sample points will be found closest to the complex plane origin.

6.1 Ships as large targets

For the purposes of our discussion we can treat a ship as a large, rigid body that has six degrees of freedom (heave, sway, surge, pitch, roll, and yaw) that describe its motion. For radar measurements of large vessels, the most important motions are:

- translation (ship center of buoyancy motion due to its propulsion system),
- heave (vertical motion due to the local wave field),
- pitch (longitudinal, vertical oscillation about the center of buoyancy due to the local wave field),
• roll (transverse oscillation due to the local wave field or systematic bias cause by a turn) and
• yaw (change in the direction of travel) terms.

The significance of the various terms for ATI motion measurements depends on the three dimensional location of the relevant scattering center on the ship.

Figure 14 shows an example image of a large container ship, moving nearly in direction of the ground projection of radar range vectors, in a calm sea, as imaged by a radar that has 3 m range resolution. In this case, the ship translation terms dominate the motion and the sea motion components are negligible. The sea surface, although not readily visible in the image intensity slice shown in Figure 14 has measurable radar returns. The ensemble motion generated from the motion estimates of the radar samples that are clearly seen in Figure 14 will be a measure of the bulk translational motion of the ship.

![Figure 14: SAR image of a container vessel moving in a calm sea.](image)

The ATI phase estimates for the ship samples are shown in the polar projection of the complex plane in Figure 15.

From the target amplitude ring labels, the ship scattering centers that generate the target points in the ship cluster ATI response can be very strong indeed and will completely dominate all other terms in equation 11. In Figure 15 it can also be seen that the greatest density of ship cluster points occurs in the vicinity of the origin. There are very many more weak scattering centers than strong scattering centers. This is illustrated by the amplitude / radial-velocity (scaled phase) plane ATI plot for a different container vessel in a more active sea shown in figure 16a and 16b..
Figure 15: ATI phase of a container vessel moving in a calm sea

Figure 16 (a) shows the ship in an image level-slice that saturates the vessel image intensity and displays the sea surface. Figure 16 (b) displays the ATI amplitude-radial-velocity plane (this is completely equivalent to the ATI amplitude-phase plane shown in Figure 8) that contains the ocean surface ATI measurements to the left of the detection boundary curve (red circles) and contains the ship returns to the right of the detection boundary curve (green circles). The target cluster representing the ship is travelling away from the radar with an ensemble ground speed of 6.3 m/s and contains 200 detected points whose amplitudes vary over a 50 dB range.

As is expected from the discussions of equation 11, the variance of the ship speed estimates is largest for the weakest detected points (the points closest to B(T)) due to the inter-modulation terms of equation 11. An examination of Figure 16 (b) suggests that the intersection of the target cluster with the detection threshold boundary curve plays a large role in the relationship between target cluster speed and the ATI variance of cluster speed estimates for two-aperture radars.
Figure 16a: Magnitude image scaled to show the sea surface

Figure 16b: Sea surface and ship in the ATI magnitude-velocity plane
6.2 Trains as large targets

Trains have been selected to represent the class of physically large moving targets that have large length-to-width and length-to-height aspect ratios. The target can be described as a string of longitudinally coupled rigid bodies (the railway cars) where each rigid body in the string has independent sway motions normal to its direction of travel due to rail bed conditions. Semi-trailer transports and road trains fall into the same target class. Figure 17 shows an example of a train travelling on a straight track through agricultural and wooded terrain near Trenton Ontario.

The contents of Figure 17 are a SAR image that contains the train and a cluster of green, superimposed dots that represent detected moving target points that have been repositioned in the image plane form their ATI velocity estimates. The cluster spread normal to the train motion away from the radar (towards the upper right in the image) is a result of the target-clutter inter-modulation terms in equation 11. Other coloured dots in the image are moving road vehicles that have been detected and repositioned from their radial-velocity estimates.

A white line has been superimposed on the displaced image of the detected train to enhance its visibility in the figure.

![Figure 17: Target samples from a moving train that have been repositioned from ATI phase estimates.](image)

Figure 18a shows signals from two trains that are travelling in the opposite directions and at different speeds from another data set acquired at a near-by location at a similar incidence angle. Instead of using the repositioned data points, Figure 18a represents each moving target point as a symbol. The red rectangles represent points that are moving to the left of the figure and the green circles represent points that are moving to the right of the figure. Included in the figure are vehicles moving on various roads in the scene.
Figure 18b displays data for the two trains as they appear in the ATI magnitude-velocity \{U, V\} plane. The two detection-thresholds, T_1 and T_2, that were used in the analysis of these data are illustrated in Figure 18b as the ATI boundary curves B(T_1) and B(T_2).

The relationship between the DPCA detection boundary, the target speed and the target cluster variance can be clearly seen in this figure. The data have been processed by first detecting the moving target points and then applying the ATI algorithm to estimate the radial speed of each detected point. This processing sequence removes all target points that were not detected by the thresholds T_1 or T_2 from the estimation process so that the velocity estimates lie primarily to the right of the detection boundary curves in the ATI \{U, V\} plane. As is expected from our previous considerations, the weaker target returns are closest to the detection boundaries and have the largest velocity estimate variation (largest standard deviation). For the slow train, the detection boundary curve and the point scatter of the train target illustrate the target point filtering effects of the detection boundary since the weaker target returns that have small ATI speed estimates have been rejected in the detection process. The weak target point rejection impacts the local target velocity estimation by biasing it towards a higher velocity to produce the secondary bias effect noted in discussions of Figure 9 in Section 5.1.1.1.

*Figure 18a: SAR scene near Trenton Ontario showing repositioned moving targets as symbols*
6.3 Estimating the bulk motions of large targets

The motion of a physically large moving object is computed from the ensemble of sample-based estimates in the cluster of moving-target points that represents the object. The discussions and experimental results reported to this point assume that all points in the sample cluster representing a large, moving object are equally weighted. This assumption ignores the correlation between point standard deviation and the SCNR and ignores the biasing effects of the detection rule as it impacts the speed and amplitude distribution of the radar samples.

Looking at the point spreading effect due to local SCR, the largest number of samples has relatively small amplitude and locally large statistical spread due to clutter contamination (intermodulation terms in equation 11). If all samples are equally weighted, simple processing of cluster properties sets the computation point for the mean to the center of the sample population and biases the standard deviation of the cluster towards the weakest radar returns.

As was noted in the discussion in Section 5.0, the PDF for the target sample ATI magnitudes is generally unknown and a rigorously defined weighting function that is based on the target point ATI amplitude distribution in not accessible. One empirical mitigation approach is to scale each cluster-point’s contribution to the target ensemble motion estimate by its relative magnitude with respect to the largest amplitude in the cluster.
Consider a cluster on N samples where each sample has an estimated ATI speed $V_i$ and an estimated ATI magnitude $U_i$. The sample set $\{V_i, U_i\}$ for $i=1:N$ represents the ATI estimate cluster. No attempt has been made to determine the best functional form of $W_i$ and so, for illustration purposes, we will assume that the sample weighting function $W_i$ is linear in ATI amplitude so that if $U_m = \max(U_i)$,

\begin{equation}
W_i = \frac{U_i}{U_m}.
\end{equation}

Assuming no knowledge of the statistics of the target cluster, the radial speed of the target cluster can be expressed as the weighted mean, $<V_{\text{Targ}}>$, of the sample speeds and so:

\begin{equation}
<V_{\text{Targ}}> = \frac{\sum_{i=1}^{N} V_i W_i}{N}.
\end{equation}

Alternatively, if we assume a Gaussian distribution of target estimates, we could use the expression:

\begin{equation}
<V_{\text{Targ}}> = \frac{\lambda V_{\text{Sat}}}{4\pi D} \arg\left(\sum_{i=1}^{N} U_i e^{i\pi V_i} <V_{\text{Amb}}^2>\right),
\end{equation}

where $V_{\text{Amb}}$ is the directional ambiguity speed.

For either case, the standard deviation of the cluster speed estimate is then:

\begin{equation}
SD = \sqrt{\frac{\sum_{i=1}^{N} (V_i^2 - <V_{\text{Targ}}>^2) W_i^2}{N-1}}.
\end{equation}

This approach has value when the large object target is moving at uniform speed over the observation period and when the clutter is not moving. Clutter from wave fields on a sea surface can be somewhat compensated and non-uniformly moving large targets can make this approach invalid. Although a linear weighting function is used as an example, the weighting function needs to be defined in a manner that respects joint amplitude, motion-phase PDF that describes the clutter and the joint amplitude, motion-phase PDF that describes the target in the context of equation 11. The simple, linear, weighting function definition has been found to be empirically useful for analysing ship motion for a limited data set.
7. Summary

The introduction outlines the principles of a two-aperture SAR-GMTI radar. The relationships between the sampling theorem requirements and the temporal displacements of the radar phase centers are discussed to introduce the concept of DPCA sampling. The concept of radar sampling ambiguities is introduced, their structure is discussed and their relationship to radar sampling and radar aperture registration is examined.

Section 2 develops a signal model for SAR-GMTI analysis. SAR-GMTI signals are defined as phasor sums of target, clutter, sampling ambiguity noise terms. The properties of each term are discussed and a signal model, equation 2, is defined for each of the two SAR-GMTI channels.

Section 3 looks at the moving target detection problem. Moving target detection by the subtraction of registered radar channels is defined as the DPCA moving target detection process. A threshold-based moving-target detection rule, equations 3 and 4, is defined and the relationship between the detection threshold and the detection false alarm rate, equation 4, is discussed.

Section 4 defines the ATI phase as a target motion measurement algorithm and defines the two channel GMTI ATI phase estimate at each sample point in terms of the phasor sum signals from each channel in equation 11. The ATI phase is related to the radial velocity of the target points in equation 12. The impact of the phasor sum on ATI motion estimates is discussed in detail and the cross-product (inter-modulation) term between the signal components is shown to be the major contributor to the variance of ATI target speed estimates.

Section 5 examines properties of the DPCA target detection rule and explores its impact on SAR-GMTI motion estimates. The DPCA detection rule is expressed in the ATI target phase-magnitude plane, equation 13, which is used to define detection boundary curves for this plane, equation 14. A simple signal model, containing only moving targets and uniform amplitude clutter is used to illustrate the gross properties of the interaction between the detection and estimation process. The relationships between target signal magnitude, target speed (ATI phase) and clutter amplitude and stationary phase are explored for four cases to show that the ATI target phase variance increases with target speed over the unambiguous portion of the ATI phase plane. The symmetry of the detection boundary curves is used to introduce the idea of directionally ambiguous targets and their gross speed/variance relationships up to the first blind speed of the radar. The minimum detectable target velocity for a two-aperture radar is shown to be a function of the strength of the target and clutter returns, equation 17, at the image location of the detected target. Statistically reasonable clutter models are introduced to illustrate the dependence of ATI phase estimates on the SCNR.

Section 6 presents RADARSAT-2 two-aperture GMTI data examples for ships and trains to illustrate the DPCA/ATI speed estimate variance relationships for real targets. An algorithm, defined by equations 18, 19, 20 and 21, is introduced for use in estimating the bulk motion of large targets.

Annex A expands the sampling ambiguity discussion to provide detailed descriptions and looks at the channel registration problem for GMTI.
Annex B examines the role of sampling in the creation of directionally ambiguous targets from adequately sampled GMTI data.

Annex C presents graphical representations of the phasor sum process to augment the discussions in Section 4.

Annex D introduces the concept of matched filter bank processing and discusses the implications of moving target focus in SAR GMTI processing.
8. References


Annex A  Sampling Ambiguities

The two dimensional signal spectrum of an orbiting earth-observation radar can be decomposed into two components: a range component whose characteristics are defined by the pulse coding used in the radar and an along-track component whose characteristics are determined by the satellite motion, the earth rotation, scene motion, radar beam pointing and the terrain reflectivity. The range-compressed signal data are considered in this discussion so that we are dealing with one-dimensional signal spectra. Since the complexities of the general radar imaging problem add little to the following discussion, we will focus on a simple case:

1. The radar under discussion is a synthetic-aperture radar.
2. The radar beam is pointed close to the broadside normal of the radar platform motion vector.
3. The along-track projection of the radar beam is concentrated over a narrow angle range.
4. The radar beam is symmetric with respect to the mechanical normal vector of the radiating face of the antenna.
5. The observed terrain is stationary with respect to the earth, has a radar scattering response that varies slowly with observation angle and is spatially incoherent.

For each radar pulse, the spectrum of the radar returns will be determined by the two-way gain, \( G(\alpha) \), the effective beam pointing squint angle, \( \gamma \), and the terrain scattering function, \( E \). The angle, \( \alpha \) is measured from the antenna bore-sight vector. The effective beam pointing angle, \( \gamma \), includes the joint effects of the actual radar beam squint from the satellite velocity vector normal and the effect of the earth rotation velocity vector at the observation point. It is estimated at the observation point.

From the viewpoint of the radar system, the along-track component of the signal spectrum seen by the radar has the form:

\[
S(f) = E(\beta)G(\beta), \quad \text{where} \quad \beta = \alpha + \gamma,
\]

when the other factors in the radar equation are ignored. \( E(\beta) \) is a complex random variable that is governed by the scattering statistics of the surface.

\( S(f) \) is internally coded by the radar range rate in wavelengths at angle \( \beta \). In our simple model, the envelope of \( \langle S(f) \rangle \) where \( \langle \rangle \) denotes and ensemble mean, is the radar Cross-range antenna pattern and the center frequency of \( S(f) \) is

\[
f_0 = \frac{2V_{sat} \sin(\gamma)}{\lambda},
\]

for an effective radar platform velocity, \( V_{Eff} \), effective squint angle, \( \gamma \), and radar wavelength, \( \lambda \). In our simple model:

\[
\beta (f - f_0) = \alpha.
\]
In the real world, $S(f)$ is defined over all observation angles $-\pi < \beta \leq \pi$ but, because of the angular filter defined by the two-way antenna pattern, is only significant about a narrow angle range centered on the radar beam pointing direction.

GMTI radars are sampled systems. The generalized Shannon sampling theorem states that if two real samples or one complex sample is taken per unit bandwidth of a signal then the signal can be reconstructed from the sample set. For radar data, the spectral filter in the direction of travel of the radar is the two-way antenna pattern. For a GMTI radar it is the two-way antenna pattern for each antenna aperture. The antenna pattern is a continuous function and thus signal bandwidth must be defined in terms of the portion of the signal spectrum that contains most of the signal information. Conventionally the $-6$ dB width of a radar signal spectrum is used to define the minimum sampling rate and thus the minimum, valid, pulse repetition frequency (PRF) is equal to the -6 dB signal bandwidth $f_{-6}$.

Define the sampling frequency as:

$$F = \kappa f_{-6} = \text{PRF} \mid \text{Shannon over sampling ratio} = \kappa$$

where the symbol, $\mid$, means “given” or “for”.

The maximum two-way, cross-range, observation angles that are adequately sampled by $F$ are:

$$\pm \alpha_0 = \sin^{-1}\left(\frac{\lambda F}{4 V_{\text{Sat}} \cos(\gamma)}\right)$$

And the adequately sampled portion of the signal spectrum, $S(f)$ is defined by:

$$\frac{-\text{PRF}}{2} < f \leq \frac{\text{PRF}}{2}$$

From a signal processing viewpoint we will call this region of the spectrum the main beam of the radar.

From this point we can set the effective squint angle, $\gamma = 0$, as it does not contribute to the essence of the argument.

Sampled radar signal spectral components that lie outside of the frequency interval $\pm F/2$ are still captured but their spectra are aliased (shifted to the opposite side of the adequately sampled spectrum and superimposed on the main beam signals) and coherently summed with main beam. The spectral interval closest to the main beam spectrum defines the first sampling ambiguity and has contributions from signal spectral components above $F/2$ and below $-F/2$. The azimuth oversampling ratio of the radar, $\kappa$, has a role in the reconstruction of sample signals and in defining the gain of a SAR processor.

The first sampling ambiguity has two components:
which is translated in the frequency domain into the spectral interval \(-\frac{F}{2} < f' \leq \frac{F}{2}\) by the transform \(f' = F + f\), and

\[
(A-9) \quad S(f) \frac{F}{\frac{3F}{2}} \left| \frac{F}{2} < f \leq \frac{3F}{2} \right.,
\]

which is translated into the frequency domain \(-\frac{F}{2} < f' \leq \frac{F}{2}\) by the transform \(f' = f - F\).

The envelope of the first ambiguity spectrum is thus:

\[
(A-10) \quad S_1(f) = G(\alpha)^{-2\alpha_0} + G(\alpha)^{3\alpha_0}
\]

The phase of the ambiguous signals is shifted by \(-\pi\) and \(\pi\) radians respectively; and the ATI phase of the two aliased clutter blocks will have moving target properties in a two aperture SAR-GMTI radar due to the squint angles \(\delta, \delta = 2\alpha_0\), that relate the centers of the main-beam and ambiguous spectra. The apparent mean ATI velocities of the ambiguities are the differences

\[
(A-11) \quad V_{Amb} = \pm \frac{\lambda V}{2d} \mp \frac{2V}{\lambda} \sin(\delta).
\]

A similar mechanism generates second, third, and higher ambiguities. All ambiguity orders always exist in radar data.

For a synthetic aperture radar, the radar ambiguities are also SAR signals and will focus under a SAR compression algorithm. The focussed, ambiguous images will be displaced in the along-track or azimuth direction from the main beam image by

\[
(A-12) \quad N = \pm \frac{2R\alpha_0 F}{V}
\]

samples where \(R\) is the range to the center of the ambiguous spectral block. Since the matched filter required to focus the main-beam signals is defined at the radar range to the center of the main-beam on the imaged terrain, the FM rate of the ambiguous signals will be somewhat mismatched and the ambiguity SAR focus will be degraded.

After SAR processing, the ambiguous signal amplitudes will be scaled by the two-way antenna gain function over the ambiguous region during the SAR focusing integration operation to generate the amplitude scaling factors:
If the two way antenna pattern is symmetric about the along-track bore-sight and if the effective antenna squint is zero, $g_{amb^+} = g_{amb^-} = g_{amb}$. Since that ambiguous terms come from signals received through the skirts of the antenna pattern and possibly part of the first side-lobe, $g_{amb} \ll 1$. For a simple, two-aperture GMTI radar that has un-weighted antenna aperture, transmits from the full antenna and receives signals on the two along-track antenna halves, $g_{amb} \approx 0.07$. Other GMTI radar configurations will have other scaling factor values.

When the ambiguous data are cast into the form that describes signal contributions to equation 2, each sample point that contains target, clutter and noise contributions also includes two clutter ambiguity contributions of the form:

$$g_{amb} \left( U_{CN} e^{j \left( \varphi_C - \pi + \frac{\varphi_{amb}}{2} \right)} + U_{C(-N)} e^{j \left( \varphi_C(-N) + \pi - \frac{\varphi_{amb}}{2} \right)} \right)$$

In the ambiguity term expressions:
1. $g_{amb}$ is described in equation A-13.
2. $U_{CN}$ is the magnitude of the clutter signal from the forward ambiguity.
3. $\varphi_C$ is the clutter phase of the forward ambiguity sample.
4. $\varphi_{amb}$ is the wrapped platform motion phase component for the forward ambiguity location.
5. $U_{C(-N)}$ is the magnitude of the clutter signal from the aft ambiguity.
6. $\varphi_C(-N)$ is the clutter phase of the aft ambiguity sample.

In equation A-14 the fore and aft ambiguity contributions represent radar returns from different terrain blocks (one preceding the adequately sampled interval and one following the adequately sampled interval with respect to the radar trajectory) with the result that the amplitudes and phases of the ambiguity samples will be uncorrelated with each other.

### A.1 Channel registration for GMTI

The previous section examined the sampling ambiguity structure for a single SAR channel. In this discussion, we noted that the full structure of the complex input signal is retained under sampling of the complex signal but that the signal components in the sampled spectrum are imaged about the sampling frequency to create main-beam and ambiguous sample components. What happens when we need to temporally register two channels for GMTI analysis?

A complex, base-band signal $s(t)$ has a complex spectrum $S(f)$ whose two-sided, adequately-sampled width is $W$ ($W = 2W'$ where $W'$ is the conventional base-band width of the real spectrum). If the complex spectrum is represented by $W$ samples, the time-domain signal is
sampled at $\delta t = 1/W$. For our two channel SAR-GMTI mode, both channels are synchronously sampled at intervals $\delta t$ under the sampling conditions discussed in the previous section. If the two GMTI channels can be temporally (spatially) registered by an integer shift (DPCA sampling), the ambiguous components of the spectra remain matched between the two channels and a DPCA channel subtraction cancels both the stationary components of the main beam signals and the ambiguous terms. If a fractional sample shift is required, things get interesting.

As discussed in the previous section, the sampling process wraps sampled signal spectra outside of the adequately sampled interval, $W$, across the $-\frac{W}{2} \leftrightarrow \frac{W}{2}$ discontinuity imposed by the sampling process so that the ambiguously sample signals are superimposed on the adequately sampled interval as illustrated in Figure A-1.
In Figure A-1, the positive ambiguity represents the aft portion of the signal spectrum and the negative ambiguity represents the fore part of the signal spectrum when we relate the spectrum to the direction of travel of the radar.
A simple visual model can be imagined if the signal spectrum is thought to be wrapped around a beer can where the seam weld on the side of the can represents the \(- \frac{W}{2} \leftrightarrow \frac{W}{2}\) discontinuity. If the spectrum is centered in the sampling interval you have the image shown in Figure A-1. If the spectrum is not centered in the interval the figure becomes asymmetric about the center of the interval. This is equivalent to sliding the spectrum around the beer can.

If GMTI channel registration requires that one of the two channel is shifted by a fractional time (space) interval, \( \varepsilon < \delta_t \), the time shifting property of the Fourier transform shows that this is equivalent to multiplying the spectrum of that channel by \( e^{j2\pi \epsilon} \) or in our beer can analogy, we rotated the spectrum through the angle \( 2\pi \epsilon \). A positive rotation decreases the aft ambiguity and increases the fore ambiguity, thus changing the ambiguity symmetry with respect to the reference channel. Referring to section 2.0, the first-order sampling ambiguities are now:

\[
S_{Af1} = U_{Af1} e^{j(\xi1 + \xi1_{u1})} \quad \text{and} \quad S_{At1} = U_{At1} e^{j(\nu1 + \nu1_{v1})}
\]

for channel 1 and

\[
S_{Af2} = U_{Af2} e^{j(\xi2 + \xi2_{u2})} \quad \text{and} \quad S_{At2} = U_{At2} e^{j(\nu2 + \nu2_{v2})}
\]

for channel 2. When we attempt channel cancellation, the ambiguity contributions:

\[
S_{Af1} - S_{Af2} = U_{Af1} e^{j(\xi1 + \xi1_{u1})} - U_{Af2} e^{j(\xi2 + \xi2_{u2})}
\]

\[
\text{and}...
\]

\[
S_{At1} - S_{At2} = U_{At1} e^{j(\nu1 + \nu1_{v1})} - U_{At2} e^{j(\nu2 + \nu2_{v2})}
\]

in the channel difference. The residual ambiguity terms exceed the system noise floor for strong, ambiguous terrain features and can limit DPCA detection effectiveness.

In equation 11, the sampling-ambiguity cross-products terms change from a doubled sum of two terms (fore and aft) to a sum of four terms (fore1, fore2, aft1, aft2).
Annex B  Moving target ambiguities

Annex A discussed the signal ambiguities that are generated when the radar returns from a stationary terrain surface are sampled by the radar pulse rate. Sections 1.2 and 1.2.2. introduce the concept that moving target signals will exhibit ambiguity effects due to the signal sampling process. This section expands the discussion of moving target ambiguities to include directional ambiguities that depend on the phase center separation of the radar apertures.

To minimize unnecessary complexities we will use the K.i.s.s. (Keep it simple, stupid!) principle and will make the following assumptions:

1. The radar data is range-compressed and we are dealing with a one dimensional signal processing problem.
2. The shape of a target signal spectrum is the shape of the two-way target illumination spectrum and is centered at the Doppler frequency corresponding to the radial motion of the target.
3. The target is not accelerating.
4. The radar PRF is chosen to correspond to the DPCA sampling condition.
5. The radar beam pointing correctly centers the spectrum of the stationary terrain a Doppler frequency of zero Hz (Hertz).
6. SAR processing uses the entire, sampled signal spectrum.
7. The matched filter spectrum used for SAR processing is unweighted (boxcar).

Most SAR processing algorithms that are used to create SAR imagery violate assumptions 6 and 7 in that spectral guard bands are chosen to reduce the visible image sampling ambiguities (the bandwidth of the processed data is less than the sampled bandwidth) and the matched filters are spectrally weighted to reduce side-lobe effects. These make pretty pictures. When we invoke assumptions 6 and 7, our radar images are not as pretty as they could be but we do retain all signal properties for moving target measurements.

Prior to any signal processing, radar systems are sublimely ignorant of the contents of the scene that they observe. Since SAR processing is based on an assumed relative motion between the radar and the imaged terrain the default processing choice is that all motion is due to the radar and that the radar spectrum of the stationary world is centered at zero Hz.

The radar spectrum of a moving target is shifted with respect to the stationary world spectrum but has the same shape. If the radar sampling is keyed to the stationary world, the moving target spectrum (including its ambiguities) is wrapped across the sampling frequency according to the beer-can model discussed in Annex A. The spectral wrapping for moving targets is discussed at length in [24].

Consider a moving target whose radial speed is less than the ATI directional ambiguity speed,

\[ |V_{\text{arg}}| < \left| \frac{\lambda V_{\text{eff}}}{4d} \right|. \]
The target spectrum will be centered at the Doppler frequency of the radial motion,

\[
\hat{f}_{\text{arg}} = \frac{2V_{\text{arg}}}{\lambda}
\]

If the adequately-sampled signal bandwidth is \( F \) and sampling is centered at zero Hz, the spectral folding will decompose the target spectrum into two components. For our target speed condition, the largest component will correspond to the unambiguous target signal and will be truncated by the sampling boundary in the direction of target motion. Let us consider a target that is moving towards the radar so that the target Doppler frequency is positive, the range rate of the target is negative and the ATI phase is negative (following the convention in Section 4.1.2). The target spectrum has bandwidth \( F \) and is shifted \( \hat{f}_{\text{arg}} \) towards the positive, stationary-world sampling boundary at \( F/2 \).

Figure B-1 shows the un-sampled clutter amplitude-spectrum envelope for one channel of an ideal two-aperture SAR-GMTI radar that has a uniformly illuminated transmit aperture and uniformly illuminated receive apertures that are half of the transmit aperture length. The lines – \( -F/2 \) and \( F/2 \) in Figure B-1 show the boundaries of the -6 dB sampling interval. The frequency scale in Figure B-1 is in arbitrary units.

![Figure B-1: Clutter amplitude spectrum envelope for an ideal two-aperture SAR-GMTI radar](image-url)
To illustrate the spectral wrapping effects for a moving target, a model target was defined as a point target that is moving towards the radar at a speed corresponding to Doppler shift $F/4$. The target cross section for the model is 12 dB above the background clutter. Figure B-2 displays the envelopes for positive part of the sampled amplitude spectra for both the clutter and target components for the case where the SAR focusing operation uses the stationary world model. For this case the clutter spectrum is centered in the sampling interval and the moving target spectrum is wrapped about the $-F/2$, $F/2$ sampling interval (the beer-can model seam).

The three curves displayed in Figure B-2 are:
- The positive envelope of the clutter spectrum at the physical location of the target,
- The positive envelop of the un-ambiguous target spectrum and
- The positive envelope of the directionally ambiguous target spectrum.

In Figure B-2 it is important to note that the ambiguous target curves represent the ATI directional ambiguity and not the base-line sampling ambiguities. The sampling ambiguities are not shown to keep the graph from being too cluttered.

![Figure B-2: Directionally ambiguous target spectra](image)

We can make a number of inferences from figure B-2:

1. If the motion-wrapped scene is SAR-processed with a stationary-world matched filter, the clutter will be well focused and the moving target will appear as two targets. The stronger of these will correspond to the real target under the target definition conditions and will be moving in the real target direction and the weaker of these will be moving in
the opposite direction. Each wrapped target has a directionally ambiguous friend that moves in the opposite direction.

2. Both the main target and its directional ambiguity will focus under a SAR processing matched filter but the quality of the focus will be degraded by shape of the corresponding signal spectrum and by the captured spectral energy.

3. When the scene containing the spectrally wrapped target is processed with a stationary-world matched filter the two target images will be displaced from their stationary-world image position by the motion-induced spectral shifts.

4. If the radar PRF is set to the DPCA sampling condition, the relationship between the adequately-sampled data and their sampling ambiguities, that allows cancellation of the sampling ambiguities, is the same for the two GMTI channels. It is preserved for the data containing moving targets and the stationary terms can be cancelled by channel subtraction as was discussed in Annex A. In our simple, visual model, channel registration by an integer number of pulses corresponds to one or more full turns of the beer can.

5. When the target amplitude spectra are used as weighting functions to estimate the Doppler centroids of the two, shifted target components, the effective positions of the two parts are not simply related to the true speed (offset frequency of the targets). For our example case the unambiguous targets will have a Doppler centroid of 0.35 on our arbitrary frequency scale, the ambiguous target will have a Doppler centroid at -0.85 on our arbitrary frequency scale and the true target shift, F/4 will be at 0.87 on the same scale. When the data are processed by a stationary world matched filter, The effective speeds for the unambiguous and ambiguous parts will not yield the true target speed. This is a limitation of GMTI systems that use moving target displacement form a known target track (road or rail line) to estimate target speed.

SAR GMTI data contains the complex signal history of every point in the imaged terrain. Instead of looking at the target and signal spectra, the temporal history of detected moving target returns can be examined in a time-frequency plane. Figure B-3 shows the time history of the Doppler frequency of detected targets (highway vehicles) passing through wooded terrain [33] from an airborne, two-aperture, SAR-GMTI radar.
The arrows in Figure B-3 connect the target time-frequency components for signals that are folded about the sampling interval ± 330 Hz.

When the moving target speed yields a Doppler shift equal to the sampling boundary frequency $F/2$, the frequency shifted target components will have equal amplitudes in the positive and negative frequency intervals of the stationary world sampled data. This corresponds to the ATI directional ambiguity condition discussed earlier when the DPCA sampling constraints are met.

Using the frequency shifting property of the Fourier transform, the SAR matched filter frequency shifted to match the moving target speed. When the SAR filter is matched to the target, the target is well focused and is shifted to its true image position but the terrain focus will degrade since the filter model no longer matches the stationary world. With reference to equation 11, the clutter terms that are significant for GMTI phase estimates correspond the clutter at the position of the velocity (frequency) shifted moving target, not to the clutter at its true position.

In SAR-GMTI processing, a matched-filter-map approach that tests a set of target motion hypotheses can be used to find the estimated target speed.
Annex C  Phasor diagrams: A Geometrical representation

C.1 Phasor diagrams

Simplified phasor sums are illustrated in the complex plane for several given clutter and target magnitudes $U_C$ and $U_T$, neglecting all other components of the composite signal model ($U_{Af} = U_{Aa} = N_1 = N_2 = 0$) and assuming no motion in the clutter component ($\theta_m = 0$). In this simple case, the fore and aft channel signals can be represented as:

$$S_1 = U_1 e^{i\zeta_1} = U_T e^{i(\varphi + \frac{\varphi_T}{2})} + U_G e^{i\theta}$$

$$S_2 = U_2 e^{i\zeta_2} = U_T e^{i(\varphi - \frac{\varphi_T}{2})} + U_C e^{i\theta}$$

In all diagrams, the clutter phasor is shown in red and the moving target phasor is shown in green. It is assumed that $U_C$, $U_T$ and $T$ have some fixed values. As a result, $U_{\text{PCA}}$ also has a fixed value and it is shown in blue. The relative phase between the clutter phasor and the target phasor, $\varphi - \theta$, is assumed random. This makes the values of $U_1$, $U_2$, $\zeta_1$ and $\zeta_2$ random within certain constraints. Random phasors are shown in black. The geometrical loci of possible end points for the combined phasors $S_1$ and $S_2$ are shown as green/red lines (circles) in all diagrams. It is important to note that any phasor diagram representing a random process illustrates a single, possible realization of the process.

C.1.1 Clutter stronger than the moving target

Figure C-1 presents the phasor sum in the complex plane for the case $U_C > U_T$. All fixed values ($U_C$, $U_T$, $\varphi_T$) in Figures C-1(a) and C-1(b) are shown in colors and the random variables (such as $\varphi$, $\theta$) are represented by black lines and symbols. The following relationships are seen from the diagrams C-1(a) and C-1(b):

- When the start points of the combined phasors $S_1$ and $S_2$ coincide, their end points must be on the red/green circle with radius $U_T$.
- The ATI phase $\chi = \arg(S_1 S_2^*) = \zeta_1 - \zeta_2 = (\zeta_1 - \theta) - (\zeta_2 - \theta)$
- The phase $|\zeta_1 - \theta| < \pi/2$.
- The phase $|\zeta_2 - \theta| < \pi/2$.
- Consequently, $|\chi| = |(\zeta_1 - \theta) - (\zeta_2 - \theta)| \leq |(\zeta_1 - \theta)| + |(\zeta_2 - \theta)| < \pi$, which means that $\chi$ is unambiguous.
For unambiguous angle values (-\pi < \chi < \pi), the average value can also be expressed as the difference of the average phases as follows:

\[
\langle \chi \rangle = \langle \zeta_1 - \theta \rangle - \langle \zeta_2 - \theta \rangle
\]

Moreover, the average value of the two phases \(\zeta_1 - \theta\) and \(\zeta_2 - \theta\) is 0. This can be proven geometrically, as demonstrated for example for \(\zeta_2 - \theta\) using Figure C-1. Noting that both \(\phi\) and \(\theta\) are uniformly distributed random variables: for any random relative phase \(\phi - \theta\), shown in figure C-1(a), which produces some allowed value of \(\zeta_2 - \theta\), there exists an equally probable relative phase, \(\phi' - \theta'\), shown in Figure C-1(b), satisfying the equation \(\phi' - \theta' = \phi_T - (\phi - \theta)\) that produces an opposite value \(\zeta_2' - \theta' = - (\zeta_2 - \theta)\). In this way, all possible values of \(\zeta_2 - \theta\) can be grouped in pairs that cancel each other, so that the average value is 0. The same logic is valid for \(\zeta_1 - \theta\).

In conclusion, given \(U_c < U_T\), the conditional expectation of the ATI angle \(\langle \chi \rangle = 0\). However, the variance of the ATI angle depends (increases) both on the ratio \(U_T / U_c\) and on \(\phi_T\).

Figure C-1: Phasor diagrams for \(U_c > U_T\) for two equally probable angles \(\phi - \theta\)

C.1.2 Clutter and moving target amplitudes are equal

Figure C-2 presents the phasor sum in the complex plane for the case \(U_c = U_T\).

Figure C-2: Phasor diagrams for \(U_c = U_T\) for two equally probable values of \(\phi - \theta\)
Figure C-2: Phasor diagrams for $U_c = U_T$ for two equally probable values of $\varphi - \theta$

C.1.3 Clutter is much weaker than the moving target

Figure C-3 presents the phasor sum in the complex plane for the case $U_c < U_T \cos(\varphi/2)$.

Figure C-3: Phasor diagram for $U_c < U_T \cos(\varphi/2)$ for two equally probable values of $\varphi - \theta$
**Annex D  ATI, DPCA and matched filter banks**

The bulk of this report uses ATI phase measurements to provide estimates of the radial speed component of moving targets. There is an alternative speed estimation approach that applies a bank of velocity-tuned SAR matched filters to DPCA-detected target signals and then searches the processed results for the best response [35].

To explore the potential role of matched filter bank (MFB) analysis, a simulation experiment was defined using terrain similar to the mixed agriculture and forest scene shown in the background of Figure 15 with superimposed lines of point targets that represent two trains.

Experimental results are shown for a case of actual background clutter acquired in MODEX1 mode with a channel coherence of 0.965 and imbedded simulated point target (PT) movers. A train of 40 PT’s was created moving in each direction, i.e. away from and towards the SAR. The receding PT’s have a radial speed VT corresponding to roughly a moving target ATI phase of $\phi_T = \pi / 6$, while the approaching PT’s are twice as fast ($\phi_T = -\pi / 3$). The target speeds correspond to those discussed in Section 5.1.1. The DPCA detection threshold is set to $T = 4.5$ dB relative to the root-mean-square (rms) signal return measured on the background clutter image, which is taken as clutter amplitude reference. All 80 PT’s have the same magnitude. With this threshold, a total of 240 mover pixels are defined, where some take their $U_T$ values from the peak of the PT response and some from the sides of the main lobe or even from the side lobes, thus providing a variation of the SCR in the simulated target set.

Full SAR-ATI processing is repeated with and without the embedded PT’s using the same processing parameters using a stationary-world SAR processing model, so that the background clutter samples are made available for each mover (the actual SCNR can be evaluated). Estimation results are shown in Figure C-1a in the form of a magnitude – speed diagram similar to Figure 16, where speed estimates $V$ are derived from the measured ATI phase, $\chi$, for each of the detections.

Figure D-1(a) presents the ATI results in the $U$, plane together with the curve B that corresponds to the threshold $T$. In this ATI diagram, points representing pure clutter (in the absence of the injected PT’s) and points representing detected movers in clutter are marked by different symbols. It can be seen that the maximum values of $U$ are the same for both target directions. Some detections are mapped to the left of B, as expected. More scattering along the $V$ axis occurs closer to the curve B as expected.

Fig. D-1(b) presents the results of DPCA processing with the DPCA detection threshold $T$ and the results of an alternative method for radial speed estimation. In this case, $U_{DPCA}$ reaches larger values for the faster target and all detections are mapped strictly to the right of the threshold line $U_{DPCA} = T$. Although the DPCA technique does not provide any speed estimates, it enables the application of the matched filter bank (MFB) that is modelled to capture the PT Doppler shift and the ideal envelope of the PT response (proportional to the antenna beam pattern as a function of the azimuth angle). Starting from the DPCA difference signal (co-registered, but not focused), speed estimation is performed by the correlation with a bank of constructed ideal responses, each tuned to a different VT. In this example, SCR was first significantly increased by the DPCA
technique (especially for the faster PT’s), then a velocity-matched processor bank was applied for focusing and the maximum PT response was sought at each pixel and finally compared to T.

The strongest mover has a relative $U_T$ value of 24 dB (relative to the reference) for both PT sets. The lowest $U_T$ values for the detected mover sets with increasing $V_T$ are: 10.2 dB and 4.5 dB. After DPCA, the lowest signal-to-noise (and residual clutter) ratio is 14.5 dB for both PT sets, while the highest ratio is: 28.3 dB and 34 dB in the order of increasing $V_T$. This ratio is calculated after azimuth compression with parameters tuned to the mover.

It is interesting to compare the accuracy trend of the two methods when $V_T$ (i.e. $\varphi_T$) increases. Applying the ATI approach, the standard deviation of the radial speed estimates increases with increasing $V_T$ from 0.67 m/s to 1.24 m/s. When MFB is applied after DPCA, the standard deviation of the speed estimates is lower and decreases from 0.65 m/s to 0.49 m/s when $V_T$ increases. This comparison is mostly of theoretical interest. ATI estimator is much more robust as it does not depend on the point-target assumption to find the best target speed by target amplitude maximization.

The MFB applied in this experiment uses more information than ATI, namely the shape of the uncompressed PT signal envelope, which is known for the simulated ideal PTs since these are simple targets (one dominant scattering center per PT response). PT envelope knowledge is commonly unknown in the real world since most targets have more than one significant scattering center per PT response and their PT envelopes are affected by target scintillation, multipath interference, glint etc. The single-dominant-scatterer assumption and thus the simple point target model becomes a more accurate representation of reality for high resolution radars.

![Figure D-1: Magnitude-speed diagrams for two simulated trains embedded in a realistic clutter background. $U_T$ is at or above the DPCA detection threshold.](image-url)
# List of symbols/abbreviations/acronyms(initialisms)

<table>
<thead>
<tr>
<th>Symbol/Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATI</td>
<td>Along Track Interferometry</td>
</tr>
<tr>
<td>AIS</td>
<td>Automatic Identification System</td>
</tr>
<tr>
<td>$\mathbb{C}^2$</td>
<td>Two-dimensional complex signal space</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramer-Rao Bound</td>
</tr>
<tr>
<td>DC</td>
<td>Doppler Centroid</td>
</tr>
<tr>
<td>DPCA</td>
<td>Displaced Phase Centre Antenna</td>
</tr>
<tr>
<td>GMTI</td>
<td>Ground Moving Target Indication</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>LOS</td>
<td>Line Of Sight</td>
</tr>
<tr>
<td>MFB</td>
<td>Matched Filter Bank</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>$P_{fa}$</td>
<td>Probability of false alarm or falsealarm rate</td>
</tr>
<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>$\mathbb{R}^3$</td>
<td>Three-dimensional real signal space</td>
</tr>
<tr>
<td>PT</td>
<td>Point Target</td>
</tr>
<tr>
<td>PTR</td>
<td>Point-Target Response</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean square</td>
</tr>
<tr>
<td>Rx</td>
<td>Receiver, Receive</td>
</tr>
<tr>
<td>SAR</td>
<td>Synthetic Aperture Radar</td>
</tr>
<tr>
<td>SCR</td>
<td>Signal to Clutter Ratio</td>
</tr>
<tr>
<td>SCNR</td>
<td>Signal to Clutter plus Noise Ratio</td>
</tr>
<tr>
<td>SD</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmitter, Transmit</td>
</tr>
</tbody>
</table>
**SAR-GMTI phasor sums and their impact on target velocity measurements**

### Technical Report

**Defence R&D Canada – Ottawa**

3701 Carling Avenue

Ottawa, Ontario K1A 0Z4

**Livingstone C.E., Dragosevic, S.V.**

---

**DATE OF PUBLICATION**

(Month and year of publication of document.)

**NO. OF PAGES**

(Total containing information, including Annexes, Appendices, etc.)

**NO. OF REFS**

(Total cited in document.)

<table>
<thead>
<tr>
<th>Month and Year</th>
<th>Total Pages</th>
<th>Total Refs</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 2011</td>
<td>82</td>
<td>35</td>
</tr>
</tbody>
</table>

---

**SPONSORING ACTIVITY**

(The name of the department project office or laboratory sponsoring the research and development – include address.)

**DRDC Ottawa**

**TR 2011-177**

---

**DOCUMENT AVAILABILITY**

(Any limitations on further dissemination of the document, other than those imposed by security classification.)

**Unlimited**

---

**DOCUMENT ANNOUNCEMENT**

(Any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in (11) is possible, a wider announcement audience may be selected.)

**Unlimited**
This tutorial on SAR GMTI signal properties was written to provide an introduction for those who are unfamiliar with the field. The main focus of this document is the simple two aperture SAR-GMTI radar. A SAR signal model is developed from a phasor sum to illustrate the roles of the major components of a SAR signal including sampling ambiguities. The signal model is expanded to include the case of a two-aperture SAR-GMTI radar. A DPCA (displaced phase center antenna) model is used to describe moving target detection and an ATI (Along-track interferometry) model is used to discuss target motion estimation.

The interactions between the moving target detection and motion estimation processes are discussed in terms of a simple target and clutter model and then discussions are generalized to more realistic cases to illustrate how the principles deduced from the simple model influence real-world observations. Radar sampling strategies are discussed in terms of sampling ambiguities and their impact on target detection. Physically large moving targets (much larger that the radar impulse response function width) are discussed as ensembles of moving point targets.

Some SAR-GMTI measurement examples are introduced to illustrate how the interactions between moving target detection and motion estimation processes influence the measurement outcomes.

An extensive list of references is provided to guide further study.

Le présent guide sur les propriétés des signaux d’indication de cibles terrestres mobiles (GMTI) fournis par un radar à synthèse d’ouverture (SAR) a été rédigé à titre d’introduction pour les novices dans le domaine. Le présent document porte surtout sur le simple SAR GMTI à deux ouvertures. Un modèle de signal SAR est développé à l’aide d’une somme des phaseurs pour montrer le rôle des principaux éléments d’un signal SAR, y compris les ambiguïtés d’échantillonnage. Ce modèle est étendu pour inclure le cas d’un SAR GMTI à deux ouvertures. De plus, un modèle d’antenne à centre de phase déplacé (DPCA) est utilisé pour décrire la détection de cibles mobiles, et un modèle d’interférométrie longitudinale (ATI) sert à examiner l’estimation des mouvements des cibles.

Les interactions entre les processus de détection de cibles mobiles et d’estimation des mouvements des cibles sont examinées en fonction d’un modèle simple de cibles et de clutters, puis l’analyse est généralisée à des cas plus réalistes pour montrer comment les principes déduits du modèle simple influent sur les observations réelles. On analyse en outre des stratégies d’échantillonnage radar en fonction des ambiguïtés d’échantillonnage et de leur effet sur la détection de cibles. Par ailleurs, on examine de grosses cibles mobiles (beaucoup plus grosses que la largeur de la fonction de réponse impulsionnelle du radar) en tant qu’ensembles de cibles ponctuelles mobiles.

Quelques exemples de mesures faites par le SAR GMTI sont présentés pour montrer comment les interactions entre les processus de détection de cibles mobiles et d’estimation des mouvements influent sur les résultats des mesures.

Une liste exhaustive de références est fournie pour guider une étude ultérieure.

SAR; GMTI; Phasor sum; Ambiguities; DPCA; Along Track Interferometry; Inter-modulation