A qualitative bidder evaluation method

A new scoring method void of cardinal weight assignment

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Abstract

Director General Major Project Delivery Project Management Offices are called upon to develop fair, transparent and defensible bidder evaluation methods for major Crown procurement tenders. The traditional evaluation approach is to use an additive weighted scoring rule where evaluation criteria are established, rating functions are determined, and relative weights are assigned to the criteria. Bidders are then rated per criterion and scored using a product-sum of the weights times the ratings. The highest scoring bidder wins. This approach requires the pre-selection of criteria weights. Ideally these weights are arithmetically or objectively measured; however in most cases this is not possible. Lacking such measurements, the determination of appropriate weights that accurately reflect the decision maker's priorities is known to be a cognitively demanding task—it is difficult to objectively quantify the relative importance of criteria. This paper presents an alternative: an additive weighted scoring method where the weights remain unexpressed. With qualitative bidder evaluation (QuBE) an ordinal ranking of the criteria is solicited and the winning option is determined by examining the sizes of the weight-subspaces associated with the set of all weight vectors respecting the criteria priority ordering. In addition to minimizing subjectivity, the method is transparent, fair, defensible, and reduces the possibility of bidder gaming.

Résumé

Les bureaux de gestion de projet du Directeur général - Réalisation des grands projets doivent concevoir des méthodes justes, transparentes et défendables d'évaluation des soumissionnaires pour les soumissionnaires associés aux acquisitions majeures de l'État. L'approche d'évaluation traditionnelle consiste à utiliser un pointage pondéré additionnel lorsque les critères d'évaluation sont établis. Les fonctions d'évaluation sont alors déterminées et les pondérations relatives sont assignées aux critères. Les soumissionnaires sont alors évalués par critères et cotés en utilisant le résultat du produit des notes pondérées multiplié par les évaluations. Le soumissionnaire obtenant le pointage le plus élevé gagne. Cette approche requiert que l'on choisisse au préalable la pondération des critères. Idéalement, cette pondération devrait être mesurée de façon arithmétique et objective ; toutefois, dans la plupart des cas, cela n'est pas possible. Sans ces mesures, la détermination d'une pondération appropriée qui reflète avec exactitude les priorités du preneur de décision constituent une tâche exigeante sur le plan cognitif - il est difficile de quantifier de manière objective l'importance relative des critères. Ce document présente une méthode ajoutée de pointage pondéré où la pondération n'est jamais exprimée. Dans le cas d'une évaluation qualitative des soumissionnaires, (QuBE) on sollicite un classement ordinal et l'option gagnante est déterminée en examinant les dimensions des pondérations associées à l'ensemble des pondérations qui répondent aux critères d'évaluation. En plus de minimiser la subjectivité, la méthode est transparente, juste, défendable et réduit la possibilité de toute ingérence de la part des soumissionnaires.

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Executive summary

A qualitative bidder evaluation method

Bohdan L. Kaluzny; DRDC CORA TM 2010–007; Defence R&D Canada – CORA; February 2010.

Background: Decision-makers within Director General Major Project Delivery (DGMPD) Project Management Offices (PMOs) are called upon to develop fair, transparent, and defensible bidder evaluation methods for major Crown procurement tenders. Fortunately, the application of multi-criteria decision analysis facilitates this process. The general form of a multi-criteria decision analysis problem is the evaluation of a number of options (bidders) against a number of criteria. A simple, commonly-used approach is to use an additive weighted scoring rule where relative weights are assigned to the criteria and the bidders are rated per criterion. The bidders are scored by taking the sum of the products (weight times rating) over all criteria. The highest scoring bidder wins.

Ideally the criteria weights are arithmetically or objectively measured; however in most cases this is not possible. This is problematic since when the criteria weights are subjectively chosen there is potential for major flaw in the additive weighted scoring method. It is known that the proper assessment of criteria weights is a cognitively demanding task—it is difficult to objectively quantify the relative importance of criteria. Weight values that are determined by subjective judgment are known to suffer from internal consistency and validity problems. It is possible to end up with an unfair evaluation system. Furthermore, many analysts argue that the only real measure of preference in evaluating criteria is the act of making a choice, in other words determining a relative order of importance of the criteria (an ordinal ranking).

Summary of principal results: This paper presents a novel qualitative additive weighted scoring method that can be used for bidder evaluation. The method does not require the pre-selection of criteria weights. Instead, the PMO decides on an ordinal ranking of the selected criteria which establishes criteria weight relationships. The set of all possible weight vectors respecting the weight relationships is then considered and each vector is used to score the bidders. The winning bidder is the one that gets the most high-scores over all satisfying weight vectors. The underlying assumption is that, without additional information, all weight vectors respecting the criteria ordinal ranking are equivalent in the sense that none is considered superior to another. In addition to minimizing subjectivity, the method is transparent, fair, defensible, and reduces the possibility of bidder gaming.

Significance of results: The methodology presented should be of use to DGMPD PMOs required to develop robust bidder evaluation methods. The Directorate Materiel Group Operational Research (DMGOR) Acquisition Support Team (AST) demonstrated the methodology on a previous options analysis problem faced by PMO Canadian Surface Combatant (CSC) and show its potential use for bidder evaluation for the Tactical Armoured Patrol Vehicle (TAPV) project.

Sommaire

A qualitative bidder evaluation method

Bohdan L. Kaluzny ; DRDC CORA TM 2010–007 ; R & D pour la défense Canada – CARO ; février 2010.

Contexte : Les preneurs de décisions travaillant au sein des bureaux de gestion de projet (BGP) du Directeur général - Réalisation des grands projets (DGRGP) doivent concevoir des méthodes justes, transparentes et défendables d'évaluation des soumissionnaires pour les grands projets de l'État. Heureusement, l'application de l'analyse de décision à l'aide de plusieurs critères facilite ce processus. La forme que prend généralement un problème d'analyse de décision à l'aide de nombreux critères consiste à évaluer un nombre d'options (soumissionnaires) par le biais d'un nombre précis de critères. Une approche simple et utilisée fréquemment consiste à utiliser une règle de pointage avec pondération ajoutée selon laquelle des pondérations relatives sont assignées aux critères et les soumissionnaires sont évalués selon ces critères. Les soumissionnaires sont évalués en établissant la somme des produits (pondération multipliée par la cote d'évaluation) en vertu de tous les critères. Le gagnant est le soumissionnaire qui obtient le pointage le plus élevé. Idéalement, la pondération des critères est mesurée de façon arithmétique ou objective ; toutefois, dans la plupart des cas, cela n'est pas possible. Cela est problématique puisque lorsque la pondération des critères est choisie de façon subjective, il y a un potentiel d'erreur importante dans la méthode de pointage avec pondération ajoutée. Il est bien connu que l'évaluation adéquate de la pondération des critères constitue une tâche exigeante sur le plan cognitif - il est difficile de quantifier de manière objective l'importance relative des critères. En outre, de nombreux analystes croient que la seule réelle mesure de préférence de l'évaluation de critères est l'action de faire un choix, en d'autres mots de déterminer un ordre relatif d'importance des critères (un classement ordinal).

Résumé des résultats principaux : Ce document présente une nouvelle méthode qualitative de pointage avec pondération ajoutée qui peut être utilisée pour l'évaluation des soumissionnaires. La méthode ne nécessite pas la pré-sélection de la pondération des critères. Le BGP décide plutôt d'un classement ordinal des critères sélectionnés qui établit les relations entre les critères et la pondération. On tient alors compte de l'ensemble des pondérations qui satisfont aux relations pré-établies, et chacune est utilisée pour établir le pointage des soumissionnaires. Le soumissionnaire gagnant est celui qui obtient le pointage le plus élevé pour tous les vecteurs de pondération. On présume ainsi que, sans renseignement additionnel, tous les vecteurs de pondération qui respectent le classement ordinal des critères sont équivalents dans le sens qu'aucun n'est préféré à un autre.

Signification des résultats : La méthodologie présentée ici devrait être utile aux BGP de DGRGP qui doivent élaborer des méthodes d'évaluation de soumissionnaires transparentes, justes et défendables. L'équipe de soutien d'acquisition (AST) du Directeur - Recherche opérationnelle (Groupe des matériels) a démontré la méthodologie lors d'un problème antérieur d'analyse d'options auquel faisait face le BGP des combattants de surface canadiens (CSC) et au cours de l'évaluation des soumissionnaires pour le BGP de véhicule blindé tactique de patrouille (VBTP).

Table of contents

List of tables

List of figures

Acknowledgments

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1 Introduction

1.1 Background

Decision-makers within Director General Major Project Delivery (DGMPD) Project Management Offices (PMOs) are required to develop fair, transparent, and defensible bidder evaluation methods for major Crown procurement tenders. Fortunately, the application of multi-criteria decision analysis facilitates this process. Evaluation criteria are established and rating functions are determined. The traditional evaluation approach is to use an additive weighted scoring rule where relative weights are assigned to the criteria and bidders are rated per criterion. The criteria weights and scoring functions are fixed and publicly announced in the PMO's Request for Proposal (RFP). After bid submission, the bidders are scored by taking the product-sum (weights times ratings) over all criteria. The highest scoring bidder wins the contract.

The additive weighted scoring method is the most common formulation of a multi-criteria decision analysis problem. The method is mathematically simple. However, Csaki and Gelleri [1] argue that *only* when the fixed weights accurately reflect the relative importance of the evaluation criteria to the decision maker (the Crown) does the method ensure objectivity and limit corruption in bidder evaluation. Unfortunately, it is known that the proper assessment of criteria weights, when quantitative measurements are not possible, is a cognitively demanding task [2]. It is difficult to objectively quantify the relative importance of criteria. Arbel and Vargas [3] and Borcherding *et al.* [4] provide evidence that weight values that are determined by subjective judgment suffer from internal consistency and validity problems. It is possible to end up with an unfair evaluation system in which too much emphasis is placed on particular evaluation criteria, favouring (intentionally or not) those bidders that rate well in these criteria.

1.2 Previous work

Numerous weight elicitation procedures based on quantifying the strength of preference of criteria have been presented in the open literature. These include Keeney and Raiffa's tradeoff method and pricing-out method [5], the ratio method and swing method of Von Winterfeldt and Edwards [6], Green and Srinivasan's conjoint methods [7], the analytical hierarchy process (AHP) of Saaty [8], the habitual domains method of Tzeng *et al.* [9], and weight elicitation methods using linear programming of Costa and Climaco [10] and Mousseau *et al.* [11].

Many analysts, including Defence Research & Development Canada (DRDC) Centre for Operational Research (CORA) experts, argue that the only real measure of preference in evaluating criteria is the act of making a choice, in other words, determining a relative order of importance of the criteria. The latter is referred to as an ordinal ranking of the criteria (see Emond [12] for further background and discussion). With this mindset, the major pitfall of all the weight elicitation procedures listed herein is that they attempt to capture the strength of preference between criteria based on subjective choice [1]. Dulmin and Mininno [14], in the context of multi-criteria supplier selection, argue that the derivation of exact weights can be legitimately challenged, but provide evidence that decision-makers can positively provide an ordinal ranking of the evaluation criteria.

1. In particular, Warren [13] argues that Saaty's AHP method is unsuitable for defence applications.

Weight elicitation schemes based purely on ordinal rankings of the criteria have also been proposed. Stillwell *et al.* [15] proposed the rank reciprocal, rank linear, and rank exponent functions to convert ordinal rankings to cardinal weights, Solymosi and Dombi [16] and Barron [17] suggested rank order centroid weights, Lootsma [18] and Lootsma and Bots [19] proposed geometric-based weights, Alfares *et al.* [20] proposed a conversion method based on a linear function derived from experimentation, and DRDC CORA's Hunter and Emond [21] presented rank-based linear and power functions. In general, as shown by Paelinck's theorem [22, 23], there can be an infinite set of weight vectors that satisfy an ordinal ranking of the criteria. While several studies have suggested that Solymosi and Dombi's centroid weight method is the most accurate and easy-to-use quantitative method (see [24–28]), the choice of appropriate method is case-dependent and the act of choosing a method (and the selection of the additional method parameters) is subjective. Given an ordinal ranking of criteria, it is quite possible that different bidders could win depending on which weight function (linear, power, centroid, etc.) is chosen. In the case where the PMO can only justify an ordinal ranking of the criteria, it may be hard to defend the evaluation method—the Crown could have to defend against a complaint that a particular weight function was advantageous for the winning bidder from the onset.

1.3 Objective

The objective of this paper is to present an alternative *qualitative* bidder evaluation method useful in those situations where fixed criterion weights are either not desirable or easily obtainable. The method does not require the pre-selection of criteria weights. In fact, fixed weights are never assigned. Instead, the PMO decides on an ordinal ranking of the selected criteria which naturally establishes criteria weight relationships. The set of all possible weight vectors satisfying the weight relationships is then considered and each vector is used to score the bidders. The winning bidder is the one that gets the most high-scores over all satisfying weight vectors [2].

1.4 Scope

Arrow's Impossibility Theorem [29], in layman's terms, states that there exists no means of ranking three or more options in a manner that will always satisfy all commonly accepted measures of "fairness". Every technique for ranking a set of options suffers from a set of mathematical drawbacks that can generate misleading results. In particular, additive weighted scoring methods are described as consensus-based electoral systems, rather than majoritarian ones. The former methods may violate the Condorcet property which stipulates that the option (or bidder) which is rated highest in all pairwise comparisons with the other options (i.e., ranked better than all other options by a majority) should be ranked first and labeled the "Condorcet winner". It is not within the scope of this paper to enter the debate on consensus ranking methods. Rather, assuming that additive weighted scoring methods have been deemed appropriate for bidder evaluation in major Crown procurements, this paper presents a variant that minimizes the impact that subjective judgments have on bidder selection.

2. The method does not actually count the number of times an option scores highest because the number is in general uncountably infinite. The winning option is determined as the one with the largest volume in multidimensional Euclidean space as determined by its scores and the relative order of the criteria. Details are provided in Section 2.

When using an additive weighted scoring method, criteria weights that are arithmetically or objectively measured are ideal, however in many cases these measurements are impossible. The next best thing is objectively chosen criteria weights that accurately reflect the Crown's priorities. Determining these weights is difficult. There are numerous weight elicitation schemes, based on either strength of preference or ordinal rankings, which a PMO can potentially choose to employ if the context is right. In this paper we present an alternative, a qualitative method that only requires that the PMO determines a priority ordering of the criteria. The underlying assumption is that, without additional information, all weight vectors respecting the criteria ordinal ranking are equivalent in the sense that none is preferred over another.

1.5 Outline

In Section 2 we formalize the proposed bidder evaluation method and present related theory. The reader is forewarned that Section 2.3 presents high-level mathematical details. The non-technically inclined reader may wish to skip this section, with limited loss of flow, and proceed to Section 3 where, to facilitate understanding, the method is applied to an example. Section 4 discusses procurement policy issues, precedent use of similar methods, and hybrid approaches. In Section 5 we show how the method could have been applied to an options analysis problem faced by PMO Canadian Surface Combatant (CSC), and how PMO Tactical Armoured Patrol Vehicle (TAPV) could employ it in their evaluation of contract bidders.

2 Methodology

In this section we first formally define the additive weighted bidder evaluation scoring method used by most PMOs. We then propose the qualitative bidder evaluation (QuBE) method and discuss mathematical details including Paelinck's fundamental theorem and its relation to the characterization of the space of all possible criteria weights.

2.1 Traditional method

DGMPD PMOs usually select an additive weighted scoring rule as their bidder evaluation function. This method is formally defined as follows. Given N bidders and M criteria, let $v_i(j)$ be the rating of bidder j relative to criterion i. Let w_i be the weight allocated to criterion i normalized such that $\sum_{i=1}^{M} w_i = 1$. The combined rating (or score) of bidder j is evaluated as $S_j = \sum_{i=1}^{M} v_i(j) w_i$. The bidder with the highest score is deemed the winner. At RFP time the PMO publishes the criteria scoring functions, $v_i(\cdot)$'s, as well as the criteria weights it has chosen. Once bids are received, the Crown rates eligible bidders (computes the $v_i(j)$'s and S_j's) and determines the winner. We henceforth refer to this method as the traditional additive weighted scoring method (or simply the traditional method).

2.2 The QuBE method

The QuBE method does not require a pre-selection of criteria weights. In fact, weights are never fixed. Instead, the PMO decides on an ordinal ranking of the selected criteria which naturally establishes criteria weight relationships. The set of all possible weights satisfying the weight relationships is then considered and each is used to score the bidders. The winning bidder is the one that gets the most high-scores over all satisfying weight vectors. Algorithm 1 provides a formal high-level description of the QuBE method.

A relevant question to ask in regard to QuBE's Step 5 is: how can one consider the set of all possible weight vectors satisfying the weight relationships established in Step 2 of the QuBE method? In fact, as subsequent sections elaborate, the method does not actually count the number of times an option scores highest because the number is in general uncountably infinite. The winning option is determined as the one with the largest volume in multidimensional Euclidean space as determined by its scores and the relative order of the criteria.

In the next section we first present Paelinck's fundamental theorem which implies that the set of satisfying weight vectors can be infinite. We then provide the mathematical details of how the QuBE method evaluates the complete set of weight vectors.

Algorithm 1 QuBE: Qualitative Bidder Evaluation

Pre-RFP:

Step 1. The PMO determines the relative preference (consensus ordinal ranking) of the rated criteria. The criteria rating functions ($v_i(\cdot)$'s) are defined.

Step 2. The ordinal ranking of criteria is converted into linear weight relationships:
Let w_i be the weight variable associated with criterion i, for $i = 1, ..., M$.
$\sum_{i=1}^{M} w_i = 1$,
$w_i > w_j$ iff criterion i is ranked higher than criterion j,
$w_i < w_j$ iff criterion i is ranked lower than criterion j,
$w_i == w_j$ iff criterion i is ranked tied with criterion j.

RFP published:

Step 3. The PMO publishes the weight relationships determined in Step 2 and also publishes the QuBE method.

Bidder evaluation:

Step 4. The PMO rates each bidder with respect to the criteria (determine all $v_i(j)$'s).

Step 5. The bidders are scored using the set of all possible weight vectors satisfying the weight relationships established in Step 2: for each $\vec{w} = (w_1, ..., w_M)$, bidder j scores $S_j = \sum_{i=1}^{M} v_i(j) w_i$.

Step 6. Final outcome: the bidder that receives the most high-scores wins.
A bidder's final score is the number of times that bidder scores highest.

2.3 Theory

2.3.1 Paelinck's theorem

Without loss of generality, let $\langle C_1 C_2 \cdots C_M \rangle$ denote the ranking of the M criteria[3]. Paelinck's theorem [22] states that there exists an infinite set of weight vectors that respect the ordinal ranking. More formally, the theorem states that all vectors $\vec{w} = (w_1, w_2, ..., w_M)$, where w_i is the weight of C_i, satisfying the rank ordering,

$$w_1 \geq w_2 \geq \cdots \geq w_m \geq 0, \tag{1}$$

$$\sum_{i=1}^{M} w_i = 1, \tag{2}$$

3. For the remainder of this paper we label options and denote a ranking of options by using angled brackets with the most preferred option listed first, second most preferred listed second, etc. Options that are tied in a ranking will be grouped and separated by the symbol '$-$'. For example, a ranking of options A,B,C and D denoted by $\langle BAC-D \rangle$ indicates that option B is most preferred, followed by option A, and finally options C and D are tied for last place. Ranking $\langle BAD-C \rangle$ is equivalent as there is no order for tied options. This follows the notational conventions established by [30].

constitute the convex hull [4] of the vectors

$$e^1 = (1,0,0,...,0)\,, \tag{3}$$

$$e^2 = \left(\frac{1}{2},\frac{1}{2},0,...,0\right), \tag{4}$$

$$e^3 = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},...,0\right), \tag{5}$$

$$\vdots$$

$$e^M = \left(\frac{1}{M},\frac{1}{M},\frac{1}{M},...,\frac{1}{M}\right). \tag{6}$$

Figure 1 depicts the weight space geometrically for three criteria C_1, C_2 and C_3. The triangle formed by the points $(1,0,0), (1/2,1/2,0)$, and $(1/3,1/3,1/3)$ contains the infinite set of weight vectors that respect the rank ordering of the criteria $\langle C_1 C_2 C_3 \rangle$.

Claessens *et al.* [23] presented an elementary proof of Paelinck's theorem and noted the following corollary: Bidder j can be declared the winner if

$$e^i \cdot (v_1(j), v_2(j), ..., v_M(j)) \geq e^i \cdot (v_1(k), v_2(k), ..., v_M(k))\,, \text{ for all } i = 1,...,M, k = 1,...,N. \tag{7}$$

The Paelinck theorem implies that the winning bidder can be identified without any numerical allocation to the criteria weights when condition (7) is satisfied (with at least one inequality being strict). Several qualitative multi-criteria decision methods are based on Paelinck's theorem, notably the Qualiflex method [22], the Dominant Regime method of Hinloopen *et al.* [31], and a method by Israels and Keller [32].

2.3.2 Mathematics of QuBE

Paelinck's theorem and the related dominance condition are the inspiration behind the QuBE method. QuBE is based on the geometric interpretation of the additive weighted scoring method presented by Kaluzny and Shaw [33]. The authors noted that the constrained region defined by the linear constraints (1), (2), and any other linear inequalities on the weights, defines a *convex polytope* P. In geometry, a polytope is a generic term that can refer to a two-dimensional polygon, a three-dimensional polyhedron, or a generalization to higher dimensions. Figure 2 depicts a two-dimensional polytope as the intersection of inequalities $x_2 \geq 0, x_1 \geq 0, x_1 \leq 6, -x_1 + x_2 \leq 5$, and $x_1 - x_2 \leq 5$ (ordered as labeled in the figure). In general, a linear inequality defines a *half space* (that divides the weight-space into two regions, one consisting of the set of weight vectors that satisfy the inequality and the other the rest) and a convex polytope P in dimension M is a bounded subset of $\mathbb{R}^M$—the set of all real number points in dimension M—which is the intersection of a finite set of half spaces. See [34] and [35] for more information on polytopes.

The set of all weight vectors resulting in a select bidder scoring highest can be defined by a polytope: assuming the rank ordering $\langle C_1 C_2 \cdots C_M \rangle$, the set of ordinal-rank-satisfying weights where bidder

4. The convex hull of a set of vectors $P = (p_1, p_2, ..., p_k)$ is the minimal convex set of the points—a set is convex if for every pair of points within the set, every point on the straight line segment that joins them is also within the set.

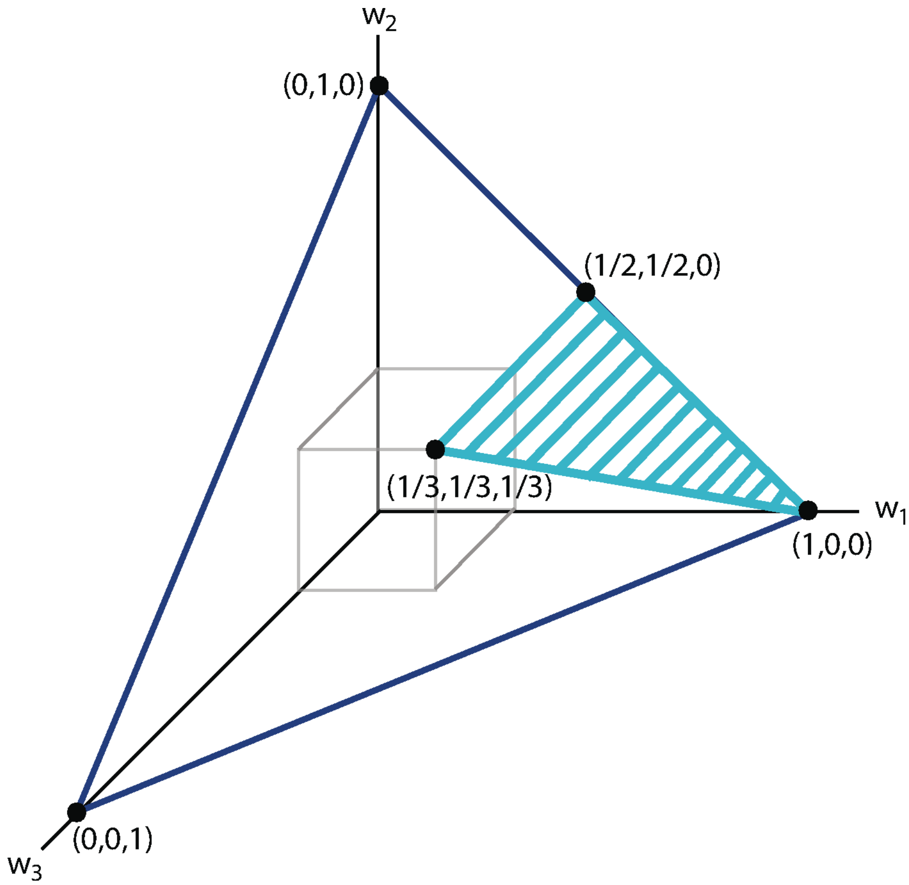

Figure 1: *Paelinck's theorem in three dimensions.*

j wins is defined by

$$\sum_{i=1}^{M} w_i = 1, \tag{8}$$

$$\sum_{i=1}^{M} v_i(j) w_i > \sum_{i=1}^{M} v_i(k) w_i \qquad \text{for all } k \neq j, \tag{9}$$

$$w_1 > w_2 > \cdots > w_m \geq 0. \tag{10}$$

The $n-1$ inequalities of (9) constrain the weight-space to the region where bidder j scores higher than all other bidders. The resulting polytope is referred to as bidder j's winning region. Since an inequality is geometrically represented by a half space, we can visualize the effect of inequalities of the form (9) on the weight-space. Figure 3 shows an example subdivision of the feasible weight space for three criteria (projected to 2D—the triangle formed by the points $(1,0,0),(1/2,1/2,0)$, and $(1/3,1/3,1/3)$ corresponds to the same triangle in Figure 1). Consider the case of three bidders, A,B, and C. Let half space H1 correspond to the inequality $\sum_{i=1}^{3} v_i(A) w_i > \sum_{i=1}^{3} v_i(B) w_i$ (the

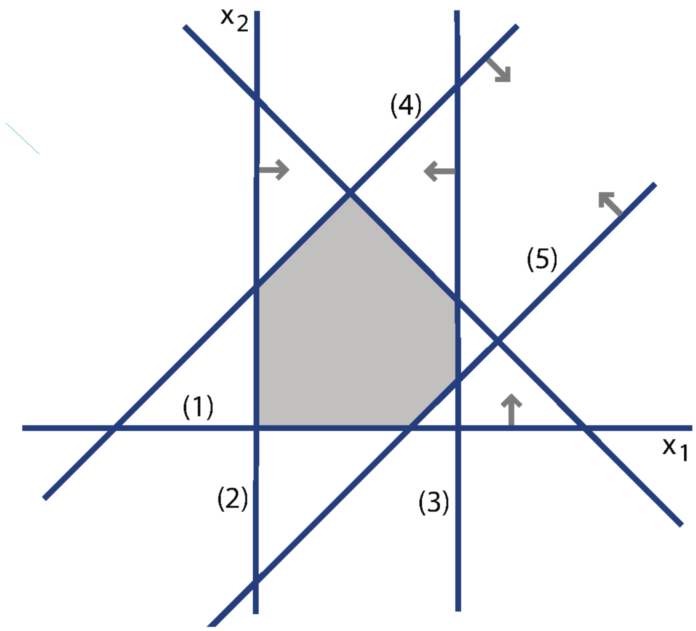

Figure 2: *An example polytope in two-dimensions.*

set of weights where bidder A scores higher than bidder B), and let half space H2 correspond to the inequality $\sum_{i=1}^{3} v_i(A)w_i > \sum_{i=1}^{3} v_i(C)w_i$ (the set of weights where bidder A scores higher than bidder C). The half spaces intersect the space of weights respecting a rank ordering of three criteria, dividing this space into four regions R1,R2,R3, and R4. In this case region R2 corresponds to the set of weight vectors where bidder A scores higher than both bidder B and bidder C.

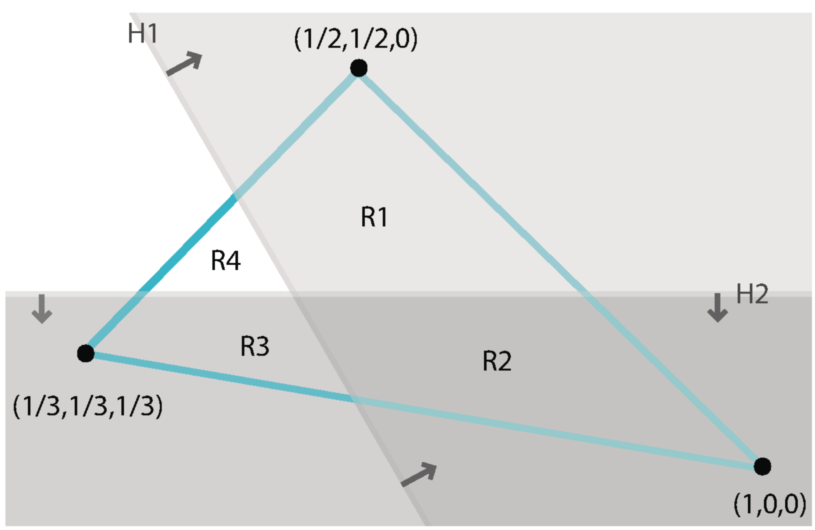

Figure 3: *Bidder regions in three dimensions.*

The geometric interpretation helps to validate the Claessens *et al.* corollary to the Paelinck theorem: if the intersection of all half spaces represented by inequalities (9) contains all the extreme points e^i for $i = 1,...,M$, then by convexity the intersection also contains all weight vectors—bidder j dominates and would win for any selection of fixed weights.

A key property of convex polytopes is that one can compute the volume of the space that they occupy. This is the essence of QuBE. If one were to select the criteria weights arbitrarily (but satisfying the weight relationships determined in Step 2 of QuBE), the probability of selecting a weight vector from a particular bidder's winning region is directly proportional to the volume of that region. The larger a bidder's region, the more likely the selection of that bidder as the winner. The underlying assumption is that, without additional information, all weight vectors satisfying the relationships established in Step 2 of QuBE are equally preferable. We can therefore evaluate the infinite set of weight vectors (satisfying the constraints of Step 2 of QuBE) and "count" the number of times each bidder wins by determining which bidder's winning region has the largest volume.

2.3.3 Implementation details

It is not within the scope of this report to dwell into the well-studied mathematics of computing volumes of polytopes. We simply note that open source software programs exist to estimate or exactly compute volumes of polytopes for practical-sized instances [5] (using *lrs* [37] the volume of a polytope generated by 30 criteria can be computed in a couple seconds) and refer the reader to [37–39] for additional information. A Wolfram *Mathematica*® implementation of QuBE based on PolySense [40] is available from the author and the Directorate Materiel Group Operational Research (DMGOR) Acquisition Support Team (AST) intends to develop a standalone application incorporating the finely-tuned volume computation algorithms of VINCI [39].

Step 2 of QuBE converts the ordinal ranking of the criteria into weight relationships using strict linear inequalities. In general, an inequality of the form $w_i \geq w_j$ implies that the weight of criterion i is greater than *or equal* to the weight of criterion j. If desired, an inequality may be strict: $w_i > w_j$. When using strict inequalities the resulting polytopes are referred to as *open convex polytopes* and otherwise referred to as *closed convex polytopes*. For our purposes, the volume of an open convex polytope is equal to the volume of its closed version. In order to remain consistent with related polyhedral theory and volume computation algorithms, for the remainder of the paper we use non-strict inequalities without loss of generality.

5. The general problem of computing the exact volume of a polytope is #P-Hard, read as "number P hard" [36]. This computational complexity class comprises of the set of counting problems associated with the set of decision problems solvable in polynomial time by a non-deterministic Turing machine. In layman terms it means that the computational time required is expected to grow exponentially as the number of dimensions (criteria) increases.

3 Example

Consider the example of buying a new car. Assume that we have a maximum budget of $50 thousand (K) dollars and require the dealership to be within ten kilometers (kms) for servicing convenience. The price, dealership proximity, fuel efficiency in miles per gallon (mpg), passenger capacity, safety rating (number of stars), and warranty length (years) are chosen as the criteria to be evaluated. Table 1 specifies the criteria scoring functions. The scoring functions assign a score between 0 and 10 points (pts) for each criterion.

Table 1: *Example: criteria and scoring functions.*

Criteria	Scoring function
C_1: Price ($K)	over $50K, 0 pts; under $20K, 10 pts; otherwise ($50K - price)/3 pts.
C_2: Fuel efficiency (mpg)	over 50 mpg, 10 pts; otherwise (fuel efficiency)/5 pts.
C_3: Passenger capacity	over 6, 10 pts; otherwise (passenger capacity)/0.6 pts.
C_4: Safety (star rating)	(number of stars)$\times$2 pts.
C_5: Servicing proximity	over 10 km away, 0 pts; otherwise (10km - distance) pts.
C_6: Warranty length (years)	over 10 years, 10 pts; otherwise (warranty length) pts.

Using the traditional bidder evaluation method we would have to assign relative criteria weights. In this case quantitative measurements are not possible—for example, it is difficult to make a direct comparison between fuel efficiency (mpg) and warranty length (years). The weights would have to be chosen with a degree of subjectivity. On the other hand, using the QuBE method we only need to prioritize the criteria. Table 2 lists a rank ordering of the criteria. For the sake of comparison the table also lists a subjectively chosen weight vector for the traditional method.

Table 2: *Example: criteria importance and weights.*

Criteria	Rank order	Weight
C_1: Price ($K)	1	35%
C_2: Fuel efficiency (mpg)	3	15%
C_3: Passenger capacity	5	10%
C_4: Safety (star rating)	2	21%
C_5: Servicing proximity	6	7%
C_6: Warranty length (years)	4	12%

At RFP time, we publish the criteria ordinal ranking which are translated into weight relationships: $w_1 \geq w_4 \geq w_2 \geq w_6 \geq w_3 \geq w_5$, $\sum_{i=1}^{6} w_i = 1$, (recall that w_i is the weight of criterion C_i).

Assume that four bidders, labeled A,B,C and D, return with contract proposals. For each criterion the bidders are evaluated and rated using the scoring functions. The bidder's criteria scores are presented in Table 3.

Table 3: *Example: bidder criteria ratings.*

Criteria	Bidder A	Bidder B	Bidder C	Bidder D
C_1 ($K)	32 (6 pts)	23 (9 pts)	26 (8 pts)	38 (4 pts)
C_2 (mpg)	30 (6 pts)	25 (5 pts)	40 (8 pts)	35 (7 pts)
C_3 (passengers)	5 (8.33 pts)	4 (6.67 pts)	5 (8.33 pts)	6 (10 pts)
C_4 (stars)	5 (10 pts)	4 (8 pts)	4.5 (9 pts)	5 (10 pts)
C_5 (kms)	10 (0 pts)	2 (8 pts)	4 (6 pts)	8 (2 pts)
C_6 (years)	3 (3 pts)	7 (7 pts)	3 (3 pts)	5 (5 pts)

Using the traditional method we apply the additive weighted scoring rule and take the product-sum of the criteria weights and bidder ratings to determine the bidder scores. In this case Bidder A obtains a score of 6.293, Bidder B scores 7.647, Bidder C scores 7.503, and Bidder D scores 6.290. Bidder B would be declared the winner. However, if the criteria weight vector was chosen to be $(31\%, 19\%, 10\%, 21\%, 7\%, 12\%)$—not a far stretch from the original weight vector and respectful of the priority ranking—then the final bidder scores would be Bidder A, 6.293; Bidder B, 7.487; Bidder C, 7.503; and Bidder D, 6.410. With this weight vector Bidder C wins. In fact, the entire ranking of bidders changes from $\langle BCAD \rangle$ to $\langle CBDA \rangle$. This possibility highlights a major flaw in using the traditional method when the criteria weights are not based on quantitative measurements but rather on subjective selection. In this case the contracting authority could have to find reasoning to defend the selection of the weight vector $(35\%, 15\%, 10\%, 21\%, 7\%, 12\%)$ over $(31\%, 19\%, 10\%, 21\%, 7\%, 12\%)$.

Using the QuBE method we can avoid this problem. To decide the winner in our example we determine, for each bidder, the size (volume) of their winning weight region as a ratio of the entire space of weights satisfying the criteria priority ranking. The entire weight space is described by the polytope:

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1, \tag{11}$$

$$w_1 \geq w_4 \geq w_2 \geq w_6 \geq w_3 \geq w_5, \tag{12}$$

$$w_i \geq 0 \quad \text{for } i = 1, \ldots, 6. \tag{13}$$

The volume of this polytope is 11.57×10^{-6}. The region of the weight space where Bidder A wins is defined as the polytope:

$$w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1, \tag{14}$$

$$w_1 \geq w_4 \geq w_2 \geq w_6 \geq w_3 \geq w_5 \tag{15}$$

$$6w_1 + 6w_2 + 8.33w_3 + 10w_4 + 0w_5 + 3w_6 \geq 9w_1 + 5w_2 + 6.67w_3 + 8w_4 + 8w_5 + 7w_6, \tag{16}$$

$$6w_1 + 6w_2 + 8.33w_3 + 10w_4 + 0w_5 + 3w_6 \geq 8w_1 + 8w_2 + 8.33w_3 + 9w_4 + 6w_5 + 3w_6, \tag{17}$$

$$6w_1 + 6w_2 + 8.33w_3 + 10w_4 + 0w_5 + 3w_6 \geq 4w_1 + 7w_2 + 10w_3 + 10w_4 + 2w_5 + 5w_6, \tag{18}$$

$$w_i \geq 0 \quad \text{for } i = 1, \ldots, 6, \tag{19}$$

where constraints (16-18) define the weight region where Bidder A dominates Bidders B, C and D respectively. The polytopes for Bidders B, C and D are defined in similar fashion (specifying the region where a bidder dominates all others) and the volumes of these polytopes are computed.

The volume of Bidder A's winning space is 0, Bidder B's volume is 7.23×10^{-6}, Bidder C's volume is 4.34×10^{-6}, and Bidder D's volume is 0. Converting the bidder volumes to percentages of the entire feasible weight space, we obtain the final bidder scores: Bidder B wins with 62.47% of the volume, Bidder C places second with 37.53% of the volume, and bidders A and D rank last with 0% of the volume. Having employed the QuBE method, the contracting authority could defensibly select Bidder B as providing the best-value-bid given the priority ranking of the criteria.

The volume results indicate that bidders A and D can never win regardless of weight selection satisfying the rank ordering of the criteria. This dominance is not obvious from the criteria scores for each bidder, but can be verified using the corollary of Paelinck's theorem presented in Section 2.3.1: Define the vectors

$$e^1 = (1,0,0,0,0,0)\,, \tag{20}$$

$$e^2 = \left(\frac{1}{2},\frac{1}{2},0,0,0,0\right), \tag{21}$$

$$e^3 = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3},0,0,0\right), \tag{22}$$

$$e^4 = \left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4},0,0\right), \tag{23}$$

$$e^5 = \left(\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5},0\right), \tag{24}$$

$$e^6 = \left(\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6},\frac{1}{6}\right). \tag{25}$$

Let $V_A = (6,10,6,3,8.33,0)$ be the criteria scores of Bidder A listed in the decreasing order of criteria priority. Similarly let $V_B = (9,8,5,7,6.67,8), V_C = (8,9,8,3,8.33,6)$, and $V_D = (4,10,7,5,10,2)$. Applying the Paelinck test we observe that $e^i \cdot V_C \geq e^i \cdot V_A$ for all $i = 1,\ldots,6$:

$$e^1 \cdot V_C = 8.00 \geq 6.00 = e^1 \cdot V_A, \tag{26}$$

$$e^2 \cdot V_C = 8.50 \geq 8.00 = e^2 \cdot V_A, \tag{27}$$

$$e^3 \cdot V_C = 8.33 \geq 7.33 = e^3 \cdot V_A, \tag{28}$$

$$e^4 \cdot V_C = 7.00 \geq 6.25 = e^4 \cdot V_A, \tag{29}$$

$$e^5 \cdot V_C = 7.27 \geq 6.67 = e^5 \cdot V_A, \tag{30}$$

$$e^6 \cdot V_C = 7.06 \geq 5.56 = e^6 \cdot V_A, \tag{31}$$

indicating that Bidder C dominates Bidder A. Similar analysis verifies that Bidder C also dominates Bidder D.

4 Discussion

4.1 Public Works and Government Services Canada policy

In this section we show that the QuBE method conforms with Public Works and Government Services Canada (PWGSC) policy. In particular, the PWGSC Supply Manual [41], used by PWGSC procurement officers, documents the departmental purchasing policies. Chapter 6—Developing the Procurement Strategy—provides guidance on the development of bidder evaluation methods:

6B.146 (1994-06-23) The basis upon which a contractor will be selected from the firms that submit responsive proposals should be indicated in the Request for Proposal (RFP). If the intent is to award the contract on the basis of best value, the criteria and the methods that will be used to determine the best value must be developed.

6B.148 (2008-12-12) The relative importance of the criteria must be clearly identified. When assigning weights to each criterion, the contracting officer and the client department should ensure that a high aggregate of points for minor criteria does not overcompensate for a low aggregate of points for major criteria.

The above paragraphs provide the only official PWGSC guidance with respect to best-value evaluation methods and criteria weighting. The QuBE method clearly obeys the first statement of Paragraph $6B.148$: the criteria are prioritized and published at RFP time. The second statement provides a guideline for the traditional additive weighted scoring method. Since the QuBE method never fixes the weights it inherently ensures that a high aggregate of points for minor criteria does not overcompensate for a low aggregate of points for major criteria.

Paragraph $6B.146$ endorses the development of evaluation methods—it does not limit the possibilities to previously-used methods. In fact, PWGSC procurement officers [42] indicate that PMOs are free to develop custom bidder evaluation methodologies as long as they are transparent, fair, defensible, and repeatable. Furthermore, the PWGSC Training and Development Division promotes objective techniques when possible:

> *"Using objective evaluation criteria that can be arithmetically or objectively measured is always preferable (easier for evaluators to determine what score should be given for a criterion, and to substantiate / justify the given score), however, in some situations they cannot easily be used. It is up to the contracting authority and technical authority at that point to determine which other scoring method is best suited for the requirement (comparative or subjective)."* - G.C. Doxtater, Professional Development Division, Policy, Risk, Integrity and Strategic Management Sector, Acquisitions Branch, Public Works and Government Services Canada [43].

The Participant's Manual [44] of the PWGSC Bid evaluation and Contractor Selection Methodologies Workshop states the following with respect to a comparative method (as oppose to a subjective point-rated method) *"Although this method would appear to be subjective, it is actually more objective since evaluators are not required to award marks... ...but only need to assess if one option is clearly better, about the same, or clearly worse than another option."*

4.2 Precedence of post-objective methods

The QuBE method is described as a post-objective evaluation model according to the classification of Pongpeng and Liston [45] [6]. Other post-objective methods for contractor selection have been proposed and accepted by the scientific community. Most recently, Lorentziadis [46] proposed a set of post-objective weight determination methods named the Least Favourable (LF), Most Favourable (MF), Average LF (ALF), and Average MF (AMF) evaluation methods. The commonality of these methods with the QuBE method is that only weight relationships are solicited prior to RFP. However where Lorentziadis' methods differ greatly from QuBE is that a weight vector is computed post process—as a function of the bidder inputs—and used for evaluation. For example, the LF method computes, for each bidder, the least favourable weights for which the proposal of the bidder can be evaluated. The contract is then awarded to the most economically advantageous proposal, determined by the LF method as the "best of the worst". The drawbacks of the Lorentziadis methods are that the post-objective weight determination method (LF, MF, ALF, or MLF) must be subjectively chosen, and that for some of his methods (LF, MF) each bidder is evaluated with a different set of weights.

The argument we share with Lorentziadis is that if a post-objective method is publicly announced with the tender request and uniformly applied to all bidders then the method is a fair evaluation process. Compared to the traditional method, post-objective evaluation methods strengthen integrity and discourage bidder gaming related to the definition of criteria weights. Since fixed criteria weights are not published, bidders cannot game the system to maximize their score. Rather, bidders must develop their optimal proposal that respects the Crown's priority ranking of the criteria.

4.3 Hybrid QuBE methods

4.3.1 Additional weight relationships

The traditional bidder evaluation method is ideal when the criteria weights are arithmetically or objectively measured. When this is not possible, then the QuBE method is recommended over subjective weight selection. However, in some cases a subset of the criteria weights can be objectively measured. The QuBE method can be adapted to take advantage of this additional information by adding the weight information in Step 2 of QuBE. In addition to the weight relationships established from the ordinal ranking of the criteria, further linear relationships can be specified. For example, decision makers can ensure that the weight of some criterion is a linear function of another, or bound the weight for a particular criterion i by specifying numerical lower and upper bounds. It is interesting to note that in the extreme case where the lower bound meets the upper bound for all criteria, the weight relationships determined in Step 2 of QuBE define a weight-space that consists of a single point. Continuing with the QuBE method in this case will yield the same outcome as the traditional additive weighted scoring method: all but the winning bidder will have zero-volume (empty) polytopes and the winning bidder's polytopal volume represents 100% of the weight-space (a single point).

The specification of additional weight relationships should be used very sparingly as without de-

6. According to this classification, the traditional method evaluation system corresponds to a pre-subjective input model.

fensible objective reasons they can incur (possibly flawed) subjectivity, diminishing the value of the QuBE method.

4.3.2 Hierarchal organization of criteria

In many cases the PMO develops a modular branching hierarchy of the criteria with the property that the weight of a parent node in the hierarchy tree is equal to the sum of the weights of its child nodes. The QuBE method can still be applied in these cases. Figure 4 depicts a simple hierarchy with ten criteria on three levels: criterion $C1$ is composed of sub-criteria $C11, C12$ and $C13$, and criterion $C3$ is composed of sub-criteria $C31$ and $C32$, where sub-criterion $C31$ is itself composed of sub-criteria $C311$ and $C312$. In this case decision makers are solicited to prioritize groups of criteria at the same level of hierarchy. Assigning weights to the criteria using matching subscript

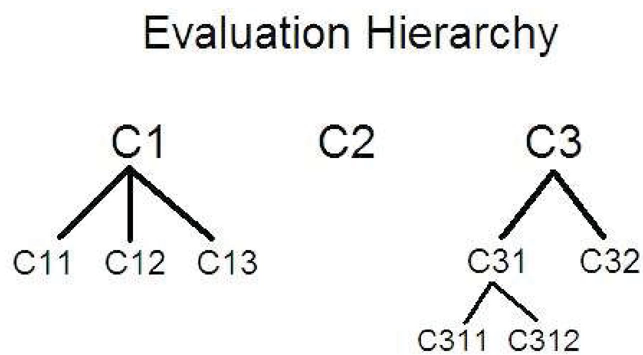

Figure 4: *A simple hierarchy.*

notation, the entire weight space is described by the polytope:

$$w_1 + w_2 + w_3 = 1, \tag{32}$$
$$w_{11} + w_{12} + w_{13} == w_1, \tag{33}$$
$$w_{31} + w_{32} == w_3, \tag{34}$$
$$w_{311} + w_{312} == w_{31}, \tag{35}$$
$$w_i \geq 0 \quad \text{for } i = 1, \ldots, 6. \tag{36}$$

Given ordinal rankings of the criteria, for example $\langle C1\, C2\, C3 \rangle$, $\langle C11\, C12\, C13 \rangle$, $\langle C31\, C32 \rangle$, and $\langle C311\, C312 \rangle$, the respective weight relationship constraints are added to the polytope defined by constraints (32)-(36):

$$w_1 \geq w_2 \geq w_3, \tag{37}$$
$$w_{11} \geq w_{12} \geq w_{13}, \tag{38}$$
$$w_{31} \geq w_{32}, \tag{39}$$
$$w_{311} \geq w_{312}. \tag{40}$$

The resulting polytope represents the entire space of weights satisfying the criteria hierarchy and priority rankings. The polytopes representing individual bidder winning regions can be defined as per Section 2.3.2.

In Section 5 we illustrate a hybrid QuBE approach for a PMO TAPV application.

5 Applications

5.1 Application to PMO CSC

The goal of the PMO CSC (Canadian Surface Combatant) project is to recapitalize the Navy's entire fleet of surface combatants. The project seeks to balance the competing objectives of minimizing overall costs (both acquisition and life-cycle) and risks to deliver warships as per the Statement of Requirements. During the options analysis period, the PMO identified and compared four recapitalization options as shown in Table 4. To evaluate each option, PMO CSC isolated thirteen evaluation criteria and assigned to them weights indicative of their relative importance (Table 5). Each of the options was subsequently scored per criterion (between 1 and 4 points). Using the traditional evaluation method, Options D, C, A and B received 3.21, 2.67, 2.11 and 2.01 points, respectively. In other words, the option to design and build new warships in Canada was most preferred, followed by purchasing a foreign design, followed by upgrading and extending the life of the IROQUOIS, and finally leasing/buying existing warships from a foreign country.

Table 4: *The four PMO CSC options.*

Option	Description
A	Upgrading and life extension of the IROQUOIS class for an additional 15 years.
B	Lease or buy existing platforms from a foreign country.
C	Purchase an existing foreign design and build in Canada.
D	Design and build warships in Canada.

Table 5: *PMO CSC criteria and option scoring.*

	Weight		Point Allocation			
Criterion	Symbol	Value	*A*	*B*	*C*	*D*
(C1) Cost: Sail-away	w_1	15%	4	2	3	1
(C2) Cost: In Service Support & Ops	w_2	8%	1	2	3	4
(C3) Capability Upgrades	w_3	8%	1	2	3	4
(C4) Growth Margin	w_4	9%	1	2	3	4
(C5) Environmental Compliance	w_5	8%	1	2	3	4
(C6) Crewing & Training (Facilities)	w_6	8%	1	2	3	4
(C7) Operations & Doctrine	w_7	7%	3	1	2	4
(C8) Schedule (Design/Build/Upgrade)	w_8	10%	4	3	2	1
(C9) Sustainment & Obsolescence	w_9	9%	1	2	3	4
(C10) Design Origin	w_{10}	7%	2	1	3	4
(C11) Economic Benefits	w_{11}	2%	2	1	3	4
(C12) Infrastructure Requirements	w_{12}	2%	4	2	2	2
(C13) Acquisition Risk	w_{13}	7%	2	3	1	4

As a post-analysis, let us examine how the QuBE method could have been used in this case. As input, QuBE requires an ordinal ranking of the criteria. In this case let us assume that the PMO's assigned weight vector

$$(15\%, 8\%, 8\%, 9\%, 8\%, 8\%, 7\%, 10\%, 9\%, 7\%, 2\%, 2\%, 7\%)$$

dictates the prioritization of the criteria: $\langle C1\ C8\ C4-C9\ C2-C3-C5-C6\ C7-C10-C13\ C11-C12\rangle$. Proceeding to Step 2, the linear weight relationships are established:

$$w_1 \geq w_8 \geq w_4 == w_9 \geq w_2 == w_3 == w_5 == w_6 \geq w_7 == w_{10} == w_{13} \geq w_{11} == w_{12}, \quad (41)$$

$$\sum_{i=1}^{13} w_i = 1, \quad (42)$$

$$w_i \geq 0 \text{ for all } i = 1, \ldots, 13. \quad (43)$$

The volume of the space of weights satisfying the relationships is 27.31×10^{-7}. The polytope defining the space of weight vectors satisfying the relationships and where option A wins is represented by the inequalities:

$$w_1 \geq w_8 \geq w_4 \geq w_2 \geq w_7 \geq w_{11} \quad (44)$$

$$w_2 == w_3 == w_5 == w_6, \quad (45)$$

$$w_4 == w_9, \quad (46)$$

$$w_7 == w_{10} == w_{13}, \quad (47)$$

$$w_{11} == w_{12}, \quad (48)$$

$$2w_1 - 2w_4 - 4w_6 + w_8 + 3w_{12} + 2w_{13} \geq 0, \quad (49)$$

$$w_1 - 4w_4 - 8w_6 + 2w_8 + w_{12} + w_{13} \geq 0, \quad (50)$$

$$3w_1 - 6w_4 - 12w_6 + 3w_8 - 5w_{13} \geq 0, \quad (51)$$

$$\sum_{i=1}^{13} w_i = 1, \quad (52)$$

$$w_i \geq 0 \text{ for all } i = 1, \ldots, 13. \quad (53)$$

The polytopes for the winning regions for options B,C, and D are defined in similar fashion. The volume of option A's polytope is 12.14×10^{-7}, option B's is 0, option C's is 8.13×10^{-7}, and options D's is 7.04×10^{-7}. Percentage-wise, option A occupies 44.46% of the weight-space, option C occupies 29.76%, option D occupies 25.78%, and option B is dominated—it occupies 0% of the weight-space. Assuming that the ranking of criteria reflects PMO CSC's priorities and that no other constraints are placed on the weights, the QuBE method outputs option A, upgrading and life extension of the IROQUOIS class for an additional 15 years, as the best recapitalization option.

In this case the QuBE method result differs from the original result determined by PMO CSC using the traditional additive weighted scoring method (where the weight vector

$$(15\%, 8\%, 8\%, 9\%, 8\%, 8\%, 7\%, 10\%, 9\%, 7\%, 2\%, 2\%, 7\%)$$

was pre-selected). It is assumed that PMO CSC had defensible reasons to select the weight vector. In fact, the AST performed a sensitivity analysis [33] on behalf of DGMPD Land & Sea to confirm that the weights were not purposely tuned to obtain a desired result. It just happens that the weight vector selected falls in the 25.78% of the weight-space where option D is preferred. A possible source of the discrepancy is the assumption that the pre-selected weight vector does indeed coincide with the priority ranking of the criteria.

This example highlights the differences between the two methods. The outcomes can differ. Hence, it is important that a PMO is able to defend its preference ranking, choice of additional weight relationships, and fixed weights regardless of which method is used. The clear benefit of using the QuBE method is that decision makers only need to defend their priority ranking of the criteria.

5.2 Application to PMO TAPV

PMO TAPV (Tactical Armoured Patrol Vehicle) has the mandate to deliver a wheeled combat vehicle that will have a high degree of tactical mobility and provide a very high degree of survivability to its crew. The vehicles will fulfill domestic and battlefield roles, including reconnaissance and surveillance, command and control, and as cargo and armoured personnel carriers. The Project is following a best-value tendering process and preparing to issue a RFP. PMO TAPV has isolated 14 criteria (including price) that are presented in Table 6 (prior to final publication of this report PMO TAPV expanded the criteria hierarchy tree). Criterion C_3 is composed of five sub-criteria, and criterion C_4 is composed of two sub-criteria. PMO TAPV has decided that price should represent 30% of the total weight and determined ordinal ranking of the remaining (technical) criteria: at the first level of the criteria hierarch the priority ranking is $\langle C_2\, C_3\, C_4\, C_5\, C_6\, C_7\, C_8\, C_9 \rangle$. At the second level. the priority rankings are $\langle C_{31}\, C_{32}\, C_{33}\, C_{34}\, C_{35} \rangle$ and $\langle C_{41}\, C_{42} \rangle$. Should PMO TAPV decide to employ

Table 6: *PMO TAPV criteria hierarchy.*

Criteria	Weight variable
C_1: Price	w_1
C_2: IED protection	w_2
C_3: Tactical mobility	w_3
C_{31}: X-country capability	w_{31}
C_{32}: Gap crossing capability	w_{32}
C_{33}: Step capability	w_{33}
C_{34}: Turning radius	w_{34}
C_{35}: Acceleration	w_{35}
C_4: Firepower	w_4
C_{41}: Hit probability	w_{41}
C_{42}: DRI performance	w_{42}
C_5: ISS	w_5
C_6: Weight	w_6
C_7: Electrical power	w_7
C_8: Seating capacity	w_8
C_9: Ballistic protection	w_9

the QuBE method (in this case a hybrid QuBE), these priority rankings would be converted into the following weight relationship constraints:

$$
\begin{aligned}
w_1 =& 0.3, && (54)\\
w_2 \geq w_3 \geq w_4 \geq & w_5 \geq w_6 \geq w_8 \geq w_9, && (55)\\
w_{31} + w_{32} + w_{33} + w_{34} + w_{35} == & w_3, && (56)\\
w_{31} \geq w_{32} \geq & w_{33} \geq w_{34} \geq w_{35}, && (57)\\
w_{41} + w_{42} == & w_4, && (58)\\
w_{41} \geq & w_{42}, && (59)\\
\sum_{i=2}^{9} w_i = & 0.7, && (60)\\
w_i \geq & 0 \text{ for all } i = 2, ..., 9. && (61)
\end{aligned}
$$

As per the QuBE method, PMO TAPV would then publish the RFP including the weight relationships established and the QuBE algorithm to be used for bidder evaluation. Without additional objective information to further constrain the criteria weights, the QuBE method is a defensible evaluation method that reduces the possibility of bidder gaming and ensures that PMO TAPV delivers a system that reflects the priorities specified by the PMO.

6 Conclusion

DGMPD PMOs are called upon to develop fair, transparent, and defensible bidder evaluation methods for major Crown procurement tenders. The traditional evaluation approach is to use an additive weighted scoring rule where evaluation criteria are established, rating functions are determined, and relative weights are assigned to the criteria. Bidders are then rated per criterion and scored using a product-sum of the weights times the ratings. However, it is known that the proper assessment of criteria weights, when quantitative measurements are not possible, is a cognitively demanding task. Weight values that are determined by subjective judgment are known to suffer from internal consistency and validity problems. It is possible to end up with an unfair evaluation system.

Many decision analysts argue that the only real measure of preference in evaluating criteria is the act of making a choice, in other words determining a relative order of importance of the criteria. In accordance, this paper presents an additive weighted scoring method where the weights remain unexpressed. It is inspired by the Paelinck theorem that implies that the winning bidder can be identified without any numerical allocation to the criteria weights. With QuBE (qualitative bidder evaluation), the ordinal ranking of criteria is solicited and the winning option is determined by examining the sizes of the weight-subspaces associated with the set of all weights respecting the priority ordering of the criteria. QuBE is useful in those situations where fixed criterion weights are either not desirable or easily obtainable.

The essence of QuBE is that if one were to select the criteria weights arbitrarily (but respecting the ordinal ranking of criteria), the probability of selecting a weight vector from a particular bidder's winning region is directly proportional to the volume of that region. The larger a bidder's region, the more likely the selection of that bidder as the winner. The underlying assumption is that, without additional information, all weight vectors respecting the criteria ordinal ranking are equivalent in the sense that none is preferred over another. QuBE "counts" the number of times each bidder wins by determining which bidder's winning region has the largest volume.

Hybrid QuBE methods are able to deal with hierarchal criteria organization and additional weight relationships (e.g., fixing a subset of the weights). The application of QuBE was demonstrated on a PMO CSC options analysis, and as a potential PMO TAPV bidder evaluation method.

In addition to minimizing subjectivity, the QuBE method is transparent, fair, defensible, and reduces the possibility of bidder gaming.

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List of symbols/abbreviations/acronyms/initialisms

AHP	Analytical Hierarchy Process
ALF	Average Least Favourable
AMF	Average Most Favourable
AST	Acquisition Support Team
BGP	bureaux de gestion de projet
BP BCSC	Bureau de projet – Bâtiment de combat de surface du Canada
BP VBTP	Bureau de projet – Véhicule blindé tactique de patrouille
CORA	Centre for Operational Research and Analysis
CSC	Canadian Surface Combatant
DGMPD	Director General Major Projects Division
DGRGP	Directeur général - Réalisation des grands projets
DMGOR	Directorate Materiel Group Operational Research
DND	Department of National Defence
DRDC	Defence Research and Development Canada
ESA	équipe de soutien des acquisitions
K	thousand
kms	kilometers
LF	Least Favourable
MF	Most Favourable
mpg	miles per gallon
PMO	Project Management Office
pts	points
PWGSC	Public Works and Government Services Canada
QuBE	Qualitative Bidder Evaluation
RFP	Request for Proposal
TAPV	Tactical Armoured Patrol Vehicle

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Director General Major Project Delivery Project Management Offices are called upon to develop fair, transparent and defensible bidder evaluation methods for major Crown procurement tenders. The traditional evaluation approach is to use an additive weighted scoring rule where evaluation criteria are established, rating functions are determined, and relative weights are assigned to the criteria. Bidders are then rated per criterion and scored using a product-sum of the weights times the ratings. The highest scoring bidder wins. This approach requires the pre-selection of criteria weights. Ideally these weights are arithmetically or objectively measured; however in most cases this is not possible. Lacking such measurements, the determination of appropriate weights that accurately reflect the decision maker's priorities is known to be a cognitively demanding task—it is difficult to objectively quantify the relative importance of criteria. This paper presents an alternative: an additive weighted scoring method where the weights remain unexpressed. With qualitative bidder evaluation (QUBE) an ordinal ranking of the criteria is solicited and the winning option is determined by examining the sizes of the weight-subspaces associated with the set of all weight vectors respecting the criteria priority ordering. In addition to minimizing subjectivity, the method is transparent, fair, defensible, and reduces the possibility of bidder gaming.

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Volume
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