Marine Builder's Risk Insurance for the Joint Support Ship Contract

A Utility Theory Approach to Risk Analysis

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Defence R&D Canada
Centre for Operational Research and Analysis

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Technical Memorandum

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Abstract

The Department of National Defence's (DND) procurement process for the Joint Support Ships (JSS) requires a quantitative understanding of the Project Management Office's (PMO) tolerance for risk. By establishing the risk tolerance of the stakeholders, we can place bounds on the requisite amount of insurance for the JSS project. In particular, the acquisition of Marine Builder's Risk Insurance (MBRI) provides protection for the shipbuilding capital investment during the construction phase. Using utility theory, we obtain decisive results regarding the risk tolerance of the PMO JSS. Our tests yield the necessary statistical significance to reject the hypothesis of risk neutrality and to accept the hypothesis of risk aversion. Using a least-squares best-fit parameterization, we quantify PMO JSS's risk tolerance which allows us to construct a series of actuarial tables in conjunction with standard loss distributions for rare events. The decision maker can use the actuarial tables with expert opinion to determine the correct level of MBRI required for the JSS project.

Résumé

Le processus d'acquisition du ministère de la Défense nationale (MDN) pour les navires de soutien interarmées (NSI) exige de bien comprendre la tolérance aux risques du bureau de gestion du projet (BGP). En définissant la tolérance aux risques des groupes d'intérêts, nous pouvons imposer des limites à la quantité d'assurances exigées pour le projet NSI. À propos, l'assurance des risques du constructeur maritime (MBRI) permet de protéger l'investissement des capitaux dans la construction navale durant la phase de la construction. La théorie de l'utilité nous permet de déterminer la véritable tolérance aux risques du BGP NSI. Nos essais fournissent la signification statistique nécessaire pour rejeter l'hypothèse d'une neutralité devant le risque et accepter celle de l'aversion pour le risque. Nous quantifions par paramétrage d'ajustement optimal des moindres carrés la tolérance aux risques du BGP NSI, ce qui nous permet de construire une série de tables actuarielles en parallèle avec les répartitions standard des pertes liées aux événements rares. Le décideur utilise les tables actuarielles et consulte des spécialistes pour déterminer la couverture de l'assurance MBRI exigée pour le projet NSI.

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Executive summary

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Background: Shipbuilding represents one of the largest risk undertakings for the Department of National Defence (DND) procurement due to the potentiality of catastrophic loss – losses in excess of hundreds of millions of dollars arising from a small number of events. In the commercial shipbuilding industry, shipbuilders use Marine Builder's Risk Insurance (MBRI) as part of their risk mitigation strategy to protect against losses during construction and sea trials. For DND, MBRI represents one of the most expensive insurance premiums in any procurement activity within the Department.

In 2004, the Government of Canada announced that the aging Protecteur Class Auxiliary Oiler Replenishment vessels would be replaced by three Joint Support Ships (JSS). Unfortunately, no Canadian contractor could bid within the project's budget; therefore, the Project Management Office JSS (PMO JSS) suspended the procurement process at the Definition Phase. The PMO JSS is now examining areas of potential cost savings, including MBRI.

Originally, PMO JSS sought the help of Marsh Canada Ltd. to assess potential insurance needs of the JSS project. Given that the PMO JSS estimated a maximum loss of 1.5 ships during the JSS construction phase (approximately a $750 million loss), Marsh Canada Ltd., in 2006, estimated that the project would require between $37 million and $55 million in MBRI premiums. The precise premium level depended chiefly upon the insurance market and the liability presented by the eventual contractor.

In early March of 2009, the National Shipbuilding Procurement Strategy Office (NSPSO) approached the Directorate Material Group Operational Research (DMGOR) for further decision support in the area of risk mitigation. NSPSO sought new insight into potential cost savings arising from the possible reductions in insurance premiums. Through interviews with subject matter experts (SME) provided by NSPSO and through the application of utility theory, we developed a quantitative understanding of the PMO JSS's view of risk. We restricted our study to the largest insurance item facing the JSS project, MBRI.

Principal results: We obtained decisive results regarding the risk tolerance of the PMO JSS. We obtained the requisite statistical significance to reject the hypothesis of risk neutrality and to accept the hypothesis of risk aversion. Using a least-squares best-fit parameterization, we quantified PMO JSS's risk tolerance. The utility function elicited from the SME allowed us to construct a set of actuarial tables in conjunction with standard loss distributions for rare events.

Significance of results: Our results provide quantitative bounds on the JSS insurance problem, which can help guide the inquiries and deliberations of decision makers. We have quantified the scale of the MBRI issue in a manner commensurate with the risk tolerance of the PMO JSS, thereby providing the decision makers with new tools to assess the MBRI premiums within the risk-benefit framework of the entire JSS project.

Summation: The determination of the correct amount of insurance coverage for a shipbuilding project requires the attention of SME, in the context of the full procurement problem. Utility theory and its practical application can help establish consistent choices on the amount of coverage required. If decision makers find the utility approach useful for the JSS, we can apply the same methodology to other shipbuilding projects, such as the Arctic/Offshore Patrol Ship and the Canadian Surface Combatant.

Sommaire

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David W. Maybury, Gregory H. van Bavel ; DRDC CORA TM 2009–41 ; R & D pour la défense Canada – CARO ; octobre 2009.

Contexte : La construction navale représente pour le ministère de la Défense nationale (MDN) l'entreprise la plus risquée à cause de la possibilité de pertes catastrophiques - pertes se chiffrant en centaines de millions de dollars engendrées par un petit nombre d'événements. Dans l'industrie de la construction navale commerciale, les constructeurs se procurent l'assurance des risques du constructeur maritime (MBRI) comme élément de leur stratégie d'atténuation des risques afin de se protéger contre les pertes durant la construction et les essais en mer. Pour le MND, les primes de l'assurance MBRI sont les plus chères de toutes les activités d'acquisition qu'il entreprend.

En 2004, le gouvernement du Canada a annoncé que les vieux pétroliers ravitailleurs d'escadre de la classe Protecteur seraient remplacés par des navires de soutien interarmées (NSI). Malheureusement, aucun entrepreneur canadien n'a pu présenter une soumission qui respectait le budget du projet ; en conséquence, le bureau de gestion du projet NSI (BGP NSI) a suspendu le processus d'acquisition dans la phase de la définition. Il en est à examiner des solutions plus économiques, y compris l'assurance MBRI.

Au début, le BGP NSI avait demandé à Marsh Canada Ltd. d'évaluer les besoins éventuels d'assurance pour le projet NSI. Puisque le BGP NSI avait prévu une perte maximale d'un navire et demi dans la phase de la construction (perte d'approximative de 750 million $), Marsh Canada Ltd., a calculé, en 2006, que les primes de l'assurance MBRI oscilleraient entre 37 et 55 million $. Le montant exact des primes dépendait principalement du marché de l'assurance et de la responsabilité de l'entrepreneur retenu.

Au début de mars 2009, le Bureau de la stratégie nationale d'acquisition en matière de construction navale (BSACN) a sollicité l'aide de la Direction de la recherche opérationnelle (Groupe des matériels) (DRO GM) pour trouver une solution d'atténuation des risques. Le BSACN cherchait à obtenir de nouvelles opinions sur les possibles économies de coûts que pourrait engendrer la réduction des primes d'assurance. Grâce aux entrevues détaillées avec les spécialistes désignés par le BSACN et à l'application de la théorie de l'utilité, nous avons été en mesure de bien cerner la façon dont le BGP NSI percevait les risques. Nous avons limité notre étude à la question de l'assurance ayant le plus d'importance pour le projet NSI, soit l'assurance MBRI.

Résultats : Nous avons obtenu des résultats définitifs sur la tolérance aux risques du BGP NSI ainsi que la signification statistique nécessaire pour rejeter l'hypothèse d'une neutralité du risque et accepter celle de l'aversion pour le risque. Nous avons quantifié par paramétrage d'ajustement optimal des moindres carrés la tolérance aux risques du BGP NSI. La fonction d'utilité, établie par suite des entrevues avec les spécialistes, nous a permis de construire un ensemble de tables actuarielles en parallèle avec les répartitions standard de pertes liées aux événements rares.

Importance : Nos résultats établissent des limites de quantité pour le problème d'assurance du NSI, que les décideurs pourront utiliser pour orienter leurs demandes de renseignements et leurs délibérations. Nous avons quantifié la portée de la question de l'assurance MBRI de façon proportionnée à la tolérance aux risques du BGP NSI, procurant ainsi aux décideurs de nouveaux outils pour évaluer les primes de l'assurance MBRI dans la structure des risques-avantages du projet NSI dans son ensemble.

Conclusions : Les spécialistes doivent se pencher sur la question de la détermination de la couverture d'assurance appropriée pour les projets de construction navale, et ce, dans le contexte du problème d'acquisition dans son ensemble. La théorie de l'utilité et son application effective peuvent aider à déterminer des choix logiques en matière de couverture exigée. Si les décideurs jugent utile la théorie de l'utilité pour le projet NSI, nous pourrons appliquer la même méthode à d'autres projets de construction de navire, nommément le projet du navire de patrouille extracôtier pour l'Arctique et celui du bâtiment canadien de guerre de surface.

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1 Introduction

There are worse things in life than death. Have you ever spent an evening with an insurance salesman? — Woody Allen

1.1 Background

Shipbuilding represents one of the most complicated risk environments for DND procurement as a result of total budget size, value at risk during the construction phase, and the inherent difficulties in assessing exogenous risk factors such as contractor competencies. During procurement activities, DND relies in part on the commercial insurance market for risk mitigation. In shipbuilding, Marine Builder's Risk Insurance (MBRI), which protects shipbuilding enterprises from losses during construction and sea trials, represents one of the most expensive insurance layouts for DND.

In 2004, the Government of Canada announced its intention to procure three Joint Support Ships (JSS) to replace the aging Protecteur Class Auxiliary Oiler Replenishment vessels [1]. Two years later, the Government announced that two teams had been selected to proceed with Project Definition for the JSS. At the end of the Definition Phase, neither contractor was able to deliver the capability within the allotted budget. The contracted portion of the Definition Phase was therefore terminated. In an effort to reduce programmatic costs, the JSS Project Office is examining areas of potential cost savings, including Marine Builder's Risk Insurance.

As part of an effort to understand insurance costs, the Project Management Office (PMO) JSS obtained the services of Marsh Canada Ltd., who produced reports [2], [3] which estimated that DND would need to pay \$37–\$55 million in MBRI insurance (approximately 3.5% of the total JSS project budget) to provide protection for 1.5 ships over the duration of the JSS contract. Marsh qualified that their estimate was contingent on several factors including a balanced insurance market, that each vessel takes no longer than 4 years to complete, and that the chosen shipbuilding yard meets the standards of the insurance company. At the time of the estimate, PMO JSS believed that the maximum value at risk during construction would be 1.5 ships.

1.2 Scope

Given that the MBRI estimate for 1.5 ships' worth of coverage represents a large portion of the entire JSS budget for a single risk mitigation tool, PMO JSS desired to know if cost savings could be obtained in insurance coverage and risk mitigation. The National Shipbuilding Procurement Strategy Office (NSPSO) approached the (Directorate Materiel Group Operational Research) DMGOR in early March of 2009 and issued a formal tasking request on 18 March 2009 [4]. While the general problem of risk mitigation during construction involves many factors – including the application of surety bonds, general liability limits, and errors and omissions insurance – we restricted our study to the following concrete issues:

- We focused our study on the single largest insurance item – MBRI. A detailed understanding of the PMO JSS's view of risk as applicable to MBRI will provide insight into the costs associated with risk mitigation tools.

- We placed our efforts into constructing a set of actuarial tables to be used in conjunction with expert opinion. Instead of trying to develop a panacea model with limited data, our results frame the problem for the decision maker by providing boundaries on a large issue.

2 Modelling and interviewing: preliminaries in risk analysis

We approached the MBRI problem from a utility theory [5] perspective. In general, utility theory provides a risk metric in a decision making process by quantitatively assessing a decision maker's possible actions through probability concepts. Before the decision process begins, the decision maker formulates a preferred set of outcomes while understanding that each decision will result in an uncertain consequence arising from a probability distribution. The decision maker desires to learn which choices will lead to the best set of outcomes according to her preferred outcomes and her tolerance for risk. In the decision process, tradeoffs between risk aversion and reward naturally emerge and utility theory provides the key quantitative measure that balances competing objectives in the risk-reward framework. Thus, while risk assessment remains a subjective problem, utility theory can help ensure that the decision maker achieves consistent solutions.

2.1 Utility theory: application to NSPSO preferences

In the context of the MBRI for the JSS project, the courses of action are related to choosing the level of MBRI coverage (including none) and the possible consequences include levels of loss – from the trivial to the catastrophic. The value of the loss can be assigned a monetary value and the likelihood of a given loss can be assigned a probability. In the MBRI problem, we assume that PMO JSS most prefers the smallest possible monetary loss in shipbuilding (zero) and least prefers the greatest possible loss (total destruction of all ships under construction with no compensation).

Utility theory helps the decision maker arrive at consistent solutions by framing decision-consequence events as a series of lotteries. To illustrate the method, suppose that the decision maker understands her preferences among consequences which she has ranked from her worst outcome W to her best outcome B. We now consider two options associated with an arbitrary outcome, C, taken from the decision maker's ranked list of consequences:

- Certain Option: Receive outcome C for certain.
- Risky Option: Receive outcome B (the best outcome) with probability P and outcome W (the worst outcome) with probability 100% - P.

We see that the decision maker has a choice between a certain option C and a risky option in which the "prizes" W and B are determined through a "lottery". The risky option involves the probability P associated with winning the best outcome B. Clearly, if P=100%, the decision maker would choose the risky option (which would no longer be risky) as she would receive her best outcome B with certainty. On the other hand if P=0% the decision maker would choose the certain option since in this case the risky option yields her worst outcome W with certainty. Notice that the decision maker's choice depends on the given probability P of winning outcome B. It stands to reason that at some intermediate probability value, our decision maker will become *indifferent* to the two different options. At the indifference probability our decision maker views both options as equivalent – this is the point at which she will switch between options if the probability, P, were to increase or decrease by a small amount. This indifference point is called a *certainty equivalent* with respect to the lottery.

We now imagine repeating the process for all outcomes associated with our decision maker's choices. The form of the two options remain the same, but we run through the entire set of outcomes for C. Since our decision maker has a well defined list of preferences, the indifference probabilities associated with each consequence will be different. Clearly if we set C equal to B (the best outcome), the indifference probability will equal 100% (why take the risky option if the certain option gives the best outcome) and if we set C equal to W, the indifference probability will equal 0% (why take the certain option if it gives the worst outcome). For most problems that involve money, we expect the indifference probability to increase monotonically[1] with C. The list of indifference probabilities constitutes our decision maker's utility function associated with her decision space.

Note that a risk averse decision maker will prefer a certain loss over a number of favourable lotteries. To elucidate this point with our two-option example, a risk averse decision maker will prefer a number of certain outcomes, C, from option (1) which are less preferable than the expectation of the lottery associated with option (2).[2] We illustrate the acceptance of a certain loss over a favourable lottery in figure 1. In the context of the JSS project, the purchase of MBRI represents a certain loss (the insurance premium) which excuses the decision maker from the financial consequences of a catastrophe during ship building (the lottery). We will extend these concepts to build a utility function for PMO JSS from the subject matter experts (SME) with the NSPSO.

By asking NSPSO SME a series of carefully constructed questions, we assessed PMO JSS's risk tolerance for the JSS project (see Annex A for the background material sent to the SME prior to the interview sessions). In two question sessions [6], [7] we elicited responses through bracketed lotteries (similar to the two options presented in the example above) from the SME. The lotteries we presented involved choices between outcomes for certain and bracketed consequences with a 50-50 chance of occurrence. We considered the absolute best outcome as zero loss (full delivery) and the absolute worst outcome as the

[1] A monotonic increase means that a quantity is always greater than or equal to all of its previous values. Consider the case where B and W are constant: if C increases, then P would have to increase in order to maintain the attractiveness of the risky option; hence, the indifference probability increases.

[2] This situation is not difficult to appreciate. Most people would not bet their house on the flip of a coin for a chance to win one million dollars (lose the house on tails, win one million dollars on heads) even though the expectation of the lottery, which is $500,000, might be greater than the value of the house. The consequences of not having a place to live outweighs the possible gain (at least in the minds of responsible people).

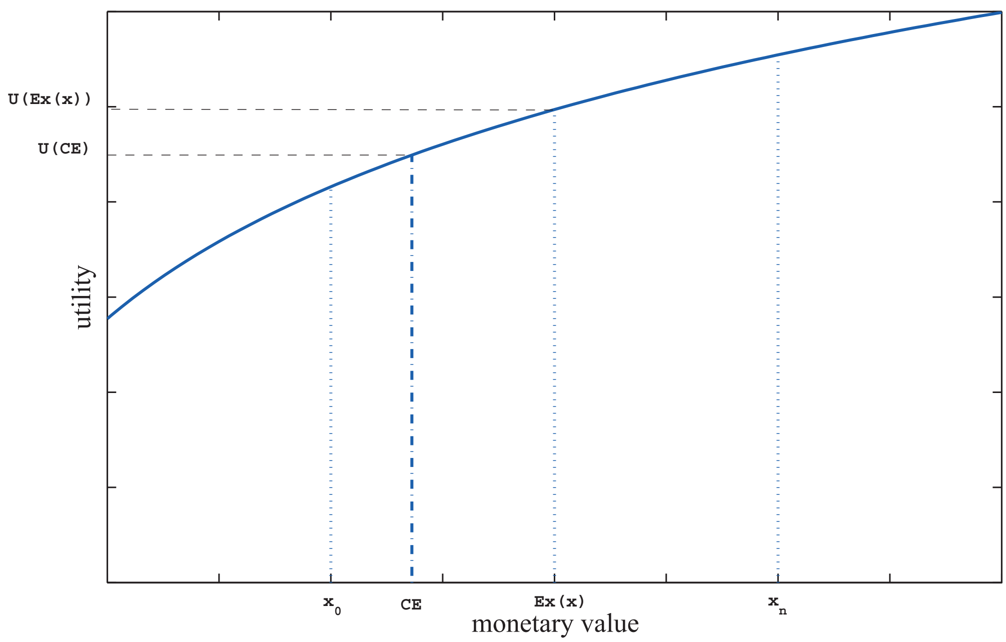

Figure 1: *A utility function of a risk averse decision maker. The certain loss is denoted by CE and is shown with respect to the expectation of the lottery Ex(x). The two reference outcomes (best and worst cases) are labeled x_n and x_0 respectively. We see that while the lottery has an expected loss, Ex(x), the decision maker is willing to accept a greater loss for certain, CE, as opposed to playing a lottery that has a chance of losing even more money. U(Ex(x)) and U(CE) denote the corresponding utility for the expected loss and for the certain loss respectively.*

loss of 1.5 ships during construction. We assumed a $500 million value per vessel.[3] We asked the panel four questions of the following form:

- **Question:** You are given two choices: a risky option and a certain option. The risky option has the construction of JSS project occurring with two outcomes with equal likelihood (i.e., a 50% chance for each): a worst-case loss (e.g., $750 million, equivalent to 1.5 ships), or a best-case loss (e.g., $0, equivalent to no loss). The certain option has the construction of the JSS project occur with a given certain loss. At what value of the certain loss would you become indifferent between the two options? We will call your indifference value for this question your certainty equivalent.

Throughout the question session, we placed the bracketing consequences (worst case, best

[3] The exact cost of each JSS vessel is not important. It is only important that the decision makers understand the consequences of a $500 million loss as pertaining to the JSS project.

Table 1: *Risky Options versus Certainty Equivalents*

	Losses in Millions		
	Risky Option (50-50 lottery)		Certain Option
Question	**Worst-Case**	**Best-Case**	**Certain Equivalent**
1	750	0	400 ± 37.5
2	400	0	250 ± 20.0
3	250	0	150 ± 7.5
4	750	400	600 ± 17.5

case) at various points of loss.

Using the SME's certainty equivalents, we constructed a quantitative measure of PMO JSS's risk tolerance. Table 1 displays the outcomes of the questions. The first two columns label the ends points of the 50-50 lottery for shipbuilding losses in the JSS project. The last column contains the SME's certainty equivalent for each lottery. In our discussions and question sessions with the SME, we discovered that each certainty equivalent response had an associated uncertainty. We indicate the level of uncertainty with each response in the last column. Using the SME's responses in conjunction with the details of the lotteries, we constructed the utility function for shipbuilding losses in Figure 2. The technical details by which we extracted the utility function from the 50-50 lotteries can be found in Annex B.

Note that in our analysis, we use the exponential function class of utility functions. This class has the property of constant risk aversion, implying that the decision maker does not become more (or less) risk averse as the lottery changes in size. In most cases that involve monetary outcomes, most decision makers show a risk profile that does in fact change over the size of the lottery [5].[4] Given the data we collected from NSPSO during our questions sessions, the level of uncertainty in SME responses precluded us from establishing a risk sensitivity other than the simplest constantly risk averse profile. Again, see Annex B.

The uncertainty in the certainty equivalents affects our evaluation of PMO JSS's utility function. Using the methodology outlined in Annex B, we reject the hypothesis of risk neutrality at the 1.3% level[5], but we accept a hypothesis of constant risk aversion at the 12.5% level[6]. Thus, we posit that PMO JSS is a risk averse decision maker. For the

[4]As an example, most individuals would regard winning $1 billion or $10 billion dollars as nearly equivalent in a lottery, while they would regard winning $5,000 and $50,000 as very different.

[5]The statistical significance of $\chi^2 = 12.74$ for $\nu = 4$ degrees of freedom is 1.27%

[6]The statistical significance of $\chi^2 = 5.74$ for $\nu = 3$ degrees of freedom is 12.50%

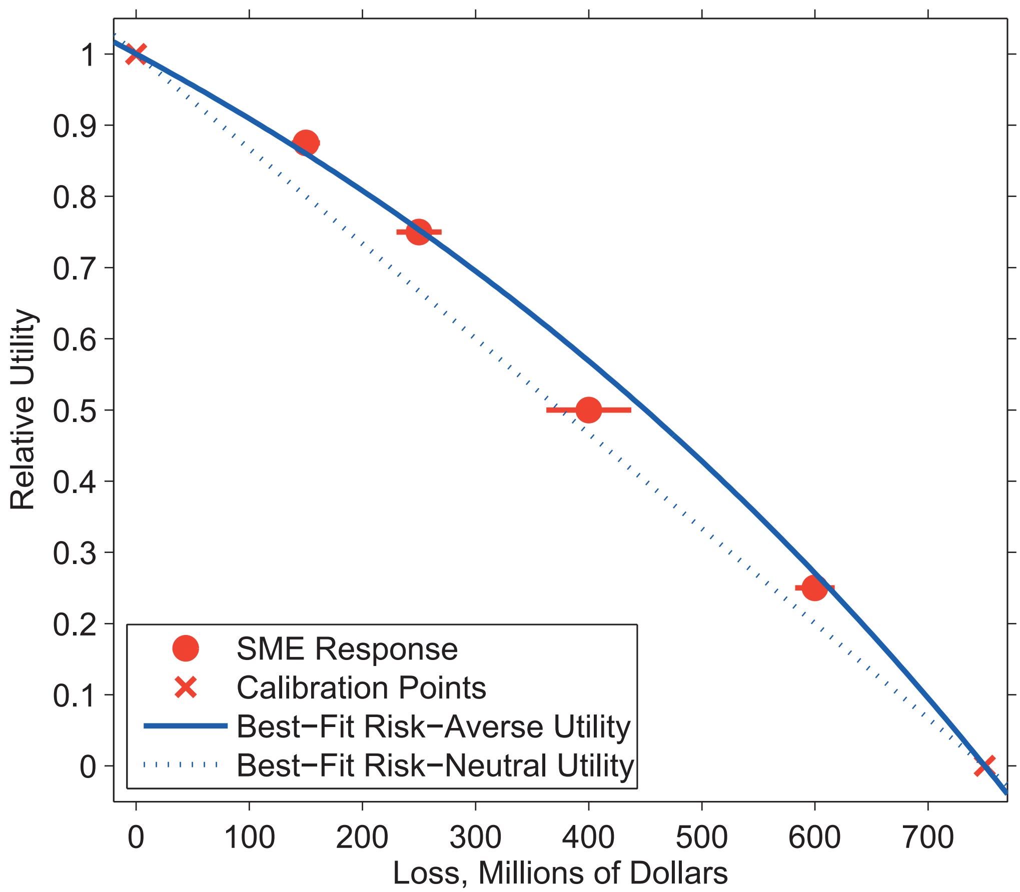

Figure 2: *Subject Matter Experts (SME) responses of certainty equivalents to 50-50 lotteries yield the best-fit utility function for Marine Builders Risk Insurance for the JSS contract. The error bars in the figure indicate the limit of consensus among the SME.*

purposes of this study, we take the 95% upper bound on the risk aversion of the PMO JSS's utility function which allows us to calculate that maximum insurance premium that PMO JSS is willing to pay at the 95% confidence level.

2.2 Loss probability analysis

As the Marsh reports [2], [3] make clear, there is little available data on which to base a detailed calculation of the real-world probability distribution of losses during ship construction. In addition, the construction of naval vessels presents shipbuilders with unique challenges, further limiting available data. Establishing the maximum insurance premium that DND is willing to pay for a given level of protection depends on two principal factors:

- the actual level of protection sought; and
- the risk premium that DND is willing to pay in addition to the expected loss based on the value at risk.

In this analysis, we also assumed that the first factor is determined by the maximum possible compensation available through the insurance provider. The Marsh reports [2, 3] indicated an initial estimate of 1.5 ships worth of coverage would be required by DND. We examined two possible scenarios: maximum compensation of 1.5 ships, and maximum compensation of 1 ship. Again, as a benchmark, we assumed that each vessel costs $500 million. The second factor requires an understanding of the probability distribution of loss. We reiterate that the Marsh reports indicate that little data is available from which to base a detailed calculation. Instead of treating the problem from a historical perspective, we use actuarial practices [8] that can provide the decision maker with a number of benchmark cases. Each benchmark case will require the decision maker's experience to place the information in the proper context of the larger insurance problem.

We assume that loss events arrive Poisson distributed and that the monetary loss is distributed as 1) an exponential distribution, and 2) two types of gamma distributions. These probability models contain reasonable features [8] that provide the decision maker with useful guidelines for the MBRI decision problem. The three distributions we use belong to the same general extreme value distribution class in that the tails of our model distributions diminish exponentially. Provided that the real-world distributions do not radically differ from our model distributions, we believe that our results establish useful benchmarks. We stress that the probability distributions that we used are meant to provide useful guidelines and do not represent actual "real-world" probabilities. However, even within this limited context, the distributions we have chosen are consistent with the arrival rate of rare events.

The choice of the Poisson distribution and the exponential distribution center on their memoryless property. In the context of shipbuilding loss incidents, the memorylessness of the distributions means that the waiting time between events does not depend on the amount of elapsed time and that the value of the loss does not depend on the amount previously lost. Thus, the expected time to a loss incident and the expected monetary loss does not change over the life of the project.

Strictly, memorylessness represents an approximation since it is reasonable to expect that construction firms will learn as they implement better practices over time. Thus, the probability of accidents and losses may diminish over the project lifetime. However, unless the learning effect is large, given that the arrival of loss events are rare, memorylessness represents a sound starting principle. Again, we stress that our methods are meant to supply benchmarks for PMO JSS decision makers based on their level of risk aversion and expert judgment.

While the memoryless distributions provide a starting point, we include two types of gamma distributions for monetary loss to show how the benchmark results change by relaxing the memoryless assumption. We feel that the gamma distribution yields a representative extension case since it is connected to the exponential distribution through a sum of independent exponentially distributed random variables. Furthermore, the gamma distribution has more freedom for tuning the expected size of the loss against the probability that the loss will exceed a critical value. Finally, the gamma distribution has an exponentially decaying tail which concords with the other distributions we use.

The central computational idea behind utility theory and the loss probability distributions concerns expected utility. In particular, we calculate the expected utility, which in words is:

> The *expected utility* is the sum of the weighted utility of losses up to the maximum insured loss plus the product of the utility of the maximum insured loss times the probability of any loss greater than the maximum insured loss.

and in mathematical symbols is:

$$\mathrm{E}(u_L) = \int_0^L u(x)p(x)\,dx + \int_L^\infty u(L)p(x)\,dx \tag{1}$$

where $\mathrm{E}(u_L)$ is the expected utility, L denotes the maximum insured loss, $p(x)$ denotes the loss distribution function, and $u(x)$ gives the utility function.

We find the insurance premium (the certainty equivalent) by using the inverse of the utility function on the expected utility, namely:

> The *insurance premium* is equal to the loss that has a utility equal to the expected utility eq.(1).

in mathematical symbols:

$$\hat{L} = u^{-1}\left[\mathrm{E}(u_L)\right]. \tag{2}$$

where $\hat{L}$ is the insurance premium. Note that in this case, the certainty equivalent represents the certain monetary loss that PMO JSS is willing to assume in order to escape the consequences of construction losses. In other words, the insurance premium represents the certain loss that PMO JSS is willing to accept. We provide the details of the calculation in Annex C.

3 Results

3.1 Benchmark cases

In this section we display the key results of our analysis in a set of actuarial tables generated from PMO JSS's utility function as elicited from the SME with NSPSO. The tables 2 to 7 are read as follows:

- Each row is specified by the expected number of claims per JSS contract against the MBRI policy (i.e., the expected number of loss events during construction). We table the results ranging from 3 to 1/5 expected claims per contract.
- Each column is specified by the expected monetary size of each claim, should a loss event occur during construction, for the type of loss distribution indicated. We table the results ranging from \$5–\$100 million in expected claim sizes.
- The values in the table correspond to the maximum premium, at the 95% confidence level as dictated by the uncertainty in SME responses, that PMO JSS is willing to pay for the loss distribution generated by the rows and columns.
- We calculate results for coverages of 1 ship and 1.5 ships.
- The two gamma distributions are distinguished by the probability of losing one or more ships during construction in a single loss event. We respectively set the probability of losing one or more ships in a single loss event at 10^{-3} (1 chance in 1,000) and 10^{-4} (1 chance in 10,000).

Four additional tables (8 to 11) are included to help the decision maker interpret the rows and columns for each probability distribution.

Tables 2 through 7 display the maximum insurance premium at the 95% confidence level for each probability distribution and coverage level. As an example, consider table 2, which displays the results for the exponential distribution with an insurance policy that provides protection of up to one ship. Suppose that the PMO JSS expects that on average they will make 2 claims against the MBRI policy and that PMO JSS also expects that on average each claim will be \$30 million. The table yields DND's maximum insurance premium at the 95% confidence level of \$64 million. The other actuarial tables are read in a similar fashion. Tables 8 through 11 provide ancillary information on the loss distributions. Table 8 gives the probability of observing a given number of loss events under the assumption of a given number of expected events. For example, if PMO JSS expects 2 claims, there is an 18% chance that PMO JSS will actually observe 3 loss events. Tables 9 through 11 indicate the skewness of the loss distributions. For example, in Table 10 (the gamma distribution with a 10^{-3} probability of losing one ship in a single event), if PMO JSS expects that on average each claim will be \$20 million, the probability that the claim will be less than \$20 million is 79%. Finally, in tables 9 through 11, the final column gives the probability that an individual claim will be less than the value of one ship (taken to be $500 million).

We stress that each table must be used in conjunction with the decision maker's best judgment. The tables do not provide the decision maker with a concrete answer as to how much insurance the JSS project requires. On the other hand, if the decision maker has an expert opinion on the expected number of claims and the expected size of each claim, then the decision maker can gain insight into the MBRI problem by using the tables across the representative probability distributions. We feel that the tables can help the decision maker understand which region within the tables best characterizes the probability of loss for the JSS project and thus provide an idea as to how much insurance would be appropriate given PMO JSS's risk tolerance.

Table 2: *Monetary losses in millions of dollars with MBRI offering one-ship coverage (exponential distribution assumed).*

Expected Number of Claims	Expected Size of a Single Claim ($ Millions)						
	5	10	20	30	40	50	100
3	15	30	62	95	129	163	290
2	10	20	42	64	87	110	212
1	5	10	21	32	43	55	115
1/2	3	5	10	16	22	28	59
1/5	1	2	4	6	9	11	24

Table 3: *Monetary losses with MBRI offering one-and-a-half ship coverage (exponential distribution assumed).*

Number of Claims	Size of a Single Claim ($ Millions)						
	5	10	20	30	40	50	100
3	15	30	62	95	130	166	338
2	10	20	42	64	87	111	235
1	5	10	21	32	43	55	121
1/2	3	5	10	16	22	28	61
1/5	1	2	4	6	9	11	25

Table 4: *Monetary losses with MBRI offering one-ship coverage (gamma distribution with* 10^{-3} *probability of losing one ship or more in a single event.)*

Number of Claims	Size of a Single Claim ($ Millions)						
	5	10	20	30	40	50	100
3	18	35	68	101	133	164	296
2	12	23	46	68	90	112	214
1	6	12	23	34	46	57	114
1/2	3	6	12	17	23	29	58
1/5	1	2	5	7	9	11	23

Table 5: *Monetary losses with MBRI offering one-ship coverage (gamma distribution with* 10^{-4} *probability of losing one ship or more in a single event.)*

Number of Claims	Size of a Single Claim ($ Millions)						
	5	10	20	30	40	50	100
3	17	33	66	99	131	163	300
2	11	22	44	66	88	110	215
1	6	11	22	33	44	56	113
1/2	3	6	11	17	22	28	57
1/5	1	2	4	7	9	11	23

Table 6: *Monetary losses with MBRI offering one-ship-and-a-half ship coverage (gamma distribution with 10^{-3} probability of losing one ship or more in a single event.)*

Number of Claims	**Size of a Single Claim ($ Millions)**						
	5	10	20	30	40	50	100
3	19	36	70	104	138	172	336
2	13	24	47	69	92	115	231
1	6	12	23	35	46	58	117
1/2	3	6	12	17	23	29	59
1/5	1	2	5	7	9	12	24

Table 7: *Monetary losses with MBRI offering one-ship-and-a-half ship coverage (gamma distribution with* 10^{-4} *probability of losing one ship or more in a single event.)*

Number of Claims	Size of a Single Claim ($ Millions)						
	5	10	20	30	40	50	100
3	17	33	66	99	133	167	334
2	11	22	44	66	89	111	228
1	6	11	22	33	44	56	115
1/2	3	6	11	17	22	28	57
1/5	1	2	4	7	9	11	23

Table 8: *Probability and the Number of Claims*

Expected Number of Claims	Probability of Occurrence for the Given Number of Loss Events			
	0	**1**	**2**	**3**
3	0.05	0.15	0.22	0.22
2	0.14	0.27	0.27	0.18
1	0.37	0.37	0.18	0.06
$1/2$	0.61	0.30	0.08	0.01
$1/5$	0.82	0.16	0.02	<0.01

Table 9: *Exponential Distribution and Claim Size*

Expected Loss ($ Millions)	Probability of a Claim Less Than the Given Amount	
	Expected Loss	**1 Ship ($500M)**
5	0.37	$1-10^{-44}$
10	0.37	$1-10^{-22}$
20	0.37	$1-10^{-11}$
30	0.37	$1-10^{-8}$
40	0.37	$1-10^{-6}$
50	0.37	$1-5\times10^{-5}$
100	0.37	$1-7\times10^{-3}$

Table 10: *Gamma distribution with* 10^{-3} *probability of losing one ship or more in a single event*

Expected Loss ($ Millions)	Probability of a Claim Less Than the Given Amount	
	Expected Loss	1 Ship ($500M)
5	0.93	$1-10^{-3}$
10	0.87	$1-10^{-3}$
20	0.79	$1-10^{-3}$
30	0.74	$1-10^{-3}$
40	0.70	$1-10^{-3}$
50	0.67	$1-10^{-3}$
100	0.60	$1-10^{-3}$

Table 11: *Gamma distribution with* 10^{-4} *probability of losing one ship or more in a single event*

Expected Loss ($ Millions)	Probability of a Claim Less Than the Given Amount	
	Expected Loss	1 Ship ($500M)
5	0.89	$1-10^{-4}$
10	0.82	$1-10^{-4}$
20	0.74	$1-10^{-4}$
30	0.69	$1-10^{-4}$
40	0.66	$1-10^{-4}$
50	0.64	$1-10^{-4}$
100	0.58	$1-10^{-4}$

4 Discussion and conclusions

While determining the correct amount of insurance coverage for a shipbuilding project represents a subjective task, utility theory can help establish consistent choices on the amount of coverage required. In our discussions with NSPSO SME, we discovered:

- We reject the hypothesis that the PMO JSS is risk-neutral (statistical significance level: 1.3%).
- We accept the hypotheses that the PMO JSS is risk-averse and that the aversion is independent of loss size (statistical significance level: 12.5%) .
- PMO JSS's utility function allows us to construct a set of actuarial tables that yield the maximum insurance premium that PMO JSS is willing to pay at the 95% confidence level for a given level of coverage. We list the tables in section 3.
- Each table requires expert opinion on the size of each claim and the number of claims expected over the life of the project. If the decision maker can delineate a portion of each table based on expert opinion, the actuarial tables will establish a range of insurance premiums consistent with PMO JSS's view of risk.

As a distribution-wide benchmark, if PMO JSS expects to make between one and three claims and PMO JSS expects that each claim size will be between \$5–\$20 million in value, then based on PMO JSS's view of risk, the maximum insurance premium that PMO JSS is willing to pay at the 95% confidence level lies in the approximate range of \$15–\$70 million. Further refinement will require expert opinion on the loss levels. Our values match the range given in the Marsh reports [2], [3], thus adding a consistency check to the earlier studies.

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Annex A: Subject matter expert preparatory documentation

This Annex contains the preparatory documents that we sent to the NSPSO senior staff prior to our risk assessment meetings. By providing a self contained introduction to utility theory, we hoped that the stakeholders would have a deeper understanding of the context of the risk assessment questions.

Subject Matter Expert Preparation Materials for Marine Builder's Risk Insurance Utility Analysis

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1 Introduction

While risk assessment often represents a highly subjective problem, techniques in utility theory can help ensure that the decision maker achieves consistent solutions. This document aims to inform the Subject Matter Expert (SME) on the central concepts of utility theory that apply to the National Shipbuilding Procurement Strategy Office's (NSPSO) insurance problem. The specific problem that we address for NSPSO centers on the level of Marine Builder's Risk Insurance (MRBI) required for the Joint Support Ship (JSS) contract. By focusing our efforts on the JSS, we can demonstrate the versatility of utility theory for a concrete problem faced by NSPSO.

1.1 Order and Value

Utility theory addresses the following problem: A decision maker is faced with several alternatives, each of which yields an outcome to which a decision maker attaches a value. Even if the decision maker is not certain of the consequences associated with each action, he must make a choice. In the context of the MBRI for the JSS, the alternatives are related to choosing the level of MBRI insurance coverage (including none) and the possible outcomes are various levels of loss – from the trivial to the catastrophic. The value of the loss can be assigned a monetary value and the likelihood of a given loss can be assigned a probability value.

The application of utility theory requires the decision maker to assign values to all possible outcomes. That valuation yields an explicit ordering of the alternative courses of action via the value of observable outcomes. We stress that no "correct" ordering exists – the ordering simply reflects the decision maker's preferences. However, if the decision maker cannot decide on his preference ordering, then utility theory cannot provide any help. Only once the decision maker has clearly stated his subjective preferences can we apply utility theory to ensure that choices among alternative courses of action remain consistent. In our insurance problem, we assume that NSPSO most prefers the smallest possible monetary loss in shipbuilding (zero) and least prefers the greatest possible loss (total destruction of the ship(s)). Between these two extremes we assume that the utility of the JSS always increases with decreasing loss.

1.2 Risky Benefit Versus Certain Cost

Suppose that the decision maker has a clear understanding of his preferred outcomes. We will have a worse outcome W and a better outcome B respectively. We now consider two options associated with an arbitrary consequence, C, taken from the decision maker's list:

- **Certain Option:** Receive consequence C for certain.
- **Risky Option:** Receive consequence B (better outcome) with probability P and consequence W (worse outcome) with probability $100\% - P$.

We see that the decision maker has a choice between a certain option C and a risky option in which the "prizes" W and B are determined through a "lottery". The risky option involves the probability P associated with the better outcome B. Clearly, if $P = 100\%$, the decision maker would choose the risky option (which would no longer be risky) as he would receive his better consequence B for sure. On the other hand, if $P = 0\%$ the decision maker would choose the certain option since in this case the risky option yields his worse consequence W for sure.

Notice that the decision maker's choice depended on the given probability P of his better outcome B. We see that there must exist some "indifference" probability between 0% and 100% at which the decision maker is indifferent between the certain option and the risky option. If we consider a list of risky versus certain options, then the resulting list of indifference probabilities allows us to construct the utility function.

In the context of the JSS scenario, the worse outcome W is a loss of 1.5 ships, the better outcome B is no loss, and the certain outcome C is the purchase of MBRI. The indifference probability corresponds to the ratio of the MBRI premium ($C \approx 45$ million dollars) to the value of the loss ($W \approx 1.5$ ships ≈ 650 million dollars).

1.3 Risk Tolerance

The indifference probability and, therefore, the value of the certain option relative to the risky option depends upon a decision maker's risk tolerance. A *risk neutral* decision maker will buy the certain option for exactly the amount of the expected loss, which is the probability of loss multiplied by the value of the loss. A *risk averse* decision maker will pay more than the expected loss to rid himself of the risky option, whereas a *risk prone* decision maker will pay less than the expected loss to do the same.

We can apply utility theory directly to the insurance problem faced by NSPSO. If we can extract NSPSO's utility function for the JSS, then we can calculate NSPSO's risk premium associated with individual loss events. This information will tell us the maximum insurance premium that NSPSO is willing to pay for specific loss amounts.

In the next section we will discuss in detail our methods for constructing NSPSO's utility function.

2 Methodology

In order to extract the JSS project-wide utility function, we will ask subject matter experts (SME) a series of questions pertaining to choices between lotteries. We will ask the questions in group/panel and individual format. It is imperative that all participants understand the purpose of the questions and that the participants feel comfortable with their responses. This section provides the details of our methods to inform the SME on what to expect.

Based on the information that we have received from NSPSO, we will assume that the cost of one Joint Support Ship is $500 million. We will also assume that the contract stipulates the construction of three ships.

2.1 Bracketed responses

In panel groups and with individual SMEs, we will begin the process by asking the following questions:

- Suppose that three ships are under construction and that we know there is a 50-50 chance that either the three vessels will be delivered without incident or there will be a $100 million dollar loss event (one fifth the amount of one ship). We assume that should the loss event occur, the loss will be exactly $100 million. What certain loss between the $100 million loss and full delivery would you accept such that you are indifferent between the lottery and the certain loss? We will call this response your first certainty equivalent.

- Consider your first certainty equivalent. Now suppose that we know for certain that there will be a loss incident (full delivery is now impossible). Suppose that there is a 50-50 chance that either the loss event will be $100 million dollars more than your first certainty equivalent or the loss event will be your first certainty equivalent exactly. What new certain loss event between the two outcomes of the 50-50 lottery would you accept that would make you indifferent between the lottery and your new certain loss. We will call this response your second certainty equivalent.

- We will repeat this process, each time considering a loss of $100 million dollars greater than your previous certainty equivalent. The questions will produce a series of certainty equivalents associated with bracketed loss events. The process will terminate when the SMEs no longer feel that there is any sense in discussing losses beyond a given amount.

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2.2 Probabilistic responses

In panel groups and with individual SMEs, we will begin the process by asking the following questions:

- Suppose that we know there is a 50-50 chance that either all ships will be delivered without incident or that one ship will be lost. What amount (or range) in insurance premium are you willing to pay to ensure the delivery of all ships?
- Suppose that we know there is a 10% chance that one ship will be lost. What amount (or range) in insurance premium are you willing to pay to ensure the delivery of all ships?
- Suppose that we know there is a 3% chance that one ship will be lost. What amount (or range) in insurance premium are you willing to pay to ensure the delivery of all ships?
- Suppose that we know there is a 1% chance that one ship will be lost. What amount (or range) in insurance premium are you willing to pay to ensure the delivery of all ships?
- We now repeat this process for a loss incident of 1.5 and 2 ships respectively. The SMEs should feel comfortable with the value or range in values of the insurance premiums that they give in each response.

3 Preparation for Analysis and Important Reminders

In preparing for the utility questioning, we have a few points that may help the SME focus during the question session and ensure consistent responses.

We must stress that in answering the utility questions, the SMEs should keep in mind all policy issues (PWGSC/DND) that apply to each question. We encourage the SMEs to familiarize themselves with all guidelines and policies that affect the risk assessment of the JSS project. Recall that we seek to find the utility function of the project as a whole, not the SME's personal value of money or loss. Thus, the SMEs should feel comfortable with their responses as applied to the shipbuilding project. If during the course of answering the questions the SMEs feel that they have learned something new about their preferences, as applied to the project, and they would like to change a previous response, please let us know. It is completely acceptable to revisit responses as the SMEs learn more about the problem. We expect that developing the utility function will be an iterative process.

If the SMEs feel uncertain about a response, it is completely acceptable for them to submit a monetary loss range as an answer. The SMEs should not feel compelled to give a definitive response if they only feel comfortable with a monetary range. Revealing the utility function will involve uncertainty in the responses and this information is important to the process. At no point in the process should the SMEs submit responses that create discomfort.

A Technical Discussion

Utility theory addresses the following problem: A decision maker is faced with several alternatives $A_1, A_2, \ldots, A_m$ each of which leads to a consequence describable in terms of a parameter X. The decision maker is not certain of the consequences associated with each action, yet he must choose among the possible alternatives. In our efforts to understand MBRI for the JSS, the parameter X is a monetary loss associated with a catastrophic event during ship construction and the possible alternatives are different insurance policies, the choice of location to build the ship, the choice of the primary contractor, etc. Ultimately, utility theory concerns choices and preferences between outcomes with certainty and "lotteries" in which the prizes are various consequences.

The application of utility theory requires the decision maker to construct an (ordinal) ranked list of the consequences. For example, if we suppose that we have n possible consequences, we assume that the decision maker understands his preferred outcomes, allowing him to write his preferences in order,

$$x_1 < x_2 < x_3 < \ldots < x_n. \tag{1}$$

We stress that no correct ordering exists – the ordering simply reflects the decision maker's preferences. However, if the decision maker cannot decide on his preference ordering, then utility theory cannot provide any help. Only once the decision maker has clearly stated his subjective preferences can we apply utility theory to ensure that choices among alternative courses of action remain consistent. In our insurance problem, we assume that NSPSO prefers the least possible monetary loss in shipbuilding. We should note that ordinal ranking does not attach a strength of preference in the ranking. For example, a consequence ranked 2 does not imply that the decision maker prefers it twice as much as a consequence that he ranked 4. The ranking only implies that the decision maker prefers the consequence ranked 2 to all other consequences except the consequence ranked 1.

Suppose that the decision maker has a clear understanding of his preferred consequences (e.g. eq.(1)). In his ranked list, he will have a worst consequence and a best consequence which we will denote as x_0 and x_n respectively. We now consider two options associated with an arbitrary consequence, x_i, taken from the decision maker's list:

- **Certain Option:** Receive consequence x_i for certain.
- **Risky Option:** Receive consequence x_n (best outcome) with probability p_i and consequence x_0 (worst outcome) with probability $1 - p_i$.

We see that the decision maker has a choice between a consequence for certain (the Certain Option) and a lottery (the Risky Option), which gives the decision maker a chance at receiving his best outcome, but a complementary chance at receiving his worst outcome. The risky option contains the probability p_i associated with the lottery. Clearly, if $p_i = 1$, the decision maker would choose the risky option (which would no longer be risky) as he would receive his best consequence for certain. On the other hand, if $p_i = 0$ the decision maker would choose the certain option since in this case the risky option yields his worst consequence. Notice that the decision maker switches his choice of option for $p_i = 1$ and $p_i = 0$ respectively. We see that there must exist some intermediate probability p_i between 0 and 1 which would cause the decision maker to become indifferent between the certain option and the lottery. Each consequence, x_i, will have a probability p_i which creates an indifference. Note that each p_i will depend on the subjective opinion of the decision maker. Consistency requires that the decision maker assigns $p_n = 1$ and $p_0 = 0$ – i.e., there is no option the decision maker would prefer if he is certain to receive his best outcome and the decision maker would prefer any option over the certainty of receiving his worst outcome. Thus the decision maker obtains a (cardinal) ranking of his respective probabilities associated with each consequence and a lottery in which he receives either his best or worst outcome:

$$0 = p_0 < p_1 < \ldots < p_n = 1. \tag{2}$$

The probability p_i associated with consequence x_i represents the *utility* of the consequence x_i with respect to the decision maker's best and worst outcomes x_n, and x_0. We will change the notation by writing p_i as u_i to explicitly denote utility.

Let us now suppose that the decision maker is faced with a number of alternatives that yield consequences $x_0, x_1, \ldots$ with uncertainty. For example, suppose an action, a, yields consequence x_0 with probability q_0, consequence x_1 with probability q_1, etc. Thinking back to our NSPSO insurance problem, we see that our consequences are monetary losses which allow us to compute the expected loss associated with the probabilities q_0, q_1, …, q_n:

$$\mathrm{E}(x_a) = q_0 x_0 + q_1 x_1 + \ldots + q_n x_n. \tag{3}$$

The subscript a in eq.(3) tells us that the probabilities of the consequences are contingent on selecting action a, and each consequence represents a loss amount associated with a loss event. On the other hand, the expected utility of the decision maker under the same action is

$$\mathrm{E}(u(x_a)) = q_0 u_0 + q_1 u_1 + \ldots + q_n u_n. \tag{4}$$

Notice that instead of weighting the probabilities in eq.(4) with each loss amount, as we did in eq.(3), we weight each probability with the decision maker's utility associated with each loss event. In general, the two expectations represented by eq.(3) and eq.(4) will not be the same. The expected utility allows us to find a *certainty equivalent* defined by the loss amount $\hat{x}$ such that the decision maker is indifferent between the uncertain outcome of the lottery created by his action a, and a certain loss of $\hat{x}$. That

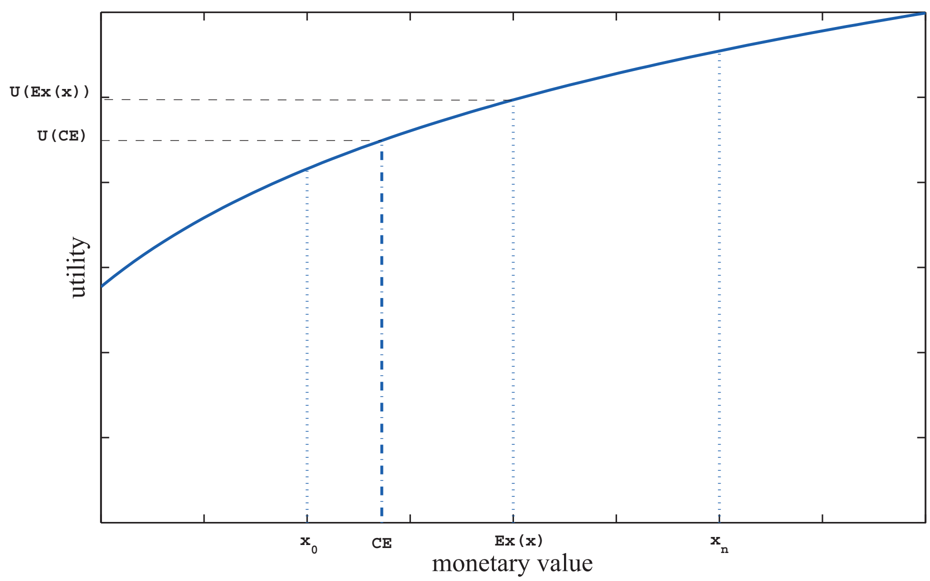

Figure 1: A utility function with the certainty equivalent (CE) shown with respect to the expectation of the lottery (Ex(x)). The two reference outcomes (best and worst cases) are x_n and x_0. The lottery has an expectation, Ex(x), for loss but the decision maker is willing to accept a greater loss for certain, CE, as opposed to playing a lottery that has a chance of losing even more money.

is, the decision maker regards the certain loss of $\hat{x}$ and the lottery as equivalent. The indifference results if the utility of $\hat{x}$ (the certain loss) matches the expected utility of the lottery,

$$u(\hat{x}) = \mathrm{E}(u(x_a)). \tag{5}$$

See figure 1 for a graphical representation of the certainty equivalent concept. The difference between the actual expectation of loss as the result of the decision maker's action and the certainty equivalent represents the risk premium of the decision maker,

$$RP(x_a) = \mathrm{E}(x_a) - \hat{x}. \tag{6}$$

A risk averse decision maker will pay more than the expected loss of the lottery, $\mathrm{E}(x_a)$, associated with his action, a, to rid himself of the financial responsibility of the uncertain outcome. The size of the decision maker's risk premium is a function of his utility. In general, the size of the risk premium also depends on the size of the expected loss.

We can interpret the risk premium from an insurance premium standpoint. Suppose that the expected loss for a project – based on empirical evidence – is $\$L$. If the decision maker is only willing to pay $\$L$ as an insurance premium, the decision maker has no

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risk premium associated with his utility function for the project, and consequently $RP = \$0$. Such a decision maker is said to be risk neutral. On the other hand, if the decision maker is risk averse, he will pay more than the expected project loss, $\$L$, to a third party to insure himself.

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Annex B: Utility function analysis

We begin our utility function analysis with establishing the distinction between risk neutrality and risk aversion. The precise mathematical formulation, derived from the associated utility functions can be found in [B.1]. For the single-variable utility function, $u(x)$, the definition of the risk aversion function $r(x)$ is given by

$$r(x) = -\frac{\frac{d^2u}{dx^2}(x)}{\frac{du}{dx}(x)}. \tag{B.1}$$

In the risk neutral case we have

$$r_N(x) \equiv 0 \tag{B.2}$$

where the subscript N denotes risk neutrality. The equation (B.1) and (B.2) yield the general form of the risk-neutral utility function

$$u_{\mathrm{N}}(x) = \alpha + \beta x \tag{B.3}$$

where α and β are constants (intercept and slope, respectively).

The constant risk averse case yields the risk aversion function

$$r_{\mathrm{A}}(x) = c \tag{B.4}$$

where c denotes a positive constant determined by the data, and the subscript A labels risk aversion. The integration of (B.1) with (B.4) yields

$$u_{\mathrm{A}} = a + be^{-cx} \tag{B.5}$$

where a and b are constants determined by the data. From (B.3) and (B.5), we see that a positive linear dependence characterizes a risk-neutral utility function, whereas an inverse exponential dependence characterizes a risk-averse utility function. We empirically test both risk-tolerance hypotheses; a standard χ^2 goodness-of-fit calculation quantitatively provided us with a test for statistical significance. We reject any hypothesis at the 5% level or less.

B.1 Calibration of utility

As the utility function does not have units, we require careful calibration. In the literature, we typically find

$$u(x_0) = 0 \text{ and } u(x_1) = 1 \ , \tag{B.6}$$

provides suitable arbitrary calibration points, where x_0 and x_1 are the boundaries of the x-interval of interest. In this study, we focus on losses L, which appear on the bounded interval zero loss to the maximum insured loss $-M$, namely

$$x = L,\ L_0 \le L \le L_1 \text{ where } L_0 = -M \text{ and } L_1 = 0\ . \tag{B.7}$$

In words, (B.7) indicates that L (the loss) has a maximum insured loss of $L_0 = -M$, which denotes the worst outcome, and L has a minimum of no loss $L_1 = 0$, which denotes the best outcome. Applying the boundary specifications, (B.7), to the calibration conditions (B.6), we have

$$u(-M) = 0 \text{ and } u(0) = 1 \; . \tag{B.8}$$

Using (B.8) in the risk neutral case with (B.3), and setting $x = L$ we obtain the calibrated form of $u_{\mathrm{N}}(L)$, namely

$$u_{\mathrm{N}}(L) = 1 + \frac{L}{M} \; . \tag{B.9}$$

Similarly, applying (B.8) to (B.5) we find the calibrated form of the risk averse utility $u_{\mathrm{A}}(L)$,

$$u_{\mathrm{A}}(L) = \frac{1 - e^{-c(L+M)}}{1 - e^{-cM}} . \tag{B.10}$$

B.2 The insurance premium

By definition [B.1], the certainty equivalent $\hat{L}_i$ solves the equation,

$$u(\hat{L}_i) = p_i \, u(L_i) \; + \; (1 - p_i) \, u(L_i') \tag{B.11}$$

where p_i denotes the probability of the loss L_i, $1 - p_i$ denotes the probability of the loss L_i', and $L_i \neq L_i'$. Since $\hat{L}_i < 0$ for all i in this study, we can interpret $\hat{L}_i$ as the insurance premium (the amount of money the client will pay to escape from the lottery). We define the Insurance Premium Function (IPF) as

$$f_u(p, L, L', \hat{L}) = p \, u(L) \; + \; (1 - p) \, u(L') - u(\hat{L}) \; , \tag{B.12}$$

which by (B.11), and (B.12) implies that

$$f_u(p, L, L', \hat{L}) \equiv 0 \; . \tag{B.13}$$

Subsituting the risk neutral utility function (B.9) into (B.12), we obtain the risk neutral IPF

$$f_N(p, L, L', \hat{L}) = p(L - L') + (L' - \hat{L}) \; , \tag{B.14}$$

where property (B.13) allows us to factor the constant $M > 0$ out of $f_N(p, L, L', \hat{L})$. Similarly, by substituting the risk averse utility function (B.10) into (B.12) we find the risk averse IPF, namely

$$f_A(p, L, L', \hat{L}) = p \left(e^{-cL'} - e^{-cL} \right) + e^{-c\hat{L}} - e^{-cL'} . \tag{B.15}$$

Again, property (B.13) allows us to factor the the non-zero constant $(1 - e^{cM})^{-1}$ out of $f_A(p, L, L', \hat{L})$.

B.3 Test of hypotheses

We test the risk-neutral and risk-averse hypotheses using the standard χ^2 goodness-of-fit test. In this case

$$\chi_u^2 = \sum_{i=1}^{4} \frac{\left[f_u(p_i, L_i, L_i', \hat{L}_i)\right]^2}{\operatorname{var}\left(f_u(p_i, L_i, L_i', \hat{L}_i)\right)} \tag{B.16}$$

where the subscript u denotes either the risk-neutral or the risk-averse IPF. The term $\operatorname{var}\left(f_u(p_i, L_i, L_i', \hat{L}_i)\right)$ gives the variance of the IPF, and the terms p_i, L_i, and L_i' for $i = 1,2,3,4$ were given to a panel of Subject Matter Experts (SME) who then provided the relevant $\hat{L}_i$. The SME also estimated the uncertainty $\operatorname{var}(\hat{L}_i)$ of $\hat{L}_i$, which allows us to estimate of the variance of the IPF [B.2], namely

$$\operatorname{var}\left(f_u(p_i, L_i, L_i', \hat{L}_i)\right) = \left(\frac{\partial f_u}{\partial \hat{L}_i}(p_i, L_i, L_i', \hat{L}_i)\right)^2 \operatorname{var}\left(\hat{L}_i\right). \tag{B.17}$$

Substituting (B.17) into (B.16) we find

$$\chi_u^2 = \sum_{i=1}^{4} \frac{1}{\operatorname{var}\left(\hat{L}_i\right)} \left[\frac{f_u(p_i, L_i, L_i', \hat{L}_i)}{\frac{\partial f_u}{\partial \hat{L}_i}(p_i, L_i, L_i', \hat{L}_i)}\right]^2. \tag{B.18}$$

Based on the SME's responses, we estimated the standard deviation (std) of $\hat{L}_i$ to be $\pm 5\%$ (ε) of the loss interval between L_i and L_i' (note that $0 \geq L_i > L_i'$)

$$\operatorname{std}\left(\hat{L}_i\right) = \varepsilon\left(L_i - L_i'\right) \tag{B.19}$$

The risk neutral χ_{N}^2 follows from the substitution of (B.9) into (B.18)

$$\chi_{\mathrm{N}}^2 = \frac{1}{\varepsilon^2} \sum_{i=1}^{4} \left[p_i + \frac{L_i' - \hat{L}_i}{L_i - L_i'}\right]^2, \tag{B.20}$$

and substituting (B.15) into (B.18), we obtain the risk-averse χ_{A}^2

$$\chi_{\mathrm{A}}^2 = \frac{1}{c^2 \varepsilon^2} \sum_{i=1}^{4} \left[1 - p_i e^{c(\hat{L}_i - L_i)} - (1 - p_i) e^{c(\hat{L}_i - L_i')}\right]^2. \tag{B.21}$$

The significance level, $\mathscr{P}$, for a χ^2 value over ν degrees of freedom is given by [B.2]

$$\mathscr{P}\left(\chi^2, \nu\right) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_{\chi^2}^{\infty} dt\, t^{(\nu-2)/2} e^{-t/2} \tag{B.22}$$

where $\Gamma(z)$ denotes the usual gamma function,

$$\Gamma(z) = \int_0^{\infty} dt\, t^{z-1} e^{-t}. \tag{B.23}$$

Table B.1 shows the data used to test the hypotheses, and Table B.2 shows the results of the tests. The test of the risk neutral hypotheses lead us to reject risk neutrality, whereas we accept the risk-averse hypothesis. Figure B.2 shows the residuals of $f_{\mathrm{A}}(p_i, L_i, L_i', \hat{L}_i)$ for $i = 1,2,3,4$ from its least squares best-fit (minimization of χ^2_{N}), which yields $c = 1.10 \times 10^{-3}$ per million lost; c has a 95% confidence interval from 0.28×10^{-3} to 1.91×10^{-3} per million lost. We use the 95% upper bound on c to calculate the insurance premiums in this study.

Table B.1: *The data used to test the hypotheses.*

		Losses in Millions		
i	p_i	L_i	L_i'	$\hat{L}_i$
1	0.5	-750	0	-400
2	0.5	-400	0	-250
3	0.5	-250	0	-150
4	0.5	-750	-400	-600

References

[B.1] Ralph L. Keeny and Howard Raiffa, *Decisions with Multiple Objectives: Preferences and Tradeoffs*, Cambridge University Press, 1993.

[B.2] R. J. Barlow, *Statistics: A Guide to the Use of Statistical Methods in the Physical Sciences*, John Wiley & Sons, Chichester, UK, 1999.

Table B.2: *Results from the tests of the hypotheses.*

	Risk Tolerance	
Result	**Neutral (N)**	**Averse (A)**
χ_u^2	12.74	5.74
ν	4	3
$\mathscr{P}$	0.0127	0.1250
Decision	Reject $u_{\mathrm{N}}(L)$	Accept $u_{\mathrm{A}}(L)$

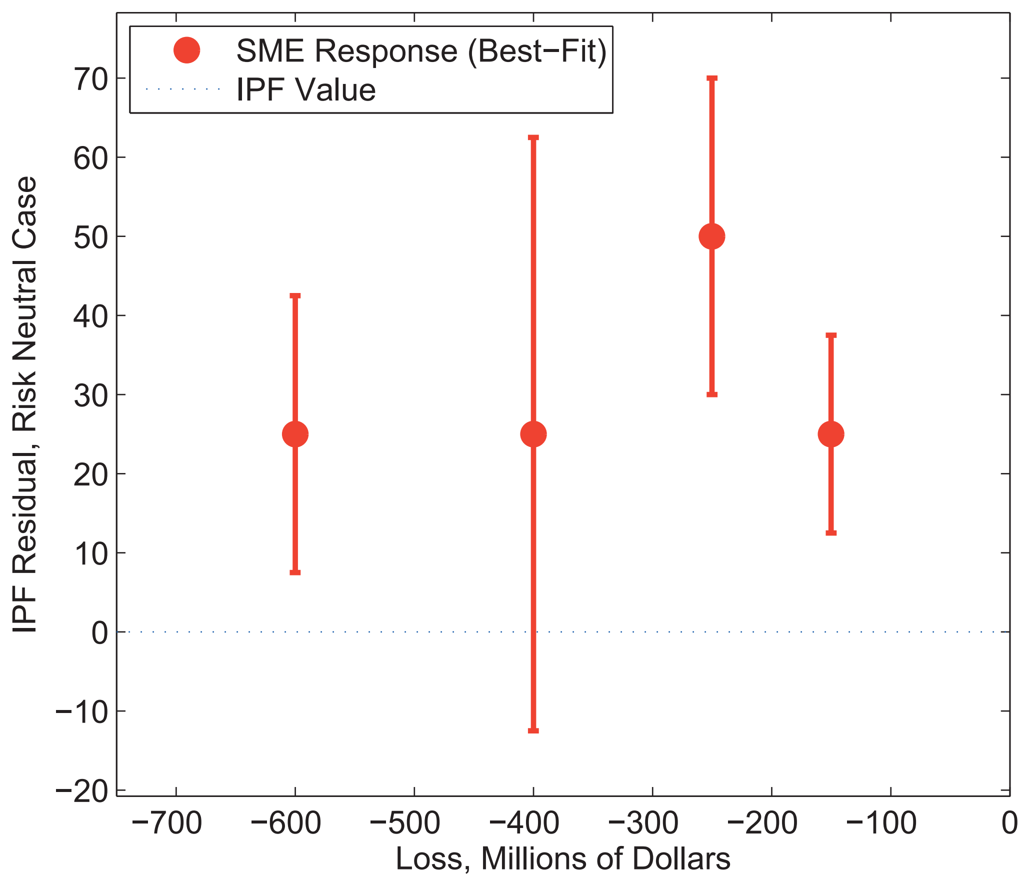

Figure B.1: *The residuals of the Insurance Premium Function (IPF) for the risk neutral case defined by equation (B.14).*

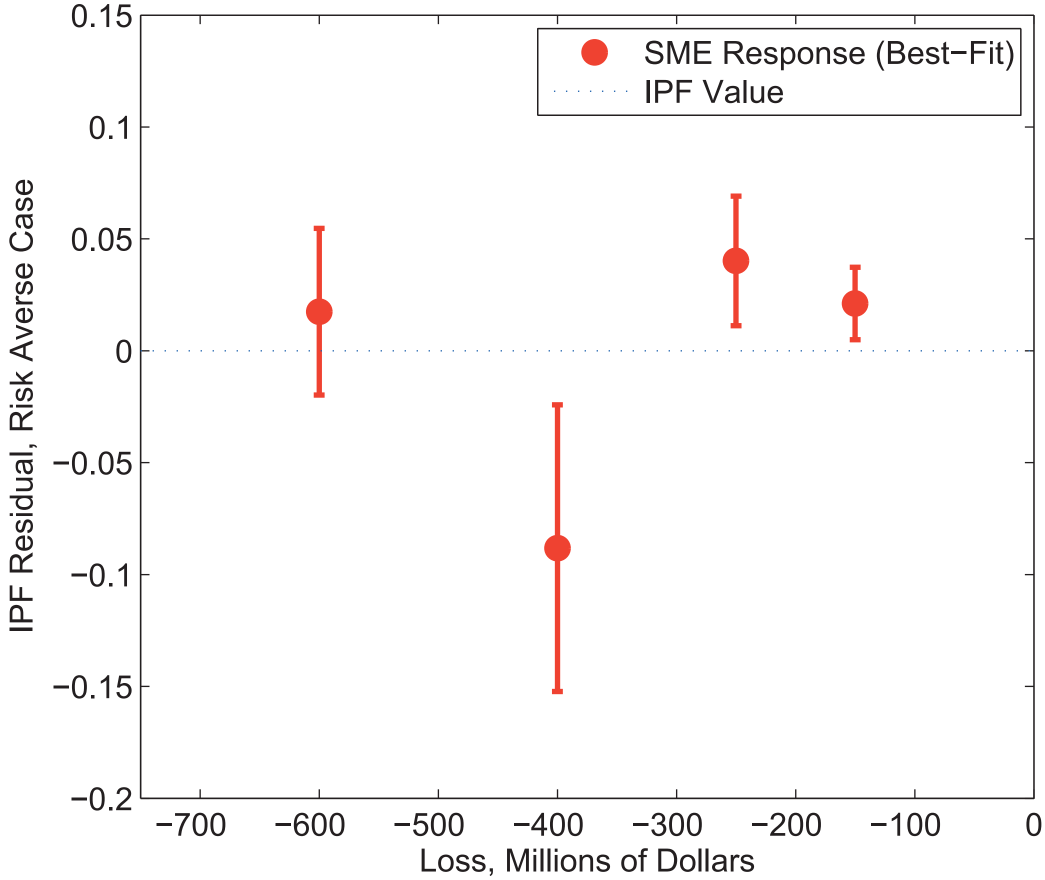

Figure B.2: *The residuals of the Insurance Premium Function (IPF) for the risk averse case defined by equation (B.15).*

Annex C: Probability distribution analysis

The Marsh reports [2], [3] indicates that little or no data exists for loss rates on naval shipbuilding enterprises. Nevertheless, we can proceed by making a series of sound actuarial assumptions. If we assume that major losses (events that would trigger a claim against the MBRI policy) follow distributions generally associated with rare events, we can construct a number of actuarial tables that, when interpreted through expert opinion, provide insight into the insurance problem.

We assume the loss rate is governed by two distributions. First, we assume that the incident arrival process obeys the Poisson distribution. By using the Poisson process, we model the incident arrivals in a memoryless fashion, implying that the expected waiting time between incidents does not depend on the amount of elapsed time. The memoryless property provides a basic starting point for rare events. Second, we assume that the monetary loss distribution follows either an exponential distribution (again memoryless but in a continuous sense) or a gamma distribution. Each case used in conjunction with the Poisson arrival process allows us to construct actuarial tables based on NSPSO's utility function. Once we construct the overall unconditional probability distribution, we can write the insurance premium as

$$\hat{x} = u^{-1}\left[\int_0^L p(x)u(x)\,dx + \int_L^\infty p(x)u(L)\,dx\right], \tag{C.1}$$

where $u(x)$ and $u^{-1}(x)$ denote the utility function and the inverse utility function respectively, $p(x)$ labels the probability distribution, and $\hat{x}$ gives the insurance premium that PMO JSS is willing to pay. The parameter L denotes the maximum coverage provided (e.g. 1.5 ships).

We construct the unconditional probability distribution from the conditional loss distributions, namely,

$$h(x) = \sum_{k=0}^{\infty} f(x|k)g(k), \tag{C.2}$$

where $g(k)$ denotes the Poisson incident arrival process and $f(x|k)$ denotes the monetary loss distribution conditioned on observing k incidents. Note that k is an integer ($k \in \{0,1,2,\ldots\}$). The Poisson distribution, $g(k)$, is given by

$$g(k) = \frac{\lambda_1^k \exp(-\lambda_1)}{k!}, \tag{C.3}$$

where λ_1 is a model parameter for the distribution. Since the Poisson distribution is a single parameter model, we can fully characterize the distribution by observing a single moment (e.g. the mean). To begin, we will assume an exponential distribution for the monetary loss associated with a single incident, namely

$$f(x|1) = \lambda_2 \exp(-\lambda_2 x). \tag{C.4}$$

As with the Poisson distribution, we can determine the parameter λ_2 from the observation of a single moment (e.g. the mean). The exponential distribution models the losses in a memoryless manner but the distribution also implies that the probability of observing a large loss diminishes exponentially. In lieu of any concrete data, we feel that the exponential distribution provides a useful benchmark distribution. Multiple loss events during construction correspond to the sum of exponential distributed random variables (each loss arrives via the Poisson process leading to the possibility of multiple claims over the life of the project) which leads to the Erlang distribution,

$$f(x|k) = \frac{\lambda_2^2 x^{k-1} \exp(-\lambda_2 x)}{(k-1)!}, \tag{C.5}$$

where $k \geq 1$ refers to the number of events. Note that when $k = 1$ we recover the exponential distribution.

We are now in a position to assemble all the parts. If no loss incident arrives, no monetary loss occurs and all ships are delivered. The no loss event scenario corresponds to the arrival of zero incidents from the Poisson distribution. Thus the sum over the conditional distributions, eq.(C.2) yields,

$$h(x) = \delta^+(x)\exp(-\lambda_1) + \sum_{k=1}^{\infty} \frac{\lambda_2^2 x^{k-1} \exp(-\lambda_2 x)}{(k-1)!} \cdot \frac{\lambda_1^k \exp(-\lambda_1)}{k!}, \tag{C.6}$$

where $\delta^+(x)$ corresponds to the Dirac-delta function with the property

$$1 = \int_0^{\infty} \delta^+(x)\, dx. \tag{C.7}$$

The Dirac-delta function places all the weight from the monetary loss distribution at zero if no loss occurs.

Eq.(C.6) gives the expression for the full unconditional probability distribution of loss derived from a Poisson arrival distribution with an exponential monetary loss distribution. Notice that each conditional distribution requires one parameter. Thus, once we know the mean of the arrival rate and the mean of the loss level, we can construct the entire distribution. We expect this behaviour since both distributions are memoryless. On the other hand, the exponential distribution for the monetary losses may underestimate the probability of large losses (the exponential may diminish too quickly in the large loss region). In order to correct for this potential shortcoming, we can extend our results to include the gamma distribution, which contains two parameters (and thus no longer memoryless). In this case,

$$\begin{aligned} f(x|1) &= \frac{x^{\alpha-1}\exp(-x/\theta)}{\theta^{\alpha}\Gamma(\alpha)}, \\ f(x|k) &= \frac{x^{k\alpha-1}\exp(-x/\theta)}{\theta^{k\alpha}\Gamma(k\alpha)}, \end{aligned}$$

where α and θ denote the parameters of the gamma distribution (which in our model become fixed by the expected number of loss events, the expected size of each claim, and the probability that a loss event exceeds a given amount), and $\Gamma(x)$ denotes the usual gamma function. Applying the conditioning approach of eq.(C.2) once more, we arrive at the full unconditional distribution for the combined Poisson incident arrival process along with the gamma function for monetary loss,

$$h(x) = \delta^{+}(x)\exp(-\lambda_1) + \sum_{k=1}^{\infty} \frac{x^{k\alpha-1}\exp(-x/\theta)}{\theta^{k\alpha}\Gamma(k\alpha)} \cdot \frac{\lambda_1^k \exp(-\lambda_1)}{k!}, \tag{C.8}$$

where λ_1 labels the parameter for the Poisson distribution. Since the gamma distribution contains two parameters, we require two pieces of information to fix the distribution. In our actuarial tables, we determine the parameters of the gamma distribution by demanding that the probability of observing an individual loss that exceeds a given critical value occurs at a given probability, and by requiring that the expectation of the size of the monetary loss is given. We vary the parameters of the model to generate tables of insurance premiums. Tables 5 to 8 contain the model details for the gamma distribution.

List of Acronyms

CE	Certainty Equivalent
DMGOR	Directorate Materiel Group Operational Research
DND	Department of National Defence
DRDC	Defence Research and Development Canada
IPF	Insurance Premium Function
JSS	Joint Support Ship
PMO JSS	Project Management Office Joint Support Ship
PWGSC	Public Works and Government Services Canada
MBRI	Marine Builder’s Risk Insurance
NSPSO	National Shipbuilding Procurement Strategy Office
SME	Subject Matter Expert

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The Department of National Defence's (DND) procurement process for the Joint Support Ships (JSS) requires a quantitative understanding of the Project Management Office's (PMO) tolerance for risk. By establishing the risk tolerance of the stakeholders, we can place bounds on the requisite amount of insurance for the JSS project. In particular, the acquisition of Marine Builder's Risk Insurance (MBRI) provides protection for the shipbuilding capital investment during the construction phase. Using utility theory, we obtain decisive results regarding the risk tolerance of the PMO JSS. Our tests yield the necessary statistical significance to reject the hypothesis of risk neutrality and to accept the hypothesis of risk aversion. Using a least-squares best-fit parameterization, we quantify PMO JSS's risk tolerance which allows us to construct a series of actuarial tables in conjunction with standard loss distributions for rare events. The decision maker can use the actuarial tables with expert opinion to determine the correct level of MBRI required for the JSS project.

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