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Abstract

In this report, first-order cyclostationarity of M-ary frequency shift keying (M-FSK) signals affected by additive Gaussian noise, phase, frequency offset and timing errors is investigated and applied to recognizing the modulation order, M. The number of first-order cycle frequencies (CFs) detected in the received FSK signal is employed as discriminating feature for modulation order recognition. Based on this feature, a recognition algorithm is proposed, which does not require timing and carrier recovery, and estimation of signal and noise powers as preprocessing tasks. In addition, the algorithm is developed with no prior information on the CFs at the receive-side. Nevertheless, the hypothetical case when knowledge of CFs is available at the receive-side is also investigated, as providing a benchmark for recognition performance evaluation. A theoretical performance analysis is carried out, and simulations are run to verify theoretical developments.

Résumé

Dans le présent rapport, on s'intéresse à la cyclostationnarité d'ordre 1 des signaux de modulation par déplacement de fréquence à base M (MDF-M) soumis à un ajout de bruit gaussien, de phase, de décalage de fréquence et d'erreurs de synchronisation comme élément de reconnaissance de l'indice de modulation (M). Le nombre de fréquences de cycle (FC) d'ordre 1 détectées dans le signal MDF reçu est employé comme élément de discrimination pour la reconnaissance de l'indice de modulation. Grâce à cet élément, on obtient un algorithme de reconnaissance qui ne nécessite aucune récupération de rythme et de porteuse ni d'estimation des puissances du signal et du bruit à l'étape de prétraitement. De plus, on obtient l'algorithme en question sans même disposer préalablement de données sur les FC à la borne réceptrice. Néanmoins, l'éventualité où l'on disposerait des données sur les FC à la borne réceptrice est également examinée afin de se donner un point de référence en vue de l'évaluation du rendement de la reconnaissance. On effectue une analyse de rendement théorique et des simulations afin de vérifier les avancées théoriques.

I. INTRODUCTION

Blind modulation classification (MC) is an important and challenging topic, which has been studied for both commercial and military applications, such as spectrum surveillance, cognitive radio, and electronic warfare. As an intermediate step between signal detection and demodulation, MC represents a major task of an intelligent receiver. Design of a modulation classifier essentially involves two steps, signal preprocessing and proper selection of a classification algorithm. Preprocessing tasks can include, but are not limited to, carrier and timing recovery, estimation of signal and noise powers, etc. Depending on the classification algorithm used in the second step, different preprocessing tasks can be required. Classification algorithms which rely less on preprocessing are of interest, especially in non-cooperative environments, where no prior information on the incoming signal is available. Two general classes of classification algorithms can be crystallized, likelihood-based (LB) and feature-based (FB) algorithms. The LB method is based on the likelihood function of the received signal and the likelihood ratio test is used for decision. With the FB approach, the decision is made based on features extracted from the incoming signal. Examples of features are moments, cumulants, cyclic moments and cyclic cumulants of the received signal, statistics of the instantaneous amplitude, frequency, and phase, signal wavelet transform (WT), and so on. An overview of MC techniques is reported in [1].

In this report, we tackle the problem of recognizing the modulation order of an FSK received signal, affected by additive Gaussian noise, phase, frequency offset and timing errors. The first-order cyclostationarity of the M-ary FSK signal is exploited for the recognition of its modulation order, M, with the number of first-order cycle frequencies employed as feature. In the open literature, signal cyclostationarity has been explored for classification of linear digital [2]-[6] and analog modulations [7]-[8]. On the other hand, classification of the modulation order of an FSK signal is also studied in the literature, as follows. In [9]-[10], Beidas and Weber explore the link between the LB and FB approaches. Starting from an expansion of the likelihood function, a correlation-based classification algorithm is proposed under the assumption of timing errors and uniformly distributed time-invariant phase over each symbol. In addition, robustness to a carrier frequency offset is claimed. However, the algorithm is developed for a specific frequency deviation of the FSK signal. Hsue and Soliman [11] propose an algorithm based on the histogram of the zero-crossing interval, whereas Ho, Prokopiw and Chan [12] exploit the number of levels in the WT magnitude to recognize the modulation order *M* of an *M*-ary FSK received signal. Both these algorithms require estimation of the signal-to-noise ratio (SNR) and timing recovery in the preprocessing step. On the other hand, the algorithm proposed in this report requires neither the SNR estimation, nor timing and frequency recovery. Furthermore, the frequency deviation is not a priori known.

The rest of the report is organized as follows. The signal model and its first-order cyclostationarity are presented in Section II. The proposed cyclostationarity-based classification algorithm is introduced in Section III. A theoretical performance analysis is performed in Section IV, and simulation results are discussed in Section V. Finally, conclusions are drawn in Section VI. In addition, cyclostationarity temporal parameters and their estimators are defined in Appendix A, mathematical derivations regarding the first-order cyclostationarity of the FSK signals are reported in Appendix B, and a cyclostationarity test and the probability to detect a cycle frequency are presented in Appendix C.

II. SIGNAL MODEL AND CORRESPONDING STATISTICAL CHARACTERIZATION

A. Signal Model

Let the baseband waveform of a received *M*-FSK signal be the sum,

$$r(t) = \sqrt{S} e^{j\theta} e^{j2\pi\Delta f t} \sum_i e^{j2\pi f_\Delta s_i (t - iT - \varepsilon T)} u_T(t - iT - \varepsilon T) + w(t), \tag{1}$$

where S is the signal power, θ is the carrier phase, Δf is the carrier frequency offset, f_Δ is the frequency deviation, T is the symbol period, $0 \le \varepsilon < 1$ represents the timing error, $u_T(t)$ is the rectangular pulse of amplitude 1 and duration T, $w(t)$ is zero-mean complex Gaussian noise of power N, s_i is the symbol transmitted within the i th period and $j = \sqrt{-1}$. The data symbols $\{s_i\}$ are assumed to be zero-mean independent and identically distributed random variables, with values drawn from the alphabet $A_{M-\text{FSK}} = \{\pm 1,\ \pm 3,\ ...,\ \pm (M\ \text{-}\ 1)\}$, with the alphabet size, M , as a power of 2.

At the receive-side, normalization of the baseband signal is carried out with respect to the received signal power to remove any scale factor from data, and then, sampling is performed at a sampling rate f_s, yielding the discrete-time normalized signal

$$r[k] = r(t)/\sqrt{S+N}\,|_{t=kf_s^{-1}} . \tag{2}$$

B. First-Order Cyclostationarity of M-FSK Signals

According to results derived in Appendix B, if $f_\Delta = lT^{-1}$, l integer, the received FSK signal exhibits first-order cyclostationarity[1]. The first-order/ zero-conjugate cyclic cumulant (CC) [1]of the discrete-time normalized signal, and the set of cycle frequencies (CFs) [1] are given respectively as

$$c_r(\beta)_{1,0} = M^{-1}\sqrt{S/(S+N)}\ e^{j\theta}e^{-j2\pi\gamma f_s T\varepsilon} , \tag{3}$$

and

$$\beta = \gamma + f_s^{-1}\Delta f \text{, with } \gamma = pT^{-1}f_s^{-1} \text{ and } p = \pm l,...,\pm(M-1)l ,\ \beta \in [-1/2, 1/2) . \tag{4}$$

According to (3), the first-order/ zero-conjugate CC at CF β, $c_r(\beta)_{1,0}$, depends on the modulation order M , signal-to-noise ratio ($\text{SNR} := S/N$), carrier phase θ, timing error ε, product $f_\Delta T$ and alphabet $A_{M-\text{FSK}}$ (both through $\gamma f_s T$). To be noted that there is no additive contribution of the noise to the first-order CC of the *M*-FSK received signal. On the other hand, the CC magnitude,

$$|\, c_r(\beta)_{1,0} \,| = M^{-1}\sqrt{S/(S+N)}, \tag{5}$$

depends only on the modulation order M and SNR, and decreases with an increase in *M* and a decrease in the SNR. In addition, one can easily notice from (4) that the number of first-order CFs is equal to the modulation order M , and for a certain M , the CFs depend on the carrier frequency offset Δf , frequency deviation f_Δ and alphabet $A_{M-\text{FSK}}$ (both through γ), and sampling frequency f_s .

[1] Signal cyclostationarity, its temporal parameters, as well as their estimators, are defined in Appendix A.

III. Cyclostationarity-Based Classification Algorithm

In the sequel, a cyclostationarity-based algorithm is proposed to identify the modulation order M of an M-ary FSK received signal, affected by affected by additive Gaussian noise, phase, frequency offset and timing errors. Classification is performed under the assumption that FSK received signals with different modulation orders have the same bandwidth. At the receive-side, the incoming signal bandwidth is roughly estimated, and a filter is used to remove the out of band noise. The signal is down-converted, normalized, sampled, and processed to extract information employed for modulation classification. Here, the number of first-order CFs is the feature extracted from the incoming signal, and exploited to decide on the modulation order, M, of an FSK received signal. With the proposed classification algorithm, information extraction is performed without a priori knowledge of received signal CFs. Furthermore, carrier and timing recovery are not required as preprocessing steps. The method is of great interest for applications in non-cooperative environments, where no prior information on the incoming signal is available. Subsequently, we also discuss the hypothetical case of known CFs, as representing a benchmark for classification performance evaluation.

A. Unknown Cycle Frequencies Scenario

The algorithm proposed to identify the modulation order of an FSK received signal consists of the following two steps.

Step I: The magnitude of the first-order/ zero-conjugate CC of the normalized received signal is estimated based on a K sample observation interval, at candidate CFs, $\beta^{'}$, over the range $[-1/2,1/2)$. According to (3)-(4), there is no noise contribution to the first-order/ zero-conjugate CC, and its magnitude is non-zero only at CFs $\{\beta \,|\, \beta \in [-1/2,1/2),\ \beta = \gamma + f_s^{-1}\Delta f, \gamma = pT^{-1}f_s^{-1},\ p = \pm l,\ldots,\pm(M-1)l\}$ [2]. However, when estimating the first-order CC from a finite length data sequence (K samples), a small, but non-zero residual additive noise contribution appears and the estimated magnitude of the CC are also non-zero at candidate

[2] Results derived in Appendix B and presented in Section II.B need to be understood as asymptotical values, which are obtained by averaging over an infinite-time interval.

CFs other than CFs[2]. These non-zero values of the CC magnitude are statistically insignificant, and appear only as a result of estimation based on a finite length data sequence[2]. As the SNR decreases, the CC magnitude at CFs, $|c_r(\beta)_{1,0}|$, decreases, and below a certain SNR, it becomes comparable to the non-zero statistically insignificant estimated CC magnitude at other candidate CFs[3].

A cut-off value, V_{co}, is set such that the non-zero statistically insignificant estimated CC magnitudes occur below V_{co}. Apparently, the longer the length of data sequence at the receive-side, the more accurate the CC estimates (asymptotically, the CC estimate at candidate CFs different than CFs are zero), and the lower the value of V_{co} than can be set. By using (5), one can easily obtain the theoretical value of the SNR for which the CC magnitude at CFs equals V_{co},

$$\mathrm{SNR}_{co} = ((MV_{co})^{-2} - 1)^{-1}. \tag{6}$$

Note that theoretically, the CC magnitudes at CFs drop below V_{co} for SNR<SNR_{co}, whereas they lie above V_{co} for SNR>SNR_{co} [2]. Practically, if the observation interval is sufficiently large, the estimated CC magnitudes at all *M* CFs lie indeed above V_{co} at high enough SNR, whereas some of them can drop below V_{co} for SNR around, but above SNR_{co}, and a few can slightly exceed V_{co} at SNR around, but below SNR_{co} [3]. Candidate CFs which correspond to CC magnitudes above or equal to V_{co} are selected for testing in the next step. It is to be noted that selection of these candidate CFs does not require knowledge of the frequency deviation, signal and noise powers, and frequency and timing recovery.

Step II: Candidate CFs selected in Step I, are verified in Step II by using the cyclostationarity test developed in [14][4]. The decision on the modulation order *M* of an FSK received signal is made based on the criterion,

If $M/2+1 \le N_{cf} \le M$, the modulation order is M [5], (7)

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[3] Examples will be given for illustration in Section V.B.
[4] The cyclostationarity test applied to first-order candidate CFs is presented in Appendix C.
[5] Note that with a sufficiently large observation interval and for high enough SNR, the number of candidate CFs which are tested in Step II is equal to *M* and all are declared to be indeed CFs. Thus, at such SNRs, a single candidate CF can be tested instead of *M*, which yields a simplification of the decision criterion. However, this does not remain valid as the SNR

where N_{cf} is the number of tested candidate CFs, which are decided to be CFs of the received signal. Note that if $N_{cf} < 2$, no decision on the modulation order is made. The reason for that is that other signals exhibit first-order cyclostationarity with a single CF ($N_{cf} = 1$), such as amplitude modulation (AM), and others, such as double side-band (DSB), single side-band (SSB), M-ary phase shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM), do not exhibit first-order cyclostationarity ($N_{cf} = 0$) [7]-[8]. Note that the algorithm can be accordingly extended to recognize AM, as well as the class of DSB, SSB, M-PSK and M-QAM signals. However, as previously mentioned, here we focus only on the recognition of the modulation order, M, of an FSK received signal.

The block diagram of the first-order cyclostationarity- based algorithm, proposed for the recognition of the modulation order of an FSK received signal, is shown in Fig. 1.

B. Known Cycle Frequencies Scenario

With known CFs, recognitionof the modulation order of a received FSK signal is formulated as a multiple hypothesis-testing problem. Under each hypothesis, the signal is assumed to have a specific modulation order, and the CFs corresponding to that modulation order (see (4) for the analytical expression) are tested as candidate CFs of the received signal. The cyclostationarity test presented in Appendix C is used for the candidate CFs testing, and the criterion in (7)[5] is employed to make a decision on the modulation order under each hypothesis. Note that if the sets of CFs corresponding to FSK signals with different modulation orders are mutually exclusive, no positive decisions on the two modulation orders (under two different hypotheses) can be simultaneously made. On the other hand, if the set of CFs of an M'-FSK signal is included into the set of CFs of an M''-FSK signal ($M'' > M'$), positive decisions on both modulation orders (under two different

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decreases, and all candidate CFs need to be tested. As the SNR is not estimated in the preprocessing step, this criterion will be applied for the whole SNR range.

hypotheses) can occur, but the positive decision on the modulation format M'' excludes that on M'. If $N_{cf} < M/2+1$ under all hypotheses, the modulation order is not identified.

IV. THEORETICAL PERFORMANCE ANALYSIS

A theoretical performance analysis of the proposed classification algorithm is subsequently performed, with the probability of correct signal classification, i.e., the probability to identify an *M*-FSK signal, when indeed such a signal is transmitted, $P_{cc}^{(M\text{FSK}|M\text{FSK})}$, as performance measure.

A. Unknown Cycle Frequencies Scenario

In this scenario, the number of CFs of the received FSK signal is determined by applying the cyclostationarity test to candidate CFs at which the magnitudes of the estimated first-order/ zero-conjugate CCs are above the cut-off value, V_{co}. Then, criterion (7)[5] is applied to decide the modulation order, *M*. Theoretically, no candidates are tested if $\text{SNR} < \text{SNR}_{co}$ and the modulation order of an FSK signal cannot be identified for this SNR range, leading to $P_{cc}^{(M\text{FSK}|M\text{FSK})} = 0$. On the other hand, if $\text{SNR} \geq \text{SNR}_{co}$, only CFs are tested as candidates. With the decision criterion (7) and under the assumption of independent detection of CFs, the expression of $P_{cc}^{(M\text{FSK}|M\text{FSK})}$ can be easily written as

$$P_{cc}^{(M\text{FSK}|M\text{FSK})} = \prod_{\nu=1}^{M} P_d^{(\nu)} + \sum_{\nu_1=1}^{M} \prod_{\substack{\nu_2=1,\\ \nu_2\neq\nu_1}}^{M} P_d^{(\nu_2)}(1-P_d^{(\nu_1)}) + \sum_{\substack{\nu_1,\nu_2=1\\ \nu_2\neq\nu_1,\\ \nu_2>\nu_1}}^{M} \prod_{\substack{\nu_3=1,\\ \nu_3\neq\nu_1,\\ \nu_3\neq\nu_2}}^{M} P_d^{(\nu_3)}(1-P_d^{(\nu_1)})(1-P_d^{(\nu_2)}) + \ldots, \quad (8)$$

where $P_d^{(\nu)}$ [6], $\nu = 1,\ldots,M$, is the probability of detection of a CF, and only the first $M/2$ terms are retained in the right hand-side of the equation for the modulation format M. For example, for a 2-FSK signal (M=2), only the first term will be kept, and (8) becomes $P_{cc}^{(2\text{FSK}|2\text{FSK})} = P_d^{(1)} P_d^{(2)}$. It is noteworthy that at high enough SNRs, at which the magnitude of estimated CC at CFs are above V_{co}, each of the M CFs is detected with probability one[5] and the probability to decide that an *M*-FSK signal is transmitted when indeed such a signal is transmitted is equal to the probability of detection of each CF, $P_d^{(\nu)}$ [6], $\nu = 1,\ldots,M$, which is equal to one.

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[6] The expression of the probability of detection of a given CF, $P_d^{(\nu)}$, $\nu = 1,\ldots,M$, is given in Appendix C.

B. Known Cycle Frequencies Scenario

In this scenario, under each hypothesis associated with a candidate modulation order, the CFs corresponding to that modulation order are tested to find the number of CFs of the FSK received signal, and then criterion (7)[5] is applied for decision-making. Apparently, the theoretical performance analysis performed under the unknown CFs scenario remains valid in this case, except that (8) also applies for $\mathrm{SNR} < \mathrm{SNR}_{co}$ (no cutoff value, V_{co}, is used under the known CFs scenario).

V. SIMULATION RESULTS

A. Simulation Setup

Simulation experiments have been performed to confirm the theoretical developments of the proposed *M*-FSK classification algorithms, by considering $M =$ 2, 4 and 8 as candidate modulation orders. The signal power is set to 1, the baseband bandwidth to 3KHz, and the frequency deviation to T^{-1} ($l = 1$). The 2-FSK, 4-FSK and 8-FSK candidate signals are assumed to have the same bandwidth, and, thus, the symbol period of 8-FSK is twice and four times higher than for 4-FSK and 2-FSK, respectively. Unless otherwise mentioned, the observation interval available at the receive-side is 1 second, which is equivalent to 1500 2-FSK symbols, 750 4-FSK symbols and 375 8-FSK symbols. The signals are affected by a carrier phase θ, uniformly distributed over $[-\pi, \pi)$, a carrier frequency offset, $\Delta f = 24\mathrm{Hz}$, and a timing error $\varepsilon = 0.6$. The received signal is sampled at a rate $f_s = 48$ KHz, which yields $K = 4.8 \times 10^4$ processed samples. The cutoff value is set to V_{co}=0.05, unless otherwise mentioned. Accordingly, SNR_{co} equals -19.9564dB, -13.8021dB and -7.2016dB for M = 2, 4, and 8, respectively. For the cyclostationarity test[4], the threshold Γ is set to 15.202. This corresponds to a probability of false alarm $P_f = 5\times10^{-4}$ [13]. A Kaiser window of length 61 and parameter 10 is used to calculate the estimates of covariances employed with the cyclostationarity test [4,7]. The probability $P_{cc}^{(M\text{FSK}|M\text{FSK})}$, used to evaluate the performance of the proposed classifier, is estimated based on 300 Monte Carlo trials.

B. Estimates of the First-Order/ Zero-Conjugate CC Magnitude

In Figs. 2, 3 and 4, the magnitude of the first-order/ zero-conjugate estimated CCs of 2-FSK, 4-FSK and 8-FSK signals, $|\hat{c}_r^{(K)}(\beta')_{1,0}|$, is respectively plotted versus candidate CFs, $\beta' \in [-1/2, 1/2)$, at different SNRs. One can notice the peaks in $|\hat{c}_r^{(K)}(\beta')_{1,0}|$ at $\beta' = \beta$, and their decrease with SNR, until they become comparable with the non-zero statistically insignificant spikes, which occur at $\beta' \neq \beta$. As expected from (5)[2], at 20dB SNR, the estimated CC magnitude at CF, β, is around 0.5, 0.25 and 0.125 for 2-FSK, 4-FSK and 8-FSK signals, respectively (see Figs 2 to 4 a). In addition, it is to be noted that at high enough SNR, the estimated CC magnitudes at all M CFs lie above V_{co} (see Figs. 2 to 4, a and b), at SNR around, but below SNR_{co}, a few can slightly exceed V_{co} (see Figs. 2 to 4 c), and for SNRs well below SNR_{co}, all drop below V_{co} (see Figs. 2 to 4 d).

C. Classification Performance

Classification performance achieved with the proposed algorithm is presented in Figs. 5 and 6, with $P_{cc}^{(M\text{FSK}|M\text{FSK})}$ plotted versus SNR. Classification results achieved under the two previously discussed scenarios, by using both theoretical analysis and simulations, are shown in Fig. 5 a, b and c, for M =2, 4, 8, respectively. The following remarks can be made: - Simulation results are in agreement with theoretical predictions under both known and unknown CFs scenarios; - For higher modulation orders, a higher SNR is required to achieve a certain classification performance. For example, recognition of 2-FSK, 4-FSK and 8-FSK signals with a probability of 0.9 under the unknown CF scenario, requires -17.31dB, -11.82dB and -4.56dB SNR, respectively; - Apparently, an SNR gain is achieved under the known CFs scenario, when compared with the unknown CFs scenario. For example, an SNR gain of 3.5B, 3.8dB and 5dB is

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[7] For the estimators of these covariances, see, for example, (48) in [14].

respectively achieved to identify 2-FSK, 4-FSK and 8-FSK signals with a probability of 0.9. It is noteworthy the increase in the SNR gain with the modulation order, M. This can be explained through a lower number of processed symbols used for higher order modulations, for the same duration of the observation interval available at the receiver (see the simulation setup in Section V.A). Hence, for higher order modulations, the CC estimate is less accurate and more CC magnitudes at CFs lie below the cutoff value as the SNR decreases, which leads in turn to more CFs missed to be selected in Step I of the algorithm, and thus, to a depreciation in performance under the unknown CFs scenario. In Fig. 6, simulation results for 2-FSK signal recognition under the unknown CF scenario are presented, with 1 and 2 second observation intervals, which correspond to 1500 and 3000 symbols, respectively. The cutoff value, V_{co}, is set to 0.05 for the former, whereas to 0.25 for the latter. As more symbols are processed in the latter case, more accurate CC estimates are achieved, and a lower cutoff value can be set. Apparently, improved classification performance is attained with a longer observation interval available at the receive-side. For example, a 3.3dB SNR gain is achieved with 3000 2-FSK symbols when compared to 1500 symbols, for a $P_{cc}^{(2\mathrm{FSK}|2\mathrm{FSK})}$ of 0.9.

VI. CONCLUSION

In this report, we propose a first-order cyclostationarity-based blind algorithm to classify the modulation order, M, of an FSK received signal, affected by additive Gaussian noise, phase, frequency offset and timing errors. The proposed algorithm has the advantage that relies less on preprocessing, i.e., it requires neither frequency and timing recovery, nor the estimation of signal and noise powers, and yet providing a good classification performance. Both theoretical analysis and simulations are used to prove the classification performance of the proposed algorithm. In a future report, we will investigate the extension of this algorithm to other modulation formats, as well as other propagation environments.

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Appendix A:

Definitions of Temporal Cyclostationarity Parameters and Their Sample Estimators

Let $r(t)$ be a generally non-stationary continuous-time complex-valued process, with the nth-order/ q-conjugate moment defined as [7], [16]

$$\tilde{m}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} := \mathrm{E}[r^{(*)_1}(t+\tilde{\tau}_1)\, r^{(*)_2}(t+\tilde{\tau}_2) \ldots r^{(*)_n}(t+\tilde{\tau}_n)], \tag{9}$$

where $\mathrm{E}[.]$ denotes statistical expectation, $\tilde{\boldsymbol{\tau}} = [\tilde{\tau}_1{=}0\ \tilde{\tau}_2 \ldots \tilde{\tau}_n]^\dagger$ is the delay-vector, with $\dagger$ as the transpose, and $(*)_u$, $u = 1,\ldots,n$, represents a possible conjugation, so that the total number of conjugations is q. The nth-order/ q-conjugate cumulant can be expressed in terms of the nth- and lower-orders moments through the moment-to-cumulant formula, as [7]

$$\tilde{c}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} := \mathrm{Cum}[r^{(*)_1}(t+\tilde{\tau}_1) r^{(*)_2}(t+\tilde{\tau}_2) \ldots r^{(*)_n}(t+\tilde{\tau}_n)] = \sum_{\{\wp_1,\ldots,\wp_Z\}} (-1)^{Z-1}(Z-1)! \prod_{z=1}^{Z} \tilde{m}_r(t;\tilde{\boldsymbol{\tau}}_z)_{n_z,q_z}, \tag{10}$$

where $\mathrm{Cum}[.]$ is the cumulant operator[8] and, $\{\wp_1,\ldots,\wp_Z\}$ is a partition of $\wp = \{1,2,\ldots,n\}$, with $\wp_z$ s, $z = 1,\ldots,Z$, as non-empty disjoint subsets of $\wp$, so that their reunion is $\wp$, Z is the number of the subsets in a partition ($1 \le Z \le n$), $\tilde{\boldsymbol{\tau}}_z$ is a delay vector whose components are elements of $\{\tilde{\tau}_u\}_{u=1}^n$, with indices specified by $\wp_z$, and n_z is the number of elements in the subset $\wp_z$, from which q_z correspond to conjugate terms. Note that $\sum_{z=1}^{Z} n_z = n$ and $\sum_{z=1}^{Z} q_z = q$.

With $r(t)$ as an nth-order cyclostationary process for a given conjugation configuration (q-conjugate), the nth-order/ q-conjugate moment, $\tilde{m}_r(t;\tilde{\boldsymbol{\tau}})_{n,q}$, is an almost periodic (AP) function of time [16]-[17]. Furthermore, for such a process, $\tilde{c}_r(t;\tilde{\boldsymbol{\tau}})_{n,q}$ turns out to be also an AP function of time [16]-[17]. As an AP function accepts a Fourier series expansion, $\tilde{m}_r(t;\tilde{\boldsymbol{\tau}})_{n,q}$ and $\tilde{c}_r(t;\tilde{\boldsymbol{\tau}})_{n,q}$ can be respectively written as [16]-[17]

$$\tilde{m}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} = \sum_{\tilde{\alpha} \in \tilde{\kappa}_{n,q}^m} \tilde{m}_r(\tilde{\alpha};\tilde{\boldsymbol{\tau}})_{n,q}\, e^{j2\pi\tilde{\alpha}t}, \tag{11}$$

and

[8] For the definition of the cumulant see, for example, [15], Ch. 2.

$$\tilde{c}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} = \sum_{\tilde{\beta}\in\tilde{\kappa}^c_{n,q}} \tilde{c}_r(\tilde{\beta};\tilde{\boldsymbol{\tau}})_{n,q} e^{j2\pi\tilde{\beta}t}, \qquad (12)$$

where $\tilde{\kappa}^m_{n,q} = \{\tilde{\alpha} : \tilde{m}_r(\tilde{\alpha};\tilde{\boldsymbol{\tau}})_{n,q} \neq 0\}$ and $\tilde{\kappa}^c_{n,q} = \{\tilde{\beta} : \tilde{c}_r(\tilde{\beta};\tilde{\boldsymbol{\tau}})_{n,q} \neq 0\}$ represent the sets of nth-order CFs, $\tilde{m}_r\left(\tilde{\alpha};\tilde{\boldsymbol{\tau}}\right)_{n,q}$ is the nth-order/ q-conjugate cyclic moment (CM) at CF α and for a delay-vector $\boldsymbol{\tau}$, and $\tilde{c}_r(\tilde{\beta};\tilde{\boldsymbol{\tau}})_{n,q}$ is the nth-order/ q-conjugate CC at CF β for a delay-vector $\tilde{\boldsymbol{\tau}}$, defined respectively as

$$\tilde{m}_r(\tilde{\alpha};\tilde{\boldsymbol{\tau}})_{n,q} := \lim_{I\to\infty} I^{-1} \int_{-I/2}^{I/2} \tilde{m}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} e^{-j2\pi\tilde{\alpha}t} dt, \qquad (13)$$

and

$$\tilde{c}_r(\tilde{\beta};\tilde{\boldsymbol{\tau}})_{n,q} := \lim_{I\to\infty} I^{-1} \int_{-I/2}^{I/2} \tilde{c}_r(t;\tilde{\boldsymbol{\tau}})_{n,q} e^{-j2\pi\tilde{\beta}t} dt. \qquad (14)$$

By using (10), (11) and (12), the nth-order/ q-conjugate CC of $r(t)$ at a CF $\tilde{\beta}$ and for a delay-vector $\tilde{\boldsymbol{\tau}}$, can be expressed as [7], [16]

$$\tilde{c}_r(\tilde{\beta};\tilde{\boldsymbol{\tau}})_{n,q} = \sum_{\{\wp_1,\ldots,\wp_Z\}} (-1)^{Z-1}(Z-1)! \sum_{\tilde{\boldsymbol{\alpha}}^\dagger \mathbf{1}=\tilde{\beta}} \prod_{z=1}^{Z} \tilde{m}_r(\tilde{\alpha}_z;\tilde{\boldsymbol{\tau}}_z)_{n_z,q_z}, \qquad (15)$$

where $\tilde{\boldsymbol{\alpha}} = [\tilde{\alpha}_1 \ldots \tilde{\alpha}_Z]^\dagger$ is a set of CFs and $\mathbf{1}$ is a Z-dimensional one vector. This is referred to as the cyclic moment-to-cumulant formula.

For the discrete-time signal $r[k] = r(t)|_{t=kf_s^{-1}}$, obtained by sampling the continuous-time signal $r(t)$ at a sampling rate f_s, the nth-order/ q-conjugate CC and the corresponding set of CFs are respectively given as (under the assumption of no aliasing) [18]

$$c_r(\beta;\boldsymbol{\tau})_{n,q} = \tilde{c}_r(\beta f_s;\boldsymbol{\tau} f_s^{-1})_{n,q}, \qquad (16)$$

and

$$\kappa^c_{n,q} = \{\beta \in [-1/2, 1/2) \ : \beta = \tilde{\beta} f_s^{-1},\ c_r(\beta;\boldsymbol{\tau})_{n,q} \neq 0\}, \qquad (17)$$

where $\boldsymbol{\tau} = \tilde{\boldsymbol{\tau}} f_s$, with components $\tau_u = \tilde{\tau}_u f_s$, $u = 1,\ldots,n$. Similar expressions can be also written for cyclic moments of the discrete-time signal [18].

The estimator of the nth-order/ q-conjugate CM at CF α ($\alpha = \tilde{\alpha} f_s^{-1}$) and for a delay vector $\boldsymbol{\tau}$, based on K samples, is given as [17]

$$\hat{m}_r(\alpha;\boldsymbol{\tau})_{n,q} = (K)^{-1}\sum_{k=1}^{K} r^{(*)_1}[k+\tau_1]\, r^{(*)_2}[k+\tau_2]\,...\, r^{(*)_n}[k+\tau_n]e^{-j2\pi\alpha k}\,. \tag{18}$$

The estimator of the nth-order/ q-conjugate CC at CF β and for a delay vector $\boldsymbol{\tau}$, based on K samples, is simply obtained by using the cyclic moment-to-cumulant formula, with the CMs replaced by their estimators [17].

Appendix B: First-Order Cyclostationarity of *M*-FSK Signals

In the sequel, the first-order cyclostationarity of the M-FSK baseband received signal is studied. By applying (9) to the signal given in (1), with $n=1$, $q=0$, and $\tilde{\boldsymbol{\tau}}=[\tilde{\tau}_1{=}0\;]^{\dagger}$ [9] one obtains

$$\tilde{m}_r(t)_{1,0} := \mathrm{E}[r(t)] = \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\sum_{m=1}^{M}\sum_i e^{j2\pi f_\Delta s_m(t-iT-\varepsilon T)}u_T(t-iT-\varepsilon T)e^{j2\pi\Delta f t}\,. \tag{19}$$

The average is performed with respect to the unknown data symbols, under the assumption that the symbol over the ith period takes equiprobable values in the signal alphabet, $A_{M-\mathrm{FSK}}$.

Eq. (19) can be further written as

$$\tilde{m}_r(t)_{1,0} = \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\sum_{m=1}^{M}(e^{j2\pi f_\Delta s_m t}u_T(t)\otimes\sum_i \delta(t-iT-\varepsilon T))\; e^{j2\pi\Delta f t}\;, \tag{20}$$

where $\otimes$ represents convolution and $\delta(t)$ is the Dirac delta function.

If $\Delta f_c = 0$, one can easily notice that $\tilde{m}_r(t)_{1,0}$ is a periodic function of t, with fundamental period T, and thus, the *M*–FSK signal exhibits first-order cyclostationarity. With $\Delta f_c \neq 0$, by Fourier transforming (20), one obtains

$$\begin{aligned}
\Im\{\tilde{m}_r(t)_{1,0}\} &= \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\int_{-\infty}^{\infty}\sum_{m=1}^{M}(e^{j2\pi f_\Delta s_m t}u_T(t)\otimes\sum_i\delta(t-iT-\varepsilon T))\; e^{j2\pi\Delta f t}e^{-j2\pi\tilde{\alpha}t}dt \\
&= \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{m=1}^{M}e^{j2\pi f_\Delta s_m \upsilon}u_T(\upsilon)\sum_i\delta(t-\upsilon-iT-\varepsilon T)\; e^{-j2\pi(\tilde{\alpha}-\Delta f)t}d\upsilon dt \\
&= \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\int_{-\infty}^{\infty}\sum_{m=1}^{M}e^{j2\pi f_\Delta s_m \upsilon}u_T(\upsilon)\int_{-\infty}^{\infty}\sum_i\delta(t-\upsilon-iT-\varepsilon T)\; e^{-j2\pi(\tilde{\alpha}-\Delta f)t}dt d\upsilon \\
&= \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\int_{-\infty}^{\infty}\sum_{m=1}^{M}e^{j2\pi f_\Delta s_m \upsilon}u_T(\upsilon)\int_{-\infty}^{\infty}\sum_i\delta(u-iT)e^{-j2\pi(\tilde{\alpha}-\Delta f)u}du\; e^{-j2\pi(\tilde{\alpha}-\Delta f)(\upsilon+\varepsilon T)}d\upsilon \\
&= \sqrt{\mathrm{S}}e^{j\theta}M^{-1}T^{-1}\int_{-\infty}^{\infty}\sum_{m=1}^{M}e^{j2\pi f_\Delta s_m \upsilon}\; e^{-j2\pi(\tilde{\alpha}-\Delta f_c)\upsilon}u_T(\upsilon)d\upsilon\sum_i\delta(\tilde{\alpha}-\Delta f-iT^{-1})\; e^{-j2\pi(\tilde{\alpha}-\Delta f)\varepsilon T}\,,
\end{aligned} \tag{21}$$

[9] As $\tilde{\boldsymbol{\tau}}=[\tilde{\tau}_1{=}0\;]^{\dagger}$, the dependency on the delay-vector is subsequently dropped for the first-order cyclostationarity temporal parameters.

where $\Im\{.\}$ denotes the Fourirer transform. Convolution and change of variables are performed at the second and fourth steps in the right hand-side of (21), respectively, and the identity $\Im\{\sum_i \delta(u-iT)\} = T^{-1}\sum_i \delta(\tilde{\alpha}-iT^{-1})$ is used in the fifth step. It is to be noted that $\Im\{\tilde{m}_r(t)_{1,0}\} \neq 0$ if

$$\tilde{\alpha} = \Delta f + iT^{-1},\ i \text{ integer}. \tag{22}$$

In other words, the $\tilde{\alpha}$ -frequency domain is discrete. By replacing (22) into (21), and using that $u_T(\upsilon)=1$ if $\upsilon \in [0,T]$ and $u_T(\upsilon)=0$ otherwise, (21) becomes

$$\Im\{\tilde{m}_r(t)_{1,0}\} = \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\sum_{m=1}^{M}\sum_i \delta(\tilde{\alpha}-\Delta f - iT^{-1})\ e^{-j2\pi i\varepsilon}\ T^{-1}\int_0^T e^{-j2\pi(f_\Delta s_m - iT^{-1})\upsilon}d\upsilon \quad . \tag{23}$$

If the product $f_\Delta s_m$ is an integer of T^{-1}, i.e., $f_\Delta s_m = pT^{-1}$, p integer, by using the identity $T^{-1}\int_0^T e^{-j2\pi(p-i)T^{-1}\upsilon}d\upsilon = \delta((p-i)T^{-1})$ in (23), one obtains

$$\Im\{\tilde{m}_r(t)_{1,0}\} = \sqrt{\mathrm{S}}e^{j\theta}M^{-1}\ \sum_{p\in P} e^{-j2\pi p\varepsilon}\ \delta(\tilde{\alpha}-\Delta f - pT^{-1}) \ , \tag{24}$$

where $P = \{p : p \text{ integer},\ p = f_\Delta s_m T,\ s_m \in A_{M-\mathrm{FSK}}\}$.

If $f_\Delta = lT^{-1}$, l integer, it is straightforward that $p = \pm l, \pm 3l \ldots, \pm(M-1)l$, the spectrum consists of M finite- strength additive components,

$$\Im\{\tilde{m}_r(t)_{1,0}\} = \sum_{p\in\{\pm l,\pm 3l,\ldots,\pm(M-1)l\}} \sqrt{S}e^{j\theta}M^{-1}e^{-j2\pi p\varepsilon}\ \delta(\tilde{\alpha}-\Delta f - pT^{-1}), \tag{25}$$

and the expression for the $\tilde{m}_r(t)_{1,0}$ becomes

$$\tilde{m}_r(t)_{1,0} = \sum_{p\in\{\pm l,\pm 3l,\ldots,\pm(M-1)l\}} \sqrt{S}e^{j\theta}M^{-1}e^{-j2\pi p\varepsilon}e^{j2\pi(\Delta f + pT^{-1})t}. \tag{26}$$

Apparently, this is equivalent to (11), with $n=1$, $q=0$, $\tilde{\boldsymbol{\tau}} = [\tilde{\tau}_1{=}0\ ]^\dagger$, $\tilde{\kappa}_{1,0}^m = \{\Delta f + \tilde{\gamma},\ \tilde{\gamma} = pT^{-1}, p = \pm l, \pm 3l \ldots, \pm(M-1)l\}$, and $\tilde{m}_r(\tilde{\alpha})_{1,0} = \sqrt{\mathrm{S}}M^{-1}e^{j\theta}e^{-j2\pi p\varepsilon}$ [9]. Note that the same expression for the first-order/ zero-conjugate CC at CF $\tilde{\alpha}$, $\tilde{m}_r(\tilde{\alpha})_{1,0}$, can easily obtained by applying (13).

According to (15), the first-order/ zero-conjugate CC and CM are identical, such that

$$\tilde{c}_r(\tilde{\beta})_{1,0} = \sqrt{\mathrm{S}}M^{-1}e^{j\theta}e^{-j2\pi\tilde{\gamma}T\varepsilon}, \tag{27}$$

with $\tilde{\beta} = \tilde{\alpha} = \Delta f + \tilde{\gamma}$, $\tilde{\gamma} = pT^{-1}$, $p = \pm l, \ldots, \pm(M-1)l$.

With results previously derived for the continuous-time signal, $r(t)$, by applying (16) and (17), and taking into account signal normalization, one can easily obtain (3)-(4) for the normalized discrete-time signal, $r[k] = r(t)/\sqrt{S+N}\,|_{t=kf_s^{-1}}$.

Appendix C: Cyclostationarity Test and Probability of CF Detection

Cyclostationarity Test

With the cyclostationarity test proposed in [14], the presence of a CF is formulated as a two hypothesis-testing problem, i.e., under hypothesis H_0 the tested candidate CF, β', is not a CF, β, and under hypothesis H_1 the tested candidate CF, β', is a CF, β. This test is applied here to first-order candidate CFs, and consists of the following three steps.

Step 1: The first-order/ zero-conjugate CC at candidate CF, β', is estimated from K samples[1]. The vector

$$\hat{\mathbf{c}}_{r,1,0}^{(K)} := [\mathrm{Re}\{\hat{c}_r^{(K)}(\beta')_{1,0}\} \;\; \mathrm{Im}\{\hat{c}_r^{(K)}(\beta')_{1,0}\}] \tag{28}$$

is formed, with $\hat{c}_r^{(K)}(\beta')_{1,0}$ as the estimate of the first-order/ zero-conjugate CC at β', from K samples, and $\mathrm{Re}\{.\}$ and $\mathrm{Im}\{.\}$ as the real and imaginary parts, respectively.

Step 2: The statistic

$$\mathcal{T}_{1,0}^{(K)} = K\hat{\mathbf{c}}_{r,1,0}^{(K)}\hat{\mathbf{\Sigma}}_{r,1,0}^{-1}\hat{\mathbf{c}}_{r,1,0}^{(K)\dagger} \tag{29}$$

is then computed for the candidate CF, β'. Here -1 denotes matrix inverse, and $\hat{\mathbf{\Sigma}}_{r,1,0}$ is an estimate of the matrix

$$\mathbf{\Sigma}_{r,1,0} = \begin{bmatrix} \mathrm{Re}\{(Q_{2,0}+Q_{2,1})/2\} & \mathrm{Im}\{(Q_{2,0}-Q_{2,1})/2\} \\ \mathrm{Im}\{(Q_{2,0}+Q_{2,1})/2\} & \mathrm{Re}\{(Q_{2,1}-Q_{2,0})/2\} \end{bmatrix}, \tag{30}$$

where $Q_{2,0} := \lim_{K\to\infty} \mathrm{Cum}[\hat{c}_r^{(K)}(\beta')_{1,0}, \hat{c}_r^{(K)}(\beta')_{1,0}]$ [8] and $Q_{2,1} := \lim_{K\to\infty} \mathrm{Cum}[\hat{c}_r^{(K)}(\beta')_{1,0}, \hat{c}_r^{(K)*}(\beta')_{1,0}]$ [8], with $*$ as complex conjugate. As shown in [14], the covariances $Q_{2,0}$ and $Q_{2,1}$ are equal to the second-order/ zero- and one-conjugate cyclic spectra, $C_r(2\beta';\beta')_{2,0}$ and $C_r(0;-\beta')_{2,0}$, respectively. These are defined as[7],

$$C_r(2\beta';\beta')_{2,0} := \lim_{K\to\infty} K^{-1} \sum_{k=0}^{K-1} \sum_{\xi=-\infty}^{\infty} \mathrm{Cum}[r[k], r[k+\xi]]\, e^{-j2\pi\beta'\xi} e^{-j2\pi 2\beta' k\ 8}, \tag{31}$$

and

$$C_r(0;-\beta')_{2,0} := \lim_{K\to\infty} K^{-1} \sum_{k=0}^{K-1} \sum_{\xi=-\infty}^{\infty} \mathrm{Cum}[r[k], r^*[k+\xi]]\, e^{-j2\pi(-\beta')\xi} e^{-j2\pi 0k\ 8}. \tag{32}$$

Step 3: The decision on the candidate CF β' is made by comparing the statistic $\mathcal{T}_{1,0}^{(K)}$ against a threshold, Γ. If $\mathcal{T}_{1,0}^{(K)} \geq \Gamma$, the tested candidate CF is decided to be a CF, otherwise not. The threshold Γ is set for a given probability of false alarm, P_f, which is defined as the probability to decide that a candidate is a CF when it is actually not, and expressed as $P_f = \Pr\{\mathcal{T}_{1,0}^{(K)} \geq \Gamma \mid H_0\}$. As the statistic $\mathcal{T}_{1,0}^{(K)}$ has an asymptotic chi-square distribution with two degrees of freedom under hypothesis H_0 [14], the threshold Γ is found from the tables of this distribution, for a given probability of false alarm, P_f [13].

Probability of CF Detection

With this cyclostationarity test, the probability to decide that a CF is indeed a CF, which is referred to as the probability of detection, is given as

$$P_d = \Pr\{\mathcal{T}_{1,0}^{(K)} \geq \Gamma \mid H_1\}. \tag{33}$$

As the distribution of $\mathcal{T}_{1,0}^{(K)}$ under the hypothesis H_1 is normal, $\mathcal{N}(m_{\mathcal{T}}, \sigma_{\mathcal{T}}^2)$, with mean $m_{\mathcal{T}} = K\mathbf{c}_{r,1,0}\boldsymbol{\Sigma}_{r,1,0}^{-1}\mathbf{c}_{r,1,0}^{\dagger}$ and variance $\sigma_{\mathcal{T}}^2 = 4K\mathbf{c}_{r,1,0}\boldsymbol{\Sigma}_{r,1,0}^{-1}\mathbf{c}_{r,1,0}^{\dagger}$ [14], the probability of CF detection can be expressed as [13]

$$P_d = \int_{\frac{\Gamma - m_{\mathcal{T}}}{\sigma_{\mathcal{T}}}}^{\infty} (2\pi)^{-1/2} e^{-y^2/2} dy. \tag{34}$$

By using (3)-(4) and (29)-(32), one can easily notice that the statistic $\mathcal{T}_{1,0}^{(K)}$, its mean and variance, and thus, the probability of detection, depend on the tested CF. The notation $P_d^{(\nu)}$, $\nu = 1,\ldots,M$, is used to show this dependency.

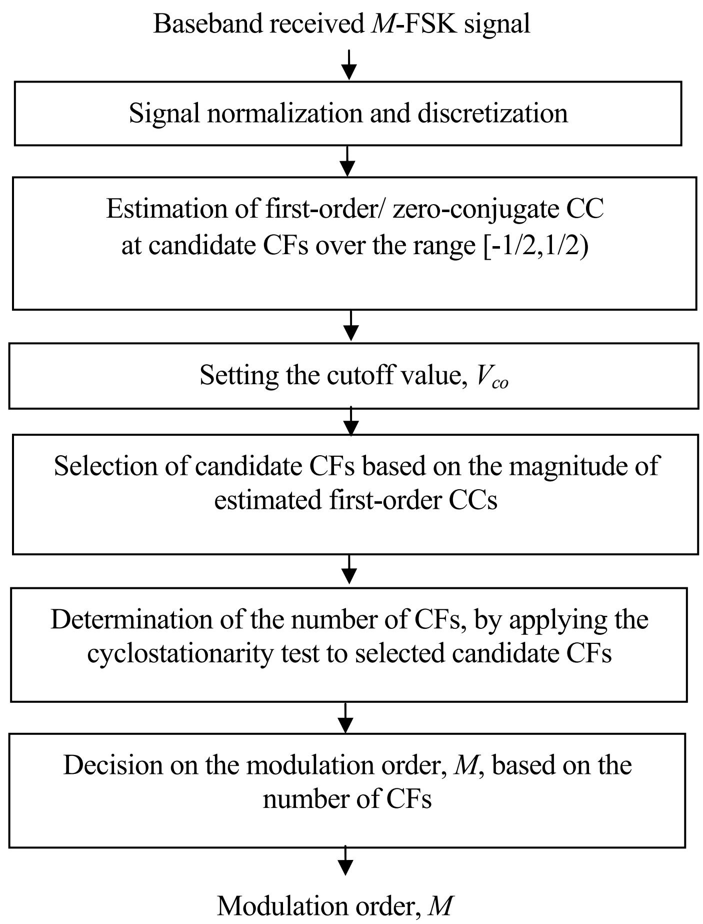


Fig. 1. Block diagram of the algorithm proposed for the recognition of the modulation order of an FSK received signal.

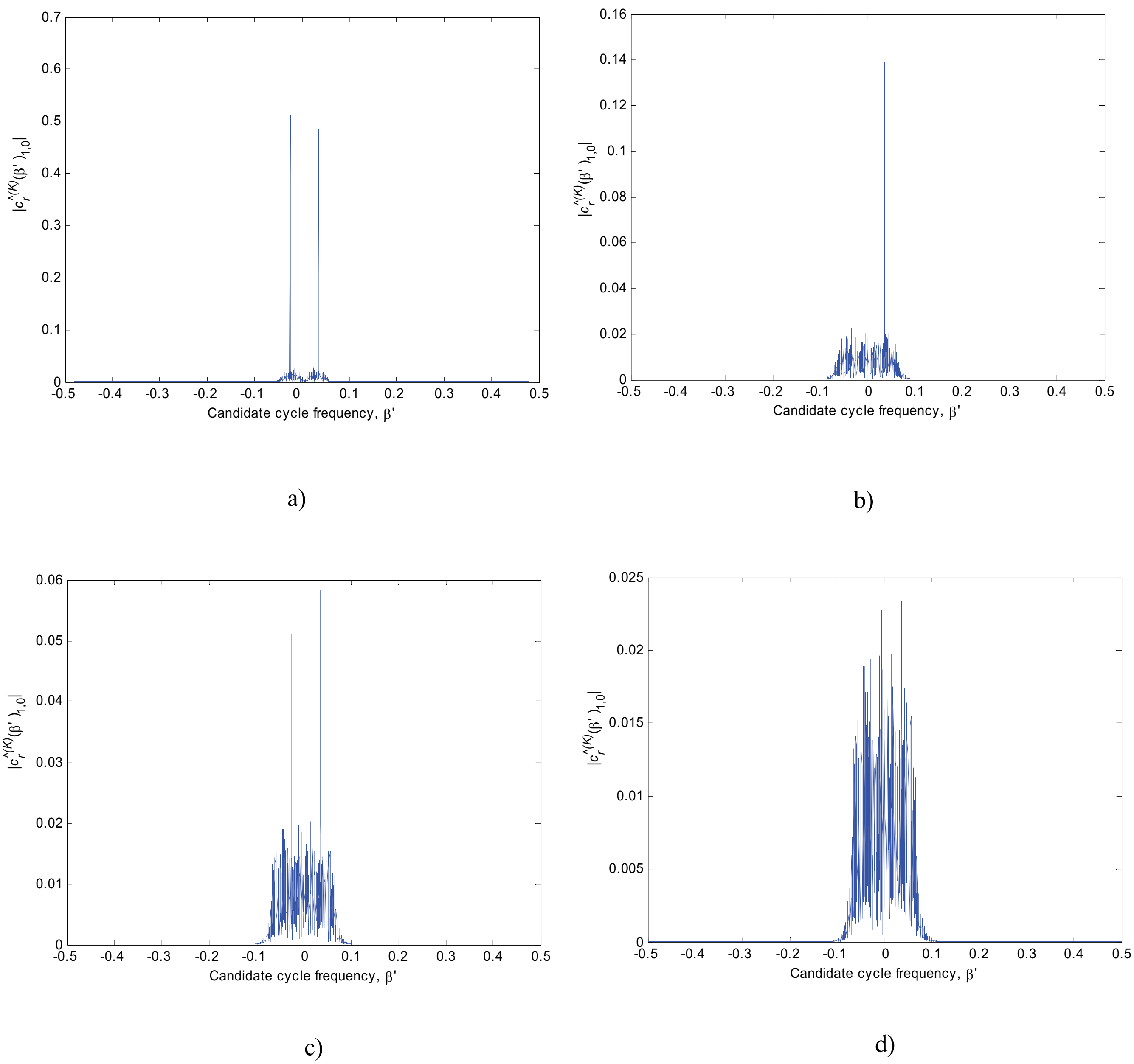


Fig. 2. Estimates of the first-order/ zero-conjugate CC magnitudes of 2-FSK, at candidate CFs β', $\beta' \in [-1/2, 1/2)$, and for a) 20dB SNR, b) -10dB SNR, c) -20dB SNR, and d) -25dB SNR.

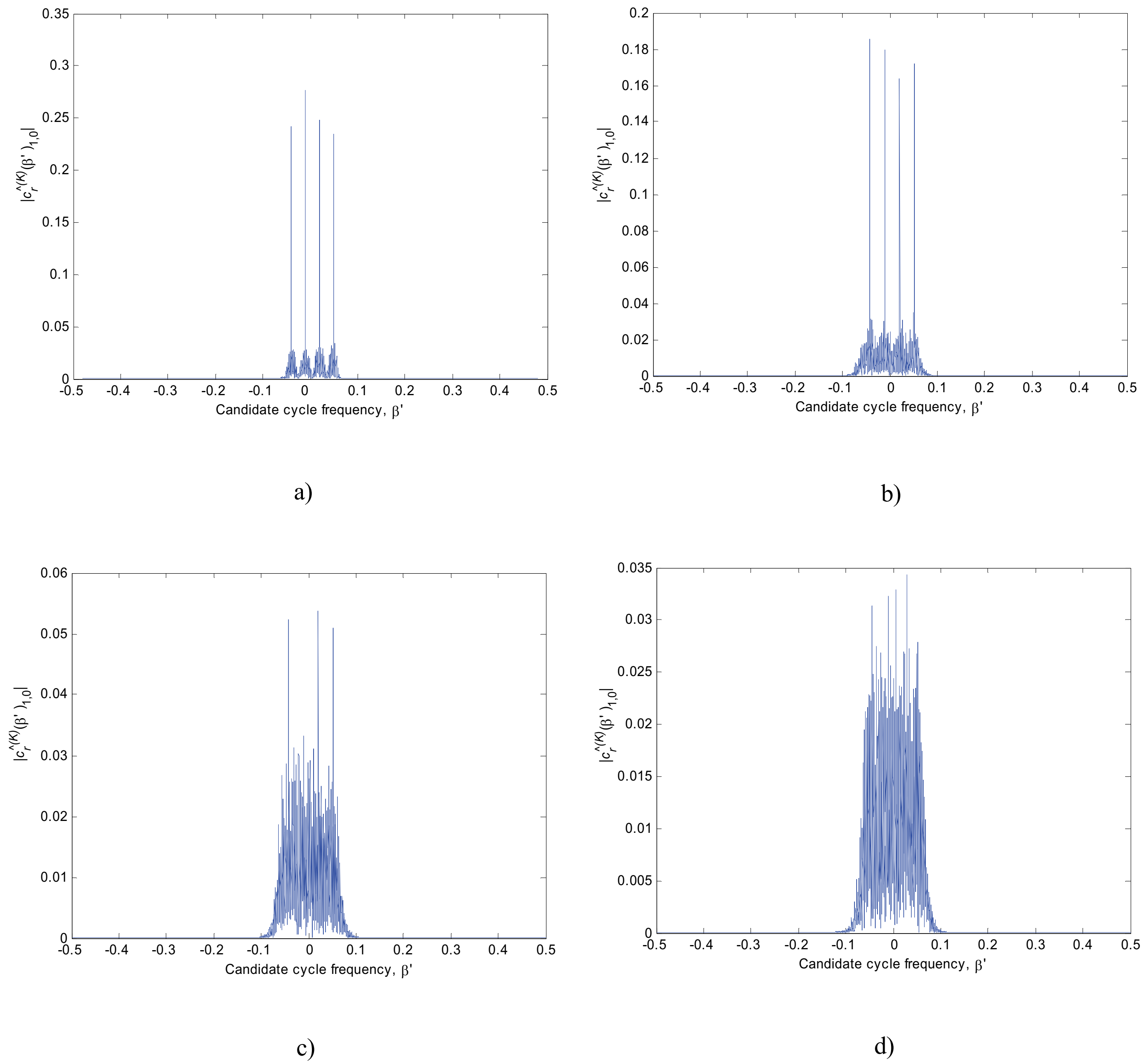


Fig. 3. Estimates of the first-order/ zero-conjugate CC magnitudes of 4-FSK, at candidate CFs β', $\beta' \in [-1/2, 1/2)$, and for a) 20dB SNR, b) 0dB SNR, c) -15dB SNR, and d) -20dB SNR.

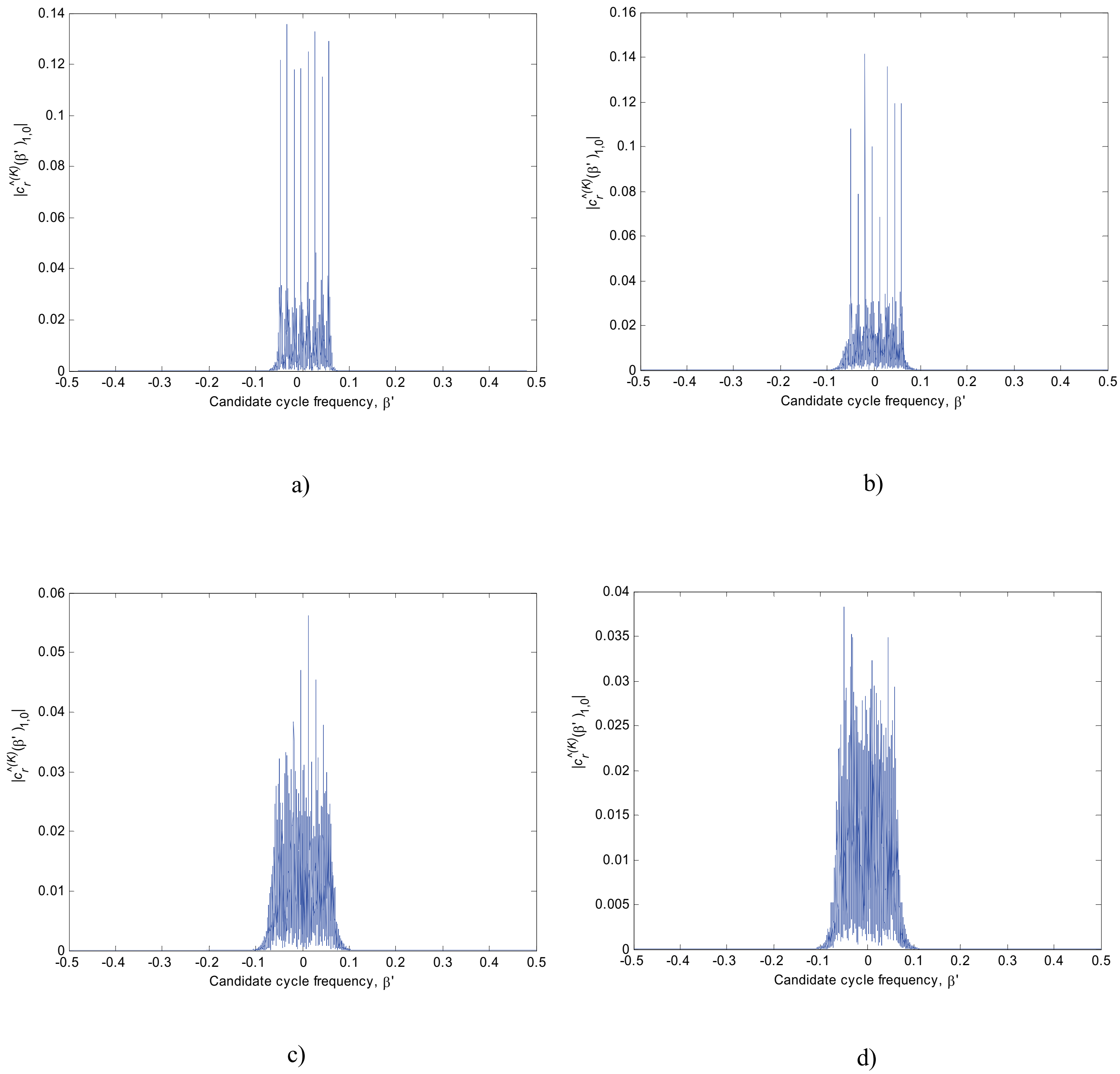
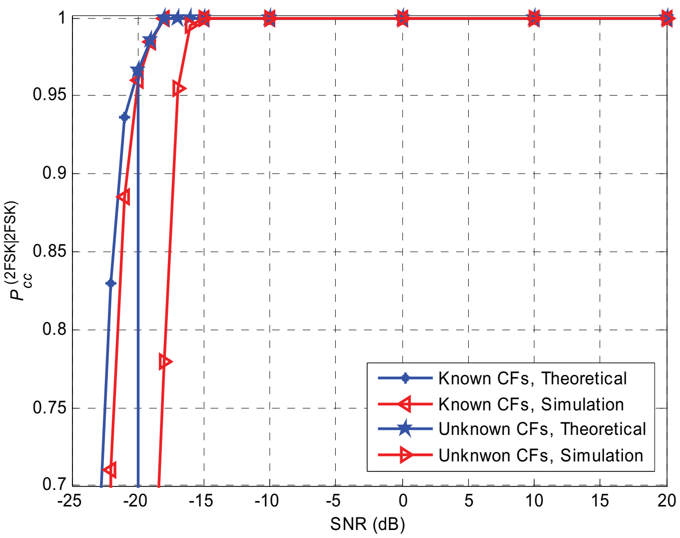
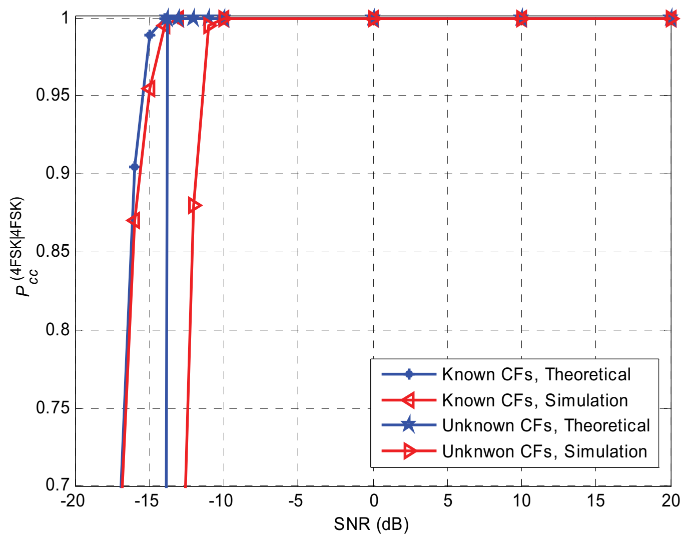


Fig. 4. Estimates of the first-order/ zero-conjugate CC magnitudes of 8-FSK, at candidate CFs β', $\beta' \in [-1/2, 1/2)$, and for a) 20dB SNR, b) 5dB SNR, c) -10dB SNR, and d) -15dB SNR.



a)



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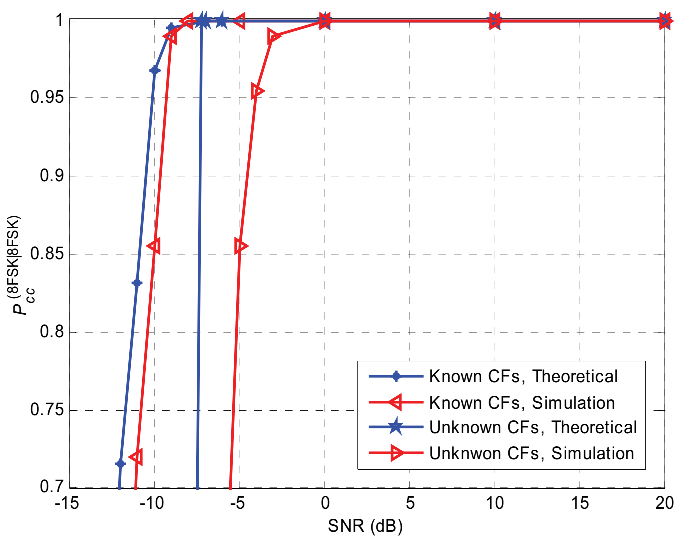

c)

Fig. 5. The probability of correct signal classification versus SNR, with 1 second observation interval and for a) *M*=2, b) *M*=4, and c) *M*=8.

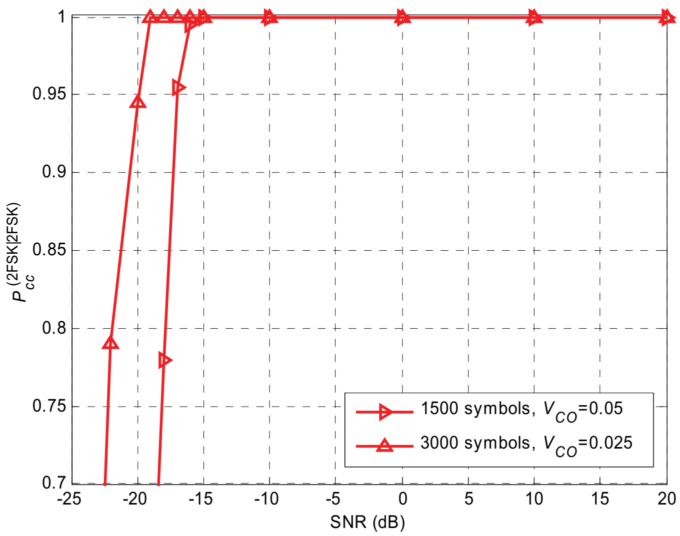

Fig. 6. The probability of correct signal classification versus SNR, under the unknown CF scenario, for M=2 and with different durations of the observation interval.

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. In this report, first-order cyclostationarity of *M*-ary frequency shift keying (*M*-FSK) signals affected by additive Gaussian noise, phase, frequency offset and timing errors is investigated and applied to recognizing the modulation order, *M*. The number of first-order cycle frequencies (CFs) detected in the received FSK signal is employed as discriminating feature for modulation order recognition. Based on this feature, a recognition algorithm is proposed, which does not require timing and carrier recovery, and estimation of signal and noise powers as preprocessing tasks. In addition, the algorithm is developed with no prior information on the CFs at the receive-side. Nevertheless, the hypothetical case when knowledge of CFs is available at the receive-side is also investigated, as providing a benchmark for recognition performance evaluation. A theoretical performance analysis is carried out, and simulations are run to verify theoretical developments.

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