

POLARIZATION IN SCATTERING

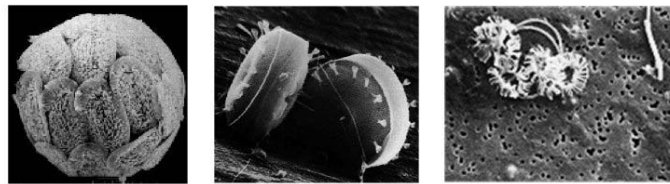
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ABSTRACT

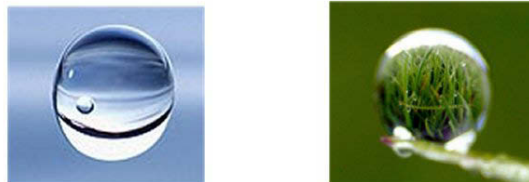
All the information available from an electromagnetic wave is contained in three variables, direction, wavelength and polarization. This is sufficient to completely characterize light. If you can't extract the information you want using these parameters, then you have reached the limit of what Electro-Optics can do for you. Historically, optical oceanography has focused on using and analyzing color and directionality first both for remote sensing and in situ imaging and detection. Even though fundamental measurements and theoretical results on the effect of polarization have been available for many years, they are only now starting to be applied to real world problems. This paper will focus on some general principles and relationships that may be of use in both remote sensing and underwater imaging and attempt to demystify the effects of polarization.

RELEVANT FEATURES OF SCATTERING IN WATER

By far the most extensive body of work to date on polarization in scattering is in the evaluation of the effect of water aerosols both in liquid and solid forms. The index of refraction is approximately 1.33 and the relevant shapes are spheres, spheroids and hexagonal crystals. The relative regularity and perfection (smoothness mainly due to surface tension effects) of the aerosols shapes allows one to effectively apply fundamental scattering theory to the problem. This is unfortunately not the case for hydrosols. The buoyancy and absence of surface tension effects leads to a great variability in shape and in very irregular or bumpy surfaces. Hydrosols can have the general shape of spheres, spheroids, plates or cylinders, however almost all are far from smooth and this small-scale shape irregularity completely modifies the effect scattering on the polarization [1]. This effect is particularly significant in the backscattering direction, which is all important for imaging and visibility applications [2].



typical hydrosols



typical aerosols (note surface smoothness)

This variability in shape and the significant surface irregularities of hydrosols seriously mitigate the usefulness of exact codes. On the other hand, the relative index of refraction is generally less than 1.1,

which allows one to use approximate relationships based on the fundamental physics and build a relatively simple framework to evaluate and predict the effect of scattering on polarization in water. This paper will therefore present and develop this simple polarization model and explore its practical consequences to both remote sensing and imaging in natural waters.


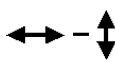

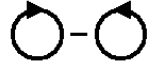
FUNDAMENTALS OF POLARIZATION

Light is composed of transverse waves. This really means that if you were a very small charged particle, as light passed over you, you would feel pulled up and down or sideways at the frequency of the wave. In fact, in the more general case, the pull you feel could change direction and strength at the frequency of the wave. If the pull is in one direction we say that the light is linearly polarized in that direction. If the pull cycles around you but stays constant in strength, we say that the light is circularly polarized. If we look at the most general pattern of transverse motion of this pull, we find that it can be represented as an ellipse at an arbitrary angle with respect to us, and around which the force vector rotates at the frequency of the light wave. Such a pattern can be described as resulting from the sum of two independent linearly polarized waves oscillating perpendicular to one another with a fixed phase difference between them. It can also equally well be represented by two circularly polarized waves whose electric vectors rotate in opposite direction (a representation directly analogous to photon spin).

Note that we can only measure the intensity of light waves, which is given by the square of the electric field. The real problem in working with polarization occurs because the frequency of light is so high that we can only measure an average over many periods. We cannot directly measure the phase difference between the independent waves or follow directly the transverse motion of the electric field vector. The first effective technique to consistently measure and describe polarized light was developed by George Stokes in 1840. He found that polarized light could be represented by a vector whose elements could all be directly measured. The first element, which he called **I**, is the total intensity of the light wave. The second element, which he called **Q**, is simply the difference between the light polarized in the horizontal direction and the light polarized in the vertical direction. The third element **U**, is simply the difference between the light linearly polarized at an angle of 45° with respect to the horizontal direction and the light linearly polarized at an angle of 135° with respect to the same horizontal direction. The final element **V**, is the difference between the circularly polarized light rotating in a clockwise direction and the circularly polarized light rotating in the counter clockwise direction. Using the difference between polarization states is a very clever way of handling the case where the incoming light is unpolarized or partially polarized. An incandescent light is an example of unpolarized light as all the photons are thermal in origin and emitted at random. In this case, as there is on average an equal amount of photons in each polarization direction, the **U**, **Q** and **V** parameters of the Stokes vector would be 0. Skylight is an example of partially polarized light.

A digression is necessary at this point to clear up some fundamental conventions which must be grasped very clearly and whose neglect is the reason why so many people have problems with this field. In general horizontal and vertical would simply refer to the laboratory frame of reference. In scattering theory however, one must reference all polarizations to the scattering plane which is the plane containing both the incoming wave and the outgoing scattered wave. In that case we refer to the linear polarization as parallel if the polarization vector is in the scattering plane or perpendicular if the polarization vector is perpendicular to the scattering plane. By convention, we assume that the scattering event has been measured by equipment on a laboratory table and in the horizontal plane. This is why in general people will refer to light polarized parallel to the scattering plane as horizontally polarized and light polarized perpendicular to the scattering plane as vertically polarized. A third very

important convention is required to measure angles and define clockwise and counter-clock wise rotation directions. It is always assumed that clockwise and counter clockwise are defined as if you are looking directly into the light beam. Note that this convention implies that a beam of light reflecting off a perfect mirror will have light originally polarized at an angle of 45° come back at an angle of 135° and vice-versa. It also implies that clockwise will come back as anti-clockwise. This means that a perfect mirror will reverse the sign of the **U** and **V** elements of the Stokes vector. Working this out is a worthwhile exercise as many authors and many textbooks are confused and sometimes wrong about their conventions [3]. What is presented here is the classic convention used by all the principal contributors to the field of polarization (Stokes, Chandrasekar, Mueller, Born and Wolf [4]).

I	Q	U	V
O	H-V	R-L	C-CC
			

From our previous discussions, it is obvious that the different polarization states can all be represented as linear combinations of any of the independent pairs of polarization states. Almost all the polarization effects in scattering from medium to large size particles come from the reflections off their surfaces. In working with reflection it turns out to be simple and convenient to evaluate the effects of the surface in terms of parallel (horizontal) and perpendicular (vertical) components. Assuming right-handed co-ordinate system with parallel polarization along the x-axis and perpendicular polarization along the y-axis [5], the electric field amplitudes can all be related to these components as follows.

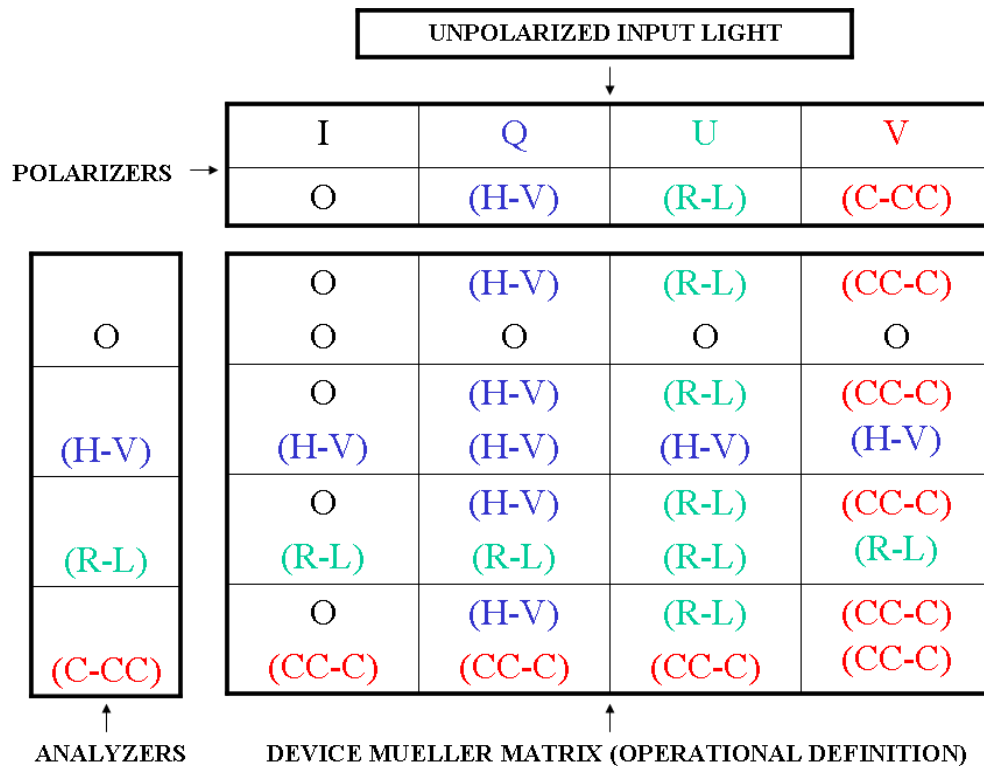
$$\begin{aligned} \vec{e}_o &= \frac{1}{\sqrt{2}} \vec{e}_1 + \frac{1}{\sqrt{2}} \vec{e}_2 \\ \vec{e}_h &= \vec{e}_1 + e^{-i\omega t} \vec{e}_2 \\ \vec{e}_v &= e^{-i\omega t} \vec{e}_1 + \vec{e}_2 \\ \vec{e}_r &= \frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_1 + \frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_2 \\ \vec{e}_l &= -\frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_1 + \frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_2 \\ \vec{e}_c &= \frac{e^{-i(\omega t + \pi/2)}}{\sqrt{2}} \vec{e}_1 + \frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_2 \\ \vec{e}_{cc} &= -\frac{e^{-i(\omega t + \pi/2)}}{\sqrt{2}} \vec{e}_1 + \frac{e^{-i\omega t}}{\sqrt{2}} \vec{e}_2 \end{aligned}$$

Where \vec{e}_1 and \vec{e}_2 refer to the perpendicular (sometimes referred to as vertical) and parallel (horizontal) components respectively. Note carefully that the scattering plane is now the horizontal plane and left-right and clockwise-anticlockwise orientations are referred to that plane. From the equations above it is obvious that in general we can express the breakdown into perpendicular and parallel components of any polarization vector as follows:

$$\vec{e}_i = a_i(\omega t) \vec{e}_1 + b_i(\omega t) \vec{e}_2$$

It turns out that we can also transform all the polarization operators (for instance reflection from a surface) in terms of the corresponding parallel (horizontal) and perpendicular (vertical) operators. We do this later for reflection from a surface as an example, since it's the term that controls polarization for large hydrosols.

The effect of any object or device on light can be described by a matrix which relates the input Stokes vector to the output Stokes vector. This matrix is called the Mueller matrix [5]. It can be most simply and generally described by the operations required to measure it [6]. The process goes as follows. Start with unpolarized incoming light. Use polarizers to produce in sequence light polarized in all the six fundamental states of the Stokes vector. Pass the resulting light through the object or device (in our case this would be a scattering by a particle). For each of the input states (one unpolarized and 6 polarized), measure the resulting unpolarized light and the 6 polarizations states. Each element of the matrix can then be represented as the direct (outer) product of the combination of the operators required to prepare/produce the input Stokes vector by the operators required to measure the output Stokes vector. This process is shown below where the input Stokes vector is laid out horizontally and the corresponding output Stokes vector is laid out vertically. Please note that the order of the operators matters and that the negative of the **V** component is used in the matrix for symmetry reasons. By convention the input operator (polarizer) is first (i.e. on the left/top) and the output operator (analyzer) comes second (i.e. on the right/bottom) in all the operator pairs.



To more easily be able to discern polarization effects from changes in overall intensity, the elements of the Mueller matrix can be normalized by dividing them by the upper leftmost $M(1,1)$ element which describes the effect of scattering on the total intensity of unpolarized light. This implies that the other elements of this normalized Mueller matrix will all conveniently range from a value of 1 to -1 .

Note that since we are assuming unpolarized light as the input to the polarizers, the polarizers reduce the total intensity by a factor of 2 and the amplitude of the input electric field vector by the corresponding factor of

$$\vec{e}_o = \frac{1}{\sqrt{2}} \vec{e}_1 + \frac{1}{\sqrt{2}} \vec{e}_2$$

Note also that the unpolarized light can obviously be represented as the sum of the output of a parallel and of a perpendicular polarizer. In order to easily use the formulas we will present below it turns out to be useful and simpler to express unpolarized light as this sum of the two polarized components operators.

$$\mathbf{O} \Rightarrow (\mathbf{H} + \mathbf{V})$$

As an example, in order to obtain expressions for all operator pairs used in the Mueller matrix in the case of reflection from a surface, we can follow a straightforward procedure.

We first express the unpolarized light initially polarized by the input polarizer in terms of its parallel and perpendicular component. Using our previous notation, this can be written as follows:

$$\vec{e}_p = a_o(\omega t) a_p(\omega t) \vec{e}_1 + b_o(\omega t) b_p(\omega t) \vec{e}_2 = \frac{a_p(\omega t)}{\sqrt{2}} \vec{e}_1 + \frac{b_p(\omega t)}{\sqrt{2}} \vec{e}_2$$

It should be noted that the absolute sum of the squares of the components is always equal to 1 since we are dealing with unit vectors by definition.

$$a_p(\omega t) a_p^*(\omega t) + b_p(\omega t) b_p^*(\omega t) = 1$$

We next compute the effect of the scattering on each of the two parallel and perpendicular components. As we are dealing here with reflection from a surface as an example, we multiply by the separate reflectance for each component.

$$\vec{e}_p = r_{11} a_o(\omega t) a_p(\omega t) \vec{e}_1 + r_{22} b_o(\omega t) b_p(\omega t) \vec{e}_2 = \frac{r_{11} a_p(\omega t)}{\sqrt{2}} \vec{e}_1 + \frac{r_{22} b_p(\omega t)}{\sqrt{2}} \vec{e}_2$$

r_{11} is the perpendicular (vertical) reflectance and r_{22} is the parallel (horizontal) reflectance.

This new output vector must now be passed through the analyzer. In order to do this we set a factor times the analyzer polarization vector as equal to the input vector modified by the scattering (in this case multiplied by the reflectance). To isolate the factor we take the scalar (dot) product of both sides of this equation by the complex conjugate of the analyzer polarization vector. As the polarization vector is a unit vector, the left hand side becomes simply the factor we were looking for and the right hand side its expression in terms of parallel and perpendicular operators (reflectances). This operation can be viewed as projecting the output vector from the scattering on the analyzer vector and obtaining its amplitude.

$$r_{p,a} [a_a(\omega t) \vec{e}_1 + b_a(\omega t) \vec{e}_2] = \left[\frac{r_{11} a_p(\omega t)}{\sqrt{2}} \vec{e}_1 + \frac{r_{22} b_p(\omega t)}{\sqrt{2}} \vec{e}_2 \right]$$

$$r_{p,a} [a_a(\omega t) \vec{e}_1 + b_a(\omega t) \vec{e}_2] \cdot [a_a^*(\omega t) \vec{e}_1 + b_a^*(\omega t) \vec{e}_2] = \left[\frac{r_{11} a_p(\omega t)}{\sqrt{2}} \vec{e}_1 + \frac{r_{22} b_p(\omega t)}{\sqrt{2}} \vec{e}_2 \right] \cdot [a_a^*(\omega t) \vec{e}_1 + b_a^*(\omega t) \vec{e}_2]$$

$$r_{p,a}(\omega t) = \frac{r_{11} a_p(\omega t) a_a^*(\omega t)}{\sqrt{2}} + \frac{r_{22} b_p(\omega t) b_a^*(\omega t)}{\sqrt{2}}$$

The final result is then obtained by transforming to intensities by multiplying this factor by its complex conjugate. At last, we take the time average of the time dependant factors over one period. This averaging operation is denoted by the angled brackets. This time averaging is required, as we cannot make any measurements at the extremely high frequencies involved when dealing with a light wave.

$$\begin{aligned} \langle r_{p,a}(\omega t) r_{p,a}^*(\omega t) \rangle &= \frac{r_{11} r_{11}^*}{2} \langle a_p(\omega t) a_p^*(\omega t) a_a(\omega t) a_a^*(\omega t) \rangle + \frac{r_{11} r_{22}^*}{2} \langle a_p(\omega t) a_a^*(\omega t) b_p^*(\omega t) b_a(\omega t) \rangle \\ &+ \frac{r_{22} r_{11}^*}{2} \langle a_p^*(\omega t) a_a(\omega t) b_p(\omega t) b_a^*(\omega t) \rangle + \frac{r_{22} r_{22}^*}{2} \langle b_p(\omega t) b_p^*(\omega t) b_a(\omega t) b_a^*(\omega t) \rangle \end{aligned}$$

The result can be used to compute all the analyzer-polarizer pairs required to evaluate operationally the elements of the Mueller matrix. It should be noted that, for the sake of simplicity and clarity, we have not included in the formulation above any cross-terms that would transform some parallel \vec{e}_2 polarization into perpendicular \vec{e}_1 polarization r_{12} and vice-versa r_{21} . As we shall mention later, such elements only occur when dealing with multiple reflections and/or transmissions from or through the surfaces of a particle in a single scattering event. These terms, when required, can be incorporated in a straightforward manner.

PHYSICS OF SCATTERING IN WATER

The basic physics of scattering occur because the periodic pull of the transverse electric field of the light wave displaces the electron cloud around the atoms and molecule a little off center from the nuclei. This gives rise to an oscillating dipole. In a particle small with respect to the wavelength of the light, all the atomic and molecular dipoles see the same uniform field and react in phase together. This gives rise to the well-known Rayleigh scattering for small particles where the scattered light polarized in the perpendicular direction is uniform as a function of angle and the scattered light with its polarization vector in the direction parallel to the scattering plane varies as a function of scattering angle as $\text{Cos}^2(\theta)$. In water the relative index of refraction is always very close to 1. This means that the phase difference across the particle is small and we can still approximate the field inside the particle as uniform. If we then assume each element scatters in a dipole pattern and integrate over these elements, accounting for the relative phase differences due to different positions relative to one another, we obtain a generalization of the Rayleigh formulas called the Rayleigh-Gans approximation which is now valid as long as $(m-1)x$ is much less than 1. In this regime, the scattering pattern becomes forward peaked for both polarizations but once again for the parallel polarization there is an additional $\text{Cos}^2(\theta)$ factor multiplying the scattering pattern common to both polarizations.

THE POLARIZED SCATTERING MODEL

Let me first start this section with an apology. As there is not enough space to include all the equations and experimental-theoretical comparisons figures, the following discussion will almost be entirely limited to simply describing the processes, methods and overall results obtained.

As the particles get even larger, other factors come into play, such as the scattered field gaining significant amplitude internal to the particles and modifying the field seen by each volume element and those elements now reacting back with a time delay due to their distance from one another.

If we assume that the particle size distributions are inverse powers ν of the particle dimension, it is possible to compute as a function of that inverse power at what value the optical differential central path length (which is the key parameter controlling the region of validity of the Rayleigh-Gans approximation) the median of the integrand for the reflection term occurs.

$$x = \frac{2\pi r}{\lambda}$$

$$\rho = 2(n-1)x$$

$$\rho_{median} = \frac{1.8}{(\nu - 3)^2}$$

Since from previous work, for ocean water the inverse power of the particle size has a mean of 3.65 with a deviation of 0.12, the contribution of the large particles overwhelmingly dominate even in the backscatter direction.

In water, even if the particles are large, the relative index of refraction stays close to unity and this allows us to assume that the rays penetrating inside the particles are not significantly deflected. This absence of optical ray deflection makes it easy to compute the scattering patterns. In this small index difference, large particle regime, it is possible to separate out the effects of diffraction around the particle, refraction through the particle and reflection from the surfaces of the particles. Such a model has been developed [1] some time ago and the concepts on which it is based will be used as the building blocks of this new polarized scattering model. One note of caution, as kindly pointed out to us by A. Gainusa-Bogdan [7] there are some typos in formulas for the reflection terms in [1] which have now been corrected and expanded.

It has also been necessary to include the Fresnel-Kirchhoff obliquity factor $(1 + \cos(\theta))$ in the formulas for diffraction [8] and to use the Generalized Eikonal Approximation [9] for the refraction term and modify the normalization of these terms appropriately. This modification not only gives a better fit in the test case of spheres, but is also absolutely required in order to properly evaluate the Mueller matrix at the larger angles. This comes about because, in the large particle limit, the effect of polarization is entirely due to the reflections from the various surfaces of the particles. Not including the obliquity factor for diffraction and the GEA correction to the refraction terms means they are much too big at moderate to large angles where the reflection terms actually dominate. These modifications bring the simple model much more closely in line with experimental results.

Using the new model, which gives the correct parallel and perpendicular polarized phase function, and the operator transform equations given above we can derive explicit formulae for the complete Mueller matrix elements by simply combining the terms according to their operational definitions.

If we only consider the reflection terms from our model, we obtain a matrix whose only non-zero elements are exactly the same as those found to be significant by Voss and Fry [10] in their classic sets of measurements of the Mueller matrix of ocean water. That the only significant terms are the same is an extremely strong piece of evidence in favor of reflection as the dominant factor.

COMPARISON WITH EXPERIMENT

The one element from this first level model which is significantly in error, is the M(2,2) element. This occurs because on a single reflection there is no possibility of inducing a rotation of the polarization vector, which would be required to transfer part of the parallel component into the perpendicular component and vice-versa. In order to induce such a rotation of polarization, we have found that we need to take into consideration either two reflections or one reflection and one transmission by surfaces that do not scatter in the same plane. A simple example is the standard two-mirror beam director one uses on a lab bench to send a beam from a fixed source in any desired direction. It has the property of changing the polarization. As a matter of fact, a little thought will make it obvious that when the beam is rotated at 90 degrees, the parallel polarization has been completely turned into perpendicular polarization. For randomly oriented pairs of surfaces, the effect is not as marked but looks more like what was seen by Voss and Fry. This effect will occur naturally if the particles are irregular. As a matter of fact, one of the models which most closely fits the experimental polarized ocean data is that of large fractal particles with a real index of 1.1 [11]

CONCLUSION AND FUTURE WORK

The key element to model and the real source of information on the nature of the scattering particles in water is the $M(2,2)$ element of the Mueller matrix. It is the real tell tale and the lynchpin to understanding and robustly and reliably modeling polarization in water. This has serious implications both for remote sensing of polarization and for underwater imaging. It seems to imply that the particles need to be modeled as having a significant fractal component. In order to adapt the model to account for this we are developing a statistical approach where we determine the distributions of the pairs of surfaces for randomly oriented particles. This approach should allow us to constrain the problem sufficiently to obtain a reliable and accurate solution. New fast devices are now becoming available to measure the Mueller matrix [12] and more data equally reliable as that of Voss and Fry should become available in the future to help constrain the theoretical options even further.

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