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**SOME REMARKS ON**

**MEASURES OF EFFECTIVENESS IN OR**

by J.W. Mayne  
 Operational Research Division  
 Canadian Forces Headquarters

02b Director General Operational Research Div (Can)  
 02c Ottawa Ont

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The prime role played by measures of effectiveness in OR studies is well known. According to one of the early text books on OR (1) a measure of effectiveness must be specified and its suitability established when formulating any OR problem. Consequently it would seem to follow that most OR publications should make use of, or at least should refer to some appropriate measure of effectiveness. A review of some of the literature reveals that such is not the case. The purposes of this paper are to direct attention to the need for further theoretical work on measures of effectiveness, criteria and objective functions. It may first be helpful to state that by a measure of effectiveness is meant a variable or function which provides quantitative expression of the degree to which the system or organization under study achieves its objective (cf. Ref. 2).

Background

Some time ago in endeavouring to decide on a suitable measure of effectiveness for a military OR problem I made a literature search in the hope of finding some guidance. However the International Abstracts (3) carried no references whatever to effectiveness, criteria or objective functions; nor were there any helpful suggestions under 'measures' although there were many references to measurements and one to 'efficiency'. I decided to check through readily available OR journals to determine to what extent the general subject of effectiveness or criteria had been treated. The results of this survey are given in the following table below:

**TABLE I**  
 DISTRIBUTION OF REFERENCES TO MEASURES OF  
 EFFECTIVENESS IN THREE OPERATIONAL RESEARCH  
 JOURNALS

Journal	Volumes	No. of Articles	Measure of Effectiveness Mentioned or Used	Measure of Effectiveness Described or Discussed	Measure of Effectiveness Discussed Generally or Theoretically
JORSA	12	825	235	83	10
OR Quarterly	12	187	77	4	3
CORS Journal	3	24	14	1	0
<b>TOTALS</b>	<b>27</b>	<b>1036</b>	<b>326</b>	<b>88</b>	<b>13</b>

In assigning articles to the first two categories the following procedure was followed: if an article mentioned or defined a measure of effectiveness without describing why or how this particular measure was selected it was listed in one category. If, in addition to being cited, the particular measure of effectiveness was described in relation to the objectives of the associated decision maker, the article was listed in both the first and second categories. If there was a more general and theoretical discussion the article was also included in a third category.

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Although it is tempting to draw general conclusions from the data of Table I the only two points that will be remarked on are the very small percentage of articles that discussed measures of effectiveness or the equivalent in a general or theoretical fashion, and the surprisingly small percentage of articles that even used or mentioned a measure of effectiveness or an equivalent concept. The results raise the interesting question: What do contributors to these journals write about? However, it may be said that many articles are mathematical in nature, many are concerned with solutions that do not explicitly involve a measure of effectiveness, and many deal with other important aspects of OR. Still, the very few which actually discuss measures of effectiveness theoretically makes one wonder why the number and proportion of such papers is so low.

### Objectives, Models, and Measures of Effectiveness

It is customary in OR practice to relate the consumer's objectives, the mathematical model, measures of effectiveness, and alternative courses of action in attempting to formulate the problem (cf. Ref. 1 Chapter 1 and Ref. 4).

The general form of such a mathematical model may be expressed in the well known form  $E = f(X_i, Y_j)$  where E represents the effectiveness,  $X_i$  the controllable variables, and  $Y_j$  the uncontrolled ones. Consequently, it would appear obvious that all OR models would automatically give the selected measure of effectiveness in terms of the units in which E is measured. However, the term 'model' is a somewhat overworked and ill-defined term in many OR papers. In many cases no distinction is made between types of models (Ref. 1 defines iconic, analogue, and mathematical models, and a problem may need more than one model.) Also, many mathematical models used in OR do not express effectiveness. As a result it is possible to find many references to models in OR publications and few to effectiveness and measures thereof.

The model should express the effectiveness of the system which, in turn, should be related to, and compared with, the objectives which the system is attempting to achieve (4). In trying to decide whether the measure of effectiveness is suitably chosen it is certainly convenient to be able to express the relative importance of the objectives in quantitative terms and, especially so, if there are two or more, as is often the case. In addition it is desirable to know what Churchman, Ackoff, and Arnoff call the 'efficiency' of the course of action. (Ref. 1 Chapters 5 & 6). In devising a measure of effectiveness for cases where there are two or more objectives it is necessary to evolve a common measure for the efficiencies of the different objectives. To do so involves the OR worker in quantifying objectives and in the theory of measurement. Consequently the procedure for defining a suitable over-all measure of effectiveness is not simple. A useful treatment of this complex problem is given in Chapters 5 and 6 of Ref. 1. Interesting discussions of measurement in OR are given in Refs. 5, 6, 7, and 8.

### Discussion

In some cases an easy way out of the attendant difficulties has been to choose what appears to be the most important objective and recommend the course of action which will maximize the effectiveness or minimize cost-effectiveness relative to this objective. The possible pitfalls in such a procedure are evident or at least fairly well-known. An important related question is raised by Ackoff who asks (9) whether it is better to devise a good 'measure of performance' and use abbreviated and qualitative procedures for estimating the efficiency of alternative courses of action, or to use an 'incomplete measure of performance' and determine quantitatively optimal courses of action relative to it? Here again there may be no simple or single answer but it might be possible to provide some guidance by determining whether or not there are certain types or classes of problems for which the first approach is better and others for which the second would be preferable.

However, in a larger sense, the choice of good criteria, payoffs, objective functions, or measures of effectiveness and cost-effectiveness should be based on what Hitch called in his now classical paper (10) 'the analysis of relations between sub-optimizations at lower and higher levels'. In this paper (10) Hitch states that for a criterion in OR to be a good one it must be consistent with a 'good' criterion at a higher level. Although it may be difficult or impossible to apply this criterion satisfactorily in practice it should always be helpful to give consideration to it. In another paper (11) Hitch points out that frequently there is no one 'right' answer to the criterion selection problem and that it is generally necessary to content ourselves with a practical solution that may be far short of perfection. These remarks suggest the query: Are there any guide lines for using the yardstick of a 'good' criterion to assist in making better approximations to perfection? Also one might wish for some measure of the accuracy, precision, or reliableness (12) of a criterion or measure of effectiveness. It would certainly be useful to know, for example, how accurate or reliable a 'partially satisfactory' measure of effectiveness is in any particular case. Can measures of the accuracy or reliableness of measures of effectiveness be developed?

The difficulties encountered in trying to combine measures of effectiveness or, more particularly, measurements of different characteristics of a system, are well known to OR workers and have been touched on above. Some writers leave the impression that a single over-all measure of effectiveness should be sought in all cases. However, it is not apparent that to do so is always the best procedure. For example the inadequacy of 'cost-per-kill' has been pointed out by Walsh (13) and Quade has written about the limitations of a cost-effectiveness approach to military decision-making (14). Hitch points out (15) that there is a general case for using money measures of effort in certain circumstances and that there is relevant theory (16) which can be used and should be developed and extended. On the other hand Enthoven states (17) that many misgivings about the application of OR or systems analysis to national policy questions appear to be based on the misapprehension that workers in these fields believe that effectiveness can be measured in terms of a single number or scale. Yet in many successful applications of OR single measures of effectiveness have been used. Hitch states (15) that we know enough in many cases to distinguish the good criteria from the bad (Cf. Refs. 10 and 18). Again questions arise: Under what circumstances, or for what classes of problems, is a single measure of effectiveness satisfactory; and when should there be more than one, and how many? How can our knowledge about good and bad criteria be extended?

Timeliness is often overlooked in setting up measures of effectiveness, although such criteria as the maximization of profits over a period of time and of the expected number of targets destroyed in a given period of time certainly take this factor into account.

#### Concluding Remarks

The data presented in Table I indicate that there have been no satisfactory answers to the pleas for better measures of effectiveness (19) and research into the theory for choosing criteria (15). No body of theory has been developed yet and few relevant articles have been published. As the foregoing discussion suggests, the development of such theory will not be easy. However, as is pointed out by Hitch (15) the only road to good theory is by way of bad theory. But, apparently, not even bad theory is being developed.

This paper has attempted to draw attention to some facts, problems, and questions relating to the development of ways and means of improving criteria and measures of effectiveness, or cost-effectiveness, in OR. Some of the several questions which have been raised may suggest starting points for attacking particular facets of the main problem of evolving some bad theory which, in turn, may lead to better theory. A less ambitious hope is that OR workers may be encouraged to give more consideration than heretofore to choosing criteria and measures of effectiveness and to be more aware of the dangers that may surround innocent-looking ones.

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