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Emitter Number Estimation by the General Information Theoretic Criterion from Pulse Trains

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Abstract

In this report, an information theoretic criterion based approach for estimating the number of emitters from a set of interleaved pulse trains is proposed. In the approach, a new pulse signal model is formulated to handle large numbers of pulses. The approach is based on the application of the general information criterion (GIC) and has the advantage of not requiring any threshold setting procedures. Compared to the classical information theoretic criterion based approaches, the general information theoretic criteria based approach is more versatile and does not involve any computationally sophisticated maximum likelihood estimator. Computer simulations are used to demonstrate the effectiveness of the proposed approach.

Résumé

Nous proposons dans ce rapport une nouvelle approche pour estimer le nombre d'émetteurs à partir d'une série de trains d'impulsions entrelacés. Nous formulons un nouveau modèle de signaux capable de traiter un grand nombre d'impulsions.

L'approche est basée sur l'application du Critère d'Information Général et a comme avantage de ne pas nécessiter de procédures d'ajustement de seuils. Notre approche est plus souple par rapport aux approches basées sur les critères classiques de la théorie de l'information, et n'implique aucun estimateur de probabilité maximum sophistiqué. L'efficacité de cette approche est démontrée par des simulations faites à l'ordinateur.

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Executive summary

Background: In a practical signal environment, a wide-open ESM system with full angular coverage will receive a sequence of signal pulses interleaved. In order to identify the emitters, the pulse trains need to be deinterleaved. Conventional approaches for deinterleaving are based on the use of pulse parameters such as DOA, RF, TOA, PW, PRI and polarization. They are efficient for simple emitters with relatively constant frequency and PRI, and well separated parameters. However, they encounter difficulties in dense signal environments where emitters have similar parameters. The presence of similar emitters plus inaccuracy in signal parameter measurement by the receivers will cause ambiguities in deinterleaving. Furthermore, as modern radar adopts more and more complex digital techniques and sophisticated waveform modulations, conventional techniques become even more vulnerable.

Recently, there has been increasing interest in developing deinterleaving techniques based on the use of intrapulse information of the pulses. Intrapulse carries rich information about the emitter characteristics, signal propagation path and the environment. The intrapulse information describes the pulse more completely and more uniquely than the conventional pulse parameters. In this report, we focus on the problem of estimating the number of emitters from a set of pulse trains based on the use of intrapulse information, which is a pre-requisite for pulse deinterleaving.

Results: Previously, the author had developed a novel approach for estimating the number of emitters based on the minimum description length (MDL) criterion. However, the approach has a drawback in handling large numbers of pulse. It would involve heavy computation when the number of pulses increases. In this report, we extended the approach to handling large numbers of pulses. The main points are as follows.

- The signal model is re-formulated for accepting large numbers of pulses.
- The general information theoretic (GIC) criteria are applied to estimating the number of emitters. Performace analysis was carried out and it was shown that the approach is statistically consistent.
- Computer simulations are used to demonstrate the effectiveness of the approach.

Significance: The approach has overcome the difficulties encountered by the previously developed method. It is capable of handling large numbers of pulses, making it attractive for real-time applications. It is shown that the GICs are generalized versions of the regular information theoretic criteria with the same advantages, i.e., they do not require any subjective threshold setting. In addition, since the GICs do not involve any computationally sophisticated maximum likelihood estimator, it is efficient in terms of computation complexity and time. In summary, the proposed approach is a good candidate for use in a modern autonomous ESM system.

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Sommaire

Contexte: Dans un bon environnement pour les signaux, un système ESM ouvert à couverture angulaire complète recevra une série d'impulsions. Afin d'identifier les émetteurs, les trains d'impulsions doivent être désentrelacés. Certaines approches conventionnelles utilisent des paramètres d'impulsion comme leur DOA, RF, TOA, PW, PRI et la polarisation. Ceux-ci sont efficaces pour des émetteurs simples ayant des fréquences et PRI relativement constants et des paramètres bien séparés. Cependant, ils ne sont pas très efficaces dans des environnements à signaux denses où les émetteurs ont des paramètres semblables. La présence d'émetteurs similaires et d'imprécision dans les mesures causent des ambiguïtés au désentrelacement. De plus, étant donné que les radars modernes adoptent des techniques de plus en plus complexes et des modulations d'ondes sophistiquées, les méthodes conventionelles deviennent encore plus vulnérables.

Le développement de techniques de désentrelacement faisant usage de l'information intra-impulsion suscite un grand intérêt depuis peu. L'information intra-impulsion est une source riche d'information des caractéristiques des émetteurs, des chemins de propagation, ainsi que de l'environnement. Elle décrit les impulsions avec beaucoup plus de précision que les méthodes conventionnelles. Dans ce rapport, nous examinons le problème de l'estimation du nombre d'émetteurs à partir des trains d'impulsions et de l'information intra-impulsion. Il s'agit là d'un pré-requis pour le désentrelacement dimpulsions.

Résultats: Dans le passé, l'auteur a développé un moyen d'estimer le nombre d'émetteurs en se basant sur le critère de la longueur de description minimale (MDL). Cependant, cette approche est limitée lorsqu'elle est doit traiter un grand nombre d'impulsions. Le nombre de calculs augmente beaucoup lorsque la quantité de signaux est élevée. Dans ce rapport, nous avons étendu l'approche afin de traiter des impulsions nombreuses. Les points principaux sont les suivants.

- Le modèle de signal est re-formulé pour accepter un grand nombre d'impulsions.
- Le critère basé sur la théorie d'information générale (GIC) est appliqué pour estimer le nombre d'émetteurs. L'analyse de la performance de cette approche fut analysée et sa consistance statistique fut démontrée.
- Des simulations sont utilisées pour démontrer l'efficacité de l'approche.

Interprétation: L'approche a surmonté les difficultés rencontrées par la méthode précédente. Elle peut traiter un grand nombre d'impulsions, ce qui la rend attrayante pour des applications en temps réel. Nous démontrons que les GICs sont des versions généralisées des critères basés sur la théorie de linformation avec les mêmes avantages de ceux-ci, c'est-à-dire qu'ils n'ont pas besoin de procédure d'ajustement de seuils. En plus, comme les GICs n'impliquent aucun calcul sophistiqué pour les estimateurs de probabilité maximale, ils sont plus rapides. En conclusion, l'approche proposée est une

possibilité que nous pourrions utiliser avec succès dans un système ESM autonome moderne.

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1. Introduction

In ESM (electronic support measures) and ELINT (electronic intelligence) applications, receivers are used to intercept radar signals and perform emitter classification and identification for electronic warfare (EW) purposes. In a typical signal environment, since there are usually more than one emitters out there transmitting pulsed signals, a wide-open EW receiver with full angular coverage will receive a sequence of signal pulses interleaved. In order to identify the emitters, the pulse trains need to be deinterleaved, i.e., to separate the pulse trains into individual emitter groups, where pulses in one group belong to same emitter [1][2]. Conventional approaches for deinterleaving are based on the use of pulse parameters such as direction-of-arrival (DOA), radio frequency (RF), time-of-arrival (TOA), pulse width (PW), pulse repetition interval (PRI) and polarization [1]. They are efficient for simple emitters, i.e., emitters with relatively constant frequency and PRI from pulse to pulse, and well separated parameters. However, they encounter difficulties in dense signal environments where emitters have similar parameters. The presence of similar emitters plus inaccuracy in signal parameter measurements by the receivers will cause ambiguities in deinterleaving. In addition, as modern radars adopt more and more complex and sophisticated digital techniques, they use complex waveform modulations, making the conventional techniques even more vulnerable.

The drawbacks of the conventional deinterleaving approaches are due to the fact that the conventional pulse parameters are not sufficient for uniquely identifying a pulse. To counter these difficulties, there has been increasing interest in developing deinterleaving techniques based on the use of intrapulse information of the pulses [2][3][4]. Intrapulse refers to the shape of the pulse which includes the pulse envelope and phase variation within a pulse. It carries rich information about the emitter characteristics, signal propagation path and the environment [1]. The intrapulse information describes the pulse more completely and more uniquely than conventional pulse parameters, and is considered to be stable in the sense that, for a short period of time (at the order of milliseconds), the shape of the pulses signals from the same emitter will not change significantly. The intrapulse approaches are sometimes called *finger printing* by EW engineers [1]. They are closely related to the unsupervised learning in pattern recognition such as cluster analysis [3], fuzzy clustering algorithms [4] and neural networks [5][6]. Note that intrapulse may also refer to features extracted from the pulse waveforms. Granger *et al.* [7] studied the application of four different self-organizing neural networks for automatic deinterleaving of radar pulses. In [8], Kamgar-Parsi *at al.* proposed a neural network approach, which they referred to as the Hopfield-Kamgar clustering technique, for sorting the pulse feature vectors using the Hopfield networks. These unsupervised algorithms are all adaptive and can autonomously analyze the pulse data and categorize them. However, the performance of the algorithms is dependent on the correct estimation of the number of emitters out there. The self-organizing algorithms tend to produce artifacts and possible false alarms when corrupted pulses are encountered. The Hopfield-Kamgar algorithm handles the problem by first partitioning the pulses into an initial number of groups, and then merging the groups by examining their pairwise normalized standard deviation. The approach requires that a threshold be set according to some subjective judgment, thus increasing the demand for interactions between command users and the system. In [9], Wong *et al.* formulated the pulse deinterleaving as a multivariate clustering problem. They used the Minimum Description Length (MDL) for cluster validation. The approach is computationally exhaustive since for each assumed number of cluster, complex clustering procedures have to be performed. Recently, Zhou and Lee [10] have developed a novel approach for estimating the number of emitters from a set of received pulse trains. In the approach, it was discovered that, by formulating the pulse data in certain ways, the number of emitters can be determined from the number of significant eigenvalues of the covariance matrix of the pulse data. The MDL criterion was derived for determining the optimal number of emitters. The approach is also capable of deinterleaving the pulses at the same time. The approach was statistically consistent and has the advantage of not requiring any subjective threshold setting. When the number of received pulses is modest, the approach is computationally efficient since it does not involve iterative procedures. However, in an EW signal environment, the data rate can be very high, and the number of intercepted radar pulses can reach up to 10^6 per second. For the approach by Zhou and Lee, this means an eigen-decomposition of a huge covariance matrix, making it intractable and even impractical.

In this report, we extend the approach by Zhou and Lee to handling large number of pulses. The contribution of this report is two-fold. First, the signal model is reformulated for accepting large numbers of pulses. Secondly, the general information theoretic criteria are applied to estimating the number of emitters. In [10], the size of the covariance matrix is determined by the number of pulses received. When the number of pulses is large, the eigen-decomposition of the covariance matrix becomes impractical. In this report, the pulse data is arranged in such a way that the size of the covariance matrix is dependent on the number of samples within pulses, and the number of pulses has no impact on the dimension of it. It is shown that the the number of significant eigenvalues of the covariance matrix is equal to that of the emitters present. The general information theoretic criterion is proposed for estimating the number of significant eigenvalues. The GIC is a generalized version of the regular information theoretic criteria. The regular criteria are usually obtained through the maximization of the likelihood function while the GICs are constructed based on the r-regulated functions. The GICs do not involve any computationally sophisticated maximum likelihood estimators. We show that the criterion in [10] is a special case of the GIC, where the 1-regulated function is used. Under certain conditions, they are strongly statistically consistent, i.e., the estimated number of emitters converges to the actual value as the number of pulses approaches infinity.

The report is organized as follows. In Section 2, the pulse signal model is formulated and discussed. In Section 3, the general information theoretic criteria are applied to estimating the number of emitters. The use of different r-regular function is discussed, and the strong statistical consistency of the criteria is shown. In Section 4, computer simulations are used to demonstrate the effectiveness of the approach.

2. PULSE SIGNAL MODEL

Assume that there are K emitters which transmit narrowband pulse signals. The receiver is assumed to be capable of separating simultaneous pulses. Assuming that the mth pulse is from the kth emitter, the analytic representation of the mth pulse signal at baseband can be written as

$$x_m(nT) = \delta_m s_k(nT - \tau_m) + w_m(nT), \tag{1}$$

where T denotes the sampling interval, $s_k(nT)$ denotes the complex pulse waveforms from the mth emitter at the receiver, δ_m represents the amplitude and phase-shift on the pulse, and τ_m is the time delays relative to a reference time origin. The parameter δ_m is complex. Its amplitude represents the gain fluctuation of the receiver and the received signal power which may vary from pulse to pulse depending on the antenna pattern and scanning mode of the emitter. The signal-to-noise ratio (SNR) of the received signal may vary from 45dB (mainbeam illumination) to 15dB (sidelobe illumination) for a circular scanning emitter. The phase of δ_m reflects the time delay on the signal caused by the receiver. It is known that the sampling of a digital receiver is usually controlled by trigger signals. When an incoming signal is corrupted with noise or its power varies, the trigger signal and its pre/post adjustment may cause different time delays on the pulses. Since $s_k(t)$ is the baseband representation of a narrowband signal, when the sampling rate is sufficiently high, the time delay τ_m can be ignored. It follows that the signal model (1) can be written as

$$x_m(n) = \delta_m s_k(n) + w_m(n). \tag{2}$$

The input noise $w_m(n)$ includes the medium ambient noise, antenna thermal noise, and the circuitry noise, *etc.*. In practice, it is usually dominated by the circuitry noise since the RF pre-amplifier of the receiver is always designed to have sufficient gain to overcome the additional noise sources in the receiver. For digital receivers, the quantization noise introduced by the analog-to-digital converter is also a source of noise. We make the following assumptions about $s_k(t)$ and $w_m(t)$.

- The signal waveforms $s_k(n)$ are unknown deterministic. Let

  $$\underline{s}_k = [s_k(1), s_k(2), \ldots, s_k(N)]^T, \tag{3}$$

  where N denotes the number of samples. The pulse vectors $\{\underline{s}_k;\ k = 1, 2, \ldots, K\}$ are linearly independent.

- The noise process $w_m(t)$ is additive and independent identically distributed (*i.i.d.*) Gaussian process with zero mean and an unknown variance of σ_w^2.

3. Emitter Number Estimation

Assume that data collected for each pulses have the same length. The mth pulse data model (2) can be written in a vector form as

$$\underline{x}_m = \delta_m \underline{s}_k + \underline{w}_m, \tag{4}$$

where $\underline{x}_m = [x_m(1), \ldots, x_m(N)]^T$ and $\underline{w}_m = [w_m(1), \ldots, w_m(N)]^T$. Denote $X = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_M]$. We have

$$X = SA + W, \tag{5}$$

where $S = [\underline{s}_1, \underline{s}_2, \ldots, \underline{s}_K]$, $W = [\underline{w}_1, \underline{w}_2, \ldots, \underline{w}_M]$ and A is of $K \times M$ given by

$$A = \begin{bmatrix} \delta_1 & 0 & \ldots & 0 & 0 \\ 0 & \delta_2 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & 1 & \delta_N \end{bmatrix}. \tag{6}$$

The matrix A is called the association matrix. Each column of A has one non-zero entry which associates the received pulse data with an underlying emitter. For example, a non-zero entry at the kth row and the mth column position would mean that the mth pulse is from the the kth emitter. Since the rows of A are linearly independent, A can be verified to have a rank of K.

3.1 Eigenvalue distribution of R_x

The covariance of X is given by

$$R_x = E\{(SA + W)(SA + W)^H\} = S(AA^H)S^H + \sigma_w^2 I, \tag{7}$$

where E is the expectation operator. Since $\{\underline{s}_k;\ k = 1, 2, \ldots, K\}$ are assumed to be linearly independent, S has a full column rank of K. In addition, since A has a rank of K, matrix $S(AA^H)S^H$ can be verified to be positive semi-definite with rank K. The eigendecomposition of R_x can be written as

$$R_x = \sum_{i=1}^{K} \lambda_i \underline{u}_i \underline{u}_i^H, \tag{8}$$

where $\{\lambda_i, \underline{u}_i\}$ denotes the eigenvalues and corresponding eigenvectors. It can be verified that the eigenvalues, when arranged in a decreasing order, are given by

$$\lambda_1 \geq \lambda_2 \ldots \geq \lambda_K \geq \lambda_{K+1} = \lambda_{K+2} = \ldots = \lambda_N. \tag{9}$$

where the last $(N - K + 1)$ smaller eigenvalues are identical. It can also be verified that the eigenvectors associated with the K largest form the basis of the subspace spanned by the columns of S.

3.2 General information theoretic criterion

From (9), it can be seen that the number of emitters can be determined from the number of large eigenvalues of R_x. In this section, we discuss the application of the general information theoretic criteria for estimating the number of large eigenvalues of R_x. In the context of array processing, there have been recently developed many different approaches [12][13]. They are often based on the application of the information theoretic criteria introduced by Akaike [14], Rissanen [15] and Schwartz [16] *et al.*. In the model selection problems, the information theoretic criteria take into account both the goodness-of-fit (likelihood) of a model and the number of parameters used to achieve that fit. The criteria take the form of a penalized likelihood function, i.e., a negative log likelihood function plus a penalty function. The general information criteria by Yin and Krishnaiah [11] are different from the regular information theoretic criteria. They are based on the use of r-regulated functions, and are considered to be a general versions of the information theoretic criteria. The GIC based approaches do not involve any computationally complex and sophisticated maximum likelihood estimators. Some of the main results in [11] are re-stated as follows.

r-regulated function Let $\mathcal{R}$ be an open set of real numbers and $\mathcal{R}^2 = \mathcal{R} \times \mathcal{R}$, $\mathcal{R}^3 = \mathcal{R}^2 \times \mathcal{R}$, ..., be the Cartesian powers of $\mathcal{R}$. Let f be a real function of finite sequences of $\mathcal{R}$. Then, f is r-regulated if the following conditions are satisfied.

1. if $z_1 = z_2 = \ldots = z_N$ then $f(z_1, z_2, \ldots, z_N) = f(z_1)$.
2. if z_1, z_2, ..., z_N are not identical, then $f(z_1, z_2, \ldots, z_N) > f(z_1)$.
3. For each k, the restrictions of f on $\mathcal{R}^k$, f_k belongs to $C^{r+1}(\mathcal{R}^k)$, and the partial derivatives of f_k up to order r are all zero on the set

$$\{(z_1, z_2, \ldots, z_N) \in \mathcal{R}^k : z_1 = z_2 = \ldots = z_k\}$$

Theorem: Let f be a r-regulated function of finite sequences of $\mathcal{R}$. Let $\{\hat{\lambda}_i,\ i = 1, 2, \ldots, N\}$ be a sequence of random variables for $i = 1, 2, \ldots, N$, and $\hat{\lambda}_i$ takes values in $\mathcal{R}$. If the following conditions exist.

1. $\mid \hat{\lambda}_i - \lambda_i \mid = 0(\alpha_M)$ as $N \to \infty$ for $i = 1, 2, \ldots, N$, where

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K = \lambda_{K+1} = \ldots \lambda_N \tag{10}$$

are non-random constants in $\mathcal{R}$, $\alpha_M > 0$ are non-random, and $\alpha_M \to 0$.

2. C_M are non-random, $C_M \to 0$, and $\alpha_M^{r+1}/C_M \to 0$.

The general information theoretic criterion takes the forms of

$$Q(k, C_M) = f(\hat{\lambda}_{k+1}, \ldots, \hat{\lambda}_N) + kC_M, \tag{11}$$

and the number K is estimated as the value of k that minimizes $Q(k, C_M)$,

$$\hat{K} = \arg\min_k Q(k, C_M). \tag{12}$$

Then, $\hat{K}$ converges to K asymptotically, i.e.,

$$\hat{K} \to K, \quad M \to \infty. \tag{13}$$

and

$$Prob(\hat{K} \neq K) \leq N \cdot Prob\left(C_M \leq \sum_{i=1}^{N} \mid \lambda_i^{(M)} - \hat{\lambda}_i^{(M)} \mid\right) \tag{14}$$

The following are some examples of the r-regulated functions (the superscript denotes the degree of regularity) [11].

$$f^{(1)}(z_1, z_2, \ldots, z_N) = -\sum_{i=1}^{N} \log z_k + N \log\left(\frac{1}{N}\sum_{i=1}^{N} z_i\right) \tag{15}$$

$$f^{(1)}(z_1, z_2, \ldots, z_N) = \frac{1}{N}\sum_{i=1}^{N} z_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} z_i\right)^2 \tag{16}$$

$$f^{(3)}(z_1, z_2, \ldots, z_N) = \frac{1}{N}\sum_{i=1}^{N} \left(z_i - \frac{1}{N}\sum_{j=1}^{N} z_j^4\right)^4 \tag{17}$$

$$f^{(\infty)}(z_1, z_2, \ldots, z_N) = \left\{N\sum_{i=1}^{N} \exp\left[-\left(z_i - \frac{1}{N\sum_{j=1}^{N} z_j}\right)^2\right]\right\}^{-1}. \tag{18}$$

3.3 Emitter number estimation

In practice, the covariance R_x is usually not available and can only be estimated by its sample covariance

$$\hat{R}_x = \frac{1}{M} X X^H = \frac{1}{M} \sum_{i=1}^{M} \underline{x}_i \underline{x}_i^H. \tag{19}$$

Lemma:([12], Appendix) The sample covariance $\hat{R}_x$ converges to R_x with probability one, i.e.,

$$\lim_{M \to \infty} \hat{R}_x = R_x, \tag{20}$$

with a convergence rate proportional to α_M given by

$$\alpha_M = \sqrt{(\log \log M)/M}. \tag{21}$$

It can be verified that $\lim_{M \to \infty} \alpha_M = 0$.

Let $\{\hat{\lambda}_i,\ i = 1, 2, \ldots, N\}$ denote the eigenvalues of $\hat{R}_x$. Due to noise and finite number of samples in estimating the covariance, $\{\hat{\lambda}_i\}$ will be the perturbed version of $\{\lambda_i\}$ and the smaller eigenvalues will no longer be identical. They tend to separate into two categories with K large eigenvalues and the rest being smaller ones. Let r_{ij} and $\hat{r}_{ij}$ denotes the ijth components of R_x and $\hat{R}_x$, respectively. According to the theorem by Bai, Miao and Rao [13], since $\hat{R}_x$ converges to R_x at a rate proportional to α_M, i.e.,

$$\mid r_{ij} - \hat{r}_{ij} \mid = \alpha_M \quad i, j = 1, 2, \ldots, N, \tag{22}$$

there exists a constant B independent of α_M, such that

$$\mid \hat{\lambda}_i - \lambda_i \mid = B \alpha_M, \tag{23}$$

which implies that the eigenvalues of $\hat{R}_x$ each converge to those of R_x with a rate dominated by α_M.

To apply the general information theoretic criterion, a non-random C_M is needed which satisfies the second condition in the *Theorem*. For criteria (16), (17) and (18), we choose

(24) $$C_M = \eta \cdot (\log M/M)^{(r+1)/2},$$

where η is a constant and r is the degree of the regularity. It can be verified that, when η is a constant,

(25) $$\lim_{M\to\infty} \eta \cdot (\log M/M)^{(r+1)/2} = 0,$$

and

(26) $$\lim_{M\to\infty} \frac{\alpha_M^{r+1}}{\eta(\log M/M)^{(r+1)/2}} = 0.$$

For criterion (18) which has an infinite regularity degree, we simply use $C_M = \eta \cdot \log M/M$. It can be verified that such a C_M satisfies the conditions in the *Theorem*.

Then, from the *Theorem*, the general information theoretic criterion can be constructed as

(27) $$Q(k) = f(\hat{\lambda}_{k+1}, \hat{\lambda}_{k+2}, \ldots, \hat{\lambda}_N) + k \cdot C_M,$$

where f denotes a r-regulated function, C_M is given by (24) and $k = 0, 1, \ldots, N$. The optimal number of emitters, $\hat{K}$, is given by the one that minimizes the GIC (27) over all possible k. The strong consistency can be established by the *Theorem*.

Asymptotically, the selection of the constant η in C_M will not affect the strong consistency of the GIC. However, for finite numbers of pulses, the performance of the estimator is dependent on η. In [18], in the context of array processing, Wu and Fuhrmann did some studies on the performance of the estimator. They provided the empirical distributions of the r-regulated functions and a strategies for selecting η. It is also noted that GIC based on the 1-regular function (16) has the same form as the MDL criterion derived in [10], which involves the ratio of the arithmetic to the geometric mean of the k smallest eigenvalues of the sample covariance of the pulse data vector. Finally, it should be mentioned that, asymptotically, the degree of regularity of the r-regular functions, in general, impacts the speed of convergence for the consistency of the GICs. Usually, the higher the degree of regularity, the faster the convergence. This is only true asymptotically. For a finite N, the convergence of the GICs may not follow this rule, as will be shown in the simulation studies.

4. Numerical Simulation Studies

Computer simulated data is used to demonstrate the effectiveness of the proposed algorithm. Four emitter are simulated. The signal waveforms are selected from the real intercepted radar signals. They have been converted into the analytic forms using in-phase and quadrature (I/Q) components. The length of the pulses are 103, 99, 103 and 104, respectively. We use the maximum $N = 104$ as the pulse data vector length. The shorter pulses are expanded to this length by appending zeros at the end. Each pulse is simulated to have a twenty percent perturbation relative in magnitude. A phase shift uniformly distributed in $[0, 2\pi]$ is also simulated. The pulses are uniformly selected among the four emitters. The additive noise is simulated as *i.i.d.* Gaussian with zero mean and different variances. The signal-to-noise ratio (SNR) is defined as the ratio of the average signal power to the noise variance

(28) $$\text{SNR} = 10 \log_{10} \frac{\overline{P}}{2\sigma_w^2}.$$

where

(29) $$\overline{P}_s = \frac{1}{2KN} \sum_{k=1}^{K} \sum_{t=1}^{N} \mid s_k(t) \mid^2,$$

denotes the averaged pulse power.

In forming the GIC for determining the number of large eigenvalues, instead of using the total N eigenvalues, we used the first L eigenvalues. In the following, we discuss its geometric interpretations. The spectral representation of $\underline{x}_m$ in terms of the eigenvectors of $\hat{R}_x$ can be written as

(30) $$\underline{x}_m = \sum_{i=1}^{N} c_{mi} \underline{u}_i,$$

where $c_{mi} =< \underline{u}_i, \underline{x}_m >$ is the product inner product of $\underline{x}_m$ and $\underline{u}_i$. It is known that $\hat{R}_x$ has K large eigenvalues and the rest are smaller eigenvalues. We take advantage of this *a priori* knowledge by retaining only the first L eigenvectors in the spectrum representation

(31) $$\tilde{x}_m = \sum_{i=1}^{L} c_{mi} \underline{u}_i.$$

It follows that $\underline{\tilde{x}}_m$ is an approximate of $\underline{x}_m$ with a reduced dimension. Let $\underline{c}_m = [c_{m1}, c_{m2}, \ldots, c_{mL}]^T$ be a truncation of the Karhunen-Loéve transform of $\underline{x}_m$, which is the most efficient transform in the mean-square-error sense. The truncated Karhunen-Loéve transform $\underline{c}_m$ can be considered as a feature extracted from $\underline{x}_m$ which has a reduced dimensionality. Let $U_L = [\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_L]$. The sample covariance matrix of the feature vector $\{\underline{c}_m;\ m = 1, 2, \ldots, M\}$ can be obtained by

$$\hat{R}_c = \frac{1}{M} \sum_{m=1}^{M} \underline{c}\underline{c}^H = U_L R_x U_L = diag(\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_L), \tag{32}$$

which has the identical eigenvalues as the first L eigenvalues of $\hat{R}_x$. Thus, if we apply the GICs to the feature vectors, it is identical to the one when applied to the first L eigenvalues of $\hat{R}_x$. That is, the use of the first L eigenvalues of $\hat{R}_x$ in forming the GICs is equivalent to applying the GICs to the feature vectors of the pulse data vectors. Note that the number of eigenvalues L should be at least greater than the actual number of emitters.

We use the Monte Carlo approach to evaluate the performance of the GIC approach. First, the performance of the GICs via different number of pulse for different SNRs is examined. Each time, the number of eigenvalues used is $L = 40$. Figures 1 to 3 show the correct detection rate via the number of pulses for SNR= 10dB, 15dB and 20dB, respectively. In the figures, notations GIC1, GIC2, GIC3 and GIC4 are used to denote the criteria with the r-regular functions given by (16), (17), (18) and (18), respectively. Each test is repeated 200 times to obtain the correct detection rate. In the examples, η is selected to be 6, 0.001, 0.02×10^{-3}, and 10^{-4}, respectively, for GIC1, GIC2, GIC3 and GIC4. These numbers certainly are not optimal. They are obtained by trying different values. It can be seen that, for different SNRs, the detection rate increases as the number of pulses increases. They gradually approaches the value 1.

Figures 4 and 5 plot the variation of correct detection rate via SNR for different number of pulses. The numbers of pulses are $M = 100$ and 200 in Figures 4 and 5, respectively. The SNR varies from 4dB to 20dB with a step of 2dB. Each test is repeated 200 times to obtain the averaged results. It is observed that the correct detection rates improves as the SNRs are increased, and they approach one when the SNR is sufficiently large.

It is also observed from the plots that the speed of convergence does not necessarily agree with the theory which says that the higher the degree of regularity, the faster the speed of convergence. For example, in Figure 1, GIC3 with a degree of regularity 3 is seen to have a slower convergence rate than GIC2 which has a degree of regularity of 1. This is due to the fact that the number of pulses in use is finite.

5. Conclusions

In this report, the GIC approach has been presented for estimating the number of emitters from a set of interleaved pulse trains. The approach is able can handle large number of pulses, and does not involve the usually lengthy and difficult maximum likelihood estimator. Computer simulations are used to demonstrate the effectiveness of the proposed approach. Simulation results showed that the proposed approach was able to provide satisfactory performance under modest SNR and finite number of pulses. Note that further investigations are still needed for the selection of the certain parameters, and will be presented in the near future.

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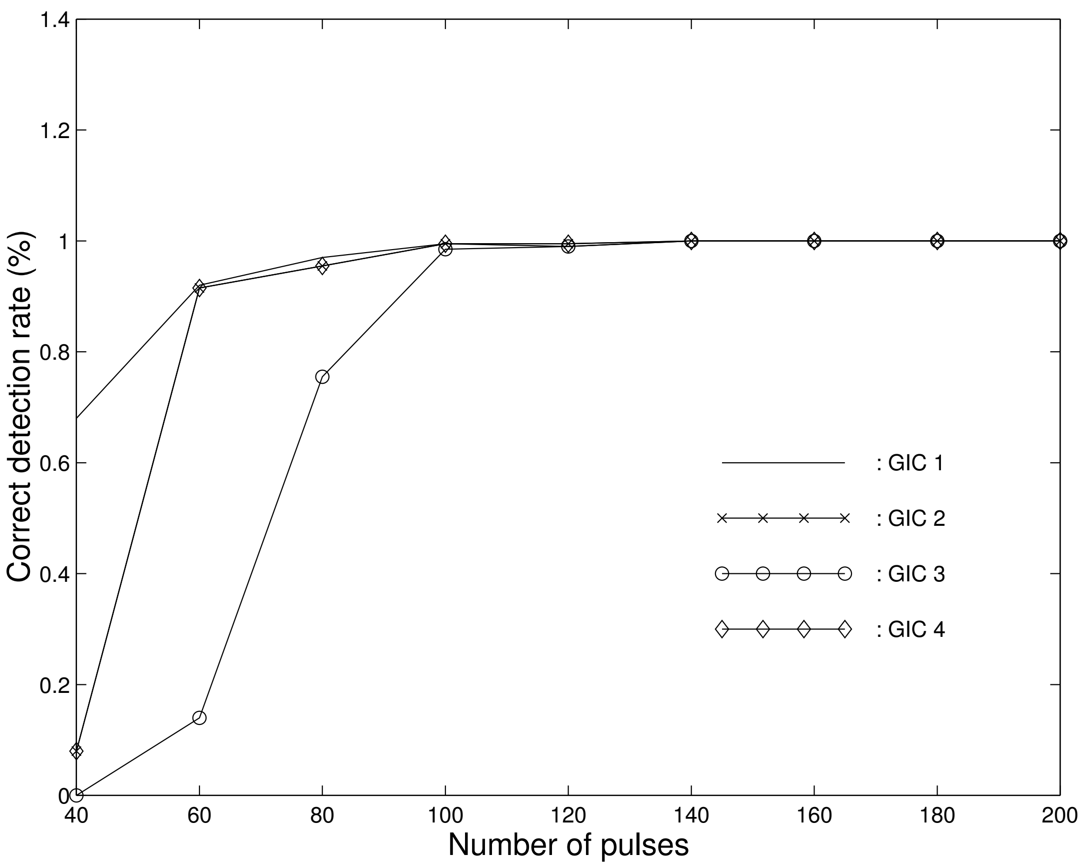

Figure 1: *Variations of the detection error rate versus the number of pulses for SNR=*10*dB.*

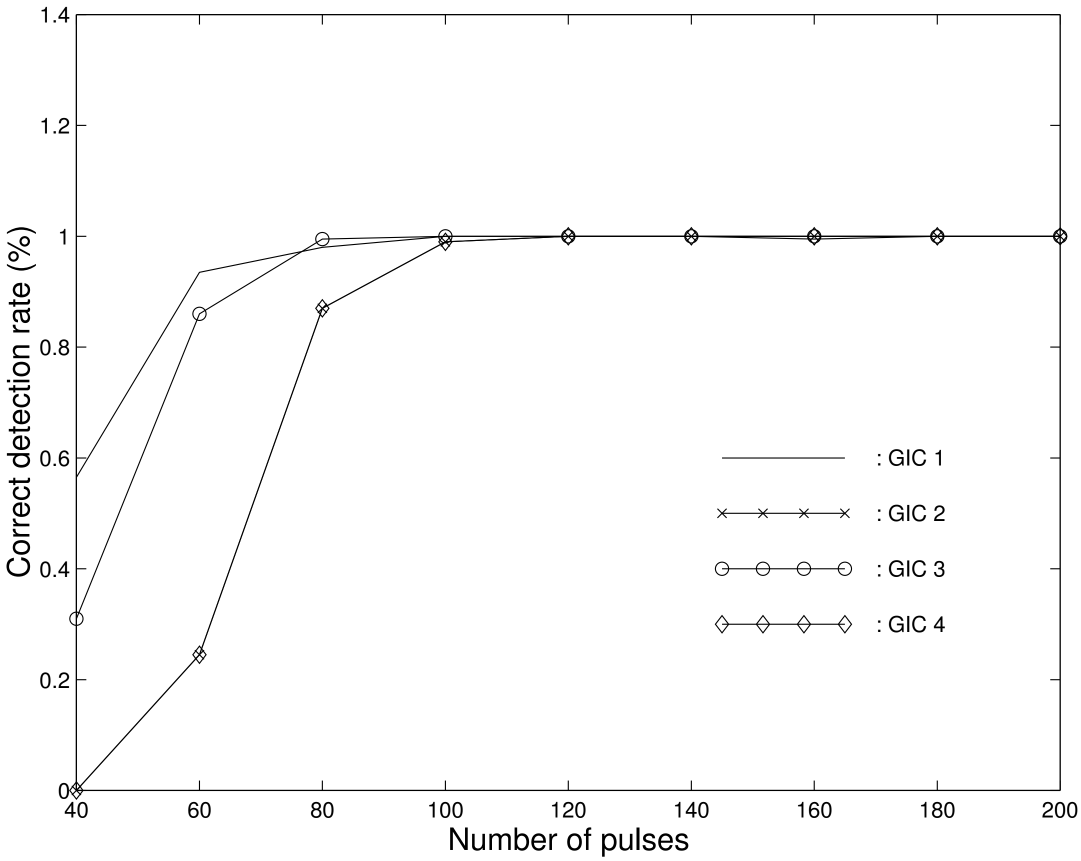

Figure 2: *Variations of the detection error rate versus the number of pulses for SNR=15dB.*

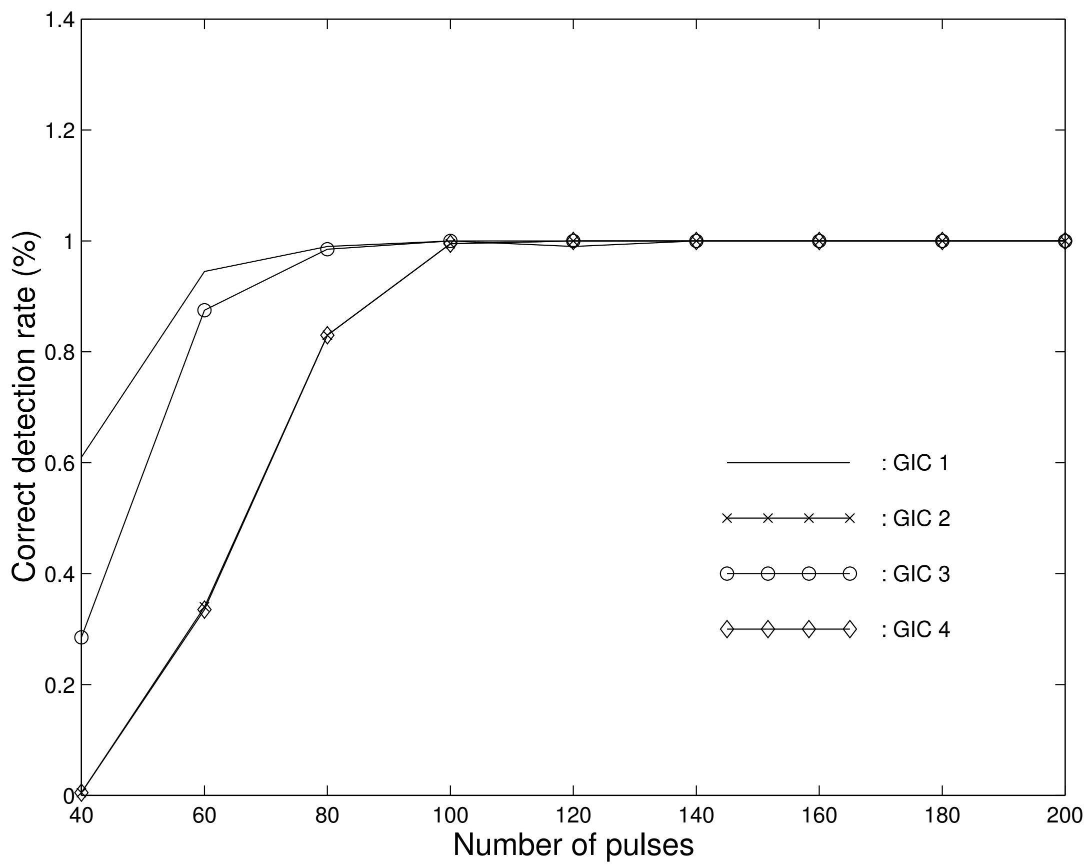

Figure 3: *Variations of the detection error rate versus the number of pulses for SNR=20dB.*

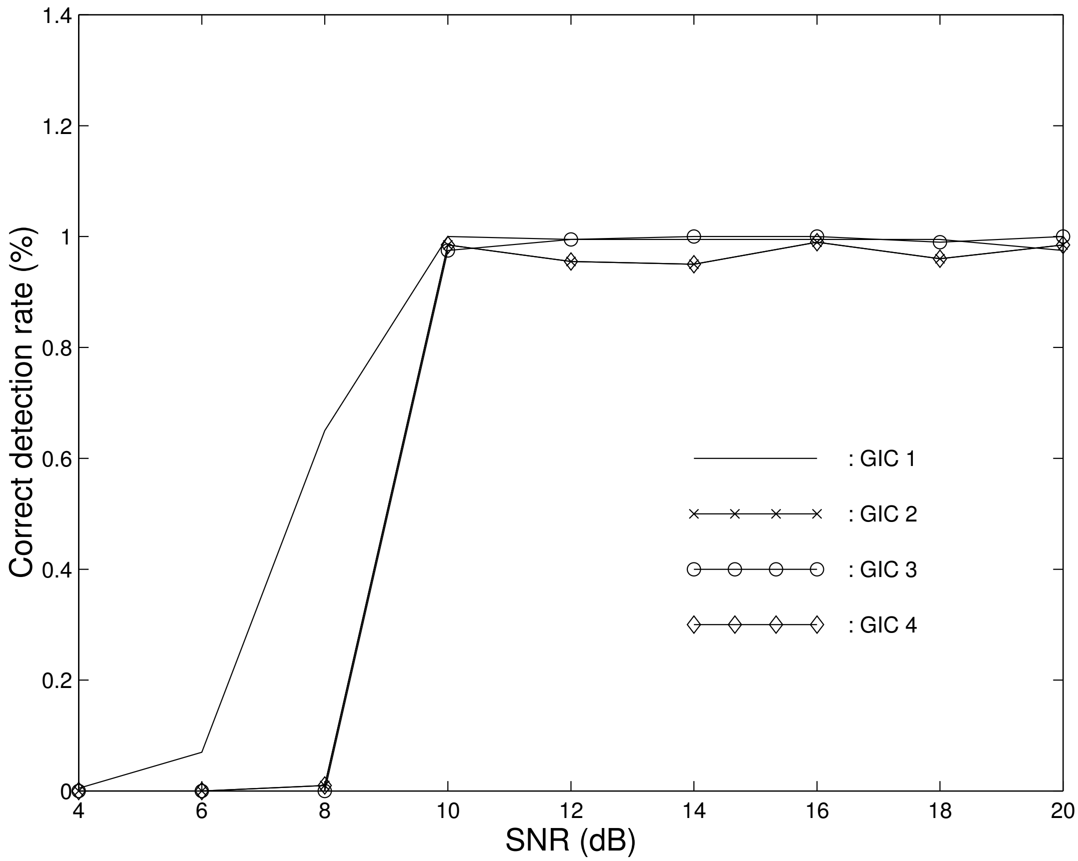

Figure 4: *Variations of the detection error rate versus SNR. The number of pulses is* $M = 100$.

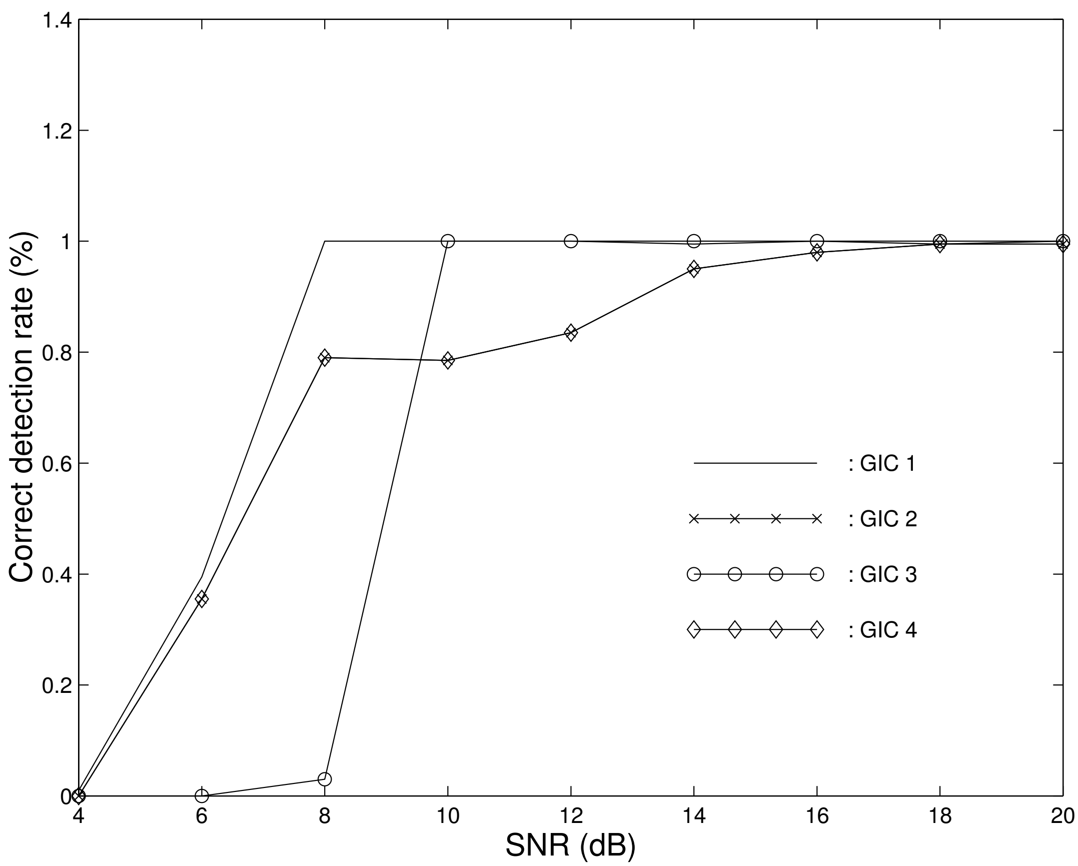

Figure 5: *Variations of the detection error rate versus SNR. The number of pulses is* $M = 200$*.*

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In this report, an information theoretic criterion based approach for estimating the number of emitters from a set of interleaved pulse trains is proposed. In the approach, a new pulse signal model is formulated to handle large numbers of pulses. The approach is based on the application of the general information criterion (GIC) and has the advantage of not
requiring any threshold setting procedures. Compared to the classical information theoretic criterion based approaches, the general information theoretic criteria based approach is more versatile and does not involve any computationally sophisticated maximum likelihood estimator. Computer simulations are used to demonstrate the effectiveness of the proposed approach.

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