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TITLE

Time-Domain Simulation of Nonlinear Ship Motions and Wave Loads

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Time-Domain Simulation of Nonlinear Ship Motions and Wave Loads

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ABSTRACT

In the assessment of the ability of a ship structure to withstand specified sea environments with an acceptable risk of failure, accurately predicted loads are vital. Of paramount importance are extreme loads on the hull structure, which usually occur when a ship undergoes large heaving and pitching motions in rough seas, sometimes accompanied by impacts between the bow and waves. In such severe motions, the instantaneous wetted part of the hull varies considerably, and linear theory's basic assumption of small-amplitude waves and ship motions is flagrantly violated. Yet it is not possible to treat the geometric nonlinearity of the wetted part of a hull and transient phenomena like slamming adequately in the frequency domain; however, they can be incorporated into time-domain analysis relatively easily. It is generally believed, therefore, that practical calculations of combined wave-induced and slamming forces may only be carried out in the time domain. This paper describes the application of strip theory to time-domain calculations of nonlinear ship motions and hull-girder loads in head seas. In this approach, the differential equations of ship motion are solved numerically by calculating hydrodynamic forces and coefficients at each time step by strip theory, taking into account nonlinearities relating to large relative motion between waves and the ship hull. The results show that this approach provides a practical means to examine the effects of nonlinear loads that cannot be adequately treated in the conventional linear frequency-domain approach.

Introduction

The linear model for ship response to ocean waves assumes that motion displacements from the equilibrium position are sufficiently small (relative to the relevant dimensions of the ship) so that the response is proportional to the amplitude of the excitation. It leads to a fairly simple model and yet often useful results. But no system in reality is exactly linear, and nonlinearities become progressively more significant as the severity of excitation increases, leading to unreliable predictions. The objective of this study, therefore, is to develop a nonlinear time-domain simulation method for ship responses to waves as a means to improve the accuracy of predicted ship motions and hydrodynamic loads in waves whose

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amplitudes are not necessarily as small as assumed in linear models. This work was carried out as part of an ongoing ship structural research program at DREA to develop an integrated reliability-based structural analysis software package for assessing the structural capabilities of Canadian Forces ships.

The time-domain approach enables us to take into account various frequency-independent nonlinearities with relative ease, a significant advantage over the conventional frequency-domain approach. One such nonlinearity is the geometric nonlinearity, which arises from large relative motion between waves and the ship hull in severe seas. Sometimes called hull-shape nonlinearity, it is a direct result of substantial changes in the instantaneous wetted hull form as a ship undergoes large motions. Geometric nonlinearity is especially significant for frigates and destroyers for two reasons. First, being of modest size, these ships are highly susceptible to extreme motions in rough seas, so fluid forces acting on frigates and destroyers can be strongly nonlinear. Second, such ships have relatively small vertical prismatic coefficients, and this accentuates geometric nonlinearities. In this study, the nonlearities in the Froude-Krylov force, the radiation force, and the buoyancy force, which stem from the geometric nonlinearities are treated.

Large-amplitude motions also give rise to other forms of nonlinearities such as bottom slam, bow-flare impact, and shipment of green water on the fore deck. The resulting nonlinear, nonstationary ship response cannot be adequately treated in the frequency domain, but can be incorporated into a time-domain simulation procedure relatively easily. It is therefore generally believed that practical computation of combined low-frequency wave-induced loads and nonperiodic transient loads may only be carried out in the time domain. Nevertheless, owing to the complexity of the hull-fluid interactions during impact, the transient loads are difficult to model mathematically and are perhaps the largest uncertainty affecting the structural analysis of ships today. The results of this study are to be extended to include transient loads in the next phase of the on-going project.

In so far as ship motions and wave loads in severe waves are concerned, comparisons with experiments clearly showed the superiority of the time-domain approach to the conventional frequency-domain approach. The strip-theoretical time-domain method described in this paper provides a practical means to examine the effects of nonlinear loads that cannot be adequately treated by frequency-domain strip theory.

Theoretical model

The ship hull is assumed slender and rigid. Water is assumed incompressible, inviscid, and deep. Deformation of the incident wave by the presence of the hull is assumed negligible. Surge motion, which has a negligible effect on wave load, is neglected in this study. When a ship is under way, it heaves and pitches about a mean position and attitude, which differ from those when the ship is at rest in calm water; the differences in these quantities are called sinkage and trim. These higher-order terms stemming from the perturbation—represented by the potential φ_s in equation (6) below—in the fluid caused by the forward motion of the ship are also neglected. The shifts in the mean values of the vertical bending moment (VBM) resulting from the ship's own wave system in calm water and from regular waves are also neglected. The three reference frames used are shown in Fig. 1: the earth-fixed frame $OXYZ$, the equilibrium translating frame $\overline{G}\overline{x}\overline{y}\overline{z}$, and the body-fixed frame $Gxyz$. The first two are inertial frames of reference in which Newton's second law holds. The XY-plane lies in the calm-water surface.

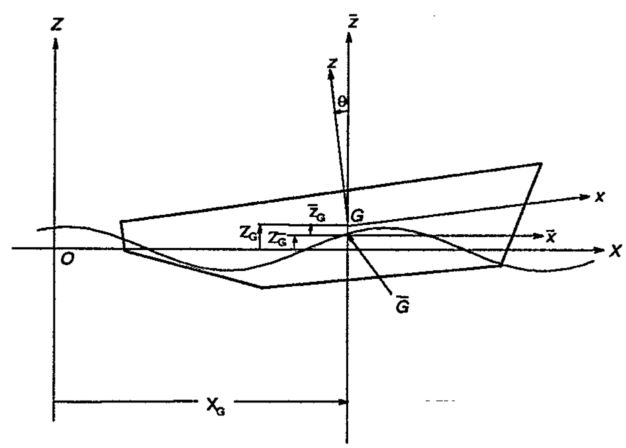


FIG. 1. Coordinate systems

The surface elevation ζ of incident regular waves is represented by

$$\zeta(X,t) = a\cos(kX + \omega t + \varepsilon), \qquad k = \omega^2 / g \quad (1)$$

where a is the wave amplitude, ω the angular frequency, ε the phase angle, k the wave number, and g the gravitational acceleration. The X-, $\overline{x}$-, and x-axes all coincide with the ship's heading. The Z- and $\overline{z}$-axes always point in the direction opposite to gravity, while the upward direction of the body-fixed z-axis changes with time. The $\overline{x}\overline{y}\overline{z}$-fame is parallel to the XYZ-frame, with origin $\overline{G}$ located at the mean position of the center of gravity, G, of the ship, and translates in the positive $\overline{x}$ direction at a constant speed equal to the mean forward speed U of the ship. At time t, the origin $\overline{G}$ is located at $(Ut, 0, Z_{\overline{G}})$ in the $OXYZ$ frame, where $Z_{\overline{G}}$ is the mean position of G. The third reference frame, the body-fixed frame $Gxyz$ is a noninertial frame. Its origin is attached to G, which is assumed to lie in the centerplane of the ship. At $t = 0$, the $Gxyz$ frame coincides with the $\overline{G}\overline{x}\overline{y}\overline{z}$ frame. At time t, G is located at $(Ut, 0, Z_G)$ in the $OXYZ$ frame. Let $\theta(t)$ denote the pitch angle and $z_G(t)$ the vertical coordinate of G relative to the $\overline{G}\overline{x}\overline{y}\overline{z}$ frame (*i.e.* the instantaneous heave displacement). Then, at a given instant t, the horizontal and vertical coordinates in the three reference frames are related by

$$\begin{bmatrix} X \\ Z \end{bmatrix} = \begin{bmatrix} Ut \\ Z_{\overline{G}} \end{bmatrix} + \begin{bmatrix} \overline{x} \\ \overline{z} \end{bmatrix}$$

$$\begin{bmatrix} \overline{x} \\ \overline{z} \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{z}_G \end{bmatrix} + \mathbf{L}_{EB} \begin{bmatrix} x \\ z \end{bmatrix} \tag{2}$$

where

$$\mathbf{L}_{EB} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

is the transformation matrix from the body-fixed frame to the earth-fixed frame.

Ship response to waves in the time domain can be dealt with by either the convolution-integral method or the direct integration method. The former is used for linear problems to which the superposition principle applies. In the latter, the differential equations of motion, which can be either nonlinear or linear, are solved by numerical integration step by step in time and nonlinear variations of parameters of the equations with time can be taken into account at each time step. The present method is based on the latter method. The force and moment equations with respect to the body-fixed frame, or Euler's equations, for a pitching and heaving rigid ship are (see, e.g. Etkin (1972))

$$\begin{aligned} m_s(\dot{w}_G - qU) &= F_z - m_s g \\ I_y \dot{q} &= M_y \end{aligned} \tag{4}$$

where m_s is the ship mass, $w_G = \dot{z}_G$ is the rectilinear velocity of CG, F_z the external force, both in the z-direction, $q = \dot{\theta}$, and M_y and I_y are the moment of external forces and the moment of inertia about the y-axis. The overdots indicate derivatives with respect to time. In this study, the external forces and moment, F_z and M_y in (4) are evaluated based on strip-theory approximations as follows:

$$F_z \approx \int_{L_w} dx \int_{C_x} p n_3 dC = \int_{L_w} \frac{dF_z}{dx} dx$$

and

$$M_y \approx \int_{L_w} x dx \int_{C_x} p n_3 dC = \int_{L_w} x \frac{dF_z}{dx} dx$$

where L_w and C_x denote the instantaneous wetted length of the hull and wetted sectional contour at x, respectively, n_3 the z-component of the unit normal, $\mathbf{n}$, to the sectional contour pointing into the hull, and

$$\frac{dF_z}{dx} = \int_{C_x} p n_3 dC$$

is the sectional external force in the z-direction. Note that, in linear theory, L_w and C_x are constant and equal to their equilibrium values. The hydrodynamic pressure p is obtained by

$$p(x, y, z; t) = -\rho \frac{D\Phi(x, y, z; t)}{Dt} \tag{5}$$

where $D/Dt = \partial/\partial t - U\partial/\partial x$ is the material derivative. The total potential Φ in (5) is assumed to be expressible as an algebraic sum as follows:

$$\begin{aligned} \Phi(x, y, z; t) &= -Ux + \varphi_s(x, y, z) + \phi_I(x, y, z; t) \\ &+ \phi_D(x, y, z; t) + \sum_{j=3,5} \xi_j(x, y, z; t)\phi_j(x, y, z; t) \end{aligned} \tag{6}$$

The terms on the right-hand of (6) represent the velocity potentials for, respectively, constant forward motion, steady perturbation caused by the constant forward motion, incident wave, diffraction of incident wave by the presence of the hull, and radiating waves. The last mentioned are generated by the ship's oscillatory motions in the jth mode (j = 3 for heave,

where p_0 denotes the combined pressure of the hydrostatic restoring pressure and the Froude-Krylov pressure in the ith component incident wave, a_{33} and b_{33} are the two-dimensional added-mass and damping coefficients for heave motion, $\omega_e = \omega + kU$ is the encounter frequency, and $\overline{T}$ is some mean value (in

in the z-direction of the hull section at x, and $(dF_z / dx)^*_M$ represents the remainder from the second line after the first term of the third line is deducted. In the time domain, added-mass and damping coefficients depend on time as well as frequency, because the wetted section changes with time. Thus, and 5 for pitch; surge motion is neglected) with ξ_j and ϕ_j representing the complex amplitude of ship motion and the velocity potential in the jth mode, respectively. By virtue of the assumed slenderness of the hull, the steady perturbation φ_s due to constant forward motion is considered negligible compared with $-Ux$. Applying strip theory, the sectional total velocity potential Φ is required to satisfy the following boundary-value problem:

(L) $$\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \quad \text{in fluid domain}$$

(FS) $$\frac{D^2\Phi}{Dt^2}+g\frac{\partial\Phi}{\partial z}=0 \quad \text{on free surface}$$

(SB) $$\frac{\partial\Phi}{\partial z}\to 0 \text{ as } z\to-\infty \quad \text{at sea bottom}$$

(H) $$\frac{\partial\Phi}{\partial n}=-\mathbf{V}\cdot\mathbf{n} \quad \text{on hull surface}$$

plus the radiation condition at infinity. In the hull condition (H), $\mathbf{V}$ is the velocity vector of the cross section.

From (5) and (6), we decompose the sectional force into the following components:

$$\frac{dF_z}{dx}=\left(\frac{dF_z}{dx}\right)_{BW}+\left(\frac{dF_z}{dx}\right)_M \tag{7}$$

The first term on the right-hand side of (7) represents the combined forces of the incident wave, diffracted wave, and hydrostatic restoring force. The second term represents the sectional motion-induced hydrodynamic force. Here, the instantaneous sectional wave-induced excitation force and restoring force in the direction of the z-axis is calculated as follows (Watanabe, 1958, Maeda, 1969, Jacobs, 1958):

$$\left(\frac{dF_z}{dx}\right)_{BW}\approx\int_{C_x}p_0n_3dC+e^{-k\overline{T}}\left\{a_{33}(\omega_e)\frac{D^2\zeta}{Dt}+\left[b_{33}(\omega_e)-U\frac{da_{33}(\omega_e)}{dx}\right]\frac{D\zeta}{Dt}\right\} \tag{8}$$

where p_0 denotes the combined pressure of the hydrostatic restoring pressure and the Froude-Krylov pressure in the ith component incident wave, a_{33} and b_{33} are the two-dimensional added-mass and damping coefficients for heave motion, $\omega_e=\omega+kU$ is the encounter frequency, and $\overline{T}$ is some mean value (in the sense of the mean-value theorem, which was used to derive the above approximate expression; see (Watanabe, 1958) of the sectional draft in the range $0\le\overline{T}\le T(x,t)$. The value of $\overline{T}$ in (8) for each section is obtained by

$$\overline{T}=T_x\sigma_x=A_x/B_x$$

where $A_x=A(x,t)$, $B_x=B(x,t)$, $T_x=T(x,t)$, and $\sigma_x=A_x(t)/[B_x(t)\,T_x]$ are the instantaneous wetted cross-sectional area, wetted sectional beam at waterline, sectional draft, and wetted sectional area coefficient, respectively. Some authors use another expression for $\overline{T}$ (see, e.g. Bishop & Price, 1979):

$$\overline{T}=\frac{1}{k}\ln\left[1-\frac{2k}{B_x}\int_{-T_x}^{0}ye^{kz}dz\right]$$

The pressure in the wave above the mean water level is approximated by hydrostatic pressure in this study. Thus, p_0 in (8) is determined by

$$p_0=-\rho g\left(Z-\sum_{i=1}^{N}\zeta_ie^{k_iZ}\right) \quad \text{for } Z\le\zeta\le0,$$
$$=-\rho g(Z-\zeta) \quad \text{for } 0<Z\le\zeta,$$

where $Z=Z(x,y,z,\overline{z}_G,\theta;t)$ and ζ is the total wave elevation defined in (1).

The sectional motion-induced hydrodynamic force in (7) is approximated by (see, e.g. Betts *et al.*, 1977; Meyerhoff & Schlachter, 1979):

$$\begin{aligned}\left(\frac{dF_z}{dx}\right)_M&\approx-\frac{D}{Dt}[a_{33}w_x]-b_{33}w_x\\&\cong-a_{33}(\dot{w}_G-x\dot{q}+2Uq)\\&\quad-\left(\frac{\partial a_{33}}{\partial T}\frac{\partial T}{\partial t}-U\frac{\partial a_{33}}{\partial x}+b_{33}\right)(w_G+xq-U\theta)\\&=-a_{33}(\dot{w}_G-x\dot{q})+\left(\frac{dF_z}{dx}\right)_M^*\end{aligned} \tag{9}$$

where a_{33} and b_{33}, in (9) are to be evaluated at the encounter frequency ω_e, $w_x=w(x,t)$ is the velocity in the z-direction of the hull section at x, and $(dF_z/dx)_M^*$ represents the remainder from the second line after the first term of the third line is deducted. In the time domain, added-mass and damping coefficients depend on time as well as frequency, because the wetted section changes with time. Thus,

$$a_{33} = a_{33}(T(x,t),\omega_e),\ \ b_{33} = b_{33}(T(x,t),\omega_e).$$

Although the method of integral equations is most commonly used to calculate added-mass and damping coefficients for a floating body of general shape, the mathematical singularity inherent to the method causes the solution to break down at a countably infinite set of frequencies called irregular frequencies. In general, we do not know in advance where these irregular frequencies lie, although they usually occur in high frequency range. In this study, DREA's computer program AMADIFF (Ando, 1995) was used to calculate added mass and damping coefficients. AMADIFF is designed to suppress the irregular frequencies by artificially extending the hull boundary to the calm waterline inside the surface-piercing two-dimensional body to obtain an integral equation on a closed surface (irregular frequencies do not occur for completely submerged bodies). This method of removing irregular frequencies is sometimes called the lid method or the extended-boundary method. At a given time step, external forces and accelerations are evaluated in terms of motion displacements and velocities, which are the solution from the previous time step. From (4), (7)–(9), we obtain the following set of equations, which describes the instantaneous motion of the ship with respect to the body-fixed frame.

$$\begin{aligned}(m_s + A_{33})\ddot{z}_G + A_{35}\ddot{\theta} &= F_z^* \\ A_{53}\ddot{z}_G + (I_y + A_{55})\ddot{\theta} &= M_y^*\end{aligned} \qquad (11)$$

where

$$A_{33} = \int_{L_w} a_{33}dx;\ \ A_{35} = A_{53} = \int_{L_w} x a_{33}dx;$$

$$A_{55} = \int_{L_w} x^2 a_{33}dx$$

$$F_z^* = \int_{L_w}\left[\left(\frac{dF_z}{dx}\right)_{BW} + \left(\frac{dF_z}{dx}\right)_M^*\right]dx - m_s g + m_s qU$$

$$M_y^* = \int_{L_w} x\left[\left(\frac{dF_z}{dx}\right)_{BW} + \left(\frac{dF_z}{dx}\right)_M^*\right]dx$$

The products of inertia $I_{xz} = I_{yz} = 0$ because of the assumed port-starboard symmetry of the ship; moreover, the integrand of the inertial coupling coefficients A_{35} and A_{53} being an odd function of x, their values are usually small for conventional hull forms. (They vanish if the hull has a fore-and-aft symmetry about the CG.) As a result, the off-diagonal terms of the inertial influence coefficient matrix of (11) are small and the diagonal terms dominate. This fact contributes to a stable inversion of the coefficient matrix of (11) in the solution process. In the body-fixed frame, the moments of inertia (and the products of inertia) are constant with respect to time. The differential equations (11) are solved by the fourth-order Runge-Kutta method subject to the following initial conditions:

$$X_G(0) = \theta(0) = 0,\ Z_G(0) = Z_{G_0},\ \dot{X}_G(0) = V, \text{ and}$$
$$\dot{Z}_G(0) = \dot{\theta}(0) = 0,$$

where Z_{G_0} is the initial position of the CG and V the mean ship speed. A ramp function is needed to avoid initial instability of the computation. The following simple linear ramp function is used:

$$R(t) = \frac{t}{t_0} \ \text{ for } 0 \le t \le t_0$$

where t_0 is the prescribed ramp period (about a few seconds).

Once the motions have been calculated, wave loads can be calculated. The shear force at a station located at x from the CG of the ship is found by:

$$Q(x) = \int_x^{x_b} m(\xi)\dot{w}_x d\xi - \int_x^{x_b} \frac{dF_z}{d\xi}d\xi \qquad (12)$$

where x_b denotes the x coordinate of the forward perpendicular. The vertical bending moment is found by integrating the shear forces from (12):

$$M_B(x) = \int_x^{x_b} Q(\xi)d\xi$$

Theoretically, shears and bending moments vanish at the ends of the hull ("closure" condition), since the ship must be in dynamic equilibrium at any time. In numerical calculation, however, there may be some nonzero values owing to round-off and other errors.

Results and discussion

The present method was used to calculate ship motions and wave loads for the Canadian Patrol Frigate (CPF) shown in Fig. 2. The ratio of the midship waterline beam B to the length between

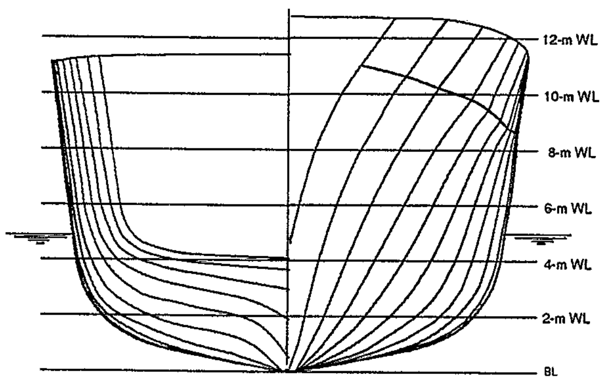
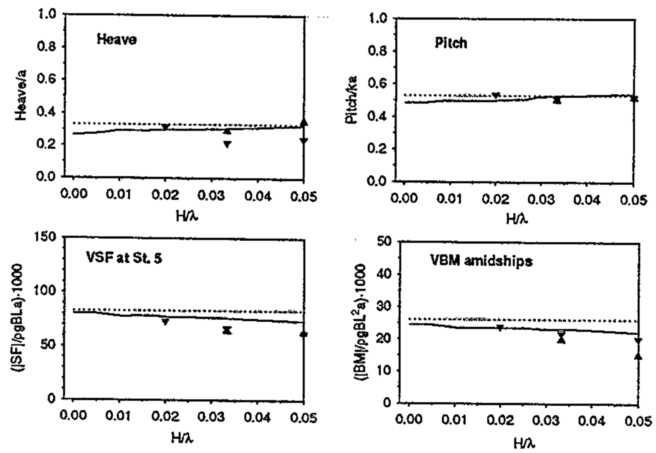
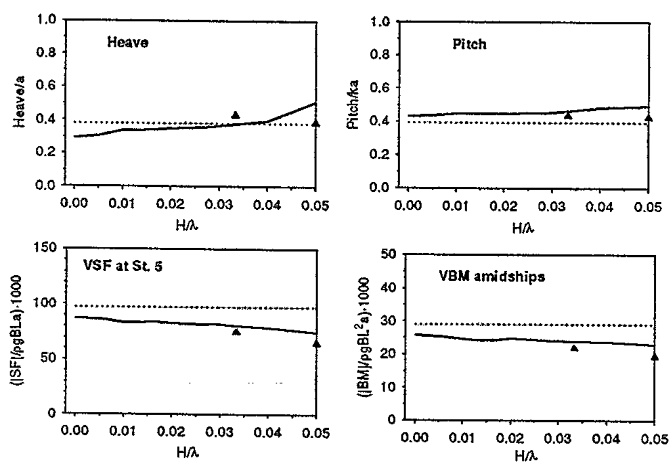


FIG. 2. Body plan



(a) Fn = 0.12, λ / L=1.01



(b) Fn = 0.20, λ / L=0.89

FIG. 3. Predicted and measured response amplitudes. Legend: — present method ; ...... SHIPMO7; ▲,▼ Experiment. In nondimensional factors, a and k are amplitude and wave number of incident wave, and B and L are beam and length between perpendiculars of the model

perpendicular L of the CPF is 0.1189, and the ratio of the draft T to B is 0.3358.

In Fig. 3, predicted and measured response amplitudes are plotted against wave steepness, H / λ, where H is the wave height and λ is the wavelength. The smallest value of wave steepness used for calculation by the present method was 0.0001. The two types of the triangular symbols in Fig. 3(a) represent the two sets of data from model tests with a 1/20-scale model of the CPF, conducted about three years apart. Data for pitch displacement and vertical shear force (VSF) at station 5 (L/4 from the forward perpendicular) show excellent repeatability, while heave displacement and VBM amidships show some variations in steep waves. The dotted line represents the predictions by the frequency-domain code SHIPMO7 (McTaggart, 1996). Although the predicted transfer functions (the ratios of response amplitudes to the incident-wave amplitude) of linear frequency-domain codes such as SHIPMO7 are independent of wave steepness for a given frequency, the measured transfer functions for VSF's and VBM's are found to decrease with increasing wave steepness. (Note: their absolute magnitudes, however, usually increase with increasing wave steepness.) Such trends for the cases of VSF at station 5 and the VBM amidships can be seen in Fig. 3. This is in good agreement with the prediction by the present method.

With increasing waveheight, the ship motions become severe enough to cause the bow bottom to emerge and green water to break over the bow deck. As it stands, the present method is not robust enough to handle such severe motions, so the computation becomes increasingly unstable and unreliable. The sudden, steep rise of the curve of the predicted heave for $H / \lambda > 0.04$ in Fig. 3(b) exhibits an incipient stage of the growing instability. Note that the height of wave of steepness $H / \lambda = 0.05$ amounts to 1.3 times the draft for waves of length $\lambda / L = 1.01$ in Fig. 3(a) and 1.1 times the draft for waves of length $\lambda / L = 0.89$ in Fig. 3(b).

For conventional ships, and the slender hulls of typical warships in particular, the VBM is usually the most important criterion for hull structure design because almost all structural failures involve this mode. The most severe VBM in ship hulls usually occurs in head seas. It is encouraging, therefore, that the present method can capture the pattern of VBM response in steep waves more accurately than the frequency-domain method. It will be useful in obtaining insight into extreme sea loads in rough seas.

Concluding remarks

Good correlation between ship motions and wave loads predicted by the present method and experiment was obtained; indeed, the more severe the wave conditions, the better the correlation between the predicted and measured amplitudes of response transfer functions. The ratio of the amplitude of the measured vertical bending moment amidships to the incident wave amplitude decreases with increasing wave slope. This was in good agreement with the prediction by the present method. Nevertheless, the theoretical model needs to be refined further, particularly with respect to robustness, before it can be used as a practical analytical tool. Such effort will be justified for two reasons. First, the most important consideration for analysis and design of hull strength is the extreme load, which usually results from the ship's large-amplitude motions in severe waves. Second, there is no established general theoretical framework to treat nonlinear problems, so predictions by a simulation method such as described in this paper are useful in assessing the accuracy of the solutions by other methods.

Possible areas of future work include (1) incorporation of bow-flare impact and green water shipped on the foredeck (it is shown by Ando (1999) that bow-flare impact is almost totally responsible for whipping of the CPF hull, while the effect of bottom slam is almost negligible); (2) more rigorous calculation of the wave diffraction; (3) incorporation of surge motion, say using Motora's approach (Motora, 1954) (although the effect of surging motion itself on hull-girder loads is negligible, occurrences of the bow emergence and water on the deck may be affected more strongly by the ship's freedom to surge); and (4) application of fuzzy logic to take into account the degree of control exercised by the ship's captain through speed reduction and/or course changes (because extreme loads are sensitive to how the ship is driven, this option will add to the usefulness of the present method).

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